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Research Article

The DEA and Intuitionistic Fuzzy TOPSIS Approach to Departments' Performances: A Pilot Study

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This paper processes a unification of Fuzzy TOPSIS and Data Envelopment Analysis (DEA) to select the units with most efficiency. This research is a two-stage model designed to fully rank the organizational alternatives, where each alternative has multiple inputs and outputs. First, the alternative evaluation problem is formulated by Data Envelopment Analysis (DEA) and separately formulates each pair of units. In the second stage, we use the opinion of experts to be applied into a model of group Decision-Making (DM) called the Intuitionistic Fuzzy TOPSIS (IFT) method. The results of both methods are then multiplied to obtain the results. DEA and Intuitionistic Fuzzy TOPSIS ranking do not replace the DEA classification model; rather, it furthers the analysis by providing full ranking in the DEA context for all units by aggregate individual opinions of decision makers for rating the importance of criteria and alternatives.

1. Introduction

Data Envelopment Analysis (DEA) measuring the relative efficiency of peer decision-making units (DMUs) with multiple inputs and multiple outputs was introduced by Charnes et al. [1]. This method is based on linear programming (LP), which gives it the ability to measure the decision units in a relative manner, though it has difficulties in measuring different scales and more than one scale, as well as in comparing entries or outputs that are in different units. Multi-Criteria Decision-Making (MCDM) is a modeling and methodological tool for dealing with complex engineering problem. However, the MCDM literature was entirely separate from DEA research until 1988, when Golany combined interactive, multiple-objective linear programming and DEA. Whilst the MCDM literature does not consider a complete ranking as their ultimate aim, they do discuss the use of preference information to further refine the discriminatory power of the DEA models. In this manner, the decision-makers could specify which inputs and outputs should lend greater importance to the model solution.

However, this could also be considered the weakness of this method, since additional knowledge on the part of the decision-makers is required. Golany [2], Kornbluth [3], Golany and Roll [4], and Halme et al. [5] each incorporated preferential information into the DEA models through, for example, a selection of preferred input/output targets or hypothetical DMUs. A separate set of papers reflected preferential information through limitations on the values of the weights, which can almost guarantee a complete DMU ranking [6].

DEA has been applied to DMUs in various forms, such as hospitals, cities, universities, business firms, and many others [7]. During the last decade, there have been attempts to fully rank units in the context of DEA. Cook and Kress [8], Cook et al. [9], and Green et al. [10] used subjective decision analysis. Norman and Stoker [11] asserted a step-by-step approach that uses the selected simple ratios between input and output couples. Ganley and Cubbin [12] improved the common weights, which maximizes the efficiency rates for all units. Sinuany-Stern et al. [13] ordered all units by using linear discriminated analysis that is based on the given DEA dichotomic classification. Friedman and Sinuany-Stern [14] used canonical correlation analysis (CCA/DEA) to order the units that are fundamental in common weights. Friedman and Sinuany-Stern [15] developed the discriminate analysis of ratios instead of traditional linear discriminate analysis. Also (DR/DEA) Oral et al. [16] used the cross-efficiency matrix for choosing R and D projects. There are deficiencies in all methods related to the nature of the methods themselves. Some of the deficiencies occur due to human faults, and some occur due to the presence of a large number of options.

Data Envelopment Analysis (DEA) as a popular method has been extensively used for ranking and classifying the decision-making units. DEA, a nonparametric technique, is an alternative to multivariate statistical methods when it is used for the data with multiple inputs and outputs. DEA provides researchers a wide usage opportunity since it does not need any assumptions, unlike the multivariate statistical methods, and it has a flexibility to add new restrictions to model according to researchers need.

The DEA is a method for mathematically comparing difference in DMUs' productivity based on multiple inputs and outputs. The ratio of weighted inputs and outputs produces a single measure of productivity called relative efficiency. The DMUs that have a ratio of 1 are referred to as "efficient", given the required inputs and produced outputs. The units that have a ratio less than 1 are "less efficient" relative to the most efficient units. Because the weights for the input and the output variables of DMU's are computed to maximize the ratio and then compared to a similar ratio of the best-performing DMU's, the measured productivity is also referred to as "relative efficiency" [17].

Intuitionistic fuzzy set (IFS) introduced by Atanassov [18] is an extension of the classical fuzzy set (FS), which is a suitable way to deal with vagueness. Intuitionistic fuzzy sets have been applied many areas such as medical diagnosis [19–21], decision-making problems [22–31], pattern recognition [32–37], supplier selection [38], personel selection [39], facility location selection [40], and evaluation of renewable energy [41].

The rest of the paper is organized as follows: Section 2 describes the literature review, Section 3 explains the materials and methods, Section 4 Applying the methodology: an Illustrative Problem, and finally Sections 4 and 5 contain discussion and conclusion.

2. Literature Review

DEA is a nonparametric approach that does not require any assumptions about the functional form of the production function. About 1000 articles have been written on the subject, providing numerous examples and further development of the model. In the simplest case of a unit having a single input and output, efficiency is defined as the ratio of output/input. DEA deals with units having multiple inputs and outputs that can be incorporated into an efficiency measure where the weighted sum of outputs is divided by the weighted sum of inputs [14].

The application of DEA to universities has generally focused on the efficiencies of university programs departments. The studies are by A. Bessent and W. Bessent [42], Tomkines and Green [43], Beasley [44], J. Johnes and G. Johnes [45], Sinuany-Stern et al [46], Leitner et al. [47], and Rayeni et al. [48].

A. Bessent and W. Bessent [42] used DEA in measuring the relative efficiency of education programs in a community college. Educational programs (DMUs) were assessed on such that outputs are revenue from state government, number of students completing a program, and employer satisfaction with training of students. These outputs represented significant planning objectives. Inputs included student contact hours, number of full-time equivalent instructors, square feet of facilities for each program, and direct instructional expenditures. The authors demonstrated how DEA can be used in improving program, terminating programs, initiating new programs, or discontinuing inefficient program.

Tomkins and Green [43] studied the overall efficiency of university accounting departments. They ran a series of six efficiency models of varying complexity where staff numbers were an input and student numbers an output. Results indicated that different configurations of multiple incommensurate inputs and outputs produced substantially stable efficiency score. On the other hand, Beasley studied chemistry and physics departments on productive efficiency where financial variables such as research income and expenditure were treated as inputs. Outputs consisted of undergraduate and postgraduate student numbers as well as research rating. İn a follow-up study, Beasley analyzed the same data set in an effort to determine the research and teaching efficiencies jointly, where weight restrictions were used.

J. Johnes and G. Johnes [45] explored various models in measuring the technical efficiency of economics department in terms of research outputs. They discuss the potential problems in choosing inputs and outputs. The authors also provide a good guide to interpreting efficiency scores. İt is interesting to note that both Beasley [44] and Johnes list research income as an input.

Sinuany-Stern et al. [46] examined the relative efficiency of 21 academic departments in Ben-Gurion University. Operating costs and salaries were entered as inputs, while grants, publications, graduate students, and contact hours comprised the outputs. Analysis suggested that the operating costs could be reduced in 10 departments. Furthermore, the authors tested for the sensitivity of efficiency score to deleting or combining variables. Their finding indicated that efficient departments may be rerated as inefficient as a result of changing the variable mix.

Leitner et al. [47] examined the measure efficiency in the university sector, as well as to apply DEA in the frame of Austrian university. DEA exceeds traditional methods analyzing university activities using simple ratio calculations. On the one hand, it determines the performance efficiency of university departments; on the other hand, it goes beyond this task and shows the improvement potential for each evaluated unit separately.

Rayeni et al. [48] explored the evolution of productivity of the university departments operating in the Islamic Azad University Zahedan Unit's education departments for the period between 2004 and 2009. Since the Islamic Azad University Zahedan Unit's education departments are part of the public sector where economic behavior is uncertain and there is no price information on the services produced, the Malmquist index based on DEA approach is well suited for productivity measurement where staff numbers (professors, assistant professor, lecture, and educational expert), number of registered student in the term of the academic year, number of presented units in each department by gust lectures were an input and number of graduates in the academic year, number of student passing to higher level, and research (books, published article or presented in authentic conferences and report and research projects) an output.

3. Materials and Methods

DEA deals with classifying the units into two categories, efficient and inefficient, based on two sets of multiple outputs contributing positively to the overall evaluation [12, 19]. The original DEA does not perform full ranking; it merely provides classification into two dichotomic groups: efficient and inefficient. It does not rank them; all efficient units are equally good in the pareto sense.

In this study, our model integrates two well-known models, DEA and Intuitionistic Fuzzy TOPSIS. The priorities obtained from DEA and Intuitionistic Fuzzy TOPSIS method are defined as a ten-step approach. The procedure for DEA and Intuitionistic Fuzzy TOPSIS ranking model has been given as follows.

Step 1. In the first step, determine the result of e_k from DEA method.

Measurement of the efficiency for a particular DMU is defined as the ratio of weighted sum of its output to weighted sum of its input. It is also defined as efficiency score of the DMU. For instance, the DMUs are used for the production of x_{ij} inputs and y_{rj} of outputs. $X(t \times n)$ and $Y(m \times n)$ are the amounts of the inputs and outputs, respectively.

3.1. Mathematical (Weighted Linear) Representation of the Problem

$$e_k = \max \sum_{r=1}^{t} u_r y_{rj} \tag{3.1}$$

subject to

$$\begin{gathered}
\sum_{i=1}^{m} v_i x_{ik} = 1, \\
\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \\
u_r \geq 0, \quad r = 1, \ldots, t, \\
v_i \geq 0, \quad i = 1, \ldots, m,
\end{gathered} \tag{3.2}$$

where e_k = efficiency score for DMU, y_{rj} = amount of input r for DMU j, x_{ij} = amount of input i for DMU j, u_r = weight attached to output r and v_i = weight attached to input i, n = number of DMUs, t = number of outputs, and m = number of inputs.

Step 2. Determine the weights of decision makers.

The IFT method, proposed by Boran et al. [38], is a suitable way to deal with MCDM problem in intuitionistic fuzzy environment. Assume that decision group contains l decision makers. The importance of the decision makers is considered as linguistic terms expressed in intuitionistic fuzzy numbers. Let $D_k = [\mu_k, \nu_k, \pi_k]$ be an intuitionistic fuzzy number for rating of kth decision maker. Then the weight of kth decision maker can be obtained as

$$\lambda_k = \frac{(\mu_k + \pi_k(\mu_k/(\mu_k + v_k)))}{\sum_{k=1}^{l}(\mu_k + \pi_k(\mu_k/(\mu_k + v_k)))}, \qquad \sum_{k=1}^{l}\lambda_k = 1. \tag{3.3}$$

Step 3. Construct aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers.

Let $R^{(k)} = (r_{ij}^{(k)})_{m\times n}$ be an intuitionistic fuzzy decision matrix of each decision maker. $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_l\}$ is the weight of decision maker and $\lambda_k \in [0,1]$. In group decision-making process, all the individual decision opinions need to be fused into a group opinion to construct aggregated intuitionistic fuzzy decision matrix. In order to do that, IFWA operator proposed by Xu [49] is used. $R = (r_{ij})_{m\times n}$, where

$$\begin{aligned} r_{ij} &= \mathrm{IFWA}_\lambda\left(r_{ij}{}^{(1)}, r_{ij}{}^{(2)}, \ldots, r_{ij}{}^{(l)}\right) \\ &= \lambda_1 r_{ij}{}^{(1)} \oplus \lambda_2 r_{ij}{}^{(2)} \oplus \lambda_3 r_{ij}{}^{(3)} \oplus \cdots \oplus \lambda_l r_{ij}{}^{(l)} \\ &= \left[1 - \prod_{k=1}^{l}\left(1-\mu_{ij}{}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(v_{ij}{}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_{ij}{}^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{l}\left(v_{ij}{}^{(k)}\right)^{\lambda_k}\right]. \end{aligned} \tag{3.4}$$

The aggregated intuitionistic fuzzy decision matrix is defined as follows:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m1} & r_{m1} & r_{mn} \end{bmatrix}. \tag{3.5}$$

Here $r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$ $(i = 1,2,\ldots,m;\ j = 1,2,\ldots,n)$ is an element of an aggregated intuitionistic fuzzy decision matrix.

Step 4. Determine the weights of criteria.

All criteria may not be assumed to be of equal importance. W represents a set of grades of importance. In order to obtain W, all the individual decision maker opinions for the importance of each criteria need to be fused. Let $w_j{}^{(k)} = (\mu_j{}^{(k)}, v_j{}^{(k)}, \pi_j{}^{(k)})$ be an intuitionistic fuzzy number assigned to criterion X_j by the kth decision maker. Then the weights of the criteria are calculated by using IFWA operator:

$$\begin{aligned} w_j &= \text{IFWA}_\lambda\left(w_j^{(1)}, w_j^{(2)}, \ldots, w_j^{(l)}\right) \\ &= \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \lambda_3 w_j^{(3)} \oplus \cdots \oplus \lambda_l w_j^{(l)} \\ &= \left[1 - \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(v_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{l}\left(v_j^{(k)}\right)^{\lambda_k}\right], \\ W &= \left[w_1, w_2, w_3, \ldots, w_j\right] \quad \text{Here } w_j = \left(\mu_j, v_j, \pi_j\right) \ (j = 1, 2, \ldots, n). \end{aligned} \tag{3.6}$$

Step 5. Construct aggregated weighted intuitionistic fuzzy decision matrix.

After the weights of criteria (W) and the aggregated intuitionistic fuzzy decision matrix are determined, the aggregated weighted intuitionistic fuzzy decision matrix is constructed according to the following definition [18]:

$$\begin{gathered} R' = R \otimes W = \left(\mu'_{ij}, v'_{ij}\right) = \left\{\left\langle x, \mu_{ij} \cdot \mu_j, v_{ij} + v_j - v_{ij} \cdot v_j \right\rangle\right\}, \\ \pi'_{ij} = 1 - v_{ij} - v_j - \mu_{ij} \cdot \mu_j + v_{ij} \cdot v_j. \end{gathered} \tag{3.7}$$

Then the aggregated weighted intuitionistic fuzzy decision matrix can be defined as follows:

$$R' = \begin{bmatrix} r'_{11} & r'_{12} & \cdots & r'_{1n} \\ r'_{21} & r'_{22} & \cdots & r'_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r'_{m1} & r'_{m1} & r'_{m1} & r'_{mn} \end{bmatrix}. \tag{3.8}$$

Here $r'_{ij} = (\mu'_{ij}, v'_{ij}, \pi'_{ij})$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is an element of the aggregated weighted intuitionistic fuzzy decision matrix.

Step 6. Obtain the intuitionistic fuzzy positive-ideal solution and intuitionistic fuzzy negative-ideal solution.

Let J_1 and J_2 be benefit criteria and cost criteria, respectively. A^* is intuitionistic fuzzy positive-ideal solution and A^- is intuitionistic fuzzy negative-ideal solution. Then A^* and A^- are obtained as

$$\begin{aligned} A^* &= \left(r_1'^*, r_2'^*, \ldots, r_n'^*\right), \qquad r_j'^* = \left(\mu_j'^*, v_j'^*, \pi_j'^*\right), \quad j = 1, 2, \ldots, n, \\ A^- &= \left(r_1'^-, r_2'^-, \ldots, r_n'^-\right), \qquad r_j'^- = \left(\mu_j'^-, v_j'^-, \pi_j'^-\right), \quad j = 1, 2, \ldots, n, \end{aligned} \tag{3.9}$$

where

$$\mu_j'^{*} = \left\{ \left(\max_i \left\{ \mu_{ij}' \right\} j \in J_1 \right), \left(\min_i \left\{ \mu_{ij}' \right\} j \in J_2 \right) \right\},$$

$$v_j'^{*} = \left\{ \left(\min_i \left\{ v_{ij}' \right\} j \in J_1 \right), \left(\max_i \left\{ v_{ij}' \right\} j \in J_2 \right) \right\},$$

$$\pi_j'^{*} = \left\{ \left(1 - \max_i \left\{ \mu_{ij}' \right\} - \min \left\{ v_{ij}' \right\} j \in J_1 \right), \left(1 - \min_i \left\{ \mu_{ij}' \right\} - \max_i \left\{ v_{ij}' \right\} j \in J_2 \right) \right\},$$

$$\mu_j'^{-} = \left\{ \left(\min_i \left\{ \mu_{ij}' \right\} j \in J_1 \right), \left(\max_i \left\{ \mu_{ij}' \right\} j \in J_2 \right) \right\},$$

$$v_j'^{-} = \left\{ \left(\max_i \left\{ v_{ij}' \right\} j \in J_1 \right), \left(\min_i \left\{ v_{ij}' \right\} j \in J_2 \right) \right\},$$

$$\pi_j'^{-} = \left\{ \left(1 - \min_i \left\{ \mu_{ij}' \right\} - \max \left\{ v_{ij}' \right\} j \in J_1 \right), \left(1 - \max_i \left\{ \mu_{ij}' \right\} - \min_i \left\{ v_{ij}' \right\} j \in J_2 \right) \right\}. \tag{3.10}$$

Step 7. Calculate the separation measures.

Separation between alternatives on intuitionistic fuzzy set, distance measures proposed by Atanassov [50], Szmidt and Kacprzyk [51], and, Grzegorzewski [52] including the generalizations of Hamming distance, Euclidean distance, and their normalized distance measures can be used. After selecting the distance measure, the separation measures, S_i^* and S_i^-, of each alternative from intuitionistic fuzzy positive-ideal and negative-ideal solutions, are calculated. In this paper, we use normalized Euclidean distance

$$S_i^* = \frac{1}{2} \sum_{j=1}^{n} \left[\left| \mu_{ij}' - \mu_j'^{*} \right| + \left| v_{ij}' - v_j'^{*} \right| + \left| \pi_{ij}' - \pi_j'^{*} \right| \right],$$

$$S_i^- = \frac{1}{2} \sum_{j=1}^{n} \left[\left| \mu_{ij}' - \mu_j'^{-} \right| + \left| v_{ij}' - v_j'^{-} \right| + \left| \pi_{ij}' - \pi_j'^{-} \right| \right]. \tag{3.11}$$

Step 8. Calculate the relative closeness coefficient to the intuitionistic ideal solution.

The relative closeness coefficient of an alternative A_i with respect to the intuitionistic fuzzy positive-ideal solution A^* is defined as follows:

$$C_i^* = \frac{S_i^-}{S_i^* + S_i^-}, \tag{3.12}$$

where $0 \le C_i^* \le 1$.

Step 9. Calculate the result of e_k from DEA and solution of C_i^*.

$$e_k' = e_k \otimes C_i^*. \tag{3.13}$$

Table 1: The DEA score.

DMU	Score
1	1
2	0.951
3	**1**
4	1
5	1
6	0.869
7	1
8	1
9	0.9200
10	0.904
11	0.834
12	0.950
13	1

Table 2: linguistic term for rating decision makers.

Linguistic terms	IFNs
Very important	(0.85, 0.10)
Important	(0.50, 0.20)
Medium	(0.50, 0.50)
Bad	(0.35, 0.60)
Very bad	(0.10, 0.85)

Step 10. Rank the alternatives.

After calculating the result of e'_k, alternatives are ranked.

4. Applying the Methodology: An Illustrative Problem

The suggested model demonstrated via an example of a selected department, supported by a university. Thirteen departments have been considered in our evaluation. In our study, we employ a six-input evaluation criteria and four-output evaluation criteria.

Inputs: Number of Professor Doctors, Associated Professors, Assistant Professors, and Instructors; Budget of departments; and Number of credits.

Outputs: Number of alumni (undergraduates and graduate students), Evaluation of instructors, Number of academic congeries, and Number of academic papers (SCI-SSCI-AHCI).

Step 1. Determine the result of e_k from DEA in Table 1. In Table 1, seven units are efficient.

Step 2. Determine the weights of the decision makers: the degree of the decision makers on group decision, shown in Table 2, and Linguistic terms used for the ratings of the decision makers and criteria, as Table 3, respectively.

Step 3. Construct the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers, the linguistic terms shown in Table 4.

Table 3: The importance of decision makers and their weights.

	DM1	DM2	DM3
Linguistic terms	Very important	Medium	Important
Weight	0.385	0.307	0.307

Table 4: Linguistic terms for rating the alternatives.

Linguistic terms	IFNs
Extremely good (EG)	[1.00; 0.00;0.00]
Very good (VG)	[0.85;0,05; 0.10]
Good (G)	[0.70; 0.20;0.10]
Medium bad (MB)	[0.50; 0.50;0.00]
Bad (B)	[0.40; 0.50;0.10]
Very bad (VB)	[0.25; 0.60;0.15]
Extremely bad (EB)	[0.00, 0.90,0.10]

The ratings given by the decision makers to 13 departments were shown in Table 5.

The aggregated intuitionistic fuzzy decision matrix based on the aggregation of decison makers' opinion was constructed in Table 6.

Step 4. Determine the weights of criteria, the linguistic terms shown in Table 7, and the importance of the criteria which was rated by three decision makers shown in Table 8.

Step 5. Construct the aggregated weighted Intuitionistic Fuzzy Decision Matrix shown in Table 9.

Finally calculate the relative closeness coefficient to the intuitionistic ideal solution shown in Table 10. Result of $e'_k = e_k \otimes C_i^*$ and rank the alternatives shown in Table 11.

5. Result and Discussion

As presented in Table 10, the third column shows the scores of the thirteen departments. The result score is always the bigger the better. As visible in Table 10, department 3 has the largest score due to its highest efficiency and performance. Department 7 has the smallest score of the thirteen departments and is ranked in the last place. The relevant results can be seen in Table 10. Obviously, the best selection is department 3. Table 10 lists the results of both models, ordered according to DEA and IFS ranks. It is several units is there no compatibility between the two models: for example, departments 7 and 8, which are efficient in DEA but are ranked by DEA and IFS worse than the inefficient department 9. Because it contains a vague perception of decision makers' opinions.

Although there is no perfect compatibility between DEA and DEA and IFS in the general case, empirically, we found many examples of complete match units. Applying the Mann-Whitney test to the above example, we found that the two methods are compatible with a P value.

Table 5: Ratings of the departments provided by three decision makers (DMs).

DMU	Criteria	DM1	DM2	DM3	DMU	Criteria	DM1	DM2	DM3
D1	C1	VG	G	G	D8	C1	VG	VG	VG
	C2	VG	VG	G		C2	VG	VG	G
	C3	G	MG	G		C3	MG	MG	MG
	C4	G	MG	VG		C4	MG	MG	MG
	C5	MG	MG	MG		C5	G	MG	G
	C6	VG	VG	VG		C6	VG	VG	VG
D2	C1	VG	VG	VG	D9	C1	VG	G	VG
	C2	G	MG	G		C2	MG	G	MG
	C3	MG	G	VG		C3	G	G	G
	C4	G	G	G		C4	MG	MG	MG
	C5	MG	VG	G		C5	VG	G	G
	C6	VG	VG	VG		C6	VG	VG	VG
D3	C1	VG	VG	VG	D10	C1	VG	VG	VG
	C2	VG	VG	VG		C2	G	VG	G
	C3	B	MG	MG		C3	G	G	G
	C4	VG	VG	VG		C4	G	MG	MG
	C5	G	VG	G		C5	G	G	G
	C6	G	VG	G		C6	VG	VG	VG
D4	C1	VG	MG	G	D11	C1	G	VG	VG
	C2	G	G	VG		C2	VG	VG	VG
	C3	B	MG	MG		C3	VG	G	G
	C4	G	G	G		C4	G	G	G
	C5	B	B	MG		C5	MG	MG	MG
	C6	VG	VG	VG		C6	VG	VG	VG
D5	C1	VG	VG	VG	D12	C1	G	VG	VG
	C2	VG	G	G		C2	G	G	G
	C3	G	G	G		C3	MG	MG	G
	C4	G	G	G		C4	G	G	G
	C5	G	MG	MG		C5	MG	MG	MG
	C6	VG	VG	VG		C6	VG	G	VG
D6	C1	VG	VG	VG	D13	C1	VG	VG	G
	C2	VG	VG	VG		C2	VG	VG	VG
	C3	B	MG	MG		C3	VG	G	G
	C4	G	G	VG		C4	G	G	G
	C5	B	B	MG		C5	MG	MG	MG
	C6	G	G	MG		C6	G	G	G
D7	C1	VG	VG	VG					
	C2	MG	MG	MG					
	C3	MG	MG	MG					
	C4	G	G	MG					
	C5	G	G	G					
	C6	VG	VG	VG					

Table 6: Aggregate intuitionistic fuzzy decision matrix.

DMU	Criteria		DMU	Criteria	
D1	C1	(0.770,0.118,0.112)	D8	C1	(0.770,0.118,0.112)
	C2	(0.814,0.077,0.109)		C2	(0.831,0.077,0.093)
	C3	(0.649,0.265,0.086)		C3	(0.500,0.500,0.000)
	C4	(0.716,0.173,0.111)		C4	(0.500,0.500,0.000)
	C5	(0.500,0.285,0.215)		C5	(0.649,0.265,0.086)
	C6	(0.850,0.050,0.100)		C6	(0.770,0.118,0.112)
D2	C1	(0.850,0.050,0.100)	D9	C1	(0.814,0.068,0.118)
	C2	(0.649,0.265,0.086)		C2	(0.572,0.378,0.050)
	C3	(0.704,0.186,0.109)		C3	(0.700,0.200,0.100)
	C4	(0.700,0.200,0.100)		C4	(0.500,0.500,0.000)
	C5	(0.704,0.186,0.109)		C5	(0.770,0.118,0.112)
	C6	(0.850,0.050,0.100)		C6	(0.850,0.051,0.100)
D3	C1	(0.850,0.050,0.100)	D10	C1	(0.850,0.050,0.100)
	C2	(0.850,0.050,0.100)		C2	(0.757,0.131,0.112)
	C3	(0.463,0.500,0.036)		C3	(0.700,0.200,0.100)
	C4	(0.850,0.500,0.100)		C4	(0.589,0.352,0.060)
	C5	(0.757,0.131,0.112)		C5	(0.700,0.200,0.100)
	C6	(0.850,0.050,0.100)		C6	(0.850,0.050,0.100)
D4	C1	(0.731,0.156,0.114)	D11	C1	(0.804,0.086,0.111)
	C2	(0.757,0.139,0.112)		C2	(0.850,0.051,0.100)
	C3	(0.463,0.500,0.037)		C3	(0.770,0.118,0.112)
	C4	(0.700,0.200,0.100)		C4	(0.700,0.200,0.100)
	C5	(0.432,0.500,0.067)		C5	(0.500,0.500,0.000)
	C6	(0.850,0.050,0.100)		C6	(0.850,0.050,0.100)
D5	C1	(0.850,0.050,0.100)	D12	C1	(0.804,0.086,0.111)
	C2	(0.770,0.118,0.112)		C2	(0.700,0.200,0.100)
	C3	(0.700,0.200,0.100)		C3	(0.572,0.378,0.050)
	C4	(0.700,0.200,0.100)		C4	(0.700,0.200,0.100)
	C5	(0.589,0.352,0.060)		C5	(0.500,0.500,0.000)
	C6	(0.850,0.050,0.100)		C6	(0.814,0.068,0.118)
D6	C1	(0.850,0.050,0.100)	D13	C1	(0.814,0.078,0.109)
	C2	(0.850,0.050,0.100)		C2	(0.850,0.051,0.100)
	C3	(0.463,0.500,0.036)		C3	(0.770,0.118,0.112)
	C4	(0.716,0.131,0.153)		C4	(0.700,0.200,0.100)
	C5	(0.432,0.500,0.067)		C5	(0.500,0.500,0.000)
	C6	(0.649,0.265,0.086)		C6	(0.700,0.200,0.100)
D7	C1	(0.770,0.118,0.112)			
	C2	(0.500,0.500,0.000)			
	C3	(0.500,0.500,0.000)			
	C4	(0.649,0.265,0.086)			
	C5	(0.700,0.200,0.100)			
	C6	(0.770,0.118,0.112)			

Table 7: Linguistic terms for rating the importance of criteria.

Linguistic terms	IFNs
Very good (VG)	[0.90;0,00]
Good (G)	[0.80; 0.10]
Medium bad (MB)	[0.70; 0.20]
Bad (B)	[0.50; 0.50]
Very bad (VB)	[0.30; 0.50]
Extremely bad (EB)	[0.20; 0.70]

Table 8: Importance weight of the criteria.

Criteria	DM1	DM2	DM3	Criteria	DM1	DM2	DM3
C1	G	MB	MB	C4	G	G	MB
C2	VG	VG	VG	D5	G	MB	B
C3	B	G	MB	C6	VG	VG	VG

Table 9: Aggregated weighted intuitionistic fuzzy decision matrix.

Alternative	Criteria		Alternative	Criteria	
D1	C1	(0.572,0.253,0.175)	D8	C1	(0.572,0.341,0.087)
	C2	(0.732,0.077,0.191)		C2	(0.744,0.077,0.176)
	C3	(0.439,0.439,0.122)		C3	(0.338,0.618,0.044)
	C4	(0.553,0.276,0.171)		C4	(0.386,0.562,0.052)
	C5	(0.349,0.430,0.221)		C5	(0.454,0.415,0.131)
	C6	(0.764,0.050,0.186)		C6	(0.692,0.118,0.190)
D2	C1	(0.631,0.195,0.174)	D9	C1	(0.605,0.210,0.185)
	C2	(0.583,0.265,0.152)		C2	(0.514,0.378,0.108)
	C3	(0.477,0.378,0.145)		C3	(0.474,0.389,0.137)
	C4	(0.541,0.299,0.160)		C4	(0.386,0.562,0.052)
	C5	(0.492,0.351,0.157)		C5	(0.538,0.297,0.165)
	C6	(0.764,0.050,0.186)		C6	(0.764,0.051,0.185)
D3	C1	(0.631,0.195,0.174)	D10	C1	(0.631,0.197,0.172)
	C2	(0.764,0.050,0.186)		C2	(0.681,0.131,0.188)
	C3	(0.314,0.618,0.068)		C3	(0.474,0.389,0.137)
	C4	(0.657,0.168,0.175)		C4	(0.455,0.432,0.113)
	C5	(0.529,0.307,0.164)		C5	(0.489,0.363,0.148)
	C6	(0.764,0.050,0.186)		C6	(0.764,0.050,0.186)
D4	C1	(0.543,0.285,0.172)	D11	C1	(0.597,0.255,0.178)
	C2	(0.681,0.139,0.180)		C2	(0.764,0.051,0.185)
	C3	(0.314,0.618,0.068)		C3	(0.521,0.326,0.153)
	C4	(0.541,0.299,0.160)		C4	(0.541,0.299,0.160)
	C5	(0.302,0.602,0.096)		C5	(0.349,0.602,0.049)
	C6	(0.764,0.050,0.186)		C6	(0.764,0.050,0.186)
D5	C1	(0.631,0.367,0.002)	D12	C1	(0.597,0.225,0.178)
	C2	(0.692,0.118,0.190)		C2	(0.629,0.200,0.171)
	C3	(0.474,0.389,0.137)		C3	(0.387,0.524,0.089)
	C4	(0.541,0.299,0.160)		C4	(0.541,0.299,0.160)
	C5	(0.412,0.483,0.105)		C5	(0.349,0.602,0.049)
	C6	(0.764,0.050,0.186)		C6	(0.732,0.068,0.200)
D6	C1	(0.631,0.195,0.174)	D13	C1	(0.605,0.218,0.177)
	C2	(0.764,0.050,0.186)		C2	(0.764,0.051,0.186)
	C3	(0.314,0.618,0.068)		C3	(0.521,0.326,0.153)
	C4	(0.553,0.239,0.208)		C4	(0.541,0.299,0.160)
	C5	(0.302,0.602,0.096)		C5	(0.349,0.602,0.049)
	C6	(0.583,0.265,0.152)		C6	(0.629,0.200,0.171)
D7	C1	(0.572,0.253,0.175)			
	C2	(0.449,0.500,0.051)			
	C3	(0.338,0.618,0.044)			
	C4	(0.501,0.356,0.143)			
	C5	(0.489,0.363,0.148)			
	C6	(0.692,0.118,0.190)			

Table 10: Separation measurement and the relative closeness coefficient of each department.

DMU	S^*	S^-	C_i^*
D1	0.562	1.518	0.730
D2	0.513	1.350	0.750
D3	0.363	1.500	0.805
D4	0.968	0.895	0.480
D5	0.685	1.322	0.659
D6	0.976	0.919	0.485
D7	1.563	0.654	0.295
D8	1.110	0.810	0.422
D9	0.849	1.003	0.542
D10	0.539	1.326	0.711
D11	0.531	1.383	0.723
D12	0.911	1.017	0.527
D13	0.674	1.240	0.648

Table 11: The DEA and intuitionistic fuzzy TOPSIS ranking score.

DMU	DEA	DEA-IFS
D1	1	0.730
D2	0.951	0.689
D3	**1**	**0.805**
D4	1	0.480
D5	1	0.659
D6	0.869	0.421
D7	1	**0.295**
D8	1	0.422
D9	0.9200	0.499
D10	0.904	0.642
D11	0.834	0.603
D12	0.950	0.500
D13	1	0.648

6. Conclusion

In this paper, we have demonstrated a simple and easy-to-use method for department comparison via DEA. Furthermore, we integrated IFS in DEA to generate a more feasible DEA result. Various types of data were adopted in DEA without any modification of the DEA formula. We have presented an effective model for rank scaling of the units with multiple inputs and multiple outputs using both DEA and IFS.

The DEA and IFS method combines the best of both models by avoiding the pitfalls of each. IFS are designed for subjective evaluation of a set of alternatives based on multiple criteria organized in a hierarchical structure. The IFS is a suitable way to deal with uncertainty. In the evaluation process, the ratings of each alternative, which were given with Intuitionistic fuzzy information, were represented as IFNs. The IFWA operator was used to aggregate the rating of decision makers. The intuitionistic fuzzy TOPSIS method is a suitable method for MCDM because it contains a vague perception of decision makers' opinions. It is important to note that DEA and IFS do not replace DEA, but rather, it provides further analysis of DEA to full ranking the units, within utilized to aggregated individual opinions of decision makers for rating the importance of criteria and alternatives. Therefore, in the future, DEA and IFS models can be used to problems such as health systems, project selection, and many other areas.

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