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Research Article

A Note on Some Properties of the Weighted q-Genocchi Numbers and Polynomials

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We consider the weighted q-Genocchi numbers and polynomials. From the construction of the weighted q-Genocchi numbers and polynomials, we investigate many interesting identities and relations satisfied by these new numbers and polynomials.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, $\mathbb{C}$, and $\mathbb{C}_p$, will, respectively, denote the ring of p-adic integers, the field, of p-adic rational numbers, the complex number field and the completion of algebraic closure of $\mathbb{Q}_p$. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ such that $|p|_p = p^{-\nu_p(p)} = 1/p$ (see [1–16]).

As well-known definition, the Euler numbers and Genocchi numbers are defined by

$$\frac{2}{e^t+1} = e^{Et} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}, \tag{1.1}$$

with the usual convention of replacing E^n by E_n and

$$\frac{2t}{e^t+1} = e^{Gt} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}, \tag{1.2}$$

with the usual convention of replacing G^n by G_n. We assume that $q \in \mathbb{C}_p$ with $|1-q|_p < 1$ and that the q-number of x is defined by

$$[x]_q = \frac{1-q^x}{1-q}, \qquad [x]_{-q} = \frac{1-(-q)^x}{1+q} \tag{1.3}$$

(see [1–19]).

In [9], Kim introduced ordinary fermionic p-adic integral on $\mathbb{Z}_p$, and he studied some interesting relations and identities related to q-extension of Euler numbers and polynomials. In [8], he also introduced the q-extension of the ordinary fermionic p-adic integral on $\mathbb{Z}_p$ and he investigated many physical properties related to q-Euler numbers and polynomials. Recently, Kim firstly introduced the meaning of the weighted q-Euler numbers and polynomials associated with the weighted q-Bernstein polynomials by using the fermionic invariant p-adic integral on $\mathbb{Z}_p$ (see [14, 15]). In [16], Ryoo tried to study the weighted q-Euler number and polynomials by the same method of Kim et al. in [14] and the q-extension of the fermionic p-adic invariant integrals on $\mathbb{Z}_p$. As well-known properties, the Genocchi numbers are integers. The first few Genocchi numbers for $n = 2, 4, \ldots$ are $-1, 1, -3, 17, -155, 2073, \ldots$. The first few prime Genocchi numbers are -3 and 17, which occur for $n = 6$ and 8. There are no others with $n < 10^5$. These properties are very important to study in the area of fermionic distribution and p-adic numbers theory. By this reason, many mathematicians and physicians have studied Genocchi and Euler numbers which are in the different areas. By the same motivation, we consider weighted q-Genocchi polynomials and numbers by using the fermionic p-adic q-integral on $\mathbb{Z}_p$ which are constructed by Kim and Ryoo (cf. [8, 16]).

In this paper, we consider the q-Genocchi numbers and polynomials with weighted α ($\alpha \in \mathbb{Q}$). From the construction of the weighted q-Genocchi numbers and polynomials, we investigate many interesting identities and relations satisfied by these new numbers and polynomials.

2. The Weighted q-Genocchi Numbers and Polynomials

Let $\mathrm{UD}(\mathbb{Z}_p)$ be the space of uniformly differentiable functions and, for $f \in \mathrm{UD}(\mathbb{Z}_p)$, the fermionic p-adic invariant integral of f on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x)\,d\mu_{-1}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-1}} \sum_{x=0}^{p^N-1} f(x)(-1)^x \tag{2.1}$$

(see [1–16]). If we take $f(x) = te^{xt}$, then we get

$$t\int_{\mathbb{Z}_p} e^{xt}\,d\mu_{-1}(x) = \frac{2t}{e^t+1}. \tag{2.2}$$

By (1.2) and (2.2), we get

$$\begin{aligned}\sum_{n=0}^{\infty} G_n \frac{t^n}{n!} &= \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) \frac{t^{n+1}}{n!} \\ &= \sum_{n=0}^{\infty} (n+1) \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) \frac{t^{n+1}}{(n+1)!} \\ &= \sum_{n=1}^{\infty} n \int_{\mathbb{Z}_p} x^{n-1} d\mu_{-1}(x) \frac{t^n}{n!}.\end{aligned} \tag{2.3}$$

From (2.3),

$$G_0 = 0, \qquad \frac{G_n}{n} = \int_{\mathbb{Z}_p} x^{n-1} d\mu_{-1}(x), \quad n \in \mathbb{N}. \tag{2.4}$$

For $f \in \mathrm{UD}(\mathbb{Z}_p)$, the fermionic p-adic q-integral of f on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x) d\mu_{-q}(x) = \lim_{N \to \infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)(-q)^x \tag{2.5}$$

(see [1–16]). From (2.5), we note that

$$q^n I_{-q}(f_n) = (-1)^n I_{-q}(f) + [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l f(l), \tag{2.6}$$

where $n \in \mathbb{N}$ and $f_n(x) = f(x+n)$.

For $\alpha \in \mathbb{Q}$, we consider the following fermionic p-adic q-integral on $\mathbb{Z}_p$:

$$t \int_{\mathbb{Z}_p} e^{[x]_{q^\alpha} t} d\mu_{-q}(x) = \sum_{n=0}^{\infty} \widetilde{G}_{n,q}^{(\alpha)} \frac{t^n}{n!}, \tag{2.7}$$

where $\widetilde{G}_{n,q}^{(\alpha)}$ are called the nth q-Genocchi numbers with weight α. From (2.7), we get

$$\begin{aligned}t \int_{\mathbb{Z}_p} e^{[x]_{q^\alpha} t} d\mu_{-q}(x) &= t \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} [x]_{q^\alpha}^n d\mu_{-q}(x) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} (n+1) \int_{\mathbb{Z}_p} [x]_{q^\alpha}^n d\mu_{-q}(x) \frac{t^{n+1}}{(n+1)!} \\ &= \sum_{n=1}^{\infty} n \int_{\mathbb{Z}_p} [x]_{q^\alpha}^{n-1} d\mu_{-q}(x) \frac{t^n}{n!}.\end{aligned} \tag{2.8}$$

By comparing the coefficients on the both sides of (2.7) and (2.8), we get

$$n\int_{\mathbb{Z}_p}[x]_{q^\alpha}^{n-1}d\mu_{-q}(x)=\widetilde{G}_{n,q}^{(\alpha)},\quad n\in\mathbb{N},\qquad \widetilde{G}_{0,q}^{(\alpha)}=0. \tag{2.9}$$

From (2.9), we obtain the following theorem.

Theorem 2.1. *For $n\in\mathbb{N}$ and $\alpha\in\mathbb{Q}$, one has*

$$\int_{\mathbb{Z}_p}[x]_{q^\alpha}^{n-1}d\mu_{-q}(x)=\frac{\widetilde{G}_{n,q}^{(\alpha)}}{n},\qquad \widetilde{G}_{0,q}^{(\alpha)}=0. \tag{2.10}$$

By the definition of fermionic p-adic q-integrals, we get

$$\begin{aligned} t\int_{\mathbb{Z}_p}[x]_{q^\alpha}^{n-1}d\mu_{-q}(x)&=\frac{1}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l\int_{\mathbb{Z}_p}q^{\alpha l x}d\mu_{-q}(x)\\ &=\frac{[2]_q}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l\frac{1}{1+q^{\alpha l+1}}. \end{aligned} \tag{2.11}$$

Therefore, we obtain the following theorem.

Theorem 2.2. *For $n\in\mathbb{N}$ and $\alpha\in\mathbb{Q}$, we have*

$$\frac{\widetilde{G}_{n,q}^{(\alpha)}}{n}=\frac{[2]_q}{(1-q)^{n-1}[\alpha]_q^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l\frac{1}{1+q^{\alpha l+1}}. \tag{2.12}$$

By Theorem 2.2, we have the generating function of $\widetilde{G}_{n,q}^{(\alpha)}$ as follows:

$$\begin{aligned} \sum_{n=0}^{\infty}\widetilde{G}_{n,q}^{(\alpha)}\frac{t^n}{n!}&=[2]_q\sum_{n=0}^{\infty}\frac{n}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l\sum_{m=0}^{\infty}(-1)^m q^{\alpha l m+m}\frac{t^n}{n!}\\ &=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=0}^{\infty}\frac{n}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l q^{\alpha l m}\frac{t^n}{n!}\\ &=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=0}^{\infty}\frac{n}{(1-q^\alpha)^{n-1}}(1-q^{\alpha m})^{n-1}\frac{t^n}{n!}\\ &=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=0}^{\infty}n[m]_{q^\alpha}^{n-1}\frac{t^n}{n!} \end{aligned}$$

$$
\begin{aligned}
&= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=1}^{\infty} [m]_{q^\alpha}^{n-1} \frac{t^n}{(n-1)!} \\
&= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} [m]_{q^\alpha}^{n} \frac{t^{n+1}}{n!} \\
&= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[m]_{q^\alpha} t}.
\end{aligned}
\tag{2.13}
$$

Let $\widetilde{F}_q^{(\alpha)}(t)$ be the generating function of $\widetilde{G}_{n,q}^{(\alpha)}$. Then, by (2.9) and (2.13), we get

$$
\begin{aligned}
\widetilde{F}_q^{(\alpha)}(t) &= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[m]_{q^\alpha} t} \\
&= \sum_{n=0}^{\infty} \widetilde{G}_{n,q}^{(\alpha)} \frac{t^n}{n!}.
\end{aligned}
\tag{2.14}
$$

The q-Genocchi polynomials with weight α are defined by

$$
\begin{aligned}
\widetilde{F}_q^{(\alpha)}(t,x) &= t \int_{\mathbb{Z}_p} e^{[x+y]_{q^\alpha} t} d\mu_{-q}(y) \\
&= \sum_{n=0}^{\infty} \widetilde{G}_{n,q}^{(\alpha)}(x) \frac{t^n}{n!}.
\end{aligned}
\tag{2.15}
$$

From (2.15), we get

$$
\begin{aligned}
t \int_{\mathbb{Z}_p} e^{[x+y]_{q^\alpha} t} d\mu_{-q}(y) &= \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n} d\mu_{-q}(y) \frac{t^{n+1}}{n!} \\
&= \sum_{n=0}^{\infty} (n+1) \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n} d\mu_{-q}(y) \frac{t^{n+1}}{(n+1)!} \\
&= \sum_{n=1}^{\infty} n \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) \frac{t^{n}}{n!}.
\end{aligned}
\tag{2.16}
$$

By (2.15) and (2.16), we obtain the following theorem.

Theorem 2.3. *For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, one has*

$$
n \int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) = \widetilde{G}_{n,q}^{(\alpha)}(x), \qquad \widetilde{G}_{0,q}^{(\alpha)}(x) = 0.
\tag{2.17}
$$

We note that

$$\begin{aligned}\int_{\mathbb{Z}_p}[x+y]_{q^\alpha}^{n-1}d\mu_{-q}(y)&=\sum_{l=0}^{n-1}\binom{n-1}{l}[x]_{q^\alpha}^{n-1-l}q^{\alpha lx}\int_{\mathbb{Z}_p}[y]_{q^\alpha}^{l}d\mu_{-q}(y)\\&=\sum_{l=0}^{n-1}\binom{n-1}{l}[x]_{q^\alpha}^{n-1-l}q^{\alpha lx}\frac{\widetilde{G}_{l+1,q}^{(\alpha)}}{l+1}.\end{aligned}\tag{2.18}$$

From (2.17) and (2.18), we obtain the following theorem.

Theorem 2.4. *For $n\in\mathbb{N}$ and $\alpha\in\mathbb{Q}$, one has*

$$\frac{\widetilde{G}_{n,q}^{(\alpha)}(x)}{n}=\sum_{l=0}^{n-1}\binom{n-1}{l}[x]_{q^\alpha}^{n-1-l}q^{\alpha lx}\frac{\widetilde{G}_{l+1,q}^{(\alpha)}}{l+1}.\tag{2.19}$$

From (2.15), we note that

$$\begin{aligned}\widetilde{F}_q^{(\alpha)}(t,x)&=\sum_{n=0}^{\infty}\widetilde{G}_{n,q}^{(\alpha)}(x)\frac{t^n}{n!}\\&=\sum_{n=0}^{\infty}n\int_{\mathbb{Z}_p}[x+y]_{q^\alpha}^{n-1}d\mu_{-q}(y)\frac{t^n}{n!}\\&=\sum_{n=0}^{\infty}n\left(\frac{1}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}q^{\alpha lx}(-1)^l\int_{\mathbb{Z}_p}q^{\alpha ly}d\mu_{-q}(y)\right)\frac{t^n}{n!}\\&=[2]_q\sum_{n=0}^{\infty}n\left(\frac{1}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}q^{\alpha lx}\frac{(-1)^l}{1+q^{\alpha l+1}}\right)\frac{t^n}{n!}\\&=[2]_q\sum_{n=0}^{\infty}n\left(\frac{1}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}q^{\alpha lx}(-1)^l\sum_{m=0}^{\infty}(-1)^m q^{\alpha lm+m}\right)\frac{t^n}{n!}\\&=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=0}^{\infty}n\left(\frac{1}{(1-q^\alpha)^{n-1}}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l q^{\alpha(x+m)l}\right)\frac{t^n}{n!}\\&=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=0}^{\infty}n\left(\frac{1}{(1-q^\alpha)^{n-1}}\left(1-q^{\alpha(x+m)}\right)^{n-1}\right)\frac{t^n}{n!}\\&=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=0}^{\infty}n[x+m]_{q^\alpha}^{n-1}\frac{t^n}{n!}\\&=[2]_q\sum_{m=0}^{\infty}(-1)^m q^m\sum_{n=1}^{\infty}[x+m]_{q^\alpha}^{n-1}\frac{t^n}{(n-1)!}\end{aligned}$$

$$
\begin{aligned}
&= [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \sum_{n=0}^{\infty} [x+m]_{q^\alpha}^n \frac{t^{n+1}}{n!} \\
&= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[x+m]_{q^\alpha} t}.
\end{aligned}
\tag{2.20}
$$

Therefore, we obtain the following theorem.

Theorem 2.5. *For $\alpha \in \mathbb{Q}$, one has*

$$
\widetilde{F}_q^{(\alpha)}(t,x) = [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{[x+m]_{q^\alpha} t}. \tag{2.21}
$$

From (2.15) and (2.21), we obtain that

$$
\begin{aligned}
\widetilde{G}_{n,q}^{(\alpha)}(x) &= \frac{d^n}{dt^n} \widetilde{F}_q^{(\alpha)}(t,x) \bigg|_{t=0} \\
&= n[2]_q \sum_{m=0}^{\infty} (-1)^m q^m [x+m]_{q^\alpha}^{m-1} \\
&= n[2]_q \frac{1}{(1-q^\alpha)^{n-1}} \sum_{l=0}^{n-1} \frac{\binom{n-1}{l} q^{\alpha l x} (-1)^l}{1+q^{\alpha l+1}} \\
&= \frac{n[2]_q}{(1-q)^{n-1} [\alpha]_q^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l q^{\alpha l x} \frac{1}{1+q^{\alpha l+1}}.
\end{aligned}
\tag{2.22}
$$

Therefore, we obtain the following theorem.

Theorem 2.6. *For $n \in \mathbb{N}$ and $\alpha \in \mathbb{Q}$, one has*

$$
\widetilde{G}_{n,q}^{(\alpha)}(x) = \frac{n[2]_q}{(1-q)^{n-1} [\alpha]_q^{n-1}} \sum_{l=0}^{n-1} \binom{n-1}{l} \frac{(-1)^l q^{\alpha l x}}{1+q^{\alpha l+1}}. \tag{2.23}
$$

From (2.6), if we take $f(x) = [x]_{q^\alpha}^m = ((1-q^{\alpha x})/(1-q^\alpha))^m$, then we get

$$
q^n \int_{\mathbb{Z}_p} [x+n]_{q^\alpha}^m d\mu_{-q}(x) = (-1)^n \int_{\mathbb{Z}_p} [x]_{q^\alpha}^m d\mu_{-q}(x) + [2]_q \sum_{l=0}^{n-1} (-1)^l q^l [l]_{q^\alpha}^m. \tag{2.24}
$$

By (2.17) and (2.24), we obtain the following theorem.

Theorem 2.7. *For $n \in \mathbb{N}$, $m \in \mathbb{Z}_+ = \mathbb{N} \cup \{0\}$, and $\alpha \in \mathbb{Q}$, one has*

$$q^n \frac{\widetilde{G}^{(\alpha)}_{m+1,q}(n)}{m+1} = (-1)^n \frac{\widetilde{G}^{(\alpha)}_{m+1,q}}{m+1} + [2]_q \sum_{l=0}^{n-1} (-1)^l q^l [l]^m_{q^\alpha}. \tag{2.25}$$

We remark that if we take $n = 2s$ $(s \in \mathbb{Z}_+)$ in Theorem 2.7, then we have

$$q^{2s} \frac{\widetilde{G}^{(\alpha)}_{m+1,q}(2s)}{m+1} = \frac{\widetilde{G}^{(\alpha)}_{m+1,q}}{m+1} + [2]_q \sum_{l=0}^{2s-1} (-1)^l q^l [l]^m_{q^\alpha} \tag{2.26}$$

and if we take $n = 2s+1$ $(s \in \mathbb{Z}_+)$ in Theorem 2.7, then we have

$$q^{2s+1} \frac{\widetilde{G}^{(\alpha)}_{m+1,q}(2s+1)}{m+1} + \frac{\widetilde{G}^{(\alpha)}_{m+1,q}}{m+1} = [2]_q \sum_{l=0}^{2s} (-1)^l q^l [l]^m_{q^\alpha}. \tag{2.27}$$

From (2.27) with $s = 0$, we obtain the following corollary.

Corollary 2.8. *For $\alpha \in \mathbb{Q}$ and $m \in \mathbb{Z}_+$, one has*

$$q \frac{\widetilde{G}^{(\alpha)}_{m+1,q}(1)}{m+1} + \frac{\widetilde{G}^{(\alpha)}_{m+1,q}}{m+1} = \begin{cases} [2]_q & \text{if } m = 0, \\ 0 & \text{if } m > 0. \end{cases} \tag{2.28}$$

From (2.19), we note that

$$\begin{aligned} \frac{\widetilde{G}^{(\alpha)}_{m+1,q}(n)}{m+1} &= \sum_{l=0}^{m} \binom{m}{l} [x]^{m-l}_{q^\alpha} \frac{\widetilde{G}^{(\alpha)}_{l+1,q}}{l+1} q^{\alpha l x} \\ &= \frac{1}{m+1} \sum_{l=0}^{m} \binom{m+1}{l+1} [x]^{m-l}_{q^\alpha} \widetilde{G}^{(\alpha)}_{l+1,q} q^{\alpha l x} \\ &= \frac{1}{m+1} \sum_{l=1}^{m} \binom{m+1}{l} [x]^{m+1-l}_{q^\alpha} \widetilde{G}^{(\alpha)}_{l,q} q^{\alpha(l-1)x} \\ &= \frac{1}{q^\alpha} \frac{1}{m+1} \sum_{l=0}^{m} \binom{m+1}{l} [x]^{m+1-l}_{q^\alpha} \widetilde{G}^{(\alpha)}_{l,q} q^{\alpha l x}. \end{aligned} \tag{2.29}$$

From (2.29), we get

$$\begin{aligned} q^\alpha \widetilde{G}^{(\alpha)}_{m+1,q}(x) &= \sum_{l=0}^{m+1} \binom{m+1}{l} [x]^{m+1-l}_{q^\alpha} \widetilde{G}^{(\alpha)}_{l,q} q^{\alpha l x} \\ &= \left([x]_{q^\alpha} + q^{\alpha x} \widetilde{G}^{(\alpha)}_q \right)^{m+1}, \end{aligned} \tag{2.30}$$

with the usual convention about replacing $(\widetilde{G}_q^{(\alpha)})^n$ by $\widetilde{G}_{n,q}^{(\alpha)}$. By (2.28) and (2.30), we get

$$\frac{q^{1-\alpha}q^{\alpha}\widetilde{G}_{m+1,q}^{(\alpha)}(1)}{m}+\frac{\widetilde{G}_{m+1,q}^{(\alpha)}}{m+1}=\frac{q^{1-\alpha}\left(1+q^{\alpha}\widetilde{G}_q^{(\alpha)}\right)^{m+1}}{m+1}+\frac{\widetilde{G}_{m+1,q}^{(\alpha)}}{m+1}. \tag{2.31}$$

From (2.28) and (2.31), we obtain the following theorem.

Theorem 2.9. *For $\alpha \in \mathbb{Q}$ and $m \in \mathbb{Z}_+$, one has*

$$q^{1-\alpha}\left(1+q^{\alpha}\widetilde{G}_q^{(\alpha)}\right)^{m+1}+\widetilde{G}_{m+1,q}^{(\alpha)}=\begin{cases}[2]_q & \text{if } m=0,\\ 0 & \text{if } m>0.\end{cases} \tag{2.32}$$

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