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Research Article

Common Fixed Point Theorems on Weakly Contractive and Nonexpansive Mappings

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A family of commuting nonexpansive self-mappings, one of which is weakly contractive, are studied. Some convergence theorems are established for the iterations of types Krasnoselski-Mann, Kirk, and Ishikawa to approximate a common fixed point. The error estimates of these iterations are also given.

1. Introduction and preliminaries

Let (X,d) be a metric space and $D \subset X$. A mapping $T : D \to X$ is said to be nonexpansive if

$$d(Tx,Ty) \le d(x,y), \quad \forall x,y \in D, \tag{1.1}$$

and it is said to be weakly contractive if

$$d(Tx,Ty) \le d(x,y) - \psi\big(d(x,y)\big), \quad \forall x,y \in D, \tag{1.2}$$

where $\psi : [0,\infty) \to [0,\infty)$ is continuous and nondecreasing such that ψ is positive on $(0,\infty)$, $\psi(0) = 0$, and $\lim_{t\to\infty}\psi(t) = \infty$.

It is evident that T is contractive if it is weakly contractive with $\psi(t) = (1-\alpha)t$, where $\alpha \in (0,1)$, and it is nonexpansive if it is weakly contractive.

As an important extension of the class of contractive mappings, the class of weakly contractive mappings was introduced by Alber and Guerre-Delabriere [1]. In Hilbert and Banach spaces, Alber et al. [1–4] and Rhoades [5] established convergence theorems on iteration of fixed point for weakly contractive single mapping.

Inspired by [2, 5, 6], the purpose of this paper is to study a family of commuting nonexpansive mappings, one of which is weakly contractive, in arbitrary complete metric spaces and Banach spaces.

We will establish some convergence theorems for the iterations of types Krasnoselski-Mann, Kirk, and Ishikawa to approximate a common fixed point and to give their error estimates.

Throughout this paper, we assume that $F(T)$ is the set of fixed points of a mapping T, that is, $F(T) = \{x : Tx = x\}$; Φ is defined by the antiderivative (indefinite integral) of $1/\psi(t)$ on $(0, +\infty)$, that is, $\Phi(t) = \int dt/\psi(t)$, and Φ^{-1} is the inverse function of Φ.

We define iterations which will be needed in the sequel.

Suppose that X is a metric space and $D \subset X$, $\{T_r\}_{r=0}^{k}$ is a family of commuting self-mappings of D and $x_0 \in D$. The iteration $\{x_n\}_{n=0}^{\infty} \subset D$ of type Krasnoselski-Mann (see [7, 8]) is cyclically defined by

$$\begin{gathered} x_1 = T_1 x_0, \dots, \qquad x_k = T_k x_{k-1}, \qquad x_{k+1} = T_0 x_k, \\ x_{k+2} = T_1 x_{k+1}, \dots, \qquad x_{2(k+1)} = T_0 x_{2k+1}, \qquad x_{2(k+1)+1} = T_1 x_{2(k+1)}, \dots. \end{gathered} \tag{1.3}$$

For convenience, we write

$$x_n = T_{n(\mathrm{mod}\ k+1)} x_{n-1}, \tag{1.4}$$

where the mod$\,k+1$ function takes values in $\{0, 1, 2, \dots, k\}$.

Let D be a closed convex subset of the normed space X. Then the iteration $\{x_n\}_{n=0}^{\infty} \subset D$ of type Kirk (see [5, 9]) is defined by

$$x_n = S^n x_0, \quad n = 1, 2, \quad S = \sum_{i=0}^{k} a_i T_i, \quad a_0 > 0, \quad a_i \geq 0 \ \ (i = 1, 2, \dots, k), \quad \sum_{i=0}^{k} a_i = 1. \tag{1.5}$$

Again, the iteration $\{x_n\}_{n=0}^{\infty} \subset D$ of type lshikawa with error (see [10–12]) is defined by

$$\begin{aligned} x_{n+1} &= (1 - a_{n1} - b_{n1}) x_n + a_{n1} T_1 y_{n1} + b_{n1} u_{n1}, \\ y_{n1} &= (1 - a_{n2} - b_{n2}) x_n + a_{n2} T_2 y_{n2} + b_{n2} u_{n2}, \\ &\ \ \vdots \\ y_{n(k-1)} &= (1 - a_{nk} - b_{nk}) x_n + a_{nk} T_k y_{nk} + b_{nk} u_{nk}, \\ y_{nk} &= (1 - a_{n0} - b_{n0}) x_n + a_{n0} T_0 x_n + b_{n0} u_{n0}, \end{aligned} \tag{1.6}$$

where $\{u_{ni}\}_{n=0}^{\infty} \subset D$ $(i = 0, 1, \dots, k)$, $\{a_{ni}\}_{n=0}^{\infty} \subset [0, 1]$, $\{b_{ni}\}_{n=0}^{\infty} \subset [0, 1]$ $(i = 0, 1, \dots, k)$, and

$$\max_{0 \leq i \leq k} (a_{ni} + b_{ni}) \leq 1 \quad (n = 0, 1, 2, \dots). \tag{1.7}$$

We will make use of following result in theproof of Theorem 2.4.

Lemma 1.1 (see [12]). *Suppose that* $\{\rho_n\}$, $\{\sigma_n\}$ *are two sequences of nonnegative numbers such that* $\rho_{n+1} \leq \rho_n + \sigma_n$, *for all* $n \geq n_0$. *If* $\sum_{n=0}^{\infty}\sigma_n < \infty$, *then* $\lim_{n\to\infty}\rho_n$ *exists.*

2. Main result

Theorem 2.1. *Let* (X,d) *be a complete metric space and let* $\{T_r\}_{r=0}^{k}$ *be a family of commuting self-mappings, where* T_i $(i = 1,2,\dots,k)$ *are all nonexpansive and* T_0 *is weakly contractive, then there is a unique common fixed point* $p \in \bigcap_{r=0}^{k} F(T_r)$ *and the iteration* $\{x_n\}$ *of type Krasnoselski-Mann generated by* (1.4) *converges in metric to* p, *with the following error estimate:*

$$d(x_n,p) \leq \Phi^{-1}\left(\Phi(d(x_0,p)) - \left[\frac{n}{k+1}\right]\right) \quad (n = 0,1,2,\dots), \tag{2.1}$$

where $[n/(k+1)]$ *is the Gauss integer of* $n/(k+1)$.

Proof. The uniqueness of fixed point of T_0 is clear from (1.2). Hence, the common fixed point of $\{T_r\}_{r=0}^{k}$ is unique. Let X_0 be an arbitrary point in X and let $\{x_n\}$ be an iteration of type Krasnoselski-Mann generated by (1.4). Since $\{T_r\}_{r=0}^{k}$ is commutative, then we have $\prod_{r=0}^{k} T_r = (\prod_{r=1}^{k} T_r)T_0$. Suppose that $n = i(\bmod k+1)$ and $[n/(k+1)] = j$. Then,

$$x_n = x_{j(k+1)+i} = \left(\prod_{r=0}^{k} T_r\right) x_{(j-1)(k+1)+i} \quad (i = 0,1,2,\dots,k,\ j = 1,2,\dots). \tag{2.2}$$

Write $y_j = x_{j(k+1)+i}$ for fixed i. Then $\{y_j\}_{j=0}^{\infty}$ is a subsequence of $\{x_n\}$. Since $\prod_{r=1}^{k} T_r$ is nonexpansive and T_0 is weakly contractive, then we obtain

$$\begin{aligned} d(y_{j+1},y_j) &= d\left(\left(\prod_{r=1}^{k} T_r\right)T_0 y_j, \left(\prod_{r=1}^{k} T_r\right)T_0 y_{j-1}\right) \\ &\leq d(T_0 y_j, T_0 y_{j-1}) \leq d(y_j,y_{j-1}) - \psi(d(y_j,y_{j-1})), \end{aligned} \tag{2.3}$$

which shows $d(y_{j+1},y_j) \leq d(y_j,y_{j-1})$, that is, $\{d(y_{j+1},y_j)\}_{j=0}^{\infty}$ is a nonincreasing sequence of nonnegative real numbers. Therefore, it tends to a limit $d \geq 0$. If $d > 0$, then, by nondecreasity of ψ, $\psi(d(y_{j+1},y_j)) \geq \psi(d)$, for all $j \geq 0$. Thus, from (2.3) it follows that

$$d(y_{j+k+1},y_{j+k}) \leq d(y_{j+1},y_j) - k\psi(d), \tag{2.4}$$

a contradiction for k large enough. Therefore,

$$\lim_{j\to\infty} d(y_{j+1},y_j) = 0. \tag{2.5}$$

By (2.5), for any given $\varepsilon > 0$, there exists N such that

$$d(y_{j+1},y_j) < \min\left\{\frac{\varepsilon}{2}, \psi\left(\frac{\varepsilon}{2}\right)\right\}, \quad \forall j \geq N. \tag{2.6}$$

We claim that

$$d(y_{j+m},y_j) < \varepsilon, \quad \forall m \geq 1,\ \forall j \geq N. \tag{2.7}$$

In fact, from (2.6) we see that (2.7) holds when $m = 1$. Suppose that $d(y_{j+m-1}, y_j) < \varepsilon$. If $d(y_{j+m-1}, y_j) < \varepsilon/2$, then from (2.6) we get

$$d(y_{j+m}, y_j) \le d(y_{j+m}, y_{j+m-1}) + d(y_{j+m-1}, y_j) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \tag{2.8}$$

If $d(y_{j+m-1}, y_j) \ge \varepsilon/2$, then $\psi(d(y_{j+m-1}, y_j)) \ge \psi(\varepsilon/2)$, we also get

$$\begin{aligned} d(y_{j+m}, y_j) &\le d(y_{j+m}, y_{j+1}) + d(y_{j+1}, y_j) \\ &= d\left(\left(\prod_{r=0}^{k} T_r\right) y_{j+m-1}, \left(\prod_{r=0}^{k} T_r\right) y_j\right) + d(y_{j+1}, y_j) \\ &\le d(y_{j+m-1}, y_j) - \psi(d(y_{j+m-1}, y_j)) + d(y_{j+1}, y_j) \\ &< \varepsilon - \psi\left(\frac{\varepsilon}{2}\right) + \psi\left(\frac{\varepsilon}{2}\right) = \varepsilon. \end{aligned} \tag{2.9}$$

Therefore, by induction we derive that (2.7) holds. Since ε is arbitrary, $\{y_j\}$ is a Cauchy sequence. As X is complete, we have

$$\lim_{j\to\infty} x_{j(k+1)+i} = p_i \in X \quad (i = 0, 1, 2, \ldots, k). \tag{2.10}$$

Observe that T_i $(i = 0, 1, 2, \ldots, k)$ are all continuous, so is $\prod_{r=0}^{k} T_r$. From (2.10), it follows that

$$\left(\prod_{r=0}^{k} T_r\right) p_i = \lim_{j\to\infty} \left(\prod_{r=0}^{k} T_r\right) x_{j(k+1)+i} = \lim_{j\to\infty} x_{(j+1)(k+1)+i} = p_i \quad (i = 0, 1, 2, \ldots, k), \tag{2.11}$$

$$T_{i+1} p_i = \lim_{j\to\infty} T_{i+1} x_{j(k+1)+i} = \lim_{j\to\infty} x_{j(k+1)+(i+1)=p_{i+1}} \quad (i = 0, 1, 2, \ldots, k;\ T_{k+1} = T_0;\ p_{k+1} = p_0) \tag{2.12}$$

By (1.1), (1.2), and (2.11), we deduce

$$\begin{aligned} d(p_s, p_t) &= d\left(\left(\prod_{r=0}^{k} T_r\right) p_s, \left(\prod_{r=0}^{k} T_r\right) p_t\right) \\ &\le d(p_s, p_t) - \psi(d(p_s, p_t)), \quad \forall t \ne s \in \{0, 1, 2, \ldots, k\}, \end{aligned} \tag{2.13}$$

which shows

$$p_s = p_t, \quad \text{that is, } p_i = p \quad (i = 0, 1, 2, \ldots, k). \tag{2.14}$$

From (2.12), it implies that p is a common fixed point of $\{T_r\}_{r=0}^{k}$, that is, $p \in \bigcap_{r=0}^{k} F(T_r)$. Hence, $\bigcap_{r=0}^{k} F(T_r) = \{p\}$. By (2.10) and (2.14), we conclude $\lim_{n\to\infty} x_n = p$. Set $\alpha_j = d(x_{j(k+1)}, p)$. From (2.3), we have

$$\alpha_j \le \alpha_{j-1} - \psi(\alpha_{j-1}), \quad \forall j \in Z^+. \tag{2.15}$$

Since ψ is continuous and nondecreasing, using (2.15), it yields

$$\begin{gathered} \Phi(\alpha_{j-1}) - \Phi(\alpha_j) = \int_{\alpha_j}^{\alpha_{j-1}} \frac{dt}{\psi(t)} \ge \frac{\alpha_j - \alpha_{j-1}}{\psi(\alpha_{j-1})} \ge 1, \\ \alpha_j \le \Phi^{-1}(\Phi(\alpha_0) - j). \end{gathered} \tag{2.16}$$

Observe that

$$d(x_n, p) = d(x_{j(k+1)+i}, p) = d(T_i \cdots T_2 T_1 x_{j(k+1)}, T_i \cdots T_2 T_1 p) \\ \leq d(x_{j(k+1)}, p), \quad 1 \leq i \leq k. \tag{2.17}$$

From (2.16) and (2.17), we obtain the error estimate (2.1). This completes the proof. □

Remark 2.2. If $T_i = I$ $(i = 1, 2, \ldots, k)$ in Theorem 2.1, where I is the identity mapping of X, then we conclude that the sequence $\{x_n\}$ converges to the unique common fixed point p of weakly contractive mapping T_0, with the error estimate $d(x_n, p) \leq \Phi^{-1}(\Phi(d(x_0, p) - n))$, where $x_n = T_0^n x_0$. Thus, our Theorem 2.1 is a generalization of the corresponding theorem of Rhoades [5].

Theorem 2.3. *Let X be a Banach space and let $D \subset X$ be a nonempty closed convex set. Let $\{T_r\}_{r=0}^k$ be a family of commuting self-mappings, where $T_i : D \to D$, $(i = 1, 2, \ldots, k)$ are all nonexpansive and $T_0 : D \to D$ is weakly contractive. Then, for any $x_0 \in X$, the iteration $\{x_n\}$ of type Kirk generated by* (1.5) *converges strongly to a unique common fixed point $p \in \bigcap_{i=0}^k F(T_r)$, with the following error estimate:*

$$\|x_{n+1} - p\| \leq a_0 \Phi^{-1}\left[\frac{1}{a_0}\Phi\left(\sum_{i=0}^k a_i \|T_i x_0 - p\|\right) - n\right] \quad (n = 1, 2, \ldots). \tag{2.18}$$

Proof. Applying Theorem 2.1, we can suppose that p is a unique common fixed point of $\{T_r\}_{r=0}^k$. Since

$$Sp = \left(\sum_{i=0}^k a_i T_i\right) p = \sum_{i=0}^k a_i (T_i p) = \sum_{i=0}^k a_i p = p, \tag{2.19}$$

we derive that p is a fixed point of S. Since T_i $(i = 1, 2, \ldots, k)$ are all nonexpansive, T_0 is weakly contractive, and $a_0 \neq 0$, then we have

$$\begin{aligned} \|Sx - Sy\| &= \left\|\sum_{i=0}^k a_i (T_i x - T_i y)\right\| \leq \sum_{i=0}^k a_i \|T_i x - T_i y\| \\ &\leq a_0 \|x - y\| - a_0 \psi(\|x - y\|) + \sum_{i=1}^k a_i \|x - y\| \\ &= \|x - y\| - a_0 \psi(\|x - y\|). \end{aligned} \tag{2.20}$$

The inequality (2.20) shows that S is weakly contractive. Thus, p is a unique fixed point of S. Set $\psi_1 = a_0 \psi$. Then,

$$\Phi_1 = \frac{1}{a_0}\Phi, \qquad \Phi_1^{-1} = a_0 \Phi^{-1}, \tag{2.21}$$

and $\{x_n\}$ converges to p with the following error estimate (see Remark 2.2):

$$\|x_{n+1} - p\| \leq \Phi_1^{-1}[\Phi_1(\|x_1 - p\|) - n]. \tag{2.22}$$

Observe that

$$\|x_1 - p\| = \|Sx_0 - p\| \leq \sum_{i=0}^k a_i \|T_i x_0 - p\|. \tag{2.23}$$

From (2.21)–(2.23), we obtain (2.18). This completes the proof. □

Theorem 2.4. *Let X be a Banach space and let $D \subset X$ be a nonempty closed convex set. Let $\{T_r\}_{r=0}^{k}$ be a family of commuting self-mappings, where $T_i : D \to D$ $(i = 1, 2, \ldots, k)$ are all nonexpansive and $T_0 : D \to D$ is weakly contractive. For any $x_0 \in D$, let $\{x_n\}$ be the iteration of type Ishikawa generated by* (1.6)*, where*

$$\sum_{n=0}^{\infty} \prod_{i=0}^{k} a_{ni} = \infty, \qquad \sum_{n=0}^{\infty} \max_{0 \le i \le k} b_{ni} < \infty, \tag{2.24}$$

and $\{u_{ni}\}_{n=0}^{\infty} \subset X$ $(i = 0, 1, \ldots, k)$ are all bounded. Then, $\{x_n\}$ converges strongly to a unique common fixed point $p \in \bigcap_{r=0}^{s} F(T_r)$ with the following estimate:

$$\|x_{n+1} - p\| \le \Phi^{-1}\left(\Phi(\|x_0 - p\|) - \sum_{j=0}^{n} \prod_{i=0}^{k} a_{ji}\right) + M \sum_{j=0}^{n} \sum_{i=0}^{k} b_{ji}, \tag{2.25}$$

where $M = \max_{0 \le i \le k} \sup_{n \ge 1} \|u_{ni} - p\|$.

Proof. Applying Theorem 2.1, we can suppose that p is a unique common fixed point of $\{T_r\}_{r=0}^{k}$. Since $\{u_{ni}\}$ $(i = 0, 1, \ldots, k)$ are all bounded, we have $M = \max_{0 \le i \le k} \sup_{n \ge 1} \|u_{ni} - p\| < \infty$. Since T_i $(i = 1, 2, \ldots, k)$ are all nonexpansive and T_0 is weakly contractive, we obtain in proper order that

$$\begin{aligned}
\|y_{nk} - p\| &\le (1 - a_{n0} - b_{n0})\|x_n - p\| + a_{n0}\|T_0 x_n - p\| + b_{n0}\|u_{n0} - p\| \\
&\le (1 - a_{n0})\|x_n - p\| + a_{n0}[\|x_n - p\| - \psi(\|x_n - p\|)] + b_{n0}M \\
&\le \|x_n - p\| - a_{n0}\psi(\|x_n - p\|) + b_{n0}M, \\
\|y_{n(k-1)} - p\| &\le (1 - a_{nk} - b_{nk})\|x_n - p\| + a_{nk}\|T_k y_{nk} - p\| + b_{nk}\|u_{nk} - p\| \\
&\le (1 - a_{nk})\|x_n - p\| + a_{nk}\|y_{nk} - p\| + b_{nk}M \\
&\le (1 - a_{nk})\|x_n - p\| + a_{nk}[\|x_n - p\| - a_{n0}\psi(\|x_n - p\|) + b_{n0}M] + b_{nk}M \\
&\le \|x_n - p\| - a_{n0}a_{nk}\psi(\|x_n - p\|) + (b_{n0} + b_{nk})M, \\
&\vdots \\
\|y_{n1} - p\| &\le \|x_n - p\| - a_{n0}\left(\prod_{i=2}^{k} a_{ni}\right)\psi(\|x_n - p\|) + \left(b_{n0} + \sum_{i=2}^{k} b_{ni}\right)M, \\
\|x_{n+1} - p\| &\le (1 - a_{n1} - b_{n1})\|x_n - p\| + a_{n1}\|T_1 y_{n1} - p\| + b_{n1}\|u_{n1} - p\| \\
&\le (1 - a_{n1})\|x_n - p\| + a_{n1}\|y_{n1} - p\| + b_{n1}M \\
&\le (1 - a_{n1})\|x_n - p\| + b_{n1}M \\
&\quad + a_{n1}\left[\|x_n - p\| - a_{n0}\left(\prod_{i=2}^{k} a_{ni}\right)\psi(\|x_n - p\|) + \left(b_{n0} + \sum_{i=2}^{k} b_{ni}\right)M\right] \\
&\le \|x_n - p\| - \left(\prod_{i=0}^{k} a_{ni}\right)\psi(\|x_n - p\|) + M\sum_{i=0}^{k} b_{ni}.
\end{aligned} \tag{2.26}$$

Write $\beta_n = \|x_n - p\|$, $\theta_n = M\sum_{i=0}^{k} b_{ni}$. Then $\sum_{n=0}^{\infty} \theta_n < \infty$, and (2.26) yields

$$\beta_{n+1} \le \beta_n + \theta_n, \tag{2.27}$$

$$\sum_{j=0}^{n} \prod_{i=0}^{k} a_{ji}\psi(\beta_j) \le \sum_{j=0}^{n} (\beta_j - \beta_{j+1}) + \sum_{j=0}^{n} \theta_j \le \beta_0 + \sum_{j=0}^{n} \theta_j. \tag{2.28}$$

From (2.27) and Lemma 1.1, it implies that $\lim_{n\to\infty}\beta_n$ exists, and so does $\lim_{n\to\infty}\psi(\beta_n)$ by the continuity of ψ. From (2.28), it implies that $\sum_{n=0}^{\infty}(\prod_{i=0}^{k}a_{ni})\psi(\beta_n) < \infty$. Since $\sum_{n=0}^{\infty}(\prod_{i=0}^{k}a_{ni}) = \infty$, we conclude that $\lim_{n\to\infty}\psi(\beta_n) = 0$. Therefore, $\lim_{n\to\infty}\beta_n = 0$, that is, x_n converges strongly to p. To establish the error estimate, we set $\sum_{j=0}^{n}\theta_j = \Gamma_n$ and $\Gamma_{-1} = 0$. Then, (2.26) yields

$$\beta_{n+1} \leq \beta_n - \left(\prod_{i=0}^{k} a_{ni}\right)\psi(\beta_n) + \Gamma_n - \Gamma_{n-1}. \tag{2.29}$$

Set $\lambda_n = \beta_n - \Gamma_{n-1}$. From (2.29) we have

$$\lambda_{n+1} \leq \lambda_n - \left(\prod_{i=0}^{k} a_{ni}\right)\psi(\lambda_n + \Gamma_{n-1}). \tag{2.30}$$

Since ψ is nondecreasing, from (2.30) we deduce

$$\Phi(\lambda_n) - \Phi(\lambda_{n+1}) = \int_{\lambda_{n+1}}^{\lambda_n} \frac{dt}{\psi(t)} \geq \frac{\lambda_n - \lambda_{n+1}}{\psi(\lambda_n)} \geq \frac{\lambda_n - \lambda_{n+1}}{\psi(\lambda_n + \Gamma_{n-1})} \geq \prod_{i=0}^{k} a_{ni}. \tag{2.31}$$

Thus,

$$\begin{gathered} \Phi(\lambda_0) - \Phi(\lambda_{n+1}) = \sum_{j=0}^{n}\left[\Phi(\lambda_j) - \Phi(\lambda_{j+1})\right] \geq \sum_{j=0}^{n}\prod_{i=0}^{k} a_{ji}, \\ \lambda_{n+1} \leq \Phi^{-1}\left(\Phi(\lambda_0) - \sum_{j=0}^{n}\prod_{i=0}^{k} a_{ji}\right). \end{gathered} \tag{2.32}$$

Hence, the estimate (2.25) holds. This completes the proof. □

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