Research Article

Contra-$\omega$-Continuous and Almost Contra-$\omega$-Continuous

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The notion of contra continuous functions was introduced and investigated by Dontchev. In this paper, we apply the notion of $\omega$-open sets in topological spaces to present and study a new class of functions called almost contra $\omega$-continuous functions as a new generalization of contra continuity.

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1. Introduction

Dontchev [1] introduced the notions of contra continuity and strong S-closedness in topological spaces. He defined a function $f : X \to Y$ is contra continuous if the preimage of every open set of $Y$ is closed in $X$. A new weaker form of this class of functions called contra semicontinuous function is introduced and investigated by Dontchev and Noiri [2]. Caldas and Jafari [3] have introduced and studied contra $\beta$-continuous function. Jafri and Noiri [4, 5] introduced and investigated the notions of contra super continuous, contra precontinuous, and contra $\alpha$-continuous functions. Almost contra precontinuous functions were introduced by Ekici [6] and recently have been investigated further by Noiri and Popa [7]. Nasef [8] has introduced and studied contra $\gamma$-continuous function. In this direction, we will introduce the concept of almost contra $\omega$-continuous functions via the notion of $\omega$-open set and study some properties of contra $\omega$-continuous and almost contra $\omega$-continuous.

All through this paper, $(X, \tau)$ and $(Y, \sigma)$ stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of $A$ and the interior of $A$ will be denoted by $\text{Cl}(A)$ and $\text{Int}(A)$, respectively. $A$ is regular open if $A = \text{Int}(\text{Cl}(A))$ and $A$ is regular closed if its complement is regular open; equivalently...
A is regular closed if \( A = \text{Cl}(\text{Int}(A)) \), see [9]. Let \((X, \tau)\) be a space and let \(A\) be a subset of \(X\). A point \(x \in X\) is called a condensation point of \(A\) if for each \(U \in \tau\) with \(x \in U\), the set \(U \cap A\) is uncountable. \(A\) is called \(\omega\)-closed [10] if it contains all its condensation points. The complement of an \(\omega\)-closed set is called \(\omega\)-open. It is well known that a subset \(W\) of a space \((X, \tau)\) is \(\omega\)-open if and only if for each \(x \in W\), there exists \(U \in \tau\) such that \(x \in U\) and \(U - W\) is countable. The family of all \(\omega\)-open subsets of a space \((X, \tau)\), denoted by \(\tau_\omega\), forms a topology on \(X\) finer than \(\tau\). We set \(\omega O(X, x) = \{ U : x \in U \text{ and } U \in \tau_\omega \}\). The \(\omega\)-closure and \(\omega\)-interior, that can be defined in a manner to \(\text{Cl}(A)\) and \(\text{Int}(A)\), respectively, will be denoted by \(\text{Cl}_\omega(A)\) and \(\text{Int}_\omega(A)\), respectively. Several characterizations and properties of \(\omega\)-closed subsets were provided in [10–12].

2. Contra \(\omega\)-continuous

**Definition 2.1.** A function \(f : X \to Y\) is called \(\omega\)-continuous [12] if for each \(x \in X\) and each open set \(V\) of \(Y\) containing \(f(x)\), there exists \(U \in \omega O(X, x)\) such that \(f(U) \subseteq V\).

**Definition 2.2.** A function \(f : X \to Y\) is called contra-\(\omega\)-continuous (resp., contra-\(\omega\)-continuous [1]) if \(f^{-1}(V)\) is \(\omega\)-closed (resp., closed) in \(X\) for each open set of \(Y\).

**Definition 2.3.** A function \(f : X \to Y\) is said to be almost continuous [13] if \(f^{-1}(V)\) is open in \(X\) for each regular open set \(V\) of \(Y\).

**Lemma 2.4** [4]. The following properties hold for subsets \(A, B\) of a space \(X\):

1. \(x \in \text{Ker}(A)\) if and only if \(A \cap F \neq \emptyset\) for any \(F \subseteq C(X, x)\);
2. \(A \subseteq \text{Ker}(A)\) and \(A = \text{Ker}(A)\) if \(A\) is open in \(X\);
3. if \(A \subseteq B\), then \(\text{Ker}(A) \subseteq \text{Ker}(B)\).

**Theorem 2.5.** The following are equivalent for a function \(f : X \to Y\):

1. \(f\) is contra-\(\omega\)-continuous;
2. for every closed subset \(F\) of \(Y\), \(f^{-1}(F) \subseteq \omega O(X)\);
3. for each \(x \in X\) and each \(F \subseteq C(Y, f(x))\), there exists \(U \subseteq \omega O(X, x)\) such that \(f(U) \subseteq F\);
4. \(f(\text{Cl}_\omega(A)) \subseteq \text{Ker}(f(A))\) for every subset \(A\) of \(X\);
5. \(\text{Cl}_\omega(f^{-1}(B)) \subseteq f^{-1}(\text{Ker}(B))\) for every subset \(B\) of \(Y\).

**Proof.** The implications (1) \(\Rightarrow\) (2) and (2) \(\Rightarrow\) (3) are obvious.

(3) \(\Rightarrow\) (2) Let \(F\) be any closed set of \(Y\) and \(x \in f^{-1}(F)\). Then \(f(x) \in F\) and there exists \(U_x \subseteq \omega O(X, x)\) such that \(f(U_x) \subseteq F\). Therefore, we obtain \(f^{-1}(F) = \cup \{ U_x \mid x \in f^{-1}(F) \}\)
and \(f^{-1}(F)\) is \(\omega\)-open, since \(\tau_\omega\) is a topological space.

(2) \(\Rightarrow\) (4) Let \(A\) be any subset of \(X\). Suppose that \(y \notin \text{Ker}(f(A))\). Then by Lemma 2.4 there exists \(F \subseteq C(Y, f(x))\) such that \(f(A) \cap F = \emptyset\). Thus, we have \(A \cap f^{-1}(F) = \emptyset\) and since \(f^{-1}(F)\) is \(\omega\)-open then we have \(\text{Cl}_\omega(A) \cap f^{-1}(F) = \emptyset\). Therefore, we obtain \(f(\text{Cl}_\omega(A)) \cap F = \emptyset\) and \(y \notin f(\text{Cl}_\omega(A))\). This implies that \(f(\text{Cl}_\omega(A)) \subseteq \text{Ker}(f(A))\).

(4) \(\Rightarrow\) (5) Let \(B\) be any subset of \(Y\). By (4) and Lemma 2.4, we have \(f(\text{Cl}_\omega(f^{-1}(B))) \subseteq \text{Ker}(f(f^{-1}(B))) \subseteq \text{Ker}(B)\) thus \(\text{Cl}_\omega(f^{-1}(B)) \subseteq f^{-1}(\text{Ker}(B))\).

(5) \(\Rightarrow\) (1) Let \(V\) be any open set of \(Y\). Then, by Lemma 2.4 we have \(\text{Cl}_\omega(f^{-1}(V)) \subseteq f^{-1}(\text{Ker}(V)) = f^{-1}(V)\) and \(\text{Cl}_\omega(f^{-1}(V)) \subseteq f^{-1}(V)\). This shows that \(f^{-1}(V)\) is \(\omega\)-closed in \(X\). \(\square\)
The following examples show that contra-ω-continuous and contra-precontinuous functions [4] (resp., contra-semicontinuous [2], contra-α-continuous [5], contra-γ-continuous [8]) are independent notions.

Example 2.6. Let \( X = \{a, b\} \) with \( \tau = \{X, \phi, \{a\}\} \) and the real number \( \mathbb{R} \) with the standard topology, consider the map \( f : \mathbb{R} \rightarrow X \) defined by \( f(x) = b \) if \( x \in \mathbb{Q} \) where \( \mathbb{Q} \) is the set of all rational numbers and \( f(x) = a \) if \( x \notin \mathbb{Q} \). Then \( f \) is contra-precontinuous but not \( f \) contra-ω-continuous since \( \{b\} \) is a closed set of \( (X, \tau) \) and \( f^{-1}(\{b\}) = \mathbb{Q} \) is not \( \omega \)-open. But \( \mathbb{Q} \) is preopen set in \( \mathbb{R} \).

Example 2.7. Let \( X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \), and \( Y = \{1, 2\} \) be the Sierpinski space with the topology \( \sigma = \{\phi, \{1\}, Y\} \). Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be defined by \( f(a) = 1 \) and \( f(b) = 2 = f(c) \). Then \( f \) is contra-ω-continuous but not contra-precontinuous, since \( \{2\} \) is a closed set of \( (Y, \sigma) \) and \( f^{-1}(\{2\}) = \{c, b\} \) is not preopen \( (X, \tau) \).

Example 2.8. Let \( X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\} \), and \( \sigma = \{\phi, \{c\}, \{b\}, \{c, b\}, X\} \). Then the identity function \( f : (X, \tau) \rightarrow (X, \sigma) \) is contra-ω-continuous but not contra-continuous.

Example 2.9. \( X = \{a, b\} \) with \( \tau = \{X, \phi, \{a\}\} \) and the real number \( \mathbb{R} \) with the standard topology, consider the map \( f : \mathbb{R} \rightarrow X \) defined by \( f(x) = b \) if \( x \in [0, 1) \) and \( f(x) = a \) if \( x \notin [0, 1) \). Then \( f \) is contra-semicontinuous but not \( f \) contra-ω-continuous since \( \{b\} \) is a closed set of \( (X, \tau) \) and \( f^{-1}(\{b\}) = [0, 1) \) is not \( \omega \)-open. But \( [0, 1) \) is semi-open set in \( \mathbb{R} \).

Example 2.10. Let \( X = \{a, b\} \) with the indiscrete topology \( \tau \) and \( \sigma = \{\phi, \{a\}, X\} \). Then the identity function \( f : (X, \tau) \rightarrow (X, \sigma) \) is contra-ω-continuous but not contra semicontinuous, since \( A = \{a\} \in \sigma \) but \( A \) is not semiclosed in \( (X, \tau) \).

Example 2.11. Let \( X = \{a, b, c, d\}, \tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}, X\} \). Define a function \( f : (X, \tau) \rightarrow (X, \tau) \) as follows: \( f(a) = b, f(b) = a, f(c) = d, \) and \( f(d) = c. \) Then \( f \) is contra-ω-continuous but not contra-α-continuous, since \( \{c, d\} \) is a closed set of \( (X, \tau) \) and \( f^{-1}(\{c, d\}) = \{c, d\} \) is not \( \alpha \)-open.

\[
\text{contra-ω-continuity} \\
\downarrow\downarrow\downarrow\downarrow
\text{contra-α-continuity} \quad \Rightarrow \quad \text{contra-percontinuity}
\]

(2.1)

\[
\text{contra-semicontinuity} \quad \Rightarrow \quad \text{contra-γ-continuity}
\]

\[
\downarrow
\text{contra-β-continuity}
\]

Theorem 2.12. If a function \( f : X \rightarrow Y \) is contra-ω-continuous and \( Y \) is regular, then \( f \) is \( \omega \)-continuous.

Proof. Let \( x \) be an arbitrary point of \( X \) and let \( V \) be an open set of \( Y \) containing \( f(x) \); since \( Y \) is regular, there exists an open set \( W \) in \( Y \) containing \( f(x) \) such that \( \text{Cl}(W) \subseteq V \).
Since \( f \) is contra-\( \omega \)-continuous, so by Theorem 2.5(3) there exists \( U \in \omega O(X, x) \) such that \( f(U) \subseteq \text{Cl}(W) \). Then \( f(U) \subseteq \text{Cl}(W) \subseteq V \). Hence, \( f \) is \( \omega \)-continuous.

**Definition 2.13.** A space \((X, \tau)\) is said to be \( \omega \)-space (resp., locally \( \omega \)-indiscrete) if every \( \omega \)-open set is open (resp., closed) in \( X \).

For any space \((X, \tau)\), we have \( \tau \subseteq \tau_\omega \). So the following results follows immediately.

**Theorem 2.14.** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is contra-\( \omega \)-continuous if and only if \( f : (X, \tau_\omega) \rightarrow (Y, \sigma) \) is contra-continuous.

**Theorem 2.15.** If a function \( f : X \rightarrow Y \) is contra-\( \omega \)-continuous and \( X \) is \( \omega \)-space, then \( f \) is contra-continuous.

**Theorem 2.16.** Let \( X \) be locally \( \omega \)-indiscrete. If a function \( f : X \rightarrow Y \) is contra-\( \omega \)-continuous, then \( f \) is continuous.

**Definition 2.17.** A function \( f : X \rightarrow Y \) is called almost-\( \omega \)-continuous if for each \( x \in X \) and each open set \( V \) of \( Y \) containing \( f(x) \), there exists \( U \in \omega O(X, x) \) such that \( f(U) \subseteq \text{Int}_w(\text{Cl}(V)) \).

**Definition 2.18.** A function \( f : X \rightarrow Y \) is said to be pre-\( \omega \)-open if the image of each \( \omega \)-open set is \( \omega \)-open.

**Theorem 2.19.** If a function \( f : X \rightarrow Y \) is a pre-\( \omega \)-open contra-\( \omega \)-continuous function, then \( f \) is almost \( \omega \)-continuous.

**Proof.** Let \( x \) be any arbitrary point of \( X \) and \( V \) be an open set containing \( f(x) \). Since \( f \) is contra-\( \omega \)-continuous, then by Theorem 2.5(3) there exists \( U \in \omega O(X, x) \) such that \( f(U) \subseteq \text{Cl}(V) \). Since \( f \) is pre-\( \omega \)-open, \( f(U) \) is \( \omega \)-open in \( Y \). Therefore, \( f(U) = \text{Int}_w f(U) \subseteq \text{Int}_w(\text{Cl}(f(U))) \subseteq \text{Int}_w(\text{Cl}(V)) \). This shows that \( f \) is almost \( \omega \)-continuous.

**Definition 2.20.** A function \( f : X \rightarrow Y \) is said to be almost weakly \( \omega \)-continuous if for each \( x \in X \) and each open \( V \) of \( f(x) \) there exists \( U \in \omega O(X, x) \) such that \( f(U) \subseteq \text{Cl}(V) \).

**Theorem 2.21.** If a function \( f : X \rightarrow Y \) is contra-\( \omega \)-continuous, then \( f \) is almost weakly \( \omega \)-continuous.

**Proof.** Let \( V \) be any open set of \( Y \). Since \( \text{Cl}(V) \) is closed in \( Y \), by Theorem 2.5(3) \( f^{-1}(\text{Cl}(V)) \) is \( \omega \)-open in \( X \) and set \( U = f^{-1}(\text{Cl}(V)) \), then we have \( f(U) \subseteq \text{Cl}(V) \). This shows that \( f \) is almost weakly \( \omega \)-continuous.

Since the family of all \( \omega \)-open subsets of a space \((X, \tau)\), denoted by \( \tau_\omega \), forms a topology on \( X \) finer than \( \tau \), then the \( \omega \)-frontier of \( A \), where \( A \subseteq X \), is defined by \( \text{Fr}_w(A) = \text{Cl}_w(A) \cap \text{Cl}_w(X - A) \).

**Theorem 2.22.** The set of all points of \( x \) of \( X \) at which \( f : X \rightarrow Y \) is not contra-\( \omega \)-continuous is identical with the union of the \( \omega \)-frontier of the inverse images of closed sets of \( Y \) containing \( f(x) \).

**Proof.** Suppose \( f \) is not contra-\( \omega \)-continuous at \( x \in X \). There exists \( F \in C(Y, f(x)) \) such that \( f(U) \cap (Y - F) \neq \emptyset \) for every \( U \in \omega O(X, x) \) by Theorem 2.5. This implies that \( U \cap f^{-1}(Y - F) \neq \emptyset \). Therefore, we have \( x \in \text{Cl}_w(f^{-1}(Y - F)) = \text{Cl}_w(X - f^{-1}(F)) \). However,
since \( x \in f^{-1}(F) \subseteq \text{Cl}_\omega(f^{-1}(F)) \), thus \( x \in \text{Cl}_\omega(f^{-1}(F)) \cap \text{Cl}_\omega(f^{-1}(Y - F)) \). Therefore, we obtain \( x \in Fr_\omega(f^{-1}(F)) \). Suppose that \( x \in Fr_\omega(f^{-1}(F)) \) for some \( F \in C(Y, f(x)) \), and \( f \) is contra-\( \omega \)-continuous at \( x \), then there exists \( U \in \omega O(X,x) \) such that \( f(U) \subseteq F \). Therefore, we have \( x \in U \subseteq f^{-1}(F) \) and hence \( x \in \text{Int}_\omega(f^{-1}(F)) \subseteq X - Fr_\omega(f^{-1}(F)) \). This is a contradiction. This mean that \( f \) is not contra-\( \omega \)-continuous.

**Theorem 2.23.** Let \( f : X \to Y \) be a function and let \( g : X \to X \times Y \) be the graph function of \( f \) defined by \( g(x) = (x, f(x)) \) for every \( x \in X \). If \( g \) is contra \( \omega \)-continuous, then \( f \) is contra \( \omega \)-continuous.

**Proof.** Let \( U \) be an open set in \( Y \), then \( X \times U \) is an open set in \( X \times Y \). Since \( g \) is contra \( \omega \)-continuous. It follows that \( f^{-1}(U) = g^{-1}(X \times U) \) is an \( \omega \)-closed in \( X \). Thus, \( f \) is contra \( \omega \)-continuous.

**Theorem 2.24.** If \( f : X \to Y \) and \( g : X \to Y \) are contra \( \omega \)-continuous and \( Y \) is Urysohn, then \( E = \{x \in X : f(x) = g(x)\} \) is \( \omega \)-closed in \( X \).

**Proof.** Let \( x \in X - E \). Then \( f(x) \neq g(x) \). Since \( Y \) is Urysohn, there exist open sets \( V \) and \( W \) such that \( f(x) \in V, g(x) \in W \), and \( \text{Cl}(V) \cap \text{Cl}(W) = \emptyset \). Since \( f \) and \( g \) is contra \( \omega \)-continuous, then \( f^{-1}(\text{Cl}(V)) \) and \( g^{-1}(\text{Cl}(W)) \) are \( \omega \)-open sets in \( X \). Let \( U = f^{-1}(\text{Cl}(V)) \) and \( G = g^{-1}(\text{Cl}(W)) \). Then \( U \) and \( V \) are \( \omega \)-open sets containing \( x \). Set \( A = U \cap G \), thus \( A \) is \( \omega \)-open in \( X \). Hence, \( f(A) \cap g(A) = f(U \cap G) \cap g(U \cap G) \subseteq f(U) \cap g(G) = \text{Cl}(V) \cap \text{Cl}(W) = \emptyset \); therefore, \( A \cap E = \emptyset \) and \( x \notin \text{Cl}_\omega(x) \). Hence, \( E \) is \( \omega \)-closed in \( X \).

A subset \( A \) of a topological space \( X \) is said to be \( \omega \)-dense in \( X \) if \( \text{Cl}_\omega(A) = X \).

**Theorem 2.25.** Let \( f : X \to Y \) and \( g : X \to Y \) be functions. If \( Y \) is Urysohn, \( f \) and \( g \) are contra \( \omega \)-continuous and \( f = g \) on \( \omega \)-dense set \( A \subseteq X \), then \( f = g \) on \( X \).

**Proof.** Since \( f \) and \( g \) are contra \( \omega \)-continuous and \( Y \) is Urysohn, by the previous theorem, \( E = \{x \in X : f(x) = g(x)\} \) is \( \omega \)-closed in \( X \). By assumption, we have \( f = g \) on \( \omega \)-dense set \( A \subseteq X \). Since \( A \subseteq E \) and \( A \) is \( \omega \)-dense set in \( X \), then \( X = \text{Cl}_\omega(A) \subseteq \text{Cl}_\omega(E) = E \). Hence, \( f = g \) on \( X \).

**Definition 2.26.** A space \( X \) is called \( \omega \)-connected provided that \( X \) is not the union of two disjoint nonempty \( \omega \)-open sets.

**Theorem 2.27.** If \( f : X \to Y \) is a contra \( \omega \)-continuous function from an \( \omega \)-connected space \( X \) onto any space \( Y \), then \( Y \) is not a discrete space.

**Proof.** Suppose that \( Y \) is discrete. Let \( A \) be a proper nonempty open and closed subset of \( Y \). Then \( f^{-1}(A) \) is a proper nonempty \( \omega \)-clopen subset of \( X \), which is a contradiction to the fact that \( X \) is \( \omega \)-connected.

**Theorem 2.28.** If \( f : X \to Y \) is contra-\( \omega \)-continuous surjection and \( X \) is \( \omega \)-connected, then \( Y \) is connected.

**Proof.** Suppose that \( Y \) is not connected space. Then there exist nonempty disjoint open sets \( V_1 \) and \( V_2 \) such that \( Y = V_1 \cup V_2 \). Therefore, \( V_1 \) and \( V_2 \) are clopen in \( Y \). Since \( f \) is contra-\( \omega \)-continuous, \( f^{-1}(V_1) \) and \( f^{-1}(V_2) \) are \( \omega \)-open in \( X \). Moreover, \( f^{-1}(V_1) \) and
Theorem 2.29. A space $X$ is $\omega$-connected, if every contra-$\omega$-continuous from a space $X$ into any $T_0$-space $Y$ is constant.

Proof. Suppose that $X$ is not $\omega$-connected and every contra-$\omega$-continuous function from $X$ into $Y$ is constant. Since $X$ is not $\omega$-connected, there exists a proper nonempty $\omega$-clopen subset $A$ of $X$. Let $Y = \{a, b\}$ and $\tau = \{Y, \phi, \{a\}, \{b\}\}$ be a topology for $Y$. Let $f : X \to Y$ be a function such that $f(A) = \{a\}$ and $f(X - A) = \{b\}$. Then $f$ is nonconstant and contra-$\omega$-continuous such that $Y$ is $T_0$ which is a contradiction. Hence, $X$ must be $\omega$-connected. $\square$

Definition 2.30. A space $X$ is said to be $\omega$-$T_2$ if for each pair of distinct points $x$ and $y$ in $X$, there exist $U \in \omega O(X, x)$ and $V \in \omega O(X, y)$ such that $U \cap V = \phi$.

Theorem 2.31. Let $X$ and $Y$ be topological spaces. If

1. for each pair of distinct points $x$ and $y$ in $X$ there exists a function $f$ of $X$ into $Y$ such that $f(x) \neq f(y)$,
2. $Y$ is an Urysohn space,
3. $f$ is contra-$\omega$-continuous at $x$ and $y$, then $X$ is $\omega$-$T_2$.

Proof. Let $x$ and $y$ be any distinct points in $X$. Then, there exists a Urysohn space $Y$ and a function $f : X \to Y$ such that $f(x) \neq f(y)$ and $f$ is contra-$\omega$-continuous at $x$ and $y$. Let $a = f(x)$ and $b = f(y)$. Then $a \neq b$. Since $Y$ is Urysohn space, there exist open sets $V$ and $W$ containing $a$ and $b$, respectively, such that $\text{Cl}(V) \cap \text{Cl}(W) = \phi$. Since $f$ is contra-$\omega$-continuous at $x$ and $y$, then there exist $\omega$-open sets $A$ and $B$ containing $a$ and $b$, respectively, such that $f(A) \subseteq \text{Cl}(V)$ and $f(B) \subseteq \text{Cl}(W)$. Then $f(A) \cap f(B) = \phi$, so $A \cap B = \phi$. Hence, $X$ is $\omega$-$T_2$. $\square$

Corollary 2.32. Let $f : X \to Y$ be contra-$\omega$-continuous injection. If $Y$ is an Urysohn space, then $X$ is $\omega$-$T_2$.

3. Almost contra $\omega$-continuous

In this section, we introduce a new type of continuity called almost contra $\omega$-continuous which is weaker than contra $\omega$-continuous.

Definition 3.1. A function $f : X \to Y$ is said to be almost contra-$\omega$-continuous (resp., almost contra-precontinuous [6]) if $f^{-1}(V) \in \omega C(X)$ (resp., $f^{-1}(V) \in PC(X)$) for every $V \in RO(X)$.

Theorem 3.2. The following are equivalents for a function $f : X \to Y$:

1. $f$ is almost contra-$\omega$-continuous;
2. $f^{-1}(F) \in \omega O(X, x)$ for every $F \in RC(Y)$;
3. for each $x \in X$ and each regular closed set $F$ in $Y$ containing $f(x)$, there exists an $\omega$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq F$;
4. for each $x \in X$ and each regular open set $V$ in $Y$ noncontaining $f(x)$, there exists an $\omega$-closed set $K$ in $X$ noncontaining $x$ such that $f^{-1}(V) \subseteq K$. 


Proof. (1) ⇔ (2). Let $F$ be any regular closed set of $Y$. Then $Y - F$ is regular open. By (1), $f^{-1}(Y - F) = X - f^{-1}(F) \in \omega C(X)$. We have $f^{-1}(F) \in \omega O(X)$. The converse is obvious.

(2) ⇒ (3). Let $F$ be any regular closed set in $Y$ containing $f(x)$. Then by (2) $f^{-1}(F) \in \omega O(X)$ and $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$. Then $f(U) \subseteq F$.

(3) ⇒ (2). Let $F$ be any regular closed set in $Y$ and $x \in f^{-1}(F)$. From (3) there exists an $\omega$-open $U_x$ in $X$ containing $x$ such that $f(U_x) \subseteq F$, thus $U_x \subseteq f^{-1}(F)$. We have $f^{-1}(F) \subseteq \cup_{x \in f^{-1}(F)} U_x$. This implies that $f^{-1}(F)$ is $\omega$-open.

(3) ⇔ (4). Let $V$ be any regular open set in $Y$ noncontaining $f(x)$. Then $Y - V$ is a regular closed set containing $f(x)$. By (3), there exists an $\omega$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq Y - V$. Hence, $U \subseteq f^{-1}(Y - V) \subseteq X - f^{-1}(V)$ and then $f^{-1}(V) \subseteq X - U$. Take $H = X - U$. We obtain that $H$ is an $\omega$-closed set in $X$ noncontaining $x$. The converse is obvious.

The following examples show that almost contra-$\omega$-continuous and almost contra-precontinuous functions are independent notions.

Example 3.3. Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $RC(X, \tau) = \{X, \phi, \{b, c\}, \{a, c\}\}$ and $\omega O(X, \tau) = \mathcal{P}(X)$, where $\mathcal{P}(X)$ is the power set of $X$, $PO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \tau)$ be the identity map. Then $f$ is almost contra-$\omega$-continuous function which is not almost contra-precontinuous, since $\{a, c\}$ is a regular closed set of $(X, \tau)$ and $f^{-1}(\{a, c\}) = \{a, c\} \in PO(X, \tau)$.

Example 3.4. Let $\mathbb{R}$ be the real number with usual topology and $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$, then $RO(X) = \{\phi, X, \{a\}, \{b\}\}$. Let $f : \mathbb{R} \to X$ be defined as $f(x) = a$ if $x \in \mathbb{Q}$ and $f(x) = c$ if $x \notin \mathbb{Q}$. Then $f$ is almost contra-precontinuous function which is not almost contra $\omega$-continuous, since $\{a\}$ is a regular closed set in $(X, \tau)$ and $f^{-1}(\{a\}) = \mathbb{Q}$ which is not $\omega$-open but preopen in $\mathbb{R}$.

\[
\begin{align*}
\text{contra-}\omega\text{-continuity} & \Rightarrow \text{almost contra-}\omega\text{-continuity} \Rightarrow \text{almost week-}\omega\text{-continuity} \\
\text{contra-continuity} & \Rightarrow (\theta, s)\text{-continuity} \Rightarrow \text{week-continuity} \\
\text{contra-percontinuity} & \Rightarrow \text{almost contra-precontinuity} \Rightarrow \text{almost week-continuity} \\
\text{contra-}\gamma\text{-continuity} & \Rightarrow \text{almost contra-}\gamma\text{-continuity} \Rightarrow \text{almost week-}\gamma\text{-continuity}
\end{align*}
\]

(3.1)

A space $(X, \tau)$ is anti-locally countable [11] if all nonempty open subsets are uncountable. Note that $\mathbb{R}$ with usual topology is anti-locally countable space.

Lemma 3.5 [11]. If $(X, \tau)$ is an anti-locally countable space, then $\text{Cl}_\omega(A) = \text{Cl}(A)$ for every $\omega$-open subset of $X$ and $\text{Int}(A) = \text{Int}_\omega(A)$ for every $\omega$-closed subset of $X$. 
Lemma 3.7 [11]. If $(X,\tau)$ is a locally countable space, then $\tau_\omega$ is the discrete topology on $X$.

Definition 3.8. A function $f : X \to Y$ is said to be regular set-connected if $f^{-1}(V)$ is clopen in $X$ for each regular open set $V$ of $Y$.

Theorem 3.9. Let $(X,\tau)$ be an anti-locally countable space, if a function $f : X \to Y$ is almost contra-$\omega$-continuous and almost continuous, then $f$ is regular set-connected.

Proof. Let $V$ be any regular open set in $Y$. Since $f$ is almost contra-$\omega$-continuous and contra continuous $f^{-1}(V)$ is $\omega$-closed and open. Thus $\text{Cl}_\omega(f^{-1}(V)) = (f^{-1}(V))$, since $(X,\tau)$ be an anti-locally countable space then by Lemma 3.5, we have $\text{Cl}_\omega(f^{-1}(V)) = \text{Cl}(f^{-1}(V))$. Hence $f^{-1}(V)$ is clopen. We obtain that $f$ is regular set-connected.

Definition 3.10 [14]. A space $X$ is said to be weakly Hausdorff if each element of $X$ is an intersection of regular closed sets.

Definition 3.11. A space $X$ is said to be $\omega$-$T_1$ if for each pair of distinct points $x$ and $y$ of $X$, there exists $\omega$-open sets $U$ and $V$ containing $x$ and $y$, respectively, such that $y \notin U$ and $x \notin V$.

Theorem 3.12. If $f : X \to Y$ is an almost contra-$\omega$-continuous injection and $Y$ is weakly Hausdorff, then $X$ is $\omega$-$T_1$.

Proof. Suppose that $Y$ is weakly Hausdorff. For any distinct points $x$ and $y$ in $X$, there exists $V, W$ which are regular closed in $Y$ such that $f(x) \in V, f(y) \notin V, f(x) \notin W$, and $f(y) \in W$. Since $f$ is almost contra-$\omega$-continuous, then $f^{-1}(V)$ and $f^{-1}(W)$ are $\omega$-open subsets of $X$ such that $x \in f^{-1}(V), y \notin f^{-1}(V), x \notin f^{-1}(W), y \in f^{-1}(W)$. This show that $X$ is $\omega$-$T_1$.

Corollary 3.13. If $f : X \to Y$ is an contra-$\omega$-continuous injection and $Y$ is weakly Hausdorff, then $X$ is $\omega$-$T_1$.

Theorem 3.14. If $f : X \to Y$ is almost contra-$\omega$-continuous surjection and $X$ is $\omega$-connected, then $Y$ is connected.

Proof. Suppose that $Y$ is not connected space. There exist nonempty disjoint open sets $V_1$ and $V_2$ such that $Y = V_1 \cup V_2$. Therefore, $V_1$ and $V_2$ are clopen sets. Thus they are regular open in $Y$. Since $f$ is almost contra-$\omega$-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $\omega$-open in $X$. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that $X$ is not $\omega$-connected. This is a contradiction. This means that $Y$ is connected.

Definition 3.15. A space $X$ is said to be

(1) $\omega$-compact if every $\omega$-open cover of $X$ has a finite subcover;
(2) countably $\omega$-compact if every countable cover of $X$ by $\omega$-open sets has a finite subcover;
(3) $\omega$-Lindelof if every $\omega$-open cover of $X$ has a countable subcover;
(4) S-Lindelof [6] if every cover of $X$ by regular closed sets has a countable subcover;
(5) countably S-closed [15] if every countable cover of $X$ by regular closed sets has a finite subcover;
(6) S-closed [16] if every regular closed cover of $X$ has a finite subcover.

**Theorem 3.16.** Let $f : X → Y$ be an almost contra-$ω$-continuous surjection. The following statements hold:

1. If $X$ is $ω$-compact, then $Y$ is S-closed;
2. If $X$ is $ω$-Lindelof, then $Y$ is S-Lindelof;
3. If $X$ is countably $ω$-compact, then $Y$ is countably S-closed.

**Proof.** We prove only (1). Let $\{ V_α : α ∈ I \}$ be any regular closed cover of $Y$. Since $f$ is almost contra-$ω$-continuous, then $\{ f^{-1}(V_α) : α ∈ I \}$ is an $ω$-open cover of $X$ and hence there exists a finite subset $I_0$ of $I$ such that $X = \bigcup \{ f^{-1}(V_α) : α ∈ I_0 \}$; therefore we have $Y = \bigcup \{ V_α : α ∈ I_0 \}$ and $Y$ is S-closed. □

**Definition 3.17.** A space $X$ is said to be

1. $ω$-closed compact if every $ω$-closed cover of $X$ has a finite subcover;
2. countably $ω$-closed compact if every countable cover of $X$ by $ω$-closed sets has a finite subcover;
3. $ω$-closed-Lindelof if every cover of $X$ by $ω$-closed sets has a countable subcover;
4. nearly compact [17] if every regular open cover of $X$ has a finite subcover;
5. nearly countably compact [17] if every countable cover of $X$ by regular open sets has a finite subcover;
6. nearly Lindelof [17] if every cover of $X$ by regular open sets has a countably subcover.

**Theorem 3.18.** Let $f : X → Y$ be an almost contra-$ω$-continuous surjection. The following statements hold:

1. If $X$ is $ω$-closed compact, then $Y$ is nearly compact;
2. If $X$ is $ω$-closed-Lindelof, then $Y$ nearly Lindelof;
3. If $X$ is countably $ω$-closed compact, then $Y$ is nearly countably compact.

**Proof.** We prove only (1). Let $\{ V_α : α ∈ I \}$ be any regular open cover of $Y$. Since $f$ is almost contra-$ω$-continuous, then $\{ f^{-1}(V_α) : α ∈ I \}$ is an $ω$-closed cover of $X$. Since $X$ is $ω$-closed compact, there exists a finite subset $I_0$ of $I$ such that $X = \bigcup \{ f^{-1}(V_α) : α ∈ I_0 \}$. Thus, we have $Y = \bigcup \{ V_α : α ∈ I_0 \}$ and $Y$ is nearly compact. □

**Definition 3.19** [14]. A space $X$ is said to be mildly compact (mildly countably compact, mildly Lindelof) if every clopen cover (resp., clopen countably cover, clopen cover) of $X$ has a finite (resp., a finite, a countable) subcover.

**Theorem 3.20.** Let $(X, τ)$ be an anti-locally countable space, if $f : X → Y$ be an almost contra-$ω$-continuous and almost continuous surjection and $X$ is mildly compact (resp., mildly countably compact, mildly Lindelof), then $Y$ is nearly compact (resp., nearly countably compact, nearly Lindelof) and S-closed (resp., countably S-closed, S-Lindelof).

**Proof.** Let $V$ be any regular closed set on $Y$. Then since $f$ is almost contra-$ω$-continuous and almost continuous, then $f^{-1}(V)$ is $ω$-open and closed in $X$. By Lemma 3.5, we have $\text{Int}(f^{-1}(V)) = \text{Int}_ω(f^{-1}(V)) = f^{-1}(V)$. Hence, $f^{-1}(V)$ is clopen. Let $\{ V_α : α ∈ I \}$ be
any regular closed (resp., regular open) cover of \( Y \). Then \( \{ F^{-1}(V_\alpha : \alpha \in I) \} \) is a clopen cover of \( X \) and since \( X \) is mildly compact, there exists a finite subset \( I_0 \) of \( I \) such that \( X = \cup \{ f^{-1}(V_\alpha) : \alpha \in I_0 \} \). Since \( f \) is surjection, we obtain \( Y = \cup \{ V_\alpha : \alpha \in I_0 \} \). This shows that \( Y \) is \( S \)-closed (resp., nearly compact). The other proofs are similar.

**Theorem 3.21.** If \( f : X \to Y \) is contra-\( \omega \)-continuous and \( A \) is \( \omega \)-compact relative to \( X \), then \( f(A) \) is strongly \( S \)-closed in \( Y \).

*Proof.* Let \( \{ V_i : i \in I \} \) be any cover of \( f(A) \), by closed sets of the subspace \( f(A) \). For \( i \in I \), there exists a closed set \( A_i \) of \( Y \) such that \( V_i = A_i \cap f(A) \). For each \( x \in A \), there exists \( i(x) \in I \) such that \( f(x) \in A_{i(x)} \) and by Theorem 2.5, there exists \( U_x \in \omega O(X, x) \) such that \( f(U_x) \subseteq A_{i(x)} \). Since the family \( \{ U_x : x \in A \} \) is a cover of \( A \) by \( \omega \)-open sets of \( X \), there exists a finite subset \( A_0 \) of \( A \) such that \( A \subseteq \cup \{ U_x : x \in A_0 \} \). Therefore, we obtain \( f(A) \subseteq \cup \{ f(U_x) : x \in A_0 \} \), which is a subset of \( \cup \{ A_{i(x)} : x \in A_0 \} \). Thus \( f(A) = \cup \{ V_{i(x)} : x \in A_0 \} \) and hence \( f(A) \) is strongly \( S \)-closed.

**Corollary 3.22.** If \( f : X \to Y \) is contra-\( \omega \)-continuous surjection and \( X \) is \( \omega \)-compacts, then \( Y \) is strongly \( S \)-closed.

4. Contra-closed graphs

Recall that for a function \( f : X \to Y \), the subset \( \{(x, f(x)) : x \in X \} \subseteq X \times Y \) is called the graph of \( f \) and is denoted by \( G(f) \).

**Definition 4.1.** The graph \( G(f) \) of a function \( f : X \to Y \) is said to be contra-\( \omega \)-closed if for each \( (x, y) \in (X \times Y) - G(f) \), there exist \( U \in \omega O(X, x) \) and \( V \in C(Y, y) \) such that \( (U \times V) \cap G(f) = \phi \).

The following results can be easily verified.

**Lemma 4.2** [6]. Let \( G(f) \) be the graph of \( f \), for any subset \( A \subseteq X \) and \( B \subseteq Y \), we have \( f(A) \cap B = \phi \) if and only if \( (A \times B) \cap G(f) = \phi \).

**Lemma 4.3.** The graph \( G(f) \) of \( f : X \to Y \) is contra-\( \omega \)-closed in \( X \times Y \) if and only if for each \( (x, y) \in (X \times Y) - G(f) \), there exist \( U \in \omega O(X, x) \) and \( V \in C(Y, y) \) such that \( f(U) \cap V = \phi \).

**Theorem 4.4.** If \( f : X \to Y \) is contra-\( \omega \)-continuous and \( Y \) is Urysohn, then \( G(f) \) is contra-\( \omega \)-closed in \( X \times Y \).

*Proof.* Let \( (x, y) \in (X \times Y) - G(f) \). Then \( y \neq f(x) \) and there exists open sets \( V, W \) such that \( f(x) \in V, y \in W \), and \( \text{Cl}(V) \cap \text{Cl}(W) = \phi \). Since \( f \) is contra-\( \omega \)-continuous, there exists \( U \in \omega O(X, x) \) such that \( f(U) \subseteq \text{Cl}(V) \). Therefore, we obtain \( f(U) \cap \text{Cl}(W) = \phi \). This shows that \( G(f) \) is contra-\( \omega \)-closed.

**Theorem 4.5.** If \( f : X \to Y \) is \( \omega \)-continuous and \( Y \) is \( T_1 \), then \( G(f) \) is contra-\( \omega \)-closed in \( X \times Y \).
Theorem 4.6. If \( f : X \to Y \) has a contra \( \omega \)-closed graph, then the inverse image of a strongly \( S \)-closed set \( A \) of \( Y \) is \( \omega \)-closed in \( X \).

Proof. Assume that \( A \) is a strongly \( S \)-closed set of \( Y \) and \( x \notin f^{-1}(A) \). For each \( a \in A, (x, a) \notin G(f) \). By Lemma 4.3 there exist \( U_a \in \omega(X, x) \) and \( V_a \in C(Y, a) \) such that \( f(U_a) \cap V_a = \phi \). Since \( \{A \cap V_a \mid a \in A\} \) is a closed cover of the subspace \( A \), there exists a finite subset \( A_0 \subseteq A \) such that \( A \subseteq \bigcup \{V_a \mid a \in A_0\} \). Set \( U = \bigcap \{U_a \mid a \in A_0\} \), and \( U \) is \( \omega \)-open since \( \tau_{\omega} \) is topology and \( f(U) \cap A = \phi \). Therefore, \( U \cap f^{-1}(A) = \phi \); and hence, \( x \notin \text{Cl}_\omega(f^{-1}(A)) \). This shows that \( f^{-1}(A) \) is \( \omega \)-closed.

Theorem 4.7. Let \( Y \) be a strongly \( S \)-closed space. If a function \( f : X \to Y \) has a contra-\( \omega \)-closed graph, then \( f \) is contra \( \omega \)-continuous.

Proof. Suppose that \( Y \) is strongly \( S \)-closed space and \( G(f) \) is contra \( \omega \)-closed. First we show that an open set of \( Y \) is strongly \( S \)-closed. Let \( U \) be an open set of \( Y \) and \( \{V_i \mid i \in I\} \) be a cover of \( U \) by closed sets \( V_i \) of \( U \). For each \( i \in I \), there exists a closed set \( K_i \) of \( X \) such that \( V_i = K_i \cap U \). Then the family \( \{K_i \mid i \in I\} \cup (Y - U) \) is a closed cover of \( Y \). Since \( Y \) is strongly \( S \)-closed, there exists a finite subset \( I_0 \subseteq I \) such that \( Y = \bigcup \{K_i \mid i \in I_0\} \cup (Y - U) \). Therefore, we obtain \( U = \bigcup \{V_i \mid i \in I_0\} \). This shows that \( U \) is strongly \( S \)-closed. Now for any open set \( U \) by Theorem 4.6 \( f^{-1}(U) \) is \( \omega \)-closed in \( X \); therefore, \( f \) is contra \( \omega \)-continuous.

Definition 4.8. The graph \( G(f) \) of a function \( f : X \to Y \) is said to be strongly contra-\( \omega \)-closed if for each \( (x, y) \in (X, Y) \) \( G(f) \), there exist \( U \in \omega O(X, x) \) and \( V \in RC(Y, y) \) such that \( (U \times V) \cap G(f) = \phi \).

Lemma 4.9. The graph \( G(f) \) of \( f : X \to Y \) is strongly contra-\( \omega \)-closed graph in \( X \times Y \) if and only if for each \( (x, y) \in (X \times Y) \) \( G(f) \), there exist \( U \in \omega O(X, x) \) and \( V \in RC(Y, y) \) such that \( f(U) \cap V = \phi \).

Theorem 4.10. If \( f : X \to Y \) is almost weakly-\( \omega \)-continuous and \( Y \) is Urysohn, then \( G(f) \) is strongly contra-\( \omega \)-closed in \( X \times Y \).

Proof. Suppose that \( (x, y) \in (X \times Y) \) \( G(f) \). Then \( y \neq f(x) \). Since \( Y \) is Urysohn, there exist open sets \( V \) and \( W \) in \( Y \) containing \( y \) and \( f(x) \), respectively, such that \( \text{Cl}(V) \cap \text{Cl}(W) = \phi \). Since \( f \) is almost weakly-\( \omega \)-continuous, by Definition 2.20 there exists \( U \in \omega(X, x) \) such that \( f(U) \subseteq \text{Cl}(W) \). This shows that \( f(U) \cap \text{Cl}(V) = f(U) \cap \text{Cl}(\text{Int}(V)) = \phi \), where \( \text{Cl}(\text{Int}(V)) \subseteq RC(Y) \) and hence by Lemma 4.9 we have \( G(f) \) is strongly contra-\( \omega \)-closed.

Theorem 4.11. If \( f : X \to Y \) is almost contra-\( \omega \)-continuous, then \( f \) is almost weakly-\( \omega \)-continuous.
Lemma 4.15. Let \( x \in X \) and \( V \) be any open set of \( Y \) containing \( f(x) \). Then \( \text{Cl}(V) \) is a regular closed set of \( Y \) containing \( f(x) \). Since \( f \) is almost contra-\( \omega \)-continuous, by Theorem 3.2 there exists \( U \in \omega O(X,x) \) such that \( f(U) \subseteq \text{Cl}(V) \). By Definition 2.20 \( f \) is almost weakly-\( \omega \)-continuous. \( \square \)

Corollary 4.12. If \( f : X \to Y \) is almost contra-\( \omega \)-continuous and \( Y \) is Urysohn, then \( G(f) \) is strongly contra-\( \omega \)-closed.

The following result can be easily verified.

Lemma 4.13. a function \( f : X \to Y \) is almost \( \omega \)-continuous, if and only if for each \( x \in X \) and each regular open set \( V \) of \( Y \) containing \( f(x) \), there exists \( U \in \omega O(X,x) \) such that \( f(U) \subseteq V \).

Theorem 4.14. If \( f : X \to Y \) is almost \( \omega \)-continuous, and \( Y \) is Hausdorff, then \( G(f) \) is strongly contra-\( \omega \)-closed.

Proof. Suppose that \((x, y) \in (X \times Y) - G(f)\). Then \( y \neq f(x) \). Since \( Y \) is Hausdorff, there exist open sets \( V \) and \( W \) in \( Y \) containing \( y \) and \( f(x) \), respectively, such that \( V \cap W = \phi \); hence, \( \text{Cl}(V) \cap \text{Int}(\text{Cl}(W)) = \phi \). Since \( f \) is almost \( \omega \)-continuous, and \( W \) is regular open by Lemma 4.13 there exists \( U \in \omega O(X,x) \) such that \( f(U) = W \subseteq \text{Int}(\text{Cl}(W)) \). This shows that \( f(U) \cap \text{Cl}(V) = \phi \) and hence by Lemma 4.9 we have \( G(f) \) is strongly contra-\( \omega \)-closed. \( \square \)

We recall that a topological space \((X, \tau)\) is said to be extremely disconnected (E.D) if the closure of every open set of \( X \) is open in \( X \).

Theorem 4.15. Let \( Y \) be E.D. Then a function \( f : X \to Y \) is almost contra-\( \omega \)-continuous if and only if it is almost \( \omega \)-continuous.

Proof. Let \( x \in X \) and \( V \) be any regular open set of \( Y \) containing \( f(x) \). Since \( Y \) is E.D then \( V \) is clopen and hence \( V \) is regular closed. By Theorem 3.2, there exists \( U \in \omega O(X,x) \) such that \( f(U) \subseteq V \). Then Lemma 4.13 implies that \( f \) is almost \( \omega \)-continuous. Conversely, let \( F \) be any regular closed set of \( Y \). Since \( Y \) is E.D, \( F \) is also regular open and \( f^{-1}(F) \) is \( \omega \)-open in \( X \). This shows that \( f \) is almost contra-\( \omega \)-continuous. \( \square \)

Theorem 4.16. If \( f : X \to Y \) is an injective almost contra-\( \omega \)-continuous function with the strongly contra-\( \omega \)-closed graph, then \((X, \tau)\) is \( \omega \)-\( T_2 \).

Proof. Let \( x \) and \( y \) be distinct points of \( X \). Then, since \( f \) is injective, we have \( f(x) \neq f(y) \). Then we have \((x, f(y)) \in (X \times Y) - G(f) \). Since \( G(f) \) is strongly contra-\( \omega \)-closed, by Lemma 4.9 there exists \( U \in \omega O(X,x) \) and a regular closed set \( V \) containing \( f(y) \) such that \( f(U) \cap V = \phi \). Since \( f \) is almost contra-\( \omega \)-continuous, by Theorem 3.2 there exists \( G \in \omega O(X,y) \) such that \( f(G) \subseteq V \). Therefore, we have \( f(U) \cap f(G) = \phi \); hence, \( U \cap G = \phi \). This shows that \((X, \tau)\) is \( \omega \)-\( T_2 \). \( \square \)

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References


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Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from “Qualitative Theory of Differential Equations,” allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the Mathematical Problems in Engineering aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

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