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Research Article

New Traveling Wave Solutions of the Higher Dimensional Nonlinear Partial Differential Equation by the Exp-Function Method

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We construct new analytical solutions of the (3 + 1)-dimensional modified KdV-Zakharov-Kuznetsev equation by the Exp-function method. Plentiful exact traveling wave solutions with arbitrary parameters are effectively obtained by the method. The obtained results show that the Exp-function method is effective and straightforward mathematical tool for searching analytical solutions with arbitrary parameters of higher-dimensional nonlinear partial differential equation.

1. Introduction

Nonlinear partial differential equations (NLPDEs) play a prominent role in different branches of the applied sciences. In recent time, many researchers investigated exact traveling wave solutions of NLPDEs which play a crucial role to reveal the insight of complex physical phenomena. In the past several decades, a variety of effective and powerful methods, such as variational iteration method [1–3], tanh-coth method [4], homotopy perturbation method [5–7], Fan subequation method [8], projective Riccati equation method [9], differential transform method [10], direct algebraic method [11], first integral method [12], Hirota's bilinear method [13], modified extended direct algebraic method [14], extended tanh method [15], Backlund transformation [16], bifurcation method [17], Cole-Hopf transformation method [18], sech-tanh method [19], (G'/G)-expansion method [20–22], modified (G'/G)-expansion method [23], multiwave method [24], extended (G'/G)-expansion method [25, 26], and others [27–33] were used to seek exact traveling wave solutions of the nonlinear evolution equations (NLEEs).

Recently, He and Wu [34] have presented a novel method called the Exp-function method for searching traveling wave solutions of the nonlinear evolution equations arising in mathematical physics. The Exp-function method is widely used to many kinds of NLPDEs, such as good Boussinesq equations [35], nonlinear differential equations [36], higher-order boundary value problems [37], nonlinear problems [38], Calogero-Degasperis-Fokas equation [39], nonlinear reaction-diffusion equations [40], 2D Bratu type equation [41], nonlinear lattice differential equations [42], generalized-Zakharov equations [43], (3 + 1)-dimensional Jimbo-Miwa equation [44], modified Zakharov-Kuznetsov equation [45], Brusselator reaction diffusion model [46], nonlinear heat equation [47], and the other important NLPDEs [48–51].

In this article, we apply the Exp-function method [34] to obtain the analytical solutions of the nonlinear partial differential equation, namely, (3 + 1)-dimensional modified KdV-Zakharov-Kuznetsov equation.

2. Description of the Exp-Function Method

Consider the general nonlinear partial differential equation

$$P(u, u_t, u_x, u_y, u_z, u_{tt}, u_{xt}, u_{xx}, u_{xy}, u_{yy}, u_{yt}, u_{zz}, u_{zt}, u_{zx}, u_{zy}, \ldots) = 0. \tag{2.1}$$

The main steps of the Exp-function method [34] are as follows.

Step 1. Consider a complex variable as

$$u(x, y, z, t) = u(\eta), \qquad \eta = x + y + z - Vt. \tag{2.2}$$

Now using (2.2), (2.1) converts to a nonlinear ordinary differential equation for $u(\eta)$

$$Q(u, u', u'', u''', \ldots) = 0, \tag{2.3}$$

where primes denote the ordinary derivative with respect to η.

Step 2. We assume that the traveling wave solution of (2.3) can be expressed in the form [34]

$$u(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)} = \frac{a_{-c}\exp(-c\eta) + \cdots + a_d \exp(d\eta)}{b_{-p}\exp(-p\eta) + \cdots + b_q \exp(q\eta)}, \tag{2.4}$$

where c, d, p, and q are positive integers to be determined later, and a_n and b_m are unknown constants. Equation (2.4) can be rewritten in the following equivalent form:

$$u(\eta) = \frac{a_c \exp(c\eta) + \cdots + a_{-d}\exp(-d\eta)}{b_p \exp(p\eta) + \cdots + b_{-q}\exp(-q\eta)}. \tag{2.5}$$

Step 3. In order to determine the values of c and p, we balance the highest order linear term with the highest order nonlinear term, and, determining the values of d and q, we balance

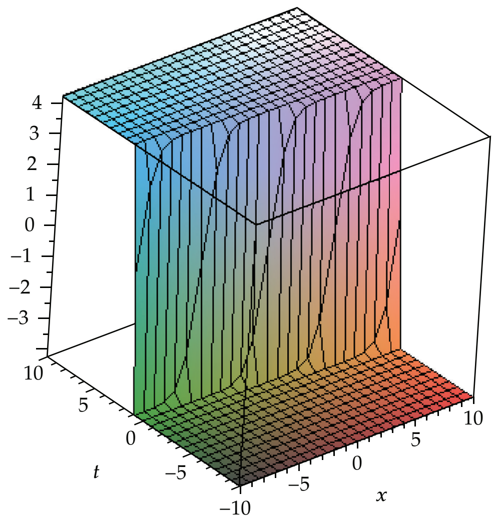

Figure 1: Periodic solution for $\alpha = -1$, $y = 0$ and $z = 0$.

the lowest order linear term with the lowest order nonlinear term in (2.3). Thus, we obtain the values of $c,\ d,\ p$, and q.

Step 4. Substituting the values of $c,\ d,\ p$, and q into (2.5), and then substituted (2.5) into (2.3) and simplifying, we obtain

$$\sum_i C_i \exp(\pm i\eta) = 0, \quad i = 0, 1, 2, 3, \ldots. \tag{2.6}$$

Then each coefficient $C_i = 0$ is to set, yields a system of algebraic equations for $a_c{}'s$ and $b_p{}'s$.

Step 5. We assume that the unknown $a_c{}'s$ and $b_p{}'s$ can be determined by solving the system of algebraic equations obtained in Step 4. Putting these values into (2.5), we obtain exact traveling wave solutions of the (2.1).

3. Application of the Method

In this section, we apply the method to construct the traveling wave solutions of the (3 + 1)-dimensional modified KdV-Zakharov-Kuznetsev equation. The obtained solutions will be displayed in Figures 1, 2, 3, 4, 5, and 6 by using the software Maple 13.

We consider the (3 + 1)-dimensional modified KdV-Zakharov-Kuznetsev equation

$$u_t + \alpha u^2 u_x + u_{xxx} + u_{xyy} + u_{xzz} = 0, \tag{3.1}$$

where α is a nonzero constant.

Zayed [52] solved (3.1) using the (G'/G)-expansion method. Later, in article [53], he solved same equation by the generalized (G'/G)-expansion method.

Here, we will solve this equation by the Exp-function method.

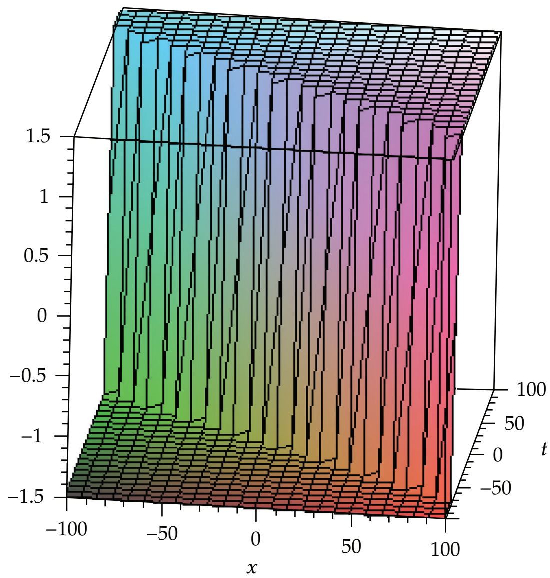

Figure 2: Periodic solution for $\beta = 2$, $y = 0$ and $z = 0$.

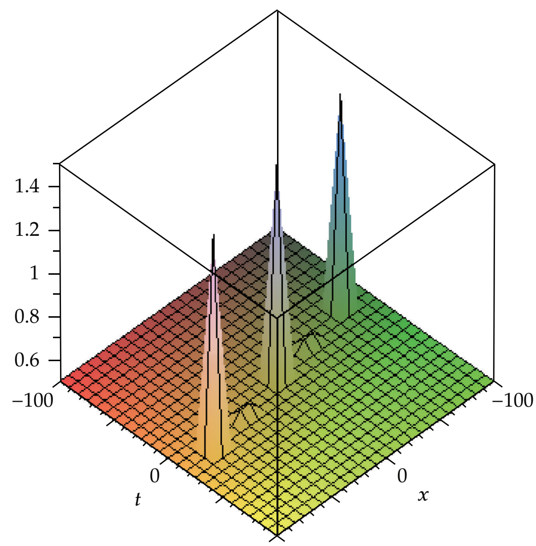

Figure 3: Solitons solution for $y = 0$ and $z = 0$.

Now, we use the transformation (2.2) into (3.1), which yields

$$-Vu' + \alpha u^2 u' + 3u''' = 0, \tag{3.2}$$

where primes denote the derivatives with respect to η.

According to Step 2, the solution of (3.2) can be written in the form of (2.5). To determine the values of c and p, according to Step 3, we balance the highest order linear term

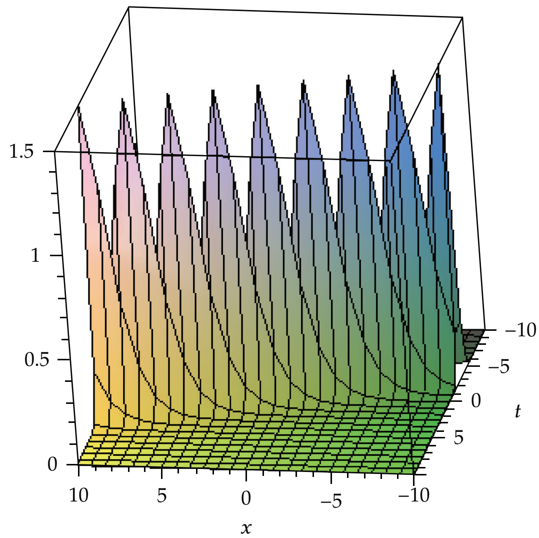

Figure 4: Solitons solution for $y = 0$ and $z = 0$.

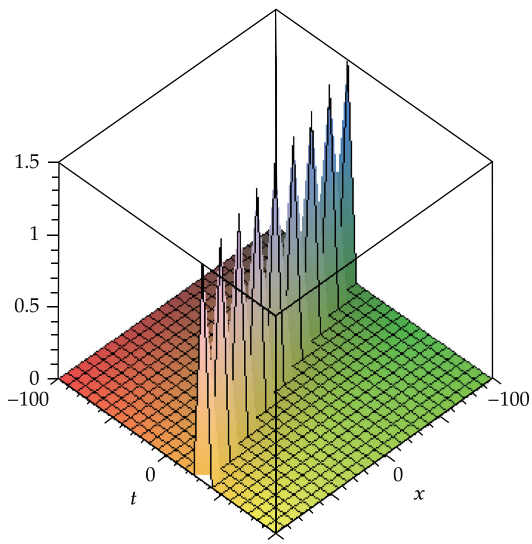

Figure 5: Solitons solution for $y = 0$ and $z = 0$.

of u''' with the highest order nonlinear term of u^2u' in (3.2), that is, u''' and u^2u'. Therefore, we have the following:

$$u''' = \frac{c_1 \exp[(3p + c)\eta] + \cdots}{c_2 \exp[4p\eta] + \cdots},$$
$$u^2u' = \frac{c_3 \exp[(p + 3c)\eta] + \cdots}{c_4 \exp[4p\eta] + \cdots}, \tag{3.3}$$

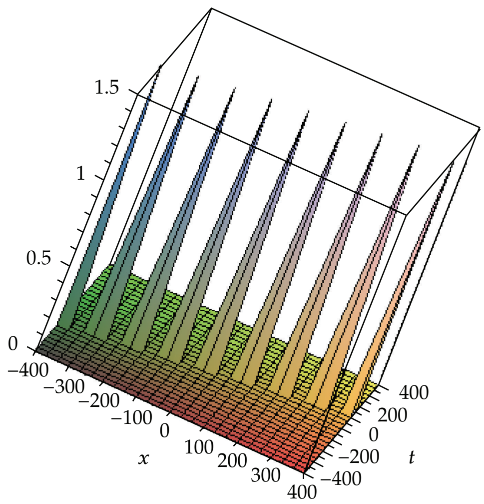

Figure 6: Solitons solution for $y = 0$ and $z = 0$.

where c_j are coefficients only for simplicity; from (3.3), we obtain that

$$3p + c = p + 3c, \quad \text{which leads } p = c. \tag{3.4}$$

To determine the values of d and q, we balance the lowest order linear term of u''' with the lowest order nonlinear term of u^2u' in (3.2). We have

$$\begin{aligned} u''' &= \frac{\cdots + d_1 \exp[-(d-q)\eta]}{\cdots + d_2 \exp[-\,4q\eta]}, \\ u^2u' &= \frac{\cdots + d_3 \exp[-3(d-q)\eta]}{\cdots + d_4 \exp[-4q\eta]}, \end{aligned} \tag{3.5}$$

where d_j are determined coefficients only for simplicity; from (3.5), we obtain

$$-(d-q) = -3(d-q), \quad \text{which leads } q = d. \tag{3.6}$$

Any real values can be considered for c and d, since they are free parameters. But the final solutions of (3.1) do not depend upon the choice of c and d.

Case 1. We set $p = c = 1$ and $q = d = 1$.

For this case, the trial solution (2.5) reduces to

$$u(\eta) = \frac{a_1 e^{\eta} + a_0 + a_{-1} e^{-\eta}}{b_1 e^{\eta} + b_0 + b_{-1} e^{-\eta}}. \tag{3.7}$$

Since, $b_1 \neq 0$, (3.7) can be simplified

$$u(\eta) = \frac{a_1 e^{\eta} + a_0 + a_{-1} e^{-\eta}}{e^{\eta} + b_0 + b_{-1} e^{-\eta}}. \tag{3.8}$$

By substituting (3.8) into (3.2) and equating the coefficients of $\exp(\pm\, n\eta)$, $n = 0, 1, 2, 3, \ldots$, with the aid of Maple 13, we obtain a set of algebraic equations in terms of $a_{-1}, a_0, a_1, b_{-1}, b_0$, and V

$$\frac{1}{A}\left(C_3 e^{3\eta} + C_2 e^{2\eta} + C_1 e^{\eta} + C_0 + C_{-1} e^{-\eta} + C_{-2} e^{-2\eta} + C_{-3} e^{-3\eta}\right) = 0. \tag{3.9}$$

And, setting each coefficient of $\exp(\pm\, n\eta)$, $n = 0, 1, 2, 3, \ldots$, to zero, we obtain

$$C_3 = 0, \quad C_2 = 0, \quad C_1 = 0, \quad C_0 = 0, \quad C_{-1} = 0, \quad C_{-2} = 0, \quad C_{-3} = 0. \tag{3.10}$$

For determining unknowns, we solve the obtained system of algebraic (3.10) with the aid of Maple 13, and we obtain four different sets of solutions.

Set 1. We obtain that

$$b_{-1} = b_{-1}, \quad a_{-1} = \mp\frac{6b_{-1}}{\sqrt{-2\alpha}}, \quad a_0 = 0, \quad a_1 = \pm\frac{6}{\sqrt{-2\alpha}}, \quad b_0 = 0, \quad V = -\,6, \tag{3.11}$$

where b_{-1} is free parameter.

Set 2. We obtain that

$$\begin{gathered} a_0 = a_0, \quad b_0 = b_0, \quad a_{-1} = \mp\frac{1}{12\sqrt{-2\alpha}}\left(2\alpha a_0^2 + 9b_0^2\right), \\ a_1 = \pm\frac{3}{\sqrt{-2\alpha}}, \quad b_{-1} = \frac{1}{18}\alpha a_0^2 + \frac{1}{4}b_0^2, \quad V = -\,\frac{3}{2}, \end{gathered} \tag{3.12}$$

where a_0 and b_0 are free parameters.

Set 3. We obtain that

$$\begin{gathered} a_1 = a_1, \quad b_0 = b_0, \quad a_{-1} = \frac{b_0^2\left(2\alpha a_1^2 + 9\right)}{8\alpha a_1}, \\ a_0 = \frac{b_0\left(\alpha a_1^2 + 9\right)}{\alpha a_1}, \quad b_{-1} = \frac{b_0^2\left(2\alpha a_1^2 + 9\right)}{8\alpha a_1^2}, \quad V = 3 + \alpha a_1^2, \end{gathered} \tag{3.13}$$

where a_1 and b_0 are free parameters.

Set 4. We obtain that

$$a_0 = a_0, \qquad a_{-1} = 0, \qquad a_1 = 0, \qquad b_{-1} = \frac{1}{72}\alpha a_0^2, \qquad b_0 = 0, \qquad V = 3, \tag{3.14}$$

where a_0 is free parameter.

Now, substituting (3.11) into (3.8), we obtain traveling wave solution

$$u(\eta) = \frac{\pm 6\ e^{\eta} \mp 6b_{-1}e^{-\eta}}{\sqrt{-2\alpha}\ (e^{\eta} + b_{-1}e^{-\eta})}. \tag{3.15}$$

Equation (3.15) can be simplified as

$$u(\eta) = \frac{\pm 6}{\sqrt{-2\alpha}}\left[1 - \frac{2b_{-1}(\cosh\eta - \sinh\eta)}{(1+b_{-1})\cosh\eta + (1-b_{-1})\sinh\eta}\right], \tag{3.16}$$

where $\eta = x + y + z + 6t$.

If $b_{-1} = 1$ from (3.16), we obtain

$$u(\eta) = \frac{\pm 6i}{\sqrt{2\alpha}}\tanh\eta. \tag{3.17}$$

Substituting (3.12) into (3.8) and simplifying, we get traveling wave solution

$$u(\eta) = \frac{\pm 3}{\sqrt{-2\alpha}}\left[1 + \frac{12\left(\pm a_0\sqrt{-2\alpha} + 3b_0\right) - 2\left(2\alpha a_0^2 + 9b_0^2\right)(\cosh\eta - \sinh\eta)}{\left(36 + 2\alpha a_0^2 + 9b_0^2\right)\cosh\eta + \left(36 - 2\alpha a_0^2 - 9b_0^2\right)\sinh\eta + 36b_0}\right], \tag{3.18}$$

where $\eta = x + y + z + (3/2)t$.

If α is negative, that is, $\alpha = -\beta$, $\beta > 0$, $b_0 = 2$ and $a_0 = 0$, then from (3.18), we obtain

$$u(\eta) = \frac{\pm 3}{\sqrt{2\beta}}\tanh\frac{\eta}{2}. \tag{3.19}$$

Substituting (3.13) into (3.8) and simplifying, we obtain

$$u(\eta) = a_1\left[1 + \frac{72b_0}{\left\{8\alpha a_1^2 + b_0^2\left(2\alpha a_1^2 + 9\right)\right\}\cosh\eta + \left\{8\alpha a_1^2 - b_0^2\left(2\alpha a_1^2 + 9\right)\right\}\sinh\eta + 8\alpha a_1^2 b_0}\right], \tag{3.20}$$

where $\eta = x + y + z - (3 + \alpha a_1^2)t$.

If $b_0 = 1$, $\alpha = 6$, and $a_1 = 1/2$, (3.20) becomes

$$u(\eta) = \frac{1}{2} + \frac{3}{1 + 2\cosh\eta}. \tag{3.21}$$

Substituting (3.14) into (3.8) and simplifying, we obtain

$$u(\eta) = \frac{72a_0}{(72 + \alpha a_0^2)\cosh\eta + (72 - \alpha a_0^2)\sinh\eta}, \tag{3.22}$$

where $\eta = x + y + z - 3t$.

If $a_0 = 3$ and $\alpha = 8$, (3.22) becomes

$$u(\eta) = \frac{3}{2}\sec h\eta. \tag{3.23}$$

Case 2. We set $p = c = 2$ and $q = d = 1$.

For this case, the trial solution (2.5) reduces to

$$u(\eta) = \frac{a_2 e^{2\eta} + a_1 e^{\eta} + a_0 + a_{-1}e^{-\eta}}{b_2 e^{2\eta} + b_1 e^{\eta} + b_0 + b_{-1}e^{-\eta}}. \tag{3.24}$$

Since, there are some free parameters in (3.24), for simplicity, we may consider that $b_2 = 1$ and $b_{-1} = 0$. Then the solution (3.24) is simplified as

$$u(\eta) = \frac{a_2 e^{2\eta} + a_1 e^{\eta} + a_0 + a_{-1}e^{-\eta}}{e^{2\eta} + b_1 e^{\eta} + b_0}. \tag{3.25}$$

Performing the same procedure as described in Case 1, we obtain four sets of solutions.

Set 1. We obtain that

$$b_0 = b_0, \quad a_{-1} = 0, \quad a_0 = \mp\frac{6b_0}{\sqrt{-2\alpha}}, \quad a_1 = 0, \quad a_2 = \pm\frac{6}{\sqrt{-2\alpha}}, \quad b_1 = 0, \quad V = -6, \tag{3.26}$$

where b_0 is free parameter.

Set 2. We obtain that

$$\begin{gathered} a_1 = a_1, \quad b_1 = b_1, \quad a_{-1} = 0, \quad a_0 = \mp\frac{1}{12\sqrt{-2\alpha}}\left(2\alpha a_1^2 + 9b_1^2\right), \\ a_2 = \pm\frac{3}{\sqrt{-2\alpha}}, \quad b_0 = \frac{1}{18}\alpha a_1^2 + \frac{1}{4}b_1^2, \quad V = \frac{-3}{2}, \end{gathered} \tag{3.27}$$

where a_1 and b_1 are free parameters.

Set 3. We obtain that

$$a_2 = a_2, \quad b_1 = b_1, \quad a_{-1} = 0, \quad a_0 = \frac{b_1^2(2\alpha a_2^2 + 9)}{8\alpha a_2},$$
$$a_1 = \frac{b_1(\alpha a_2^2 + 9)}{\alpha a_2}, \quad b_0 = \frac{b_1^2(2\alpha a_2^2 + 9)}{8\alpha a_2^2}, \quad V = \alpha a_2^2 + 3, \tag{3.28}$$

where a_2 and b_1 are free parameters.

Set 4. We obtain that

$$a_1 = a_1, \quad a_{-1} = 0, \quad a_0 = 0, \quad a_2 = 0, \quad b_0 = \frac{\alpha a_1^2}{72}, \quad b_1 = 0, \quad V = 3, \tag{3.29}$$

where a_1 is a free parameter.

Using (3.26) into (3.25) and simplifying, we obtain that

$$u(\eta) = \frac{\pm 6}{\sqrt{-2\alpha}}\left[1 - \frac{2b_0(\cosh\eta - \sinh\eta)}{(1+b_0)\cosh\eta + (1-b_0)\sinh\eta}\right]. \tag{3.30}$$

If $b_0 = 1$, from (3.30), we obtain that

$$u(\eta) = \frac{\pm 6i}{\sqrt{2\alpha}}\tanh\eta, \tag{3.31}$$

where $\eta = x + y + z + 6t$.

Substituting (3.27) into (3.25) and simplifying, we obtain that

$$u(\eta) = \frac{\pm 3}{\sqrt{-2\alpha}}\left[1 + \frac{12\left(\pm a_1\sqrt{-2\alpha} + 3b_1\right) - 2(2\alpha a_1^2 + 9b_1^2)(\cosh\eta - \sinh\eta)}{(36 + 2\alpha a_1^2 + 9b_1^2)\cosh\eta + (36 - 2\alpha a_1^2 - 9b_1^2)\sinh\eta + 36b_1}\right]. \tag{3.32}$$

If α is negative, that is, $\alpha = -\beta$, $\beta > 0$, $b_1 = 2$ and $a_1 = 0$, (3.32) can be simplified as

$$u(\eta) = \frac{\pm 3}{\sqrt{2\beta}}\tanh\frac{\eta}{2}, \tag{3.33}$$

where $\eta = x + y + z + (3/2)t$.

Substituting (3.28) into (3.25) and simplifying, we obtain that

$$u(\eta) = a_2\left[1 + \frac{72b_1}{\{8\alpha a_2^2 + b_1^2(2\alpha a_2^2 + 9)\}\cosh\eta + \{8\alpha a_2^2 - b_1^2(2\alpha a_2^2 + 9)\}\sinh\eta + 8\alpha a_2^2 b_1}\right]. \tag{3.34}$$

If $b_1 = 1,\ \alpha = 6$, and $a_2 = 1/2$, (3.34) becomes

$$u(\eta) = \frac{1}{2} + \frac{3}{1 + 2\cosh\,\eta}, \tag{3.35}$$

where $\eta = x + y + z - (3 + \alpha\ a_2^2)t$.

Using (3.29) into (3.25) and simplifying, we obtain that

$$u(\eta) = \frac{72a_1}{(72 + \alpha a_1^2)\cosh\,\eta + (72 - \alpha a_1^2)\sinh\,\eta}. \tag{3.36}$$

If $a_1 = 3$, and $\alpha = 8$, (3.36) becomes

$$u(\eta) = \frac{3}{2}\sec h\eta, \tag{3.37}$$

where $\eta = x + y + z - 3t$.

Case 3. We set $p = c = 2$ and $q = d = 2$.

For this case, the trial solution (2.5) reduces to

$$u(\eta) = \frac{a_2e^{2\eta} + a_1e^{\eta} + a_0 + a_{-1}e^{-\eta} + a_{-2}e^{-2\eta}}{b_2e^{2\eta} + b_1e^{\eta} + b_0 + b_{-1}e^{-\eta} + b_{-2}e^{-2\eta}}. \tag{3.38}$$

Since, there are some free parameters in (3.38), we may consider $b_2 = 1,\ a_{-2} = 0,\ b_{-2} = 0$, and $b_{-1} = 0$ so that the (3.38) reduces to the (3.25). This indicates that the Case 3 is equivalent to the Case 2. Equation (3.38) can be rewritten as

$$u(\eta) = \frac{a_2e^{\eta} + a_1 + a_0e^{-\eta} + a_{-1}e^{-2\eta} + a_{-2}e^{-3\eta}}{b_2e^{\eta} + b_1 + b_0e^{-\eta} + b_{-1}e^{-2\eta} + b_{-2}e^{-3\eta}}. \tag{3.39}$$

If we put $a_{-2} = 0,\ \ a_{-1} = 0,\ \ b_2 = 1,\ \ b_{-2} = 0,$ and $b_{-1} = 0$ into (3.39), we obtain the solution form as (3.8). This implies that the Case 3 is equivalent to the Case 1.

Also, if we consider $p = c = 3$ and $q = d = 3$, it can be shown that this Case is also equivalent to the Cases 1 and 2.

Therefore, we think that no need to find the solutions again.

It is noted that the solution (3.17) and (3.31) are identical, solution (3.19) and (3.33) are identical, solution (3.21) and (3.35) are identical, and solution (3.23) and (3.37) are identical.

Beyond Table 1, Zayed [52] obtained another solution (3.39). But, we obtain two more new solutions (3.21) and (3.23).

Graphical Representations of the Solutions

The above solutions are shown with the aid of Maple 13 in the graphs.

Table 1: Comparison between Zayed [52] solutions and our solutions.

Zayed [52] solutions	Our solutions
(i) If $\lambda = 2,\ \mu = 0$, equation (3.40) becomes $u(\eta) = \pm \frac{6i}{\sqrt{2\alpha}} \tanh \eta$.	(i) Solution (3.17) is $u(\eta) = \pm \frac{6i}{\sqrt{2\alpha}} \tanh \eta$.
(ii) If $1 + \lambda^2 = 4\mu$ and α is replaced with β, equation (3.38) becomes $u(\eta) = \pm \frac{3i}{\sqrt{2\beta}} \tan \frac{\eta}{2}$.	(ii) If η is replaced with $i\eta$, solution (3.19) becomes $u(\eta) = \pm \frac{3i}{\sqrt{2\beta}} \tan \frac{\eta}{2}$.

4. Conclusions

Using the Exp-function method, with the aid of symbolic computation software Maple 13, new exact traveling wave solutions of the (3 + 1)-dimensional modified KdV-Zakharov-Kuznetsev equation are constructed. It is important that some of the obtained solutions are identical to the solutions available in the literature and some are new. These solutions can be used to describe the insight of the complex physical phenomena.

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