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p-REGULAR CAUCHY COMPLETIONS

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ABSTRACT. This paper gives a further development of p-regular completion theory, including a study of p-regular Reed completions, the role of diagonal axioms, and the relationship between p-regular and p-topological completions.

Keywords and phrases. p-regular completion, p-topological completion, Reed completion, diagonal axiom, property S.

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1. Introduction. Since a paper of p-regular completions [5] first appeared in 1991, some recent and relevant developments have occurred which provide the motivation for this paper. The discovery of a dual relationship between the properties "p-regular" and "p-topological" in convergence space theory led to an investigation of p-topological convergence spaces [11] and p-topological Cauchy completions [10]. These, in turn, raised some questions about p-regular completion theory that had not been previously considered, such as the possible duality between p-regular and p-topological Cauchy spaces, the role of diagonal axioms in the study of p-regular completions, the existence of p-topological Reed completions, and the relevance of p-regular completions to the study of regular completions. These topics, along with a comparison of the behavior of p-regular and p-topological completions, form the subject matter of this paper.

2. The fine p-regular completion. For background information on convergence or Cauchy spaces, the reader is referred to [1] or [5]. Additional information about p-regular and p-topological Cauchy and convergence spaces may be found in [4, 10, 11]. If $(X,\mathscr{C})$ is a Cauchy space, the associated convergence structure is denoted by $q_{\mathscr{C}}$. *We shall assume that all convergence spaces, Cauchy spaces, and Cauchy completions discussed in this paper are T_2 (i.e., Hausdorff) unless otherwise indicated.*

If X is any set, then $\mathbb{F}(X)$ denotes the set of all filters on X, and if $x \in X$, then $\dot{x}$ denotes the ultrafilter generated by $\{x\}$. Let $\mathbb{C}(X)$ be the set of all convergence structures on X.

Let $(X,\mathscr{C})$ be a Cauchy space, X^* the set of all Cauchy equivalence classes, and $X' = \{[\mathscr{F}] \in X^* : \mathscr{F}$ is not $q_{\mathscr{C}}$-convergent$\}$. The canonical map $j : X \to X^*$, given by $j(x) = [\dot{x}]$, is an injection. It is proved in [8] that every completion of $(X,\mathscr{C})$ is equivalent to one in *standard form*, which means that X^* is the underlying set for the completion, j the Cauchy embedding map, and $j(\mathscr{F}) \cap [\dot{\mathscr{F}}]$ is a Cauchy filter in the completion structure for every $\mathcal{F} \in \mathcal{C}$. Consequently, we shall limit our attention in this paper to completions in standard form.

If $(X,\mathcal{C})$ is a Cauchy space and $p \in \mathbb{C}(X)$, then $(X,\mathcal{C})$ is defined to be *p-regular* if $\mathcal{F} \in \mathcal{C}$ implies $\mathrm{cl}_p \mathcal{F} \in \mathcal{C}$, and *p-topological* if $\mathcal{F} \in \mathcal{C}$ implies there is $\mathcal{G} \in \mathcal{C}$ such that $\mathcal{F} \geq \mathrm{I}_p \mathcal{G}$ (where cl_p and I_p are the closure and interior operators for p). $(X,\mathcal{C})$ is *regular* (respectively, *topological*) if it is $q_{\mathcal{C}}$-regular (respectively, $q_{\mathcal{C}}$-topological). The convergence structure p is extended to a convergence structure p^* on X^* by defining p^* to be the finest convergence structure on X^* satisfying:

(1) $j(\mathcal{F}) \xrightarrow{p^*} [\dot{x}]$ in X^* if and only if $\mathcal{F}$ p-converges to x in X;

(2) if $\mathcal{F} \in \mathcal{C}$ is non-$q_{\mathcal{C}}$-convergent, $j(\mathcal{F}) \xrightarrow{p^*} [\mathcal{F}]$ in X^*.

A completion of $(X,\mathcal{C})$ is *p-regular* (respectively, *p-topological*) if the completion space $(X^*,\mathcal{D})$ is p^*-regular (respectively, p^*-topological). Note that a $q_{\mathcal{C}}$-regular (or $q_{\mathcal{C}}$-topological) completion does not have the same meaning as a regular (or topological) completion.

We next introduce some additional notation and terminology relevant to the study of p-regular completions. Given a Cauchy space $(X,\mathcal{C})$, $p \in \mathbb{C}(X)$, and $A \subseteq X$, we define

$\theta_{\mathcal{C}} A = \{z \in X' : \exists \mathcal{G} \in z \text{ such that } A \in \mathcal{G}\}$,

$\Delta_p A = j(\mathrm{cl}_p A) \cup \theta_{\mathcal{C}} A$.

If $\mathcal{F} \in \mathbb{F}(X)$, $\Delta_p \mathcal{F}$ is the filter on X^* generated by $\{\Delta_p F : F \in \mathcal{F}\}$, and $\theta_{\mathcal{C}} \mathcal{F}$ is defined analogously if $\theta_{\mathcal{C}} \mathcal{F} \neq \phi$, for all $F \in \mathcal{F}$.

Lemma 2.1 [5]. *If $(X,\mathcal{C})$ is a Cauchy space, $p \in \mathbb{C}(X)$, and $A \subseteq X$, then:*

(a) $\mathrm{cl}_{p^*} j(A) = \Delta_p A$;

(b) $\mathrm{cl}_{p^*}^n j(A) \subseteq \Delta_p(\mathrm{cl}_p^n A) \subseteq \mathrm{cl}_{p^*}^{n+1} j(A)$.

A Cauchy space $(X,\mathcal{C})$ is said to have *property S* if, whenever $\mathcal{F}$, $\mathcal{G} \in \mathcal{C}$ and $[\mathcal{F}] \neq [\mathcal{G}]$, $\theta_{\mathcal{C}} \mathcal{F} \vee \theta_{\mathcal{C}} \mathcal{G}$ fails to exist. Note that $\theta_{\mathcal{C}} \mathcal{F} \vee \theta_{\mathcal{C}} \mathcal{G}$ fails to exist if either of these filters fails to exist or if both exist but contain mutually disjoint sets. Property S is a separation property stronger than T_2 (since $(X,\mathcal{C})$ is additionally assumed to be T_2), but weaker than Cauchy separated, which requires the existence of a Cauchy-continuous, real-valued function which separates the non-equivalent Cauchy filters.

Given a Cauchy space $(X,\mathcal{C})$ and $p \in \mathbb{C}(X)$, let $\mathcal{C}_p^*$ be the Cauchy structure on X^* generated by $\{\Delta_p \mathcal{F} : \mathcal{F} \in \mathcal{C}\}$. Although $\mathcal{C}_p^*$ is generally not T_2 we have the following result.

Proposition 2.2. *If $(X,\mathcal{C})$ is p-regular, then $(X^*,\mathcal{C}_p^*)$ is T_2 if and only if $(X,\mathcal{C})$ has the property S. If $(X,\mathcal{C})$ is p-regular and satisfies S, then $(X^*,\mathcal{C}_p^*)$ is a completion of $(X,\mathcal{C})$.*

Proof. If $(X,\mathcal{C})$ satisfies S and $[\mathcal{F}] \neq [\mathcal{G}]$, but $[\dot{\mathcal{F}}] \cap [\dot{\mathcal{G}}] \in \mathcal{C}_p^*$, then there are filters $\mathcal{H}_1,\ldots,\mathcal{H}_n$ in $\mathcal{C}$ such that $\Delta_p \mathcal{H}_1,\ldots,\Delta_p \mathcal{H}_n$ are linked and $[\dot{\mathcal{F}}] \cap [\dot{\mathcal{G}}] \geq \cap\{\Delta_p \mathcal{H}_i : i = 1,\ldots,n\}$. From the given assumptions about $(X,\mathcal{C})$, we can deduce that $\Delta_p \mathcal{H}_1,\ldots,\Delta_p \mathcal{H}_n$ being linked implies that $[\mathcal{H}_1] = [\mathcal{H}_2] = \cdots = [\mathcal{H}_n]$, and so there is $\mathcal{H} \in \mathcal{C}$ such that $[\dot{\mathcal{F}}] \cap [\dot{\mathcal{G}}] \geq \Delta_p \mathcal{H}$. The latter result implies that $[\mathcal{F}] = [\mathcal{H}] = [\mathcal{G}]$, and this contradiction shows that $(X^*,\mathcal{C}_p^*)$ is T_2.

Conversely, assume $\mathcal{C}_p^*$ is T_2 and let $\mathcal{F},\mathcal{G} \in \mathcal{C}$, so that $\Delta_p \mathcal{F}$ and $\Delta_p \mathcal{G}$ are in $\mathcal{C}_p^*$. If $[\mathcal{F}] \neq [\mathcal{G}]$, then $\Delta_p \mathcal{F} \vee \Delta_p \mathcal{G}$ must fail to exist, and by p-regularity this can happen only if $\theta_{\mathcal{C}} \mathcal{F} \vee \theta_{\mathcal{C}} \mathcal{G}$ fails to exist. Thus S is satisfied.

To prove the last statement, note that $j : (X,\mathscr{C}) \to (X^*,\mathscr{C}_p^*)$ is obviously a Cauchy-continuous bijection, and Cauchy-continuity of j^{-1} follows by Lemma 2.1. Also, $j(X)$ is obviously dense in X^*. □

Recall that a completion $(X^*,\mathscr{D})$ of a Cauchy space $(X,\mathscr{C})$ is *strict* if, for each $\mathscr{H} \in \mathscr{D}$, there is $\mathscr{F} \in \mathscr{C}$ such that $\mathrm{cl}_{q_{\mathscr{D}}} j\mathscr{F} \le \mathscr{H}$.

Proposition 2.3. *If $(X,\mathscr{C})$ is a p-regular Cauchy space satisfying S, then $(X^*,\mathscr{C}_p^*)$ is the finest p-regular completion of $(X,\mathscr{C})$. This completion is strict, and is in the fact the only strict p-regular completion $(X,\mathscr{C})$.*

Proof. If $\mathscr{F} \in \mathscr{C}$, it is clear that $\Delta_p\mathscr{F}$ must belong to every p-regular completion of $(X,\mathscr{C})$, which proves the first assertion. The second statement is proved in [5, Propositions 2.6 and 2.7]. □

In view of Proposition 2.3, we shall call $(X^*,\mathscr{C}_p^*)$ the *fine* p-regular completion of a p-regular Cauchy space $(X,\mathscr{C})$ with property S.

Corollary 2.4. *The following statements about a Cauchy space $(X,\mathscr{C})$ are equivalent:*

(1) $(X,\mathscr{C})$ *is p-regular and satisfies S.*
(2) $(X,\mathscr{C})$ *has a p-regular completion.*
(3) $(X,\mathscr{C})$ *has a strict p-regular completion.*
(4) $(X^*,\mathscr{C}_p^*)$ *is p-regular completion of $(X,\mathscr{C})$.*

Proposition 2.5. *If $(X,\mathscr{C})$ is a regular Cauchy space which satisfies S, then $(X^*,\mathscr{C}_{q_{\mathscr{C}}}^*)$ is the coarsest strict completion of $(X,\mathscr{C})$.*

Proof. Let $(X^*,\mathscr{D})$ be any other strict completion. Then $\mathscr{H} \in \mathscr{D}$ implies there is $\mathscr{F} \in \mathscr{C}$ such that $\mathscr{H} \ge \mathrm{cl}_{q_{\mathscr{D}}} j(\mathscr{F}) = j(\mathrm{cl}_{q_{\mathscr{D}}} \mathscr{F}) \cap \theta_{\mathscr{C}}\mathscr{F}$. The latter filter is in $\mathscr{C}_{q_{\mathscr{C}}}^*$, so $\mathscr{C}_{q_{\mathscr{C}}}^* \le \mathscr{D}$. □

3. p-regular Reed completions. In 1971, Reed [9], defined a family of completions of a Cauchy space $(X,\mathscr{C})$, which we shall now describe. By a *Reed selection function*, we shall mean a function $\lambda : X^* \to \mathbb{F}(X)$ which satisfies the following conditions: (1) $\lambda([\dot{x}]) = \dot{x}$, for all $x \in X$; (2) if $z \in X'$, then $\lambda(z) \le \mathscr{G}$, for some $\mathscr{G} \in z$.

Let Λ be the set of all Reed selection functions on $(X,\mathscr{C})$, and for arbitrary $\lambda \in \Lambda$, $F \subseteq X$, and $\mathscr{F} \in \mathbb{F}(X)$, let $F^\lambda = \{z \in X^* : F \in \lambda(z)\}$, and let $\mathscr{F}^\lambda$ be the filter on X^* generated by $\{F^\lambda : F \in \mathscr{F}\}$. For $\lambda \in \Lambda$ and $\Gamma \subseteq \Lambda$, the following complete Cauchy structures are defined on $X^* : \mathscr{C}_\lambda = \{\mathscr{A} \in \mathbb{F}(X^*) : \exists z \in X^* \text{ such that } \mathscr{A} \ge \mathscr{F}^\lambda \cap \dot{z}\}$ and $\mathscr{C}_\Gamma = \cap\{\mathscr{C}_\lambda : \lambda \in \Gamma\}$. Reed showed in [9] that for any $\Gamma \subseteq \Lambda$, $(X^*,\mathscr{C}_\Gamma)$ is a completion of $(X,\mathscr{C})$ with an extension property relative to Cauchy-continuous maps into complete, regular Cauchy spaces. The set $\{(X^*,\mathscr{C}_\Gamma) : \Gamma \subseteq \Lambda\}$ (including those of the form $(X^*,\mathscr{C}_\lambda)$, where $\Gamma = \{\lambda\}$, $\lambda \in \Lambda$) is called the *Reed family of completions* for $(X,\mathscr{C})$. By their construction, it is clear that all Reed completions are strict.

If $\lambda,\mu \in \Lambda$, $\lambda \le \mu$ means $\lambda(z) \le \mu(z)$ for all $z \in X^*$. Let q_λ (respectively, q_Γ) denote the convergence structure on X^* associated with $\mathscr{C}_\lambda$ (respectively, $\mathscr{C}_\Gamma$).

Proposition 3.1. *Let $(X,\mathscr{C})$ be a Cauchy space, let $\lambda,\mu \in \Lambda$ with $\lambda \le \mu$, and let $\Gamma \subseteq \Delta \subseteq \Lambda$. Then*

(a) $\mathscr{C}_\mu \le \mathscr{C}_\lambda$ *and* $q_\mu \le q_\lambda$*;*
(b) $\mathscr{C}_\Gamma \le \mathscr{C}_\Delta$ *and* $q_\Gamma \le q_\Delta$*;*
(c) *if* $\gamma \in \Gamma$ *and* $\lambda \le \gamma$ *for all* $\lambda \in \Gamma$*, then* $\mathscr{C}_\gamma = \mathscr{C}_\Gamma$*.*

Lemma 3.2. *If* $(X,\mathscr{C})$ *is a Cauchy space,* $A \subseteq X$*, and* $\lambda \in \Lambda$*, then* $A^\lambda \subseteq j(A) \cup \theta_{\mathscr{C}} A$*.*

Proof. If $x \in X$ and $[\dot{x}] \in A^\lambda$ then clearly $x \in A$, and so, $[\dot{x}] \in j(A)$. If $z \in A^\lambda \cap X'$, then $A \in \lambda(z)$, and $\lambda(z) \le \mathscr{H}$ for some $\mathscr{H} \in z$. Thus $A \in \mathscr{H}$ implies $z \in \theta_{\mathscr{C}} A$. □

Proposition 3.3. *Let* $(X,\mathscr{C})$ *be a Cauchy space,* $p \in \mathbb{C}(X)$*, and* $\Delta \subseteq \Gamma \subseteq \Lambda$*.*
(a) $\mathscr{C}_p^* \le \mathscr{C}_\Gamma$*.*
(b) $(X^*,\mathscr{C}_\Gamma)$ *is a* p*-regular completion of* $(X,\mathscr{C})$ *if and only if* $\mathscr{C}_p^* = \mathscr{C}_\Gamma$*.*
(c) *If* $(X^*,\mathscr{C}_\Gamma)$ *is a* p*-regular completion of* $(X,\mathscr{C})$*, so is* $(X^*,\mathscr{C}_\Delta)$*.*
(d) *Let* $\Gamma' \subseteq \Lambda$ *have the property that for each* $\lambda \in \Gamma$*, there is* $\gamma \in \Gamma$ *such that* $\gamma \le \lambda$*. If* $(X^*,\mathscr{C}_\Gamma)$ *is a* p*-regular completion of* $(X,\mathscr{C})$*, so is* $(X^*,\mathscr{C}_{\Gamma'})$*.*

Proof. If $\mathscr{F} \in \mathscr{C}$, then $\Delta_p \mathscr{F} \le \mathscr{F}^\lambda \cap [\dot{\mathscr{F}}]$ follows by Lemma 3.2, which establishes (a). The remaining results follow from (a) along with Propositions 2.3 and 3.1. □

Certain Reed completions of an arbitrary Cauchy space $(X,\mathscr{C})$ deserve special attention. Two selection functions of particular interest are ω, defined by $\omega(z) = \{X\}$, for all $z \in X'$, and σ, defined by $\sigma(z) = \mathscr{M}_z = \cap\{\mathscr{F} : \mathscr{F} \in z\}$, for all $z \in X'$. The completion $(X^*,\mathscr{C}_\omega)$ is called *Wyler completion*, often denoted in the literature by $(X^*,\mathscr{C}^*)$; this is the finest completion of $(X,\mathscr{C})$. For a p-regular Cauchy space with property S, $(X^*,\mathscr{C}_p^*)$ can be described (see [5]) as the "lower p-regular modification" of $(X^*,\mathscr{C}_\omega)$. The *Kowalsky completion* [6] is $(X^*,\mathscr{C}_\Sigma)$, where $\Sigma = \{\lambda \in \Lambda : \lambda(z) \in z,\ \text{for all } z \in X'\}$. Between the Kowalsky completion and the Wyler completion lies the, as yet, unnamed completion $(X,\mathscr{C}_\sigma)$. In [10], a Cauchy space is defined to be *cushioned* if $\mathscr{M}_z \in z$, for all $z \in X'$. For a cushioned Cauchy space, $\sigma \in \Sigma$, and by Proposition 3.1(c), $\mathscr{C}_\Sigma = \mathscr{C}_\sigma$.

In 1984, Colebunders [7], studied conditions under which Kowalsky completion and other Reed completions are regular. We shall apply her approach to the study of p-regular Reed completions. Given a Cauchy space $(X,\mathscr{C})$, $p \in \mathbb{C}(X)$, $\lambda \in \Sigma$, and F, $A \subseteq X$, we proceed to define some relevant notation and terminology.

$F <_\lambda A$ if and only if for all $z \in X^*$, $A \in \lambda(z)$ or $X \backslash F \in \lambda(z)$.
$F <^\lambda A$ if and only if $F \in \mathscr{H}$ for some $\mathscr{H} \in z$ implies $A \in \lambda(z)$.
$F <_p^\lambda A$ if and only if $\mathrm{cl}_p F \subseteq A$ and $\theta_{\mathscr{C}} F \subseteq A^\lambda$.
If $\mathscr{F} \in \mathbb{F}(X)$, let
$r_\lambda \mathscr{F} = \{A \subseteq X : F <_\lambda A,\ \text{for some } F \in \mathscr{F}\}$;
$s_\lambda \mathscr{F} = \{A \subseteq X : F <^\lambda A,\ \text{for some } F \in \mathscr{F}\}$;
$s_\lambda^p \mathscr{F} = \{A \subseteq X : F <_p^\lambda A,\ \text{for some } F \in \mathscr{F}\}$.

$(X,\mathscr{C})$ is *relatively round* if, for all $\lambda \in \Sigma$, $\mathscr{F} \in \mathscr{C}$ implies $r_\lambda \mathscr{F} \in \mathscr{C}$. Reed showed in [9] that for a relatively round space, $\mathscr{C}_\Gamma = \mathscr{C}_\Sigma$, for all $\Gamma \subseteq \Sigma$.

Proposition 3.4. *Let* $(X,\mathscr{C})$ *be a Cauchy space,* $p \in \mathbb{C}(X)$*,* $\lambda \in \Sigma$*, and* $\mathscr{F} \in \mathbb{F}(X)$*, then*
(a) $s_\lambda^p \mathscr{F} \le r_\lambda \mathscr{F}$*;*
(b) *if* $p = q_{\mathscr{C}}$*, then* $s_\lambda \mathscr{F} = s_\lambda^p \mathscr{F}$*.*

Proof of (b). Let F and A be subsets of X; it suffices to show $F <^{\lambda} A$ if and only if $F <^{\lambda}_{q_{\mathscr{C}}} A$. Assuming $F <^{\lambda} A$ and $x \in \mathrm{cl}_{q_{\mathscr{C}}} F$, there is $\mathscr{H} \xrightarrow{q_{\mathscr{C}}} x$ such that $F \in \mathscr{H}$, which implies $A \in \lambda([\dot{x}]) = \dot{x}$; thus $x \in A$. If $z \in \theta_{\mathscr{C}} F$, there is $\mathscr{H} \in z$ such that $F \in \mathscr{H}$, which implies by assumption that $A \in \lambda(z)$ and hence $z \in A^{\lambda}$. Thus $F <^{\lambda}_{q_{\mathscr{C}}} A$. Conversely, let $z \in X^*$ be such that there is $\mathscr{H} \in z$ with $F \in \mathscr{H}$. It remains to show that $A \in \lambda(z)$. If $z = [\dot{x}]$ for $x \in X$, $\mathscr{H} \xrightarrow{q_{\mathscr{C}}} x$ and $F \in \mathscr{H}$ implies that $x \in \mathrm{cl}_{q_{\mathscr{C}}} F \subseteq A$, so $A \in \dot{x} = \lambda([\dot{x}])$. If $z \in X'$ then $F \in \mathscr{H} \in z$ implies $z \in \theta_{\mathscr{C}} \mathscr{F} \subseteq A^{\lambda}$, and hence $A \in \lambda(z)$. □

Theorem 3.5. *Let $(X,\mathscr{C})$ be a p-regular Cauchy space, $p \in \mathbb{C}(X)$, and $\lambda \in \Sigma$. $(X^*,\mathscr{C}_{\lambda})$ is p-regular if and only if $\mathscr{F} \in \mathscr{C}$ implies $s^p_{\lambda}\mathscr{F} \in \mathscr{C}$.*

Proof. Assume $(X,\mathscr{C})$ is p-regular. By Proposition 3.3(b), $\Delta_p \mathscr{F} = j(\mathrm{cl}_p \mathscr{F}) \cap \theta_{\mathscr{C}} \mathscr{F} \in \mathscr{C}_{\lambda}$, and so there is $\mathscr{G} \in [\mathscr{F}]$ such that $\Delta_p \mathscr{F} \geq \mathscr{G}^{\lambda}$. Thus for any $G \in \mathscr{G}$, there is $F \in \mathscr{F}$ such that $\Delta_p F \subseteq G^{\lambda}$, which implies $F <^p_{\lambda} G$. Thus $G \in s^p_{\lambda}\mathscr{F}$, and so $\mathscr{G} \leq s^p_{\lambda}\mathscr{F}$, whence $s^p_{\lambda}\mathscr{F} \in \mathscr{C}$.

Conversely, the assumed condition implies property S, so it remains to show that $\mathscr{C}_{\lambda} \leq \mathscr{C}^*_p$. If $\mathscr{F} \in \mathscr{C}$, then $s^p_{\lambda}\mathscr{F} \in \mathscr{C}$ by assumption. Let $A \in s^p_{\lambda}\mathscr{F}$. Then there is $F \in \mathscr{F}$ such that $F <^{\lambda}_p A$, implying $\Delta_p G \subseteq A^{\lambda}$. Thus $\Delta_p \mathscr{F} \geq (s^p_{\lambda}\mathscr{F})^{\lambda} \in \mathscr{C}_{\lambda}$, and therefore $\mathscr{C}^*_p \subseteq \mathscr{C}_{\lambda}$. □

Proposition 3.6. *Let $(X,\mathscr{C})$ be a p-regular Cauchy space, $\Gamma \subseteq \Sigma$. Then $(X^*,\mathscr{C}_{\Gamma})$ is p-regular if and only if $(X^*,\mathscr{C}_{\lambda})$ is p-regular, for all $\lambda \in \Gamma$.*

Proof. If $(X^*,\mathscr{C}_{\Gamma})$ is p-regular, then $\mathscr{C}_{\Gamma} = \mathscr{C}^*_p$ by Proposition 3.3(b), and each $(X^*,\mathscr{C}_{\lambda})$ is p-regular since $\mathscr{C}^*_p \leq \mathscr{C}_{\lambda} \leq \mathscr{C}_{\Gamma}$ by Proposition 3.3(a). The converse holds because the supremum of a set of p-regular Cauchy structures is p-regular (see [5]). □

Corollary 3.7. *If $(X,\mathscr{C})$ is a Cauchy space with a p-regular completion, $(X^*,\mathscr{C}^*_p)$ is a Reed completion if and only if there is $\lambda \in \Sigma$ such that $\mathscr{F} \in \mathscr{C}$ implies $s^p_{\lambda}\mathscr{F} \in \mathscr{C}$.*

It was shown in [7] that for a regular Cauchy space $(X,\mathscr{C})$ and $\lambda \in \Lambda$, $(X^*,\mathscr{C}^*_{\lambda})$ is a regular completion if and only if $\mathscr{F} \in \mathscr{C}$ implies $s_{\lambda}\mathscr{F} \in \mathscr{C}$. Combining this result with Proposition 3.4 and Theorem 3.5, we obtain the next corollary.

Corollary 3.8. *If $(X,\mathscr{C})$ is regular Cauchy space and $\Gamma \subseteq \Sigma$, then $(X^*,\mathscr{C}_{\Gamma})$ is regular if and only if $(X^*,\mathscr{C}_{\Gamma})$ is $q_{\mathscr{C}}$-regular.*

From Propositions 3.4 and 3.6 and Theorem 3.5, we obtain the next result.

Corollary 3.9. *If $(X,\mathscr{C})$ is a Cauchy space whose Kowalsky completion is p-regular for any $p \in \mathbb{C}(X)$, then $(X,\mathscr{C})$ is relatively round.*

Lemma 3.10. *Let $(X,\mathscr{C})$ be cushioned and $p \in \mathbb{C}(X)$. Then for all $\mathscr{F} \in \mathbb{F}(X)$, $r_{\sigma}(\mathrm{cl}_p \mathscr{F}) \leq s^p_{\sigma}\mathscr{F}$.*

Proof. Recall that $\sigma(z) = \mathscr{M}_z = \cap z$, for all $z \in X'$, and, since $(X,\mathscr{C})$ is cushioned, $\sigma \in \Sigma$. Let $A \in r_{\sigma}(\mathrm{cl}_p \mathscr{F})$. Then there is $F \in \mathscr{F}$ such that, for all $z \in X^*$, $A \in \sigma(z)$ or $X \setminus \mathrm{cl}_p F \in \sigma(z)$. Since $\mathrm{cl}_p F <_{\sigma} A$, $\mathrm{cl}_p F \subseteq A$, and to show that $A \in s^p_{\sigma}\mathscr{F}$, it suffices to show that $\theta_{\mathscr{C}}\mathscr{F} \subseteq A^{\sigma}$.

If $z \in \theta_{\mathscr{C}} F$ and $z \notin A^{\sigma}$, then $X \setminus \mathrm{cl}_p F \in \sigma(z) = \mathscr{M}_z$, and so $X \setminus F \in \mathscr{M}_z$. But $z \in \theta_{\mathscr{C}} F$ implies there is $\mathscr{G} \in z$ such that $F \in \mathscr{G} \geq \mathscr{M}_z$, contrary to $X \setminus F \in \mathscr{M}_z$. Thus $\theta_{\mathscr{C}} F \subseteq A^{\sigma}$. □

Theorem 3.11. *If $(X,\mathscr{C})$ is p-regular and cushioned, then $(X^*,\mathscr{C}_\sigma)$ is p-regular if and only if $(X,\mathscr{C})$ is relatively round.*

4. Diagonal axioms. A form of "duality" between the properties "regular" and "topological" in convergence space theory was first observed by Cook and Fischer [2] in 1967. Both properties have "diagonal" characterizations and if the concluding implication in the characterizations of either property is reserved, the resulting axiom characterizes the other property. This dual behaviour is shown in [11] to extend to the properties "p-regular" and "p-topological" in the convergence space setting. We begin this section by showing that properties are likewise dual in the setting of Cauchy spaces.

Let $(X,\mathscr{C})$ be a Cauchy space, J an arbitrary set, and let $\rho : J \to \mathbb{F}(X)$ be a "selection function." Then $\kappa\rho\mathscr{F}$ is defined to be the filter $\bigcup_{F\in\mathscr{F}} \bigcap_{x\in F}$ in $\mathbb{F}(X)$. Next, for a Cauchy space $(X,\mathscr{C})$ and $p \in \mathbb{C}(X)$, we define the following dual axioms.

Δ_p: Let J be any set, $\psi : J \to X^*$, and $\rho : J \to \mathbb{F}(X)$ be such that $\rho(y) \xrightarrow{p} j^{-1}\psi(y)$ if $\psi(y) \in j(X)$ and $\rho(y) \in \psi(y)$ if $\psi(y) \in X'$. If $\mathscr{F} \in \mathbb{F}(J)$ is such that $j^{-1}\psi\mathscr{F} \in \mathscr{C}$, then $\kappa\rho\mathscr{F} \in \mathscr{C}$.

$\hat{\Delta}_p$: Let J be any set, $\psi : J \to X^*$, and $\rho : J \to \mathbb{F}(X)$ be such that $\rho(y) \xrightarrow{p} j^{-1}\psi(y)$ if $\psi(y) \in j(X)$ and $\rho(y) \in \psi(y)$ if $\psi(y) \in X'$. If $\mathscr{F} \in \mathbb{F}(J)$ is such that $\kappa\rho\mathscr{F} \in \mathscr{C}$, then $j^{-1}\psi\mathscr{F} \in \mathscr{C}$.

Let $\mathbb{U}(X)$ denote the set of all ultrafilters on X.

Theorem 4.1. *Let $(X,\mathscr{C})$ be a Cauchy space and $p \in \mathbb{C}(X)$.*
(1) *$(X,\mathscr{C})$ is p-regular if and only if $\hat{\Delta}_p$ is satisfied.*
(2) *$(X,\mathscr{C})$ is p-topological if and only if Δ_p is satisfied.*

Proof. (1) Assume $(X,\mathscr{C})$ is p-regular, and let J,ρ, and ψ be as specified in $\hat{\Delta}_p$. One can verify that for any $\mathscr{F} \in \mathbb{F}(J)$, $\mathrm{cl}_p(\kappa\rho\mathscr{F}) \le j^{-1}\psi\mathscr{F}$. Thus $\kappa\rho\mathscr{F} \in \mathscr{C}$ implies $\mathrm{cl}_p(\kappa\rho\mathscr{F}) \in \mathscr{C}$ by p-regularity, and hence $j^{-1}\psi\mathscr{F} \in \mathscr{C}$. Conversely, assume $\hat{\Delta}_p$, and let $\mathscr{F} \in \mathscr{C}$ and $J = \{(\mathscr{G},y) : \mathscr{G} \in \mathbb{U}(X),\ y \in X,\ \mathscr{G} \xrightarrow{p} y\}$. Define $\psi : J \to X^*$ by $\psi(\mathscr{G},y) = j(y)$ and $\rho : J \to \mathbb{F}(X)$ by $\rho(\mathscr{G},y) = \mathscr{G}$; note that these definitions are compatible with the assumptions of $\hat{\Delta}_p$. If $F \in \mathscr{F}$, let $H_F = \{(\mathscr{G},y) \in J : F \in \mathscr{G}\}$ and let $\mathscr{H}$ be the filter on J generated by $\{H_F : F \in \mathscr{F}\}$. One easily verifies that $\mathscr{F} \le \kappa\rho\mathscr{H}$ and, for any $F \in \mathscr{F}$, $j^{-1}\psi(H_F) \subseteq \mathrm{cl}_p F$. Thus $\kappa\rho\mathscr{H} \in \mathscr{C}$ and by $\hat{\Delta}_p$, $j^{-1}\psi\mathscr{H} \in \mathscr{C}$. But $j^{-1}\psi\mathscr{H} \le \mathrm{cl}_p\mathscr{F}$, so $\mathrm{cl}_p\mathscr{F} \in \mathscr{C}$ and $\mathscr{C}$ is regular.

(2) Assume $(X,\mathscr{C})$ is p-topological, and let J,ψ,ρ, and $\mathscr{F}$ be as described in Δ_p. Assume $j^{-1}\psi\mathscr{F} \in \mathscr{C}$. Since $(X,\mathscr{C})$ is p-topological, there is $\mathscr{G} \in \mathscr{C}$ such that $j^{-1}\psi\mathscr{F} \ge \mathrm{I}_p\mathscr{G}$. Let $G \in \mathscr{G}$ and choose $F \in \mathscr{F}$ such that $j^{-1}\psi F \subseteq \mathrm{I}_p G$. One may verify that $G \in \kappa\rho\mathscr{F}$ and hence $\mathscr{G} \le \kappa\rho\mathscr{F}$. thus $\kappa\rho\mathscr{F} \in \mathscr{C}$, and Δ_p holds. Conversely, assume Δ_p, and let $\mathscr{F} \in \mathscr{C}$ and $J = \{(\mathscr{G},x) : \mathscr{G} \in \mathbb{U}(X),\ \mathscr{G} \xrightarrow{p} x\}$. Let $\psi : J \to X^*$ be defined by $\psi((\mathscr{G},x)) = j(x)$ and $\rho : J \to \mathbb{F}(X)$ be defined by $\rho(\mathscr{G},x) = \mathscr{G}$. Note that ψ maps J onto $j(X)$; thus $\mathscr{H} = \psi^{-1}j\mathscr{F} \in \mathbb{F}(J)$. Thus $j^{-1}\psi\mathscr{H} = \mathscr{F} \in \mathscr{C}$ and, by Δ_p, $\kappa\rho\mathscr{H} \in \mathscr{C}$. One may verify that $\mathscr{F} \ge \mathrm{I}_p(\kappa\rho\mathscr{H})$, and thus $(X,\mathscr{C})$ is p-topological. □

Combining the preceding result with [10, Proposition 1.2 and Corollary 2.8], we obtain the next corollary.

Corollary 4.2. *Let $(X,\mathscr{C})$ be a Cauchy space, $p \in \mathbb{C}(X)$.*

(1) *$(X,\mathscr{C})$ has a p-regular completion if and only if $(X,\mathscr{C})$ has property S and satisfies $\hat{\Delta}_p$.*

(2) *$(X,\mathscr{C})$ has a p-topological completion if and only if $(X,\mathscr{C})$ is cushioned and satisfies Δ_p.*

Two additional diagonal axioms of interest are stated below. Let $(X,\mathscr{C})$ be a Cauchy space, $p \in \mathbb{C}(X)$, and let $\lambda \in \Lambda$, where Λ denotes the set of all Reed selection functions.

$\hat{\mathbb{D}}_p^\lambda$: Let J be any set, $\psi : J \to X^*$, and $\rho : J \to \mathbb{F}(X)$ be such that $\rho(y) \xrightarrow{p} j^{-1}\psi(y)$ if $\psi(y) \in j(X)$, and $\rho(y) \in \psi(y)$ if $\psi(y) \in X'$. If $\mathscr{F} \in \mathbb{F}(J)$ is such that $\kappa\rho\mathscr{F} \in \mathscr{C}$, then there is $\mathscr{G} \in \mathscr{C}$ such that $\kappa\rho\mathscr{F} \geq \mathscr{G}^\lambda \cap [\dot{\mathscr{G}}]$.

$\mathbb{D}_p^\lambda$: Let J be any set, $\psi : J \to X^*$, and $\rho : J \to \mathbb{F}(X)$ be such that $\rho(y) \xrightarrow{p} j^{-1}\psi(y)$ if $\psi(y) \in j(X)$, and $\rho(y) \in \psi(y)$ if $\psi(y) \in X'$. If $\mathscr{F} \in \mathbb{F}(J)$ is such that $\kappa\rho\mathscr{F} \geq \mathscr{G}^\lambda \cap [\dot{\mathscr{G}}]$ for some $\mathscr{G} \in \mathscr{C}$, then $\kappa\rho\mathscr{F} \in \mathscr{C}$.

Special cases of these two dual axioms occur elsewhere in the literature. For instance $\hat{\mathbb{D}}_p^\lambda$ is called $\mathbb{D}'_\lambda$ in [1] in the special case where $p = q_{\mathscr{C}}$, and it is proved in [1] that the Reed completion $(X^*,\mathscr{C}_\lambda)$ of $(X,\mathscr{C})$ is regular if and only if $(X,\mathscr{C})$ satisfies $\hat{\mathbb{D}}_{q_{\mathscr{C}}}^\lambda$. In [10], $\mathbb{D}_p^\lambda$ is called $\mathbb{D}_p$ when $\lambda = \omega$, and it is proved that $(X,\mathscr{C})$ has a p-topological completion if and only if $(X,\mathscr{C})$ satisfies $\mathbb{D}_p^\omega$. It is also shown in [10] that the Wyler completion $(X^*,\mathscr{C}_\omega)$ is p-topological if and only if $(X,\mathscr{C})$ satisfies $\mathbb{D}_p^\omega$. These results are special cases of the next theorem, whose proof we shall omit for the sake of brevity.

Theorem 4.3. *Let $(X,\mathscr{C})$ be a Cauchy space, $p \in \mathbb{C}(X)$, and $\lambda \in \Lambda$.*

(1) *$(X^*,\mathscr{C}_\lambda)$ is p-regular if and only if $\hat{\mathbb{D}}_p^\lambda$ holds.*

(2) *$(X^*,\mathscr{C}_\lambda)$ is p-topological if and only if $\mathbb{D}_p^\lambda$ holds.*

We next characterize those Cauchy spaces which allow a completion which is both p-regular and p-topological.

Theorem 4.4. *For a Cauchy space $(X,\mathscr{C})$ which is both p-regular and p-topological, the following statements are equivalent:*

(1) *$(X,\mathscr{C})$ has a completion which is p-regular and p-topological.*

(2) *$(X^*,\mathscr{C}_\Sigma)$ is p-regular and p-topological.*

(3) *$(X,\mathscr{C})$ is cushioned and relatively round.*

(4) *$(X,\mathscr{C})$ satisfies $\hat{\mathbb{D}}_p^\sigma$ and $\mathbb{D}_p^\sigma$.*

Proof. Note that (2) and (4) each implies that $(X,\mathscr{C})$ is cushioned and hence $\mathscr{C}_\Gamma = \mathscr{C}_\sigma$, as noted in Section 3. By Theorem 4.3, it follows that (2) and (4) are equivalent. Equivalence of (2) and (3) follows by Theorem 3.11, and (2) obviously implies (1). It is shown in [10] that if $(X,\mathscr{C})$ has p-topological completions, then $(X,\mathscr{C}_\sigma)$ is the coarsest such completion, and by Proposition 3.3(a), $(X^*,\mathscr{C}_\sigma)$ is finer than the finest p-regular completion. Thus $\mathscr{C}_\sigma$is the only candidate for a completion which is both p-regular and p-topological, and since $\mathscr{C}_\sigma = \mathscr{C}_\Sigma$, it follows that (1) implies (2). □

The duality revealed in Theorem 4.1 leads to some interesting similarities between p-regular and p-topological completions, as illustrated by the first two propositions of the next section, but there are also some important differences. Whereas all p-

topological completions are strict, p-regular completions are generally not, although every Cauchy space which has a p-regular completion also has a strict one, namely $(X^*, \mathscr{C}_p^*)$. We should also keep in mind that every p-topological Cauchy space must satisfy $q_{\mathscr{C}} \leq p$ (see [10]), whereas no such restriction applies to p-regular Cauchy spaces or completions.

5. Regular versus $q_{\mathscr{C}}$-regular completions. We first note the obvious fact that a Cauchy space is regular (respectively, topological) if and only if it is $q_{\mathscr{C}}$-regular (respectively, $q_{\mathscr{C}}$-topological). However this observation does not extend to Cauchy completions.

Proposition 5.1. *Let $(X, \mathscr{C})$ be a Cauchy space.*

(1) *A regular completion of $(X, \mathscr{C})$ is also a $q_{\mathscr{C}}$-regular completion, but the converse is generally false.*

(2) *A topological completion of $(X, \mathscr{C})$ is also $q_{\mathscr{C}}$-topological completion, but the converse is generally false.*

Proof. (1) If $(X^*, \mathfrak{D})$ is a regular completion of $(X, \mathscr{C})$, then $\mathscr{H} \in \mathfrak{D}$ implies $\mathrm{cl}_{q_{\mathfrak{D}}} \mathscr{H} = \mathrm{cl}_{q^*_{\mathscr{C}}} \mathscr{H} \in \mathfrak{D}$, and $\mathrm{cl}_{q_{\mathfrak{D}}} j(X) = \mathrm{cl}_{q^*_{\mathscr{C}}} j(X) = X^*$, so $(X^*, \mathfrak{D})$ is also a $q_{\mathscr{C}}$-regular completion. An example is given in [3] of a Cauchy space $(X, \mathscr{C})$ with a regular completion but no strict regular completion. The existence of a regular completion implies that $(X^*, \mathscr{C}^*_{q_{\mathscr{C}}})$ is a $q_{\mathscr{C}}$-regular completion which, being strict, cannot be regular.

(2) The first part of the assertion is easily verified, and an example is found in [10] of a $q_{\mathscr{C}}$-topological completion which is not topological. □

However, for Reed completion, we obtain the following result by [10, Theorem 4.7 and Corollary 3.8].

Proposition 5.2. *Let $(X^*, \mathscr{C}_\Gamma)$ be an arbitrary Reed completion of $(X, \mathscr{C})$.*

(a) *$(X^*, \mathscr{C}_\Gamma)$ is a regular completion if and only if it is a $q_{\mathscr{C}}$-regular completion.*

(b) *$(X^*, \mathscr{C}_\Gamma)$ is a topological completion if and only if it is a $q_{\mathscr{C}}$-topological completion.*

The next proposition is a restatement of [7, Theorem 3.4(b)]; it is repeated here in contrast to the corresponding situation for regular completions, as shown in Example 5.4

Proposition 5.3 [10]**.** *A topological Cauchy space $(X, \mathscr{C})$ has a topological completion if and only if it has a $q_{\mathscr{C}}$-topological completion.*

In the example that follows, we construct a regular Cauchy space $(X, \mathscr{C})$ with property S, which therefore has $(X^*, \mathscr{C}^*_{q_{\mathscr{C}}})$ as a strict, $q_{\mathscr{C}}$-regular completion. Let $(X^*, \mathscr{C}^*_r)$ denote the regular modification of the Wyler completion of $(X, \mathscr{C})$. We shall show that $(X^*, \mathscr{C}^*_r)$ is not T_2, and it follows by [3, Proposition 1.2], that $(X, \mathscr{C})$ has no regular completion (with or without the T_2 property). For readers acquainted with the *regularity series* (see [9]) of a convergence space, it is of interest to note that $(X^*, \mathscr{C}^*_{q_{\mathscr{C}}})$ is the first term of the regularity series of the Wyler completion. In the next example, it turns out that the T_2 property is “lost” in the transition between the first and the second terms of the regularity series of the Wyler completion, and consequently $(X^*, \mathscr{C}_r^*)$ is not T_2.

Example 5.4. Let X be an infinite set. Let $\{A_k : k \in N\}$, $\{B_k : k \in N\}$, $\{C_k : k \in N\}$, and $\{D_k : k \in N\}$ be denumerable collections of pairwise disjoint, denumerable subsets of X, such that each member of each collection has an empty intersection with every member of each of the other collections. We first define some subsets of X:

$$A = \bigcup_{k \in N} A_k, \qquad A'_n = \bigcup_{k=1}^{n} A_k, \qquad F_n = A \backslash A'_n, \tag{5.1}$$

$$B = \bigcup_{k \in N} B_k, \qquad B'_n = \bigcup_{k=1}^{n} B_k, \qquad G_n = B \backslash B'_n. \tag{5.2}$$

Assume further that each A_k is partitioned into a denumerable number of denumerable sets $\{A_{kj} : j \in \mathbb{N}\}$, and carry out the same partitions with the corresponding notation for B_k, C_k, and D_k, for all $k \in \mathbb{N}$.

We next define the following filters on X:

$\mathscr{F}$ is generated by $\{F_n : n \in \mathbb{N}\}$;

$\mathscr{G}$ is generated by $\{G_n : n \in \mathbb{N}\}$;

$\mathscr{A}_{kj}$ is generated by $\{A_{kj} \backslash F : F \text{ finite}\}$, for all $k, j \in \mathbb{N}$;

$\mathscr{B}_{kj}$ is generated by $\{B_{kj} \backslash F : F \text{ finite}\}$, for all $k, j \in \mathbb{N}$;

$\mathscr{C}_{kj}$ is generated by $\{C_{kj} \backslash F : F \text{ finite}\}$, for all $k, j \in \mathbb{N}$;

$\mathscr{D}_{kj}$ is generated by $\{D_{kj} \backslash F : F \text{ finite}\}$, for all $k, j \in \mathbb{N}$;

$\mathscr{H}_k$ is generated by $\{C_k \backslash C_{kj} : j \in \mathbb{N}\}$, for all $k \in \mathbb{N}$;

$\mathscr{K}_k$ is generated by $\{D_k \backslash D_{kj} : j \in \mathbb{N}\}$, for all $k \in \mathbb{N}$.

Let $\mathscr{C}$ be the Cauchy structure on X generated by: $\{\dot{x} : x \in X\} \cup \{\mathscr{F}\} \cup \{\mathscr{G}\} \cup \{\mathscr{A}_{kj} \cap \mathscr{C}_{kj} : k, j \in \mathbb{N}\} \cup \{\mathscr{B}_{kj} \cap \mathscr{D}_{kj} : k, j \in \mathbb{N}\} \cup \{\mathscr{H}_k \cap \mathscr{K}_k : k \in \mathbb{N}\}$. Since $q_{\mathscr{C}}$ is obviously discrete, $(X, \mathscr{C})$ is regular. The proof that $(X, \mathscr{C})$ satisfies S is tedious and requires examining a number of cases; we omit these details. From these observations, we conclude that $(X^*, \mathscr{C}^*_{q_{\mathscr{C}}})$ is a $q_{\mathscr{C}}$-regular completion of $(X, \mathscr{C})$.

Let q_r^* be the convergence structure on X^* derived from $\mathscr{C}_r^*$, the regular modification of $\mathscr{C}_w$. It is clear that $[\mathscr{F}]$ and $[\mathscr{G}]$ are distinct elements of X^*, but one can show that $\mathrm{cl}_{p_r^*} j\mathscr{F}$ p_r^*-converges to both of these elements, so that $(X^*, \mathscr{C}_r^*)$ is not T_2. From the remarks preceding the example, it follows that $(X, \mathscr{C})$ has a regular completion.

The fact that the analogue of Proposition 5.3 for regular Cauchy spaces fails, as evidenced by Example 5.4, is perhaps related to another problem: that of finding a diagonal axiom which characterizes Cauchy spaces having a regular (or strict, regular) completion. An effort to solve this problem for strict, regular completion was recently made by Brock and Richardson, [1], but the diagonal condition that they found has to be combined with two additional conditions in order to characterize Cauchy spaces having strict, regular completions. Indeed, this problem remains unsolved even for p-regular completions, since our results in Corollary 4.2(a), like that of [1], requires combining an additional property along with $\hat{\Delta}_p$ in order to characterize Cauchy spaces allowing p-regular completions.

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