Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2008, Article ID 318582, 9 pages
doi:10.1155/2008/318582

Research Article

Some New Inclusion and Neighborhood Properties for Certain Multivalent Function Classes Associated with the Convolution Structure

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Received 13 August 2007; Accepted 7 January 2008

Recommended by Brigitte Forster-Heinlein

We use the familiar convolution structure of analytic functions to introduce two new subclasses of multivalently analytic functions of complex order, and prove several inclusion relationships associated with the (n, δ)-neighborhoods for these subclasses. Some interesting consequences of these results are also pointed out.

1. Introduction and preliminaries

Let $\mathcal{A}_p(n)$ denote the class of functions of the form

$$f(z) = z^p + \sum_{k=n}^{\infty} a_k z^k \quad \big(p < n; n, p \in \mathbb{N} = \{1, 2, \dots\}\big), \tag{1.1}$$

which are analytic and p-valent in the open unit disk

$$\mathbb{U} = \{z;\ z \in \mathbb{C} : |z| < 1\}. \tag{1.2}$$

If $f \in \mathcal{A}_p(n)$ is given by (1.1) and $g \in \mathcal{A}_p(n)$ is given by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k, \tag{1.3}$$

then the Hadamard product (or convolution) $f * g$ of f and g is defined (as usual) by

$$(f*g)(z) := z^p + \sum_{k=n}^{\infty} a_k b_k z^k := (g*f)(z). \tag{1.4}$$

We denote by $\mathcal{T}_p(n)$ the subclass of $\mathcal{A}_p(n)$ consisting of functions of the form

$$f(z) = z^p - \sum_{k=n}^{\infty} a_k z^k \quad (p < n; a_k \geq 0 (k \geq n); n, p \in \mathbb{N}), \tag{1.5}$$

which are p-valent in $\mathbb{U}$.

For a fixed function $g(z) \in \mathcal{A}_p(n)$ defined by

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k \quad (p < n; b_k \geq 0 (k \geq n); n, p \in \mathbb{N}), \tag{1.6}$$

we introduce a new class $\mathcal{S}_p^{\lambda}(g; n, b, m)$ of functions belonging to the subclass of $\mathcal{T}_p(n)$, which consists of functions $f(z)$ of the form (1.5), satisfying the following inequality:

$$\left| \frac{1}{b} \left(\frac{z(f*g)^{(m+1)}(z) + \lambda z^2 (f*g)^{(m+2)}(z)}{\lambda z (f*g)^{(m+1)}(z) + (1-\lambda)(f*g)^{(m)}(z)} - (p - m) \right) \right| < 1$$
$$\left(z \in \mathbb{U};\ p \in \mathbb{N};\ m \in \mathbb{N}_0;\ p > m;\ 0 \leq \lambda \leq 1;\ b \in \mathbb{C} \setminus \{0\} \right). \tag{1.7}$$

We note that there exist several interesting new (or known) subclasses of our function class $\mathcal{S}_p^{\lambda}(g; n, b, m)$. For example, if $\lambda = 0$ in (1.7), we obtain the class $\mathcal{S}_p(g; n, b, m)$ studied very recently by Prajapat et al. [1]. On the other hand, if the coefficients b_k in (1.6) are chosen as follows:

$$b_k = \left(\frac{k + \mu}{p + \mu} \right)^r \quad (\mu \geq 0;\ k \geq n;\ r, p, n \in \mathbb{N}), \tag{1.8}$$

and n is replaced by $n+p$ in (1.4) and (1.5), then we obtain the class $\mathcal{S}_{n,m}^{p}(\mu, r, \lambda, b)$ of p-valently analytic functions (involving the multiplier transformation operator $I_p(r, \mu)$ defined in [2]) which was studied recently by Srivastava et al. [3]. Also, if we set $\lambda = 0$ in (1.7) and if the arbitrary sequence b_k in (1.6) is selected as follows:

$$b_k = \binom{\mu + k - 1}{k - p} \quad (\mu > -p;\ k \geq n; p, n \in \mathbb{N}), \tag{1.9}$$

also if n is replaced by $n+p$ in (1.4) and (1.5), then we obtain the class $\mathcal{H}_{n,m}^{p}(\mu, b)$ of p-valently analytic functions (involving the familiar Ruscheweyh derivative operator) investigated by Raina and Srivastava [4]. Further, when

$$\lambda = 0, \qquad m = 0, \qquad b = p(1 - \alpha) \quad (p \in \mathbb{N};\ 0 \leq \alpha < 1), \tag{1.10}$$

in (1.7), then $\mathcal{S}_p^{\lambda}(g;n,b,m)$ reduces to the class studied recently by Ali et al. [5]. Moreover, when

$$g(z) = z^p + \sum_{k=n}^{\infty} \frac{(\alpha_1)_{k-p}\cdots(\alpha_q)_{k-p}}{(\beta_1)_{k-p}\cdots(\beta_s)_{k-p}(k-p)!} z^k,$$
$$\left(\alpha_j \in \mathbb{C}\ (j=1,2,\ldots,q),\ \beta_j \in \mathbb{C} \setminus \{0,-1,-2,\ldots\}\ (j=1,2,\ldots,s)\right) \tag{1.11}$$

with the parameters

$$\alpha_1,\ldots,\alpha_q, \qquad \beta_1,\ldots,\beta_s \tag{1.12}$$

being so chosen that the coefficients b_k in (1.6) satisfy the following condition:

$$b_k = \frac{(\alpha_1)_{k-p}\cdots(\alpha_q)_{k-p}}{(\beta_1)_{k-p}\cdots(\beta_s)_{k-p}(k-p)!} \geq 0, \tag{1.13}$$

then the class $\mathcal{S}_p^{\lambda}(g;n,b,m)$ transforms into a (presumably) new class $\mathcal{S}_p^{\lambda}(n,b,m)$ defined by

$$\mathcal{S}_p^{\lambda}(n,b,m) = \left\{ f \in \mathcal{T}_p(n) : \left| \frac{1}{b}\left(\frac{z\left(H_s^q[\alpha_1]f\right)^{(m+1)}(z) + \lambda z^2\left(H_s^q[\alpha_1]f\right)^{(m+2)}}{\lambda z\left(H_s^q[\alpha_1]f\right)^{(m+1)}(z) + (1-\lambda)\left(H_s^q[\alpha_1]f\right)^{(m)}} - (p-m)\right)\right| < 1 \right\}$$
$$\left(z \in \mathbb{U};\ q \leq s+1;\ m,q,s \in \mathbb{N}_0;\ 0 \leq \lambda \leq 1,\ p \in \mathbb{N};\ b \in \mathbb{C} \setminus \{0\}\right). \tag{1.14}$$

The operator

$$\left(H_s^q[\alpha_1]f\right)(z) := H_s^q(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_s)f(z), \tag{1.15}$$

involved in (1.14), is the Dziok-Srivastava linear operator (see for details [6]; see also [7, 8]) which contains such well-known operators as the Hohlov linear operator, Saitoh generalized linear operator, Carlson-Shaffer linear operator, Ruscheweyh derivative operator as well as its generalized version, the Bernardi-Libera-Livingston operator, and the Srivastava-Owa fractional derivative operator. One may refer to [7] or [6] for further details and references for these operators. The Dziok-Srivastava linear operator defined in [6] has further been generalized by Dziok and Raina [7] (see also [8, 9]).

Following a recent investigation by Frasin and Darus [10], let $f(z) \in \mathcal{T}_p(n)$, $\delta \geq 0$, then a (q,δ)-neighborhood of the function $f(z)$ is defined by

$$\mathcal{N}_{n,\delta}^{q}(f) = \left\{ h : h \in \mathcal{T}_p(n) : h(z) = z^p - \sum_{k=n}^{\infty} c_k z^k,\ \sum_{k=n}^{\infty} k^{q+1}\left|a_k - c_k\right| \leq \delta \right\}. \tag{1.16}$$

It follows from the definition (1.16) that if

$$e(z) = z^p \quad (p \in \mathbb{N}), \tag{1.17}$$

then

$$\mathcal{N}_{n,\delta}^{q}(e) = \left\{ h : h \in \mathcal{T}_p(n) : h(z) = z^p - \sum_{k=n}^{\infty} c_k z^k,\ \sum_{k=n}^{\infty} k^{q+1} |c_k| \le \delta \right\}. \tag{1.18}$$

We observe that

$$\begin{aligned} \mathcal{N}_{2,\delta}^{0}(f) &= \mathcal{N}_{\delta}(f), \\ \mathcal{N}_{2,\delta}^{1}(f) &= \mathcal{M}_{\delta}(f), \end{aligned} \tag{1.19}$$

where $\mathcal{N}_{\delta}(f)$ and $\mathcal{M}_{\delta}(f)$ denote, respectively, the δ-neighborhoods of the function

$$g(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (a_k \ge 0,\ z \in \mathbb{U}), \tag{1.20}$$

defined by Ruscheweyh [11] and Silverman [12].

Finally, for a fixed function

$$g(z) = z^p + \sum_{k=n}^{\infty} b_k z^k \in \mathcal{A}_p(n) \quad (p < n;\ b_k > 0\,(k \ge n);\ n, p \in \mathbb{N}), \tag{1.21}$$

let $\mathcal{P}_p^{\lambda}(g; n, b, m)$ denote the subclass of $\mathcal{T}_p(n)$ consisting of functions $f(z)$ of the form (1.5) which satisfy the following inequality:

$$\begin{gathered} \left| \frac{1}{b} \left\{ \left[1 - \lambda(p - m - 1)\right] (f * g)^{(m+1)}(z) + \lambda z (f * g)^{(m+2)}(z) - (p - m) \right\} \right| < p - m \\ \left(z \in \mathbb{U},\ m \in \mathbb{N}_0;\ p \in \mathbb{N};\ p > m;\ 0 \le \lambda \le 1,\ b \in \mathbb{C} \setminus \{0\}\right). \end{gathered} \tag{1.22}$$

The object of the present paper is to investigate the various properties and characteristics of functions belonging to the above-defined subclasses

$$\mathcal{S}_p^{\lambda}(g; n, b, m), \qquad \mathcal{P}_p^{\lambda}(g; n, b, m) \tag{1.23}$$

of p-valently analytic functions in $\mathbb{U}$. Apart from deriving coefficient inequalities for each of these function classes, we establish several inclusion relationships involving the (n, δ)-neighborhoods of functions belonging to these subclasses.

2. Coefficient bound inequalities

We begin by proving a necessary and sufficient condition for the function $f(z) \in \mathcal{T}_p(n)$ to be in each of the classes

$$\mathcal{S}_p^{\lambda}(g; n, b, m), \qquad \mathcal{P}_p^{\lambda}(g; n, b, m). \tag{2.1}$$

Theorem 2.1. *Let $f(z) \in \mathcal{T}_p(n)$ be given by* (1.5). *Then $f(z)$ is in the class $\mathcal{S}_p^{\lambda}(g; n, b, m)$ if and only if*

$$\sum_{k=n}^{\infty} a_k b_k \left[\lambda(k - m - 1) + 1\right] (k - p + |b|) \binom{k}{m} \le |b| \left[\lambda(p - m - 1) + 1\right] \binom{p}{m}. \tag{2.2}$$

Proof. Assume that $f(z) \in S_p^{\lambda}(g;n,b,m)$. Then, in view of (1.5)–(1.7), we get

$$\Re\left(\frac{\lambda z^2 (f*g)^{(m+2)}(z) + z\left[1-\lambda(p-m)\right](f*g)^{(m+1)}(z) - (1-\lambda)(p-m)(f*g)^{(m)}(z)}{\lambda z (f*g)^{(m+1)}(z) + (1-\lambda)(f*g)^{(m)}(z)}\right) > -|b| \tag{2.3}$$

which yields

$$\Re\left(\frac{\sum_{k=n}^{\infty} a_k b_k \binom{k}{m}(k-p)\left[\lambda(k-m-1)+1\right] z^{k-p}}{\binom{p}{m}\left[\lambda(p-m-1)+1\right] z^{p-m} - \sum_{k=n}^{\infty} a_k b_k \binom{k}{m}\left[\lambda(k-m-1)+1\right] z^{k-p}}\right) < |b| \quad (z \in \mathbb{U}). \tag{2.4}$$

Putting $z = r\,(0 \le r < 1)$ in (2.4), the denominator expression on the left-hand side of (2.4) remains positive for $r = 0$, and also for all $r \in (0,1)$. Hence, by letting $r \to 1^-$, through real values, inequality (2.4) leads to the desired assertion (2.2) of Theorem 2.1.

Conversely, by applying the hypothesis (2.2) of Theorem 2.1, and letting $|z| = 1$, we find that

$$\begin{aligned} &\left|\frac{z(f*g)^{(m+1)}(z) + \lambda z^2 (f*g)^{(m+2)}(z)}{\lambda z (f*g)^{(m+1)}(z) + (1-\lambda)(f*g)^{(m)}(z)} - (p-m)\right| \\ &\quad \le \frac{|b|\left\{\binom{p}{m}\left[\lambda(p-m-1)+1\right] - \sum_{k=n}^{\infty} a_k b_k \left[\lambda(k-m-1)+1\right]\binom{k}{m}\right\}}{\binom{p}{m}\left[\lambda(p-m-1)+1\right] - \sum_{k=n}^{\infty} a_k b_k \left[\lambda(k-m-1)+1\right]\binom{k}{m}} = |b|. \end{aligned} \tag{2.5}$$

Hence, by the *maximum modulus principle*, we infer that $f(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$, which completes the proof of Theorem 2.1. □

Remark 2.2. In the special case when

$$\text{(i)} \quad b_k = \left(\frac{k+\mu}{p+\mu}\right)^r \quad (\mu \ge 0;\ k \ge n;\ r,p,n \in \mathbb{N};\ n \mapsto n+p). \tag{2.6}$$

Theorem 2.1 corresponds to a result given recently by Srivastava et al. [3, Theorem 1, page 3]:

$$\text{(ii)} \quad \lambda = 0; \qquad b_k = \binom{\mu+k-1}{k-p} \quad \left(\mu > -p;\ k \ge n;\ n,p \in \mathbb{N};\ n \mapsto n+p\right). \tag{2.7}$$

Theorem 2.1 yields the result given recently by Raina and Srivastava [4, Theorem 1, page 3]:

$$\text{(iii)} \quad m = 0; \qquad p = 1, \qquad b_k = k^{\Omega} \quad (\Omega \in \mathbb{N}_0;\ k \ge n;\ n \in \mathbb{N};\ n \mapsto n+p). \tag{2.8}$$

Theorem 2.1 reduces to the result of Orhan and Kamali [13, Lemma 1, page 57]:

$$\text{(iv)} \quad \lambda = 0, \qquad m = 0, \qquad b = p(1-\alpha) \quad (p \in N;\ 0 \le \alpha < 1). \tag{2.9}$$

Theorem 2.1 gives a recently established result due to Ali et al. [5, Theorem 1, page 181].

The following results concerning the class of functions $\mathcal{P}_p^{\lambda}(g;n,b,m)$ can be proved on similar lines as given above for Theorem 2.1.

Theorem 2.3. *Let $f(z) \in \mathcal{T}_p(n)$ be given by (1.5). Then $f(z)$ is in the class $\mathcal{P}_p^{\lambda}(g;n,b,m)$ if and only if*

$$\sum_{k=n}^{\infty} \left[\lambda(k-p)+1\right](k-m)\binom{k}{m} a_k b_k \le (p-m)\left[\frac{|b|-1}{m!} + \binom{p}{m}\right]. \tag{2.10}$$

Remark 2.4. Making use of the same substitutions as mentioned above in (2.6), Theorem 2.3 yields another known result due to Srivastava et al. [3, Theorem 2, page 4]. Also, using the same substitutions as mentioned above in (2.8), we get the result of Orhan and Kamali [13, Lemma 2, page 58].

3. Inclusion properties

We now obtain some inclusion relationships for the function classes

$$\mathcal{S}_p^{\lambda}(g;n,b,m), \qquad \mathcal{P}_p^{\lambda}(g;n,b,m), \tag{3.1}$$

involving the (n,δ)-neighborhood defined by (1.18).

Theorem 3.1. *If $b_k \ge b_n$ $(k \ge n)$ and*

$$\delta := \frac{n\left[\lambda(p-m-1)+1\right]|b|\binom{p}{m}}{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n} \quad (p > |b|), \tag{3.2}$$

then

$$\mathcal{S}_p^{\lambda}(g;n,b,m) \subset \mathcal{N}_{n,\delta}^{0}(e). \tag{3.3}$$

Proof. Let $f(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$. Then, in view of assertion (2.2) of Theorem 2.1, and the given condition $b_k \ge b_n$ $(k \ge n)$, we get

$$\begin{aligned}
&\left[\lambda(n-m-1)+1\right](n-p+|b|)\binom{n}{m} b_n \sum_{k=n}^{\infty} a_k \\
&\quad \le \sum_{k=n}^{\infty} a_k b_k \left[\lambda(k-m-1)+1\right](k-p+|b|)\binom{k}{m} \le |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m},
\end{aligned} \tag{3.4}$$

which implies that

$$\sum_{k=n}^{\infty} a_k \leq \frac{|b|[\lambda(p-m-1)+1]\binom{p}{m}}{(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n}. \tag{3.5}$$

Applying the assertion (2.2) of Theorem 2.1 again (in conjunction with (3.5)), we obtain

$$\begin{aligned}
&\binom{n}{m}\left[\lambda(n-m-1)+1\right]b_n\sum_{k=n}^{\infty} k a_k \\
&\quad\leq |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m} + (p-|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m} b_n \sum_{k=n}^{\infty} a_k \\
&\quad\leq |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m} + (p-|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m} b_n \\
&\qquad\cdot\frac{|b|\left[\lambda(p-m-1)+1\right]\binom{p}{m}}{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n} \\
&\quad= \frac{n|b|\left[\lambda(p-m-1)+1\right]\binom{p}{m}}{(n-p+|b|)}.
\end{aligned} \tag{3.6}$$

Hence,

$$\sum_{k=n}^{\infty} k a_k \leq \frac{n[\lambda(p-m-1)+1]|b|\binom{p}{m}}{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n} := \delta \quad (p > |b|), \tag{3.7}$$

which by virtue of (1.18) establishes the inclusion relation (3.3) of Theorem 3.1. □

In the analogous manner, by applying the assertion (2.10) of Theorem 2.3 instead of the assertion (2.2) of Theorem 2.1 to the functions in the class $\mathcal{P}_p^{\lambda}(g;n,b,m)$, we can prove the following inclusion relationship.

Theorem 3.2. *If* $b_k \geq b_n$ $(k \geq n)$ *and*

$$\delta := \frac{(p-m)\left[(|b|-1)/m! + \binom{p}{m}\right]}{\left[\lambda(n-p)+1\right]\binom{n-1}{m}b_n}, \tag{3.8}$$

then

$$\mathcal{P}_p^{\lambda}(g;n,b,m) \subset N_{n,\delta}^{0}(e). \tag{3.9}$$

Remark 3.3. Applying the parametric substitutions listed in (2.6), Theorems 3.1 and 3.2 would yield the known results due to Srivastava et al. [3, Theorem 3, page 4; Theorem 4, page 5]. Also, using substitutions (as mentioned above in (2.8)) in Theorems 3.1 and 3.2, we get the results due to Orhan and Kamali [13, Theorem 1, page 58; Theorem 2, page 59].

4. Neighborhood properties

This concluding section determines the neighborhood properties for each of the classes

$$\mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m), \qquad \mathcal{P}_p^{(\lambda,\alpha)}(g;n,b,m) \tag{4.1}$$

which are defined as follows.

A function $f(z) \in \mathcal{T}_p(n)$ is said to be in the class $\mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m)$ if there exists a function $h(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$ such that

$$\left|\frac{f(z)}{h(z)} - 1\right| < p - \alpha \quad (z \in \mathbb{U};\ 0 \le \alpha < p). \tag{4.2}$$

Analogously, a function $f(z) \in \mathcal{T}_p(n)$ is said to be in the class $\mathcal{P}_p^{(\lambda,\alpha)}(g;n,b,m)$ if there exists a function $h(z) \in \mathcal{P}_p^{\lambda}(g;n,b,m)$ such that inequality (4.2) holds true.

Theorem 4.1. *If* $h(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$ *and*

$$\alpha = p - \frac{\delta}{n^{q+1}} \cdot \frac{(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n}{\left[(n-p+|b|)\left[\lambda(n-m-1)+1\right]\binom{n}{m}b_n - |b|\left[\lambda(p-m-1)+1\right]\binom{p}{m}\right]}, \tag{4.3}$$

then

$$\mathcal{N}_{n,\delta}^{q}(h) \subset \mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m). \tag{4.4}$$

Proof. Suppose that $f(z) \in \mathcal{N}_{n,\delta}^{q}(h)$. We then find from (1.16) that

$$\sum_{k=n}^{\infty} k^{q+1}\left|a_k - c_k\right| \le \delta, \tag{4.5}$$

which readily implies that

$$\sum_{k=n}^{\infty} |a_k - c_k| \le \frac{\delta}{n^{q+1}} \quad (n \in \mathbb{N}). \tag{4.6}$$

Next, since $h(z) \in \mathcal{S}_p^{\lambda}(g;n,b,m)$, we have in view of (3.5) that

$$\sum_{k=n}^{\infty} c_k \le \frac{|b|[\lambda(p-m-1)+1]\binom{p}{m}}{(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n}, \tag{4.7}$$

so that

$$\begin{aligned}\left|\frac{f(z)}{h(z)} - 1\right| &\le \frac{\sum_{k=n}^{\infty}|a_k - c_k|}{1 - \sum_{k=n}^{\infty}c_k} \\ &\le \frac{\delta}{n^{q+1}} \frac{1}{1 - |b|[\lambda(p-m-1)+1]\binom{p}{m}/(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n} \\ &\le \frac{\delta}{n^{q+1}} \frac{(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n}{[(n-p+|b|)[\lambda(n-m-1)+1]\binom{n}{m}b_n - |b|[\lambda(p-m-1)+1]\binom{p}{m}]} \\ &= p - \alpha,\end{aligned} \tag{4.8}$$

provided that α is given by (4.3). Thus, by the above definition, $f \in \mathcal{S}_p^{(\lambda,\alpha)}(g;n,b,m)$ where α is given by (4.3), which proves Theorem 4.1. □

The proof of Theorem 4.2 below is similar to that of Theorem 4.1 above, and its proof details are, therefore, omitted here.

Theorem 4.2. *If* $h(z) \in \mathcal{P}_p^{\lambda}(g; n, b, m)$ *and*

$$\alpha = p - \frac{\delta}{n^{q+1}} \frac{\left[\lambda(n-p)+1\right](n-m)\binom{n}{m}b_n}{\left[\left[\lambda(n-p)+1\right](n-m)\binom{n}{m}b_n - (p-m)\left\{(|b|-1)/m! + \binom{p}{m}\right\}\right]}, \tag{4.9}$$

then

$$\mathcal{N}_{n,\delta}^{q}(h) \subset \mathcal{P}_p^{(\lambda,\alpha)}(g; n, b, m). \tag{4.10}$$

Remark 4.3. Applying the parametric substitutions listed in (2.6), Theorems 4.1 and 4.2 would yield the corresponding results of Srivastava et al. [3, Theorems 5 and 6, page 6]. Also using substitutions as mentioned above in (2.8), we get the results due to Orhan and Kamali [13, Theorem 3, page 60; Theorem 4, page 61].

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