Research Article

Gregus-Type Common Fixed Point Theorems for Tangential Multivalued Mappings of Integral Type in Metric Spaces

W. Sintunavarat and P. Kumam

Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangmod, Thrungkru, Bangkok 10140, Thailand

Correspondence should be addressed to P. Kumam, poom.kum@kmutt.ac.th

The concept of tangential for single-valued mappings is extended to multivalued mappings and used to prove the existence of a common fixed point theorem of Gregus type for four mappings satisfying a strict general contractive condition of integral type. Consequently, several known fixed point results generalized and improved the corresponding recent result of Pathak and Shahzad (2009) and many authors.

1. Introduction

The first important result on fixed points for contractive-type mappings was the well-known Banach contraction principle, published for the first time in 1922 in [1] (see also [2]). Banach contraction principle has been extended in many different directions, see [3–5], and so forth. Many authors in [3, 5–12] established fixed point theorems involving more general contractive conditions. In 1969, Nadler [13] combines the ideas of set-valued mapping and Lipschitz mapping and prove some fixed point theorems about multivalued contraction mappings. Afterward, the study of fixed points for multivalued contractions using the Hausdorff metric was initiated by Markin [14]. Later, an interesting and rich fixed point theory for such maps was developed (see [15–18]). The theory of multivalued maps has applications in optimization problems, control theory, differential equations, and economics.

Sessa [19] introduced the concept of weakly commuting maps. Jungck [20] defined the notion of compatible maps in order to generalize the concept of weak commutativity and showed that weakly commuting mappings are compatible but the converse is not true.

This concept was further improved by Jungck and Rhoades [21] with the notion of weakly compatible mappings. In 2002, Aamri and Moutawakil [22] defined property (E.A). This concept was frequently used to prove existence theorems in common fixed point theory. Three years later, Liu et al. [23] introduced common property (E.A). The class of (E.A) maps contains the class of noncompatible maps. Branciari [3] studied contractive conditions of integral type, giving an integral version of the Banach contraction principle, that could be extended to more general contractive conditions. Recently, Pathak and Shahzad [24] introduced the new concept of weak tangent point and tangential property for single-valued mappings and established common fixed point theorems. Very recently, Vetro [25] obtained an interesting theorem for mappings satisfying a contractive condition of integral type which is a generalization of Branciari [3, Theorem 2].

The aim of this paper is to define a tangential property for multivalued mappings which generalize the concept of tangential property for single-valued mappings of Pathak and Shahzad [24] and prove a common fixed point theorem of Gregus type for four mappings satisfying a strict general contractive condition of integral type.

2. Preliminary

Throughout this paper, (X, d) denotes a metric space. We denote by $CB(X)$, the class of all nonempty bounded closed subsets of X. The Hausdorff metric induced by d on $CB(X)$ is given by

$$H(A, B) = \max\left\{ \sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A) \right\},\tag{2.1}$$

for every $A, B \in CB(X)$, where $d(a, B) = d(B, a) = \inf\{d(a, b) : b \in B\}$ is the distance from a to $B \subseteq X$. Let $f : X \to X$ and $T : X \to CB(X)$. A point $x \in X$ is a *fixed point* of f (resp. T) if $fx = x$ (resp. $x \in Tx$). The set of all fixed points of f (resp. T) is denoted by $F(f)$ (resp. $F(T)$). A point $x \in X$ is a *coincidence point* of f and T if $fx \in Tx$. The set of all coincidence points of f and T is denoted by $C(f, T)$. A point $x \in X$ is a *common fixed point* of f and T if $x = fx \in Tx$. The set of all common fixed points of f and T is denoted by $F(f, T)$.

Definition 2.1. The maps $f : X \to X$ and $g : X \to X$ are said to be commuting if $fgx = gfx$, for all $x \in X$.

Definition 2.2 (see [19]). The maps $f : X \to X$ and $g : X \to X$ are said to be weakly commuting if $d(fgx, gfx) \leq d(fx, gx)$, for all $x \in X$.

Definition 2.3 (see [20]). The maps $f : X \to X$ and $g : X \to X$ are said to be compatible if $\lim_{n \to \infty} d(fgx_n, gfx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z$, for some $z \in X$.

Definition 2.4 (see [26]). The maps $f : X \to X$ and $g : X \to X$ are said to be weakly compatible $fgx = gfx$, for all $x \in C(f, g)$.

Definition 2.5 (see [22]). Let $f : X \to X$ and $g : X \to X$. The pair (f, g) satisfies property (E.A) if there exist the sequence $\{x_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z \in X. \tag{2.2}$$

See example of property (E.A) in Kamran [27, 28] and Sintunavarat and Kumam [11].

Definition 2.6 (see [23]). Let $f, g, A, B : X \to X$. The pair (f, g) and (A, B) satisfy a common property (E.A) if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = \lim_{n \to \infty} Ay_n = \lim_{n \to \infty} By_n = z \in X. \tag{2.3}$$

Remark 2.7. If $A = f$, $B = g$, and $\{x_n\} = \{y_n\}$ in (2.3), then we get the definition of property (E.A).

Definition 2.8 (see [24]). Let $f, g : X \to X$. A point $z \in X$ is said to be a weak tangent point to (f, g) if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gy_n = z \in X. \tag{2.4}$$

Remark 2.9. If $\{x_n\} = \{y_n\}$ in (2.4), we get the definition of property (E.A).

Definition 2.10 (see [24]). Let $f, g, A, B : X \to X$. The pair (f, g) is called tangential with respect to the pair (A, B) if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gy_n = \lim_{n \to \infty} Ax_n = \lim_{n \to \infty} By_n = z \in X. \tag{2.5}$$

3. Main Results

In this section, we first introduce the notion of tangential property for two single-valued and two multivalued mappings. Throughout this section, $\mathbb{R}_+$ denotes the set of nonnegative real numbers.

Definition 3.1. Let $f, g : X \to X$ and $A, B : X \to CB(X)$. The pair (f, g) is called tangential with respect to the pair (A, B) if

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} By_n = D \in CB(X), \tag{3.1}$$

whenever sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gy_n = z \in D, \tag{3.2}$$

for some $z \in X$.

Example 3.2. Let $(\mathbb{R}_+, d)$ be a metric space with usual metric d. Let $f, g : \mathbb{R}_+ \to \mathbb{R}_+$ and $A, B : \mathbb{R}_+ \to CB(\mathbb{R}_+)$ be mappings defined by $fx = x + 1$, $gx = x + 2$, $Ax = \{x^2/2 + 1\}$, and $Bx = \{x^2 + 2\}$, for all $x \in \mathbb{R}_+$. Clearly, there exists two sequences $\{x_n = 2 + 1/n\}$ and $\{y_n = 1 + 1/n\}$ such that

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} By_n = \{3\} \in CB(\mathbb{R}_+) \tag{3.3}$$

whenever

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gy_n = 3 \in \mathbb{R}_+. \tag{3.4}$$

So, the pair (f, g) is tangential with respect to the pair (A, B).

Definition 3.3. Let $f : X \to X$ and $A : X \to CB(X)$. The mapping f is called tangential with respect to the mapping A if

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Ay_n = D \in CB(X), \tag{3.5}$$

whenever sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} fy_n = z \in D, \tag{3.6}$$

for some $z \in X$.

Example 3.4. Let $(\mathbb{R}_+, d)$ be a metric space with usual metric d. Let $f : \mathbb{R}_+ \to \mathbb{R}_+$ and $A : \mathbb{R}_+ \to CB(\mathbb{R}_+)$ be mappings defined by

$$fx = x + 1, \qquad Ax = \left\{x^2 + 1\right\}. \tag{3.7}$$

Clearly, there exist two sequences $\{x_n = 1 + 1/n\}$ and $\{y_n = 1 - 1/n\}$ such that

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Ay_n = \{2\} \in CB(\mathbb{R}_+) \tag{3.8}$$

whenever

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} fy_n = 2 \in \mathbb{R}_+. \tag{3.9}$$

So, the mapping f is tangential with respect to the mapping A.

Now, we state and prove our main result.

Theorem 3.5. *Let $f, g : X \to X$ and $A, B : X \to CB(X)$ satisfy*

$$\left(1 + \alpha \int_0^{d(fx,gy)} \psi(t)dt\right) \int_0^{H(Ax,By)} \psi(t)dt$$

$$< \alpha \left(\int_0^{d(Ax,fx)} \psi(t)dt \int_0^{d(By,gy)} \psi(t)dt + \int_0^{d(Ax,gy)} \psi(t)dt \int_0^{d(fx,By)} \psi(t)dt\right)$$

$$+ a \int_0^{d(fx,gy)} \psi(t)dt + (1-a) \max\left\{\int_0^{d(Ax,fx)} \psi(t)dt, \int_0^{d(By,gy)} \psi(t)dt,\right.$$

$$\left(\int_0^{d(Ax,fx)} \psi(t)dt\right)^{1/2} \left(\int_0^{d(Ax,gy)} \psi(t)dt\right)^{1/2},$$

$$\left.\left(\int_0^{d(fx,By)} \psi(t)dt\right)^{1/2} \left(\int_0^{d(Ax,gy)} \psi(t)dt\right)^{1/2}\right\},$$

$$(3.10)$$

for all $x, y \in X$ for which the right-hand side of (3.10) is positive, where $0 < a < 1$, $\alpha \geq 0$ and $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a Lebesgue integrable mapping which is a summable nonnegative and such that

$$\int_0^\epsilon \psi(t)dt > 0, \qquad (3.11)$$

for each $\epsilon > 0$. If the following conditions (a)–(d) hold:

(a) *there exists a point $z \in f(X) \cap g(X)$ which is a weak tangent point to (f, g),*

(b) *(f, g) is tangential with respect to (A, B),*

(c) *$ffa = fa$, $ggb = gb$, and $Afa = Bgb$ for $a \in C(f, A)$ and $b \in C(g, B)$,*

(d) *the pairs (f, A) and (g, B) are weakly compatible.*

Then, f, g, A, and B have a common fixed point in X.

Proof. Since $z \in f(X) \cap g(X)$, $z = fu = gv$ for some $u, v \in X$. It follows from a point z which is a weak tangent point to (f, g) that there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gy_n = z. \qquad (3.12)$$

Because the pair (f, g) is tangential with respect to the pair (A, B), we get

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} By_n = D, \tag{3.13}$$

for some $D \in CB(X)$. Since $z = fu = gv$ and (3.12) and (3.13) are true, we have

$$z = fu = gv = \lim_{n \to \infty} fx_n = \lim_{n \to \infty} gy_n \in \lim_{n \to \infty} Ax_n = \lim_{n \to \infty} By_n = D. \tag{3.14}$$

We claim that $z \in Bv$. If not, then condition (3.10) implies

$$\left(1 + \alpha \int_0^{d(fx_n,gv)} \psi(t)dt\right) \int_0^{H(Ax_n,Bv)} \psi(t)dt$$

$$< \alpha \left(\int_0^{d(Ax_n,fx_n)} \psi(t)dt \int_0^{d(Bv,gv)} \psi(t)dt + \int_0^{d(Ax_n,gv)} \psi(t)dt \int_0^{d(fx_n,Bv)} \psi(t)dt\right)$$

$$+ a \int_0^{d(fx_n,gv)} \psi(t)dt + (1-a)\max\left\{\int_0^{d(Ax_n,fx_n)} \psi(t)dt, \int_0^{d(Bv,gv)} \psi(t)dt, \right.$$

$$\left(\int_0^{d(Ax_n,fx_n)} \psi(t)dt\right)^{1/2}\left(\int_0^{d(Ax_n,gv)} \psi(t)dt\right)^{1/2},$$

$$\left.\left(\int_0^{d(fx_n,Bv)} \psi(t)dt\right)^{1/2}\left(\int_0^{d(Ax_n,gv)} \psi(t)dt\right)^{1/2}\right\}. \tag{3.15}$$

Letting $n \to \infty$, we get

$$\int_0^{H(D,Bv)} \psi(t)dt \le (1-a)\int_0^{d(Bv,z)} \psi(t)dt. \tag{3.16}$$

Since

$$\int_0^{d(z,Bv)} \psi(t)dt < \int_0^{H(D,Bv)} \psi(t)dt$$

$$\le (1-a)\int_0^{d(Bv,z)} \psi(t)dt \tag{3.17}$$

$$< \int_0^{d(z,Bv)} \psi(t)dt,$$

which is a contradiction, then $z \in Bv$.

Again, we claim that $z \in Au$. If not, then condition (3.10) implies

$$
\left(1 + \alpha \int_0^{d(fu,gy_n)} \psi(t)dt\right) \int_0^{H(Au,By_n)} \psi(t)dt
$$

$$
< \alpha\left(\int_0^{d(Au,fu)} \psi(t)dt \int_0^{d(By_n,gy_n)} \psi(t)dt + \int_0^{d(Au,gy_n)} \psi(t)dt \int_0^{d(fu,By_n)} \psi(t)dt\right)
$$

$$
+ a\int_0^{d(fu,gy_n)} \psi(t)dt + (1-a)\max\left\{\int_0^{d(Au,fu)} \psi(t)dt, \int_0^{d(By_n,gy_n)} \psi(t)dt,\right.
$$

$$
\left(\int_0^{d(Au,fu)} \psi(t)dt\right)^{1/2}\left(\int_0^{d(Au,gy_n)} \psi(t)dt\right)^{1/2},
$$

$$
\left.\left(\int_0^{d(fu,By_n)} \psi(t)dt\right)^{1/2}\left(\int_0^{d(Au,gy_n)} \psi(t)dt\right)^{1/2}\right\}.
$$

(3.18)

Letting $n \to \infty$, we get

$$
\int_0^{H(Au,D)} \psi(t)dt \leq (1-a)\int_0^{d(Au,z)} \psi(t)dt.
$$

(3.19)

Since

$$
\int_0^{d(z,Au)} \psi(t)dt < \int_0^{H(Au,D)} \psi(t)dt
$$

$$
\leq (1-a)\int_0^{d(Au,z)} \psi(t)dt
$$

(3.20)

$$
< \int_0^{d(z,Au)} \psi(t)dt,
$$

which is a contradiction, then $z \in Au$.

Now, we conclude $z = gv \in Bv$ and $z = fu \in Au$. It follows from $v \in C(g,B)$, $u \in C(f,A)$ that $ggv = gv$, $ffu = fu$, and $Afu = Bgv$. Hence, $gz = z$, $fz = z$ and $Az = Bz$.

Since the pair (g,B) is weakly compatible, $gBv = Bgv$. Thus $gz \in gBv = Bgv = Bz$. Similarly, we can prove that $fz \in Az$. Consequently, $z = fz = gz \in Az = Bz$. Therefore the maps f, g, A, and B have a common fixed point. □

If $\alpha = 0$ in Theorem 3.5, we get the following corollary.

Corollary 3.6. *Let $f, g : X \to X$ and $A, B : X \to CB(X)$ satisfy*

$$
\int_0^{H(Ax,By)} \psi(t)dt < a \int_0^{d(fx,gy)} \psi(t)dt
$$

$$
+ (1-a) \max \left\{ \int_0^{d(Ax,fx)} \psi(t)dt, \int_0^{d(By,gy)} \psi(t)dt, \right.
$$

$$
\left(\int_0^{d(Ax,fx)} \psi(t)dt \right)^{1/2} \left(\int_0^{d(Ax,gy)} \psi(t)dt \right)^{1/2},
$$

$$
\left. \left(\int_0^{d(fx,By)} \psi(t)dt \right)^{1/2} \left(\int_0^{d(Ax,gy)} \psi(t)dt \right)^{1/2} \right\},
$$

(3.21)

for all $x, y \in X$ for which the right-hand side of (3.21) is positive, where $0 < a < 1$ and $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a Lebesgue integrable mapping which is a summable nonnegative and such that

$$
\int_0^{\epsilon} \psi(t)dt > 0,
$$

(3.22)

for each $\epsilon > 0$. If the following conditions (a)–(d) hold:

    (a) *there exists a point $z \in f(X) \cap g(X)$ which is a weak tangent point to (f, g),*

    (b) *(f, g) is tangential with respect to (A, B),*

    (c) *$ffa = fa$, $ggb = gb$ and $Afa = Bgb$ for $a \in C(f, A)$ and $b \in C(g, B)$,*

    (d) *the pairs (f, A) and (g, B) are weakly compatible.*

Then, f, g, A, and B have a common fixed point in X.

If $\alpha = 0$, $g = f$, and $B = A$ in Theorem 3.5, we get the following corollary.

Corollary 3.7. *Let $f : X \to X$ and $A : X \to CB(X)$ satisfy*

$$
\int_0^{H(Ax,Ay)} \psi(t)dt < a \int_0^{d(fx,fy)} \psi(t)dt
$$

$$
+ (1-a) \max \left\{ \int_0^{d(Ax,fx)} \psi(t)dt, \int_0^{d(Ay,fy)} \psi(t)dt, \right.
$$

$$
\left(\int_0^{d(Ax,fx)} \psi(t)dt \right)^{1/2} \left(\int_0^{d(Ax,fy)} \psi(t)dt \right)^{1/2},
$$

$$
\left. \left(\int_0^{d(fx,Ay)} \psi(t)dt \right)^{1/2} \left(\int_0^{d(Ax,fy)} \psi(t)dt \right)^{1/2} \right\},
$$

(3.23)

for all $x, y \in X$ for which the right-hand side of (3.23) is positive, where $0 < a < 1$ and $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a Lebesgue integrable mapping which is a summable nonnegative and such that

$$\int_0^\epsilon \psi(t) dt > 0 \tag{3.24}$$

for each $\epsilon > 0$. If the following conditions (a)–(d) hold:

(a) *there exists sequence $\{x_n\}$ in X such that $\lim_{n \to \infty} f x_n \in X$,*

(b) *f is tangential with respect to A,*

(c) *$ffa = fa$ for $a \in C(f, A)$,*

(d) *the pair (f, A) is weakly compatible.*

Then, f and A have a common fixed point in X.

If $\psi(t) = 1$ in Theorem 3.5, we get the following corollary.

Corollary 3.8. *Let $f, g : X \to X$ and $A, B : X \to CB(X)$ satisfy*

$$\begin{aligned}
(1 + \alpha d(fx, gy)) & H(Ax, By) \\
& < \alpha(d(Ax, fx)d(By, gy) + d(Ax, gy)d(fx, By)) + ad(fx, gy) \\
& + (1 - a) \max\left\{ d(Ax, fx), d(By, gy), (d(Ax, fx))^{1/2}(d(Ax, gy))^{1/2}, \right. \\
& \left. (d(fx, By))^{1/2}(d(Ax, gy))^{1/2} \right\}
\end{aligned} \tag{3.25}$$

for all $x, y \in X$ for which the right-hand side of (3.25) is positive, where $0 < a < 1$ and $\alpha \geq 0$. If the following conditions (a)–(d) holds:

(a) *there exists a point $z \in f(X) \cap g(X)$ which is a weak tangent point to (f, g),*

(b) *(f, g) is tangential with respect to (A, B),*

(c) *$ffa = fa$, $ggb = gb$ and $Afa = Bgb$ for $a \in C(f, A)$ and $b \in C(g, B)$,*

(d) *the pairs (f, A) and (g, B) are weakly compatible.*

Then, f, g, A, and B have a common fixed point in X.

If $\psi(t) = 1$ and $\alpha = 0$ in Theorem 3.5, we get the following corollary.

Corollary 3.9. *Let $f, g : X \to X$ and $A, B : X \to CB(X)$ satisfy*

$$\begin{aligned}
H(Ax, By) & < ad(fx, gy) \\
& + (1 - a) \max\left\{ d(Ax, fx), d(By, gy), (d(Ax, fx))^{1/2}(d(Ax, gy))^{1/2}, \right. \\
& \left. (d(fx, By))^{1/2}(d(Ax, gy))^{1/2} \right\},
\end{aligned} \tag{3.26}$$

for all $x, y \in X$ for which the right-hand side of (3.26) *is positive, where $0 < a < 1$. If the following conditions (a)–(d) hold:*

(a) *there exists a point $z \in f(X) \cap g(X)$ which is a weak tangent point to (f, g),*

(b) *(f, g) is tangential with respect to (A, B),*

(c) *$ffa = fa$, $ggb = gb$ and $Afa = Bgb$ for $a \in C(f, A)$ and $b \in C(g, B)$,*

(d) *the pairs (f, A) and (g, B) are weakly compatible.*

Then, f, g, A, and B have a common fixed point in X.

If $\psi(t) = 1$, $\alpha = 0$, $g = f$ and $B = A$ in **Theorem 3.5**, we get the following corollary.

Corollary 3.10. *Let $f : X \rightarrow X$ and $A : X \rightarrow CB(X)$ satisfy*

$$
\begin{aligned}
H(Ax, Ay) &< ad(fx, fy) \\
&\quad + (1 - a)\max\Big\{ d(Ax, fx), d(Ay, fy), (d(Ax, fx))^{1/2}(d(Ax, fy))^{1/2}, \\
&\qquad\qquad\qquad (d(fx, Ay))^{1/2}(d(Ax, fy))^{1/2} \Big\}
\end{aligned}
$$

(3.27)

for all $x, y \in X$ for which the right-hand side of (3.27) *is positive, where $0 < a < 1$. If the following conditions (a)–(d) holds:*

(a) *there exists sequence $\{x_n\}$ in X such that $\lim_{n \to \infty} f x_n \in X$,*

(b) *f is tangential with respect to A,*

(c) *$ffa = fa$ for $a \in C(f, A)$,*

(d) *the pairs (f, A) is weakly compatible.*

Then, f and A have a common fixed point in X.

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