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CORRIGENDUM

ON LINEAR ALGEBRAIC SEMIGROUPS III

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There are some errors in the above paper. There is a line missing at the bottom of page 672. Also, pages 681-683 are organized incorrectly.

These errors are corrected as follows:

Replace the last sentence on page 672 with:

" $\bigcup_{g \in G} gE(Y)g^{-1} = E(S)$. In particular $E(Y) \cap J \neq \emptyset$ for all $J \in \mathcal{U}(S)$. Moreover the length of any maximal chain in $\mathcal{U}(S)$ equals dim Y. "

Replace page 681 beginning from line 14 (from the top), the entire page of 682 and the first seven lines (from the top) of page 683 with:

"PROOF. We can assume that e is the identity element of S (otherwise we work with eSe). By Lemma 1.1 we are reduced to the case when f is the zero of S. By Corollary 1.5, we are reduced to the case when S is also a d-semigroup. By Lemma 2.2 and Theorem 2.7, we can assume that S is as in Theorem 2.7, with $e = (1,\dots,1)$, $f = (0,\dots,0)$. Let $V_1 = \{(\omega_1(a,\dots,a),\dots,\omega_n(a,\dots,a)) \mid a \in K\}$, $S_1 = \overline{V}_1$. Then $e, f \in S_1$, $\dim S_1 = 1$, $S_1 \subseteq S$. Define $\theta: K \to S_1$ as $\theta(a) = (\omega_1(a,\dots,a),\dots,\omega_n(a,\dots,a))$. Then θ is a *-homomorphism. So S_1 is connected. This proves the theorem.

3. POLYTOPES

If $X \subseteq \mathbb{R}^n$, then we let $C(X)$ denote the convex hull of X (see[4]). The convex hull of a finite set in $\mathbb{R}^n$ is called a <u>polytope</u> [4]. If the vertices of P are rational, then P is said to be a <u>rational polytope</u>. If $X \subseteq P$, then X is said to be a <u>face</u> of P [4; p. 25] if for all a, $b \in P$, $\alpha \in (0,1)$, $\alpha a + (1 - \alpha)a \in X$ if and only if a, $b \in X$. Let $X(P)$ denote the set of all faces of P. Then [4; p. 21], $(X(P), \subseteq)$ is a finite lattice. Dimension of P is defined to be the dimension of the affine hull of P [4; p.3]. Then dimension of P = (length of any maximal chain in $X(P)$) - 1. Two polytopes P_1, P_2 have the same <u>combinatorial type</u> if $X(P_1) \cong X(P_2)$ (see [4; p. 38]). By [4; p. 244], every polytope of dimension ≤ 3 has the same combinatorial type as some rational polytope. However this is not true in general [4: p. 94]. If $u = (\alpha_1, \ldots, \alpha_n)$, $v = (\beta_1, \ldots, \beta_n) \in \mathbb{R}^n$ then let $u \cdot v = \sum_{i=1}^{n} \alpha_i \beta_i$ denote the inner product of u and v.

Let S be a semigroup. An ideal I of S is said to be <u>semiprime</u> if for all $a \in S$, $a^2 \in I$ implies $a \in I$. I is <u>prime</u> if for all a, $b \in S$, $ab \in I$ implies $a \in I$ or $b \in I$. Let

$I(S)$ = {All ideals of S}

$A(S)$ = {All principal ideals of S}

$\Gamma(S)$ = {All semiprime ideals of S} $\cup$ {$\emptyset$}

$\Lambda(S)$ = {All prime ideals of S} $\cup$ {$\emptyset$}.

$X(S) = \{S \setminus I \mid I \in \Lambda(S)\}$.

$\Omega(S)$ = Maximal semilattice image of S.

It is easy to see that $(\Lambda(S), \subseteq) \cong (\Lambda(\Omega(S)), \subseteq)$ is a complete lattice. If S is finitely generated, then $\Omega(S)$ is finite and so $(\Lambda(S), \subseteq)$ is a finite lattice.

THEOREM 3.1. Let S be a connected d-semigroup with zero. Define $\alpha: I(S) \to \Gamma(\Phi(S))$ as $\alpha(I) = \{\chi \mid \chi \in \Phi(S), \chi(a) = 0 \text{ for all } a \in I\}$. Define $\beta: \Gamma(\Phi(S)) \to I(S)$ as $\beta(W) = \{a \mid a \in S, \chi(a) = 0 \text{ for all } \chi \in W\}$. Then α, β are inclusion reversing bijections and $\beta = \alpha^{-1}$. Moreover $\alpha(A(S)) = \Lambda(\Phi(S))$.

PROOF. Clearly α,β are inclusion reversing. Let $I \in A(S)$. Then $I = eS$ for some $e \in E(S)$. So $\alpha(I) = \{\chi | \chi \in \Phi(S),\ \chi(e) = 0\}$. It follows that $\alpha(I) \in \Lambda(\Phi(S))$. Clearly $I \subseteq \beta(\alpha(I))$. We claim that $I = \beta(\alpha(I))$. Suppose not. Then there exists $a \in \beta(\alpha(I))$ such that $a \notin I$. Let $a\ Hf$, $f \in E(S)$. Then $f \in I$, $f \in \beta(\alpha(I))$. So $e \not\geq f$. By Lemma 2.1 (2), there exists $\chi \in \Phi(S)$ such that $\chi(f) = 1$, $\chi(e) = 0$. So $\chi \in \alpha(I)$ and $f \notin \beta(\alpha(I))$, a contradiction. So

$$\text{for all } I \in A(S),\ \alpha(I) \in \Lambda(\Phi(S)) \text{ and } \beta(\alpha(I)) = I \tag{12}$$

Let $P \in \Lambda(\Phi(S))$. We calim that $\beta(P) \in A(S)$ and $\alpha(\beta(P)) = P$. By Lemma 2.1, this is true for $P = \Phi(S)$. So assume $P \neq \Phi(S)$. Then $F = \Phi(S)\backslash P$ is a subsemigroup of $\Phi(S)$. By Lemma 2.2 we can assume that S is a closed submonoid of some $(K^n,\cdot)$, $0 = (0,\dots,0) \in S$ and that $\Phi(S) = < \chi_1,\dots,\chi_n >$ where χ_i is the i^{th} projection of S into K, $i = 1, \dots,n$. Let $A = \{\chi_i | \chi_i \in F\}$. Then $<A> = F$. Let $e = (e_1,\dots,e_n)$ where $e_i = 1$ if $\chi_i \in A$, $e_i = 0$ if $\chi_i \notin A$. We claim that $e \in S$. Suppose not. Then by Lemma 2.3, there exist $u, v \in F(X_1,\dots,X_n)$ such that $u(a) = v(a)$ for all $a \in S$ and $u(e) \neq v(e)$. Since $u(e)^2 = u(e)$ and $v(e)^2 = v(e)$ we can assume that $u(e) = 1$, $v(e) = 0$. Clearly $u(\chi_1,\dots,\chi_n) = v(\chi_1,\dots,\chi_n)$. Since $u(e) = 1$, $u(X_1,\dots,X_n)$