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Erratum

Erratum to "Iterative Methods for Variational Inequalities over the Intersection of the Fixed Points Set of a Nonexpansive Semigroup in Banach Spaces"

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In my recent published paper [1] to prove Lemmas 3.1 and 5.1, an inequality involving the single-valued normalized duality mapping J from X into 2^{X^*} has been used that generally turns out there is no certainty about its accuracy. In this erratum we fix this problem by imposing additional assumptions in a way that the proofs of the main theorems do not change.

We recall that a uniformly smooth Banach space X is q-uniformly smooth for $q > 1$ if and only if there exists a constant $\beta_q > 0$ such that, for all $x, y \in X$,

$$\|x + y\|^q \leq \|x\|^q + q\|x\|^{q-2}\langle y, J(x)\rangle + 2\beta_q\|y\|^q, \tag{1}$$

for more details see [2]. Therefore, if $q = 2$, then there exists a constant $\beta > 0$ such that

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, J(x)\rangle + 2\beta\|y\|^2. \tag{2}$$

It is well known that Hilbert spaces, l_p and L_p for $p \geq 2$, are 2-uniformly smooth.

Throughout the paper we suggest to impose one of the following conditions:

(a) the Banach space X is 2-uniformly smooth;

(b) there exists a constant $\beta \in \mathbb{R}^+$ for which J satisfies the following inequality:

$$\langle y, J(x+y)\rangle \leq \langle y, J(x)\rangle + \beta\|y\|^2, \tag{3}$$

for all $x, y \in X$.

Remark 1.1. If J is β-Lipschitzian, then J satisfies (3) and is norm-to-norm uniformly continues that suffices to guarantee that X is 2-uniformly smooth. For more results concerning β-Lipschitzian normalized duality mapping see [3].

Note that since every uniformly smooth Banach space X has a Gateaux differentiable norm and each nonempty, bounded, closed, and convex subset of X has common fixed point property for nonexpansive mappings, we have $D(x_n) \cap C \neq \emptyset$ in [1]. So, when X is 2-uniformly smooth, we can remove these two conditions from Theorems 3.2, 4.2, and 5.2 in [1].

Considering the above discussion to complete our paper, we reprove Lemmas 3.1 and 5.1 of [1] here with some little changes.

Lemma 3.1 (see [1]). *Either let X be a real Banach space, and let J be the single-valued normalized duality mapping from X into 2^{X^*} satisfing* (3) *or let X be a 2-uniformly smooth real Banach space. Assume that $F : X \to X$ is η-strongly monotone and κ-Lipschitzian on X. Then*

$$\psi(x) = I(x) - \mu F(x) \tag{4}$$

is a contraction on X for every $\mu \in (0, \eta/\beta\kappa^2)$.

Proof. If J satisfies (3), considering the inequality

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, J(x+y)\rangle, \tag{5}$$

for all $x, y \in X$, we have

$$\begin{aligned}
\|\psi x - \psi y\|^2 &\leq \|(I-\mu F)x - (I-\mu F)y\|^2 = \|(x-y) + \mu(Fy - Fx)\|^2 \\
&\leq \|x-y\|^2 + 2\langle \mu(Fy-Fx), J((x-y) + \mu(Fy-Fx))\rangle \\
&\leq \|x-y\|^2 + 2\mu\langle Fy - Fx, J(x-y)\rangle + 2\beta\mu^2\langle Fy-Fx, J(Fy-Fx)\rangle \\
&\leq \|x-y\|^2 - 2\mu\langle Fx - Fy, J(x-y)\rangle + 2\beta\mu^2\|Fy-Fx\|\|J(Fy-Fx)\| \\
&\leq \|x-y\|^2 - 2\mu\eta\|x-y\|^2 + 2\beta\mu^2\|Fy-Fx\|^2 \\
&\leq \|x-y\|^2 - 2\mu\eta\|x-y\|^2 + 2\mu^2\beta\kappa^2\|x-y\|^2 \\
&\leq \left(1 - 2\mu\eta + 2\mu^2\beta\kappa^2\right)\|x-y\|^2.
\end{aligned} \tag{6}$$

Clearly, the same inequality holds if X is a 2-uniformly smooth real Banach space. Thus, we obtain

$$\|\psi x - \psi y\| \leq \sqrt{1 - 2\mu(\eta - \mu\beta\kappa^2)}\,\|x - y\|. \tag{7}$$

With no loss of generality we can take $\beta \geq 1/2$; therefore, if $\mu \in (0, \eta/\beta\kappa^2)$, then we have $\sqrt{1 - 2\mu(\eta - \mu\beta\kappa^2)} \in (0, 1)$; that is, ψ is a contraction, and the proof is complete. □

Also Lemma 5.1, which is easily proved in the same way as Lemma 3.1, will be as follows.

Lemma 5.1 (see [1]). *Either let X be a real Banach space, and let J be the single-valued normalized duality mapping from X into 2^{X^*} satisfing* (3)*, or let X be a 2-uniformly smooth real Banach space. Assume that $F : X \to X$ is η-strongly monotone and κ-Lipschitzian on X. If $\mu \in (0, \eta/\sigma^2)$, where $\sigma = \sqrt{\beta}(\kappa + 2)$, then*

$$\psi(x) = I(x) - \mu(F + I - T)(x) \tag{8}$$

is a contraction on X.

With the new imposed conditions and considering the above lemmas, the following corrections should be done in [1]:

(1) in Theorem 3.2 and Theorem 4.2, $\mu \in (0, \eta/\beta k^2)$;

(2) in Theorem 5.2, $\mu \in (0, \eta/(\sigma^2 + 1))$, where $\sigma = \sqrt{\beta}(\kappa + 2)$;

(3) in Remark 5.3, $\mu \in (0, 2(\eta - 1)/(2\sigma^2 - 1))$, where $\sigma = \sqrt{\beta}(\kappa + 2)$.

Also in [1, Corollary 4.3] the real Banach space X does not necessarily need to have a uniformly Gateaux differentiable norm.

To avoid any ambiguity in terminology note also that η-strongly monotone mappings in Banach spaces are usually called η-strongly accretive.

References

[1] I. Mohamadi, "Iterative methods for variational inequalities over the intersection of the fixed points set of a nonexpansive semigroup in Banach spaces," *Fixed Point Theory and Applications*, vol. 2011, Article ID 620284, 17 pages, 2011.

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