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Research Article

New Jacobi Elliptic Function Solutions for the Zakharov Equations

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A generalized (G'/G)-expansion method is proposed to seek the exact solutions of nonlinear evolution equations. Being concise and straightforward, this method is applied to the Zakharov equations. As a result, some new Jacobi elliptic function solutions of the Zakharov equations are obtained. This method can also be applied to other nonlinear evolution equations in mathematical physics.

1. Introduction

In recent years, with the development of symbolic computation packages like Maple and Mathematica, searching for solutions of nonlinear differential equations directly has become more and more attractive [1–7]. This is because of the availability of computers symbolic system, which allows us to perform some complicated and tedious algebraic calculation and help us find new exact solutions of nonlinear differential equations.

In 2008, Wang et al. [8] introduced a new direct method called the (G'/G)-expansion method to look for travelling wave solutions of nonlinear evolution equations (NLEEs). The method is based on the homogeneous balance principle and linear ordinary differential equation (LODE) theory. It is assumed that the traveling wave solutions can be expressed by a polynomial in (G'/G), and that $G = G(\xi)$ satisfies a second-order LODE $G'' + \lambda G' + \mu G = 0$. The degree of the polynomial can be determined by the homogeneous balance between the highest order derivative and nonlinear terms appearing in the given NPDEs. The coefficients of the polynomial can be obtained by solving a set of algebraic equations. Many literatures have shown that the (G'/G)-expansion method is very effective, and many nonlinear equations have been successfully solved. Later, the further developed methods named the generalized (G'/G)-expansion method [9], the modified (G'/G)-expansion method [10], the extended (G'/G)-expansion method [11], the improved (G'/G)-expansion method [12], and the $(G'/G, 1/G)$-expansion method [13] have been proposed.

As we know, when using the direct method, the choice of an appropriate auxiliary LODE is of great importance. In this paper, by introducing a new auxiliary LODE of different literature [8], we propose the generalized (G'/G)-expansion method, which can be used to obtain travelling wave solutions of NLEEs.

In our contribution, we will seek exact solutions of the Zakharov equations [14]:

$$n_{tt} - c_s^2 n_{xx} = \beta\left(|E|^2\right)_{xx}, \tag{1.1}$$

$$iE_t + \alpha E_{xx} = \delta n E, \tag{1.2}$$

which are one of the classical models on governing the dynamics of nonlinear waves and describing the interactions between high- and low-frequency waves, where n is the perturbed number density of the ion (in the low-frequency response), E is the slow variation amplitude of the electric field intensity, c_s is the thermal transportation velocity of the electron ion, and $\alpha \neq 0$, $\beta \neq 0$, $\delta \neq 0$ and c_s are constants.

Recently, many exact solutions of (1.1)-(1.2) have been successfully obtained by using the extended tanh-expansion method, the extended hyperbolic function method, the F-expansion method, the $(G'/G, 1/G)$-expansion method [13–19], and so on.

In this paper, we construct the exact solutions to (1.1)-(1.2) by using the generalized (G'/G)-expansion method. Furthermore, we show that the exact solutions are expressed by the Jacobi elliptic function.

2. The Generalized (G'/G)-Expansion Method

Suppose that we have a nonlinear partial differential equation (PDE) for $u(x, t)$ in the form

$$N(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0, \tag{2.1}$$

where N is a polynomial in its arguments.

Step 1. By taking $u(x, t) = u(\xi)$, $\xi = x - ct$, we look for traveling wave solutions of (2.1) and transform it to the ordinary differential equation (ODE)

$$N\left(u, -cu', u', c^2u'', -cu'', u'', \ldots\right) = 0. \tag{2.2}$$

Step 2. Suppose the solution u of (2.2) can be expressed as a finite series in the form

$$u = a_0 + \sum_{i=1}^{m} a_i \left(\frac{f'}{f}\right)^i, \tag{2.3}$$

where a_0, a_i $(i = 1, 2, \ldots, m)$ are constants to be determined later; $f = f(\xi)$ is a solution of the auxiliary LODE

$$f'^2 = Pf^4 + Qf^2 + R, \tag{2.4}$$

where P, Q, and R are constants.

Step 3. Determine the parameter m by balancing the highest order nonlinear term and the highest order partial derivative of u in (2.2).

Step 4. Substituting (2.3) and (2.4) into (2.2), setting all the coefficients of all terms with the same powers of $(f'/f)^k$ $(k = 1, 2, \ldots)$ to zero, we obtain a system of nonlinear algebraic equations (NAEs) with respect to the parameters c, a_0, a_i $(i = 1, 2, \ldots, m)$. By solving the NAEs if available, we can determine those parameters explicitly.

Step 5. Assuming that the constants c, a_0, a_i $(i = 1, 2, \ldots, m)$ can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions into (2.3), we can obtain the explicit solutions of (2.1) immediately.

3. Exact Solutions of the Zakharov Equations

In this section, we mainly apply the method proposed in Section 2 to seek the exact solutions of the Zakharov equations.

Since $E(x,t)$ in (1.2) is a complex function and we are looking for the traveling wave solutions, thus we introduce a gauge transformation:

$$E(x,t) = \varphi(\xi)e^{[i(kx-\omega t+\xi_0)]}, \quad n = n(\xi), \quad \xi = x - c_g t + \xi_1, \tag{3.1}$$

where $\varphi(x,t)$ is a real-valued function, c_g, k, and ω are constants to be determined later, and ξ_0 and ξ_1 are constants. Substituting (3.1) into (1.1)-(1.2), we have

$$\left(c_g^2 - c_s^2\right)n'' = \beta\left(\varphi^2\right)'', \tag{3.2}$$

$$\alpha\varphi'' + i(2\alpha k - c_g)\varphi' + \left(\omega - \alpha k^2\right)\varphi - \delta n\varphi = 0. \tag{3.3}$$

Integrating (3.2) twice with respect to ξ, we have

$$n = \frac{\beta}{c_g^2 - c_s^2}\varphi^2 + C_1\xi + C_2, \quad c_g^2 - c_s^2 \neq 0, \tag{3.4}$$

where C_1 and C_2 are integration constants. Substituting (3.4) into (3.3), we have

$$\alpha\varphi'' + i(2\alpha k - c_g)\varphi' + \left(\omega - \alpha k^2\right)\varphi - \delta\left(\frac{\beta}{c_g^2 - c_s^2}\varphi^2 + C_1\xi + C_2\right)\varphi = 0. \tag{3.5}$$

In (3.5), we assume that

$$c_g = 2\alpha k, \qquad k_1 = \frac{\beta}{c_g^2 - c_s^2}. \tag{3.6}$$

Then (3.5) becomes the nonlinear ODE

$$\alpha\varphi'' + \left(\omega - \alpha k^2\right)\varphi - \delta k_1 \varphi^3 + \delta(C_1\xi + C_2)\varphi = 0. \tag{3.7}$$

According to the homogeneous balance between φ'' and φ^3 in (3.7), we obtain $m = 1$. So we assume that φ can be expressed as

$$\varphi(\xi) = a_0 + a_1\left(\frac{f'}{f}\right), \tag{3.8}$$

where $f = f(\xi)$ satisfies (2.4). By using (2.4) and (3.8), it is easily derived that

$$\varphi'' = 2a_1\left(\frac{f'}{f}\right)\left(\left(\frac{f'}{f}\right)^2 - Q\right). \tag{3.9}$$

Substituting (3.8) and (3.9) into (3.7), the left-hand side of (3.7) becomes a polynomial in (f'/f) and ξ. Setting their coefficients to zero yields a system of algebraic equations in a_0, a_1, k, k_1, and ω. Solving the overdetermined algebraic equations by Maple, we can obtain the following results:

$$a_0 = 0, \qquad a_1 = \sqrt{\frac{2\alpha}{k_1\delta}}, \qquad \omega = \alpha\left(2Q + k^2\right) + \delta C_2, \qquad C_1 = 0. \tag{3.10}$$

Substituting (3.10) into (3.8), we obtain

$$\varphi = \sqrt{\frac{2\alpha}{k_1\delta}}\left(\frac{f'}{f}\right). \tag{3.11}$$

Substituting (3.11) into (3.1) and (3.4), we have the following formal solution of (1.1)-(1.2):

$$\begin{gathered} E = \sqrt{\frac{2\alpha}{k_1\delta}}\left(\frac{f'}{f}\right)e^{[i(kx-\omega t+\xi_0)]}, \\ n = \frac{2\alpha}{\delta}\left(\frac{f'}{f}\right)^2 + C_2, \end{gathered} \tag{3.12}$$

where $\xi = x - 2\alpha k t + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

With the aid of the appendix [20] and from the formal solution (3.12), we get the following set of exact solutions of (1.1)-(1.2).

Case 1. Choosing $P = m^2$, $Q = -(1+m^2)$, $R = 1$, and $f(\xi) = \mathrm{sn}(\xi)$ and inserting them into (3.12), we obtain the Jacobi elliptic function solution of (1.1)-(1.2)

$$E_1 = \sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{cs}(\xi)\mathrm{dn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_1 = \frac{2\alpha}{\delta}\mathrm{cs}^2(\xi)\mathrm{dn}^2(\xi) + C_2, \tag{3.13}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(-2(1+m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 2. Choosing $P = m^2$, $Q = -(1+m^2)$, $R = 1$, and $f(\xi) = \mathrm{cd}(\xi)$ and inserting them into (3.12), we obtain the Jacobi elliptic function solution of (1.1)-(1.2)

$$E_2 = -\sqrt{\frac{2\alpha}{k_1\delta}}\left(1-m^2\right)\mathrm{sd}(\xi)\mathrm{nc}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_2 = \frac{2\alpha\left(1-m^2\right)^2}{\delta}\mathrm{sd}^2(\xi)\mathrm{nc}^2(\xi) + C_2, \tag{3.14}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(-2(1+m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 3. Choosing $P = -m^2$, $Q = 2m^2 - 1$, $R = 1 - m^2$, and $f(\xi) = \mathrm{cn}(\xi)$, we obtain

$$E_3 = -\sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{dc}(\xi)\mathrm{sn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_3 = \frac{2\alpha}{\delta}\mathrm{dc}^2(\xi)\mathrm{sn}^2(\xi) + C_2, \tag{3.15}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2-1)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 4. Choosing $P = -1$, $Q = 2 - m^2$, $R = m^2 - 1$, and $f(\xi) = \mathrm{dn}(\xi)$, we obtain

$$E_4 = -\sqrt{\frac{2\alpha}{k_1\delta}}m^2\mathrm{cd}(\xi)\mathrm{sn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_4 = \frac{2\alpha m^4}{\delta}\mathrm{cd}^2(\xi)\mathrm{sn}^2(\xi) + C_2, \tag{3.16}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2-m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 5. Choosing $P = 1$, $Q = -(1+m^2)$, $R = m^2$, and $f(\xi) = \text{ns}(\xi)$, we obtain

$$E_5 = -\sqrt{\frac{2\alpha}{k_1\delta}}\text{cs}(\xi)\text{dn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_5 = \frac{2\alpha}{\delta}\text{cs}^2(\xi)\text{dn}^2(\xi) + C_2, \tag{3.17}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(-2(1+m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 6. Choosing $P = 1$, $Q = -(1+m^2)$, $R = m^2$, and $f(\xi) = \text{dc}(\xi)$, we obtain

$$E_6 = \sqrt{\frac{2\alpha}{k_1\delta}}\left(1-m^2\right)\text{sc}(\xi)\text{nd}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_6 = \frac{2\alpha\left(1-m^2\right)^2}{\delta}\text{sc}^2(\xi)\text{nd}^2(\xi) + C_2, \tag{3.18}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(-2(1+m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 7. Choosing $P = 1-m^2$, $Q = 2m^2-1$, $R = -m^2$, and $f(\xi) = \text{nc}(\xi)$, we obtain

$$E_7 = \sqrt{\frac{2\alpha}{k_1\delta}}\text{dc}(\xi)\text{sn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_7 = \frac{2\alpha}{\delta}\text{dc}^2(\xi)\text{sn}^2(\xi) + C_2, \tag{3.19}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2-1)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 8. Choosing $P = m^2-1$, $Q = 2-m^2$, $R = -1$, and $f(\xi) = \text{nd}(\xi)$, we obtain

$$E_8 = \sqrt{\frac{2\alpha}{k_1\delta}}m^2\text{cd}(\xi)\text{sn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_8 = \frac{2\alpha m^4}{\delta}\text{cd}^2(\xi)\text{sn}^2(\xi) + C_2, \tag{3.20}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2-m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 9. Choosing $P = 1-m^2$, $Q = 2-m^2$, $R = 1$, and $f(\xi) = \text{sc}(\xi)$, we obtain

$$E_9 = \sqrt{\frac{2\alpha}{k_1\delta}}\text{dc}(\xi)\text{ns}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_9 = \frac{2\alpha}{\delta}\text{dc}^2(\xi)\text{ns}^2(\xi) + C_2, \tag{3.21}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2-m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 10. Choosing $P = -m^2(1-m^2)$, $Q = 2m^2 - 1$, $R = 1$, and $f(\xi) = \mathrm{sd}(\xi)$, we obtain

$$E_{10} = \sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{cd}(\xi)\mathrm{ns}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{10} = \frac{2\alpha}{\delta}\mathrm{cd}^2(\xi)\mathrm{ns}^2(\xi) + C_2, \tag{3.22}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2-1)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 11. Choosing $P = 1$, $Q = 2 - m^2$, $R = 1 - m^2$, and $f(\xi) = \mathrm{cs}(\xi)$, we obtain

$$E_{11} = \sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{ds}(\xi)\mathrm{nc}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{11} = \frac{2\alpha}{\delta}\mathrm{ds}^2(\xi)\mathrm{nc}^2(\xi) + C_2, \tag{3.23}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2-m^2)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 12. Choosing $P = 1$, $Q = 2m^2 - 1$, $R = -m^2(1-m^2)$, and $f(\xi) = \mathrm{ds}(\xi)$, we obtain

$$E_{12} = -\sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{cs}(\xi)\mathrm{nd}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{12} = \frac{2\alpha}{\delta}\mathrm{cs}^2(\xi)\mathrm{nd}^2(\xi) + C_2, \tag{3.24}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2-1)+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 13. Choosing $P = 1/4$, $Q = (1-2m^2)/2$, $R = 1/4$, and $f(\xi) = \mathrm{ns}(\xi) \pm \mathrm{cs}(\xi)$, we obtain

$$E_{13} = \mp\sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{ds}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{13} = \frac{2\alpha}{\delta}\mathrm{ds}^2(\xi) + C_2, \tag{3.25}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(1-2m^2+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 14. Choosing $P = (1-m^2)/4$, $Q = (1+m^2)/2$, $R = (1-m^2)/4$, and $f(\xi) = \mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)$, we obtain

$$E_{14} = \pm\sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{dc}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{14} = \frac{2\alpha}{\delta}\mathrm{dc}^2(\xi) + C_2, \tag{3.26}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(1+m^2+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 15. Choosing $P = 1/4$, $Q = (m^2 - 2)/2$, $R = m^2/4$, and $f(\xi) = \mathrm{ns}(\xi) \pm \mathrm{ds}(\xi)$, we obtain

$$E_{15} = \mp\sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{cs}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{15} = \frac{2\alpha}{\delta}\mathrm{cs}^2(\xi) + C_2, \tag{3.27}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 16. Choosing $P = m^2/4$, $Q = (m^2 - 2)/2$, $R = m^2/4$, and $f(\xi) = \mathrm{sn}(\xi) \pm i\mathrm{cn}(\xi)$, we obtain

$$E_{16} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\mathrm{dn}(\xi)(\mathrm{cn}(\xi) \mp i\mathrm{sn}(\xi))e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{sn}(\xi) \pm i\mathrm{cn}(\xi)},$$
$$n_{16} = \frac{2\alpha\mathrm{dn}^2(\xi)(\mathrm{cn}(\xi) \mp i\mathrm{sn}(\xi))^2}{\delta(\mathrm{sn}(\xi) \pm i\mathrm{cn}(\xi))^2} + C_2, \tag{3.28}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 17. Choosing $P = m^2/4$, $Q = (m^2 - 2)/2$, $R = m^2/4$, and $f(\xi) = \sqrt{m^2 - 1}\,\mathrm{sd}(\xi) \pm \mathrm{cd}(\xi)$, we obtain

$$E_{17} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\left(\sqrt{m^2-1}\mathrm{cn}(\xi) \pm m^2\mathrm{sn}(\xi) \mp \mathrm{sn}(\xi)\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{dn}(\xi)\left(\sqrt{m^2-1}\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)\right)},$$
$$n_{17} = \frac{2\alpha\left(\sqrt{m^2-1}\mathrm{cn}(\xi) \pm m^2\mathrm{sn}(\xi) \mp \mathrm{sn}(\xi)\right)^2}{\delta\mathrm{dn}^2(\xi)\left(\sqrt{m^2-1}\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)\right)^2} + C_2, \tag{3.29}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 18. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = m\mathrm{cd}(\xi) \pm i\sqrt{1-m^2}\mathrm{nd}(\xi)$, we obtain

$$E_{18} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{m\mathrm{sn}(\xi)\left(\pm m^2 \mp 1 + im\sqrt{1-m^2}\mathrm{cn}(\xi)\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{dn}(\xi)\left(i\sqrt{1-m^2} \pm m\mathrm{cn}(\xi)\right)},$$
$$n_{18} = \frac{2\alpha m^2\mathrm{sn}^2(\xi)\left(\pm m^2 \mp 1 + im\sqrt{1-m^2}\mathrm{cn}(\xi)\right)^2}{\delta\mathrm{dn}^2(\xi)\left(i\sqrt{1-m^2} \pm m\mathrm{cn}(\xi)\right)^2} + C_2, \tag{3.30}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 19. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = m\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi)$, we obtain

$$E_{19} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{m\mathrm{cn}(\xi)(\mathrm{dn}(\xi) \mp im\mathrm{sn}(\xi))e^{[i(kx-\omega t+\xi_0)]}}{m\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi)},$$
$$n_{19} = \frac{2\alpha m^2\mathrm{cn}^2(\xi)(\mathrm{dn}(\xi) \mp im\mathrm{sn}(\xi))^2}{\delta(m\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi))^2} + C_2, \tag{3.31}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 20. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = \sqrt{m^2-1}\mathrm{sc}(\xi) \pm i\mathrm{dc}(\xi)$, we obtain

$$E_{20} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\left(\sqrt{m^2-1}\mathrm{dn}(\xi) \mp im^2\mathrm{sn}(\xi) \pm i\mathrm{sn}(\xi)\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{cn}(\xi)\left(\sqrt{m^2-1}\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi)\right)},$$
$$n_{20} = \frac{2\alpha\left(\sqrt{m^2-1}\mathrm{dn}(\xi) \mp im^2\mathrm{sn}(\xi) \pm i\mathrm{sn}(\xi)\right)^2}{\delta\mathrm{cn}^2(\xi)\left(\sqrt{m^2-1}\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi)\right)^2} + C_2, \tag{3.32}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 21. Choosing $P = (m^2-1)/4$, $Q = (m^2+1)/2$, $R = (m^2-1)/4$, and $f(\xi) = m\,\mathrm{sd}(\xi) \pm \mathrm{nd}(\xi)$, we obtain

$$E_{21} = \pm\sqrt{\frac{2\alpha}{k_1\delta}}m\mathrm{cd}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{21} = \frac{2\alpha m^2}{\delta}\mathrm{cd}^2(\xi) + C_2, \tag{3.33}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 22. Choosing $P = m^2/4$, $Q = (m^2 - 2)/2$, $R = 1/4$, and $f(\xi) = \mathrm{sn}(\xi)/(1 \pm \mathrm{dn}(\xi))$, we obtain

$$E_{22} = \pm\sqrt{\frac{2\alpha}{k_1\delta}}\mathrm{cs}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{22} = \frac{2\alpha}{\delta}\mathrm{cs}^2(\xi) + C_2, \tag{3.34}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 23. Choosing $P = -1/4$, $Q = (m^2+1)/2$, $R = (1-m^2)^2/4$, and $f(\xi) = m\mathrm{cn}(\xi) \pm \mathrm{dn}(\xi)$, we obtain

$$E_{23} = \mp\sqrt{\frac{2\alpha}{k_1\delta}}\, m\mathrm{sn}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{23} = \frac{2\alpha m^2}{\delta}\mathrm{sn}^2(\xi) + C_2, \tag{3.35}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2+1+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 24. Choosing $P = (1-m^2)^2/4$, $Q = (m^2+1)/2$, $R = 1/4$, and $f(\xi) = \mathrm{ds}(\xi) \pm \mathrm{cs}(\xi)$, we obtain

$$E_{24} = \mp\sqrt{\frac{2\alpha}{k_1\delta}}\,\mathrm{ns}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{24} = \frac{2\alpha}{\delta}\mathrm{ns}^2(\xi) + C_2, \tag{3.36}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2+1+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 25. Choosing $P = 1/4$, $Q = (m^2-2)/2$, $R = m^4/4$, and $f(\xi) = \mathrm{dc}(\xi) \pm \sqrt{1-m^2}\,\mathrm{nc}(\xi)$, we obtain

$$E_{25} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\mathrm{sn}(\xi)\left(\mp m^2 \pm 1 + \sqrt{1-m^2}\,\mathrm{dn}(\xi)\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{cn}(\xi)\left(\sqrt{1-m^2} \pm \mathrm{dn}(\xi)\right)},$$
$$n_{25} = \frac{2\alpha\mathrm{sn}^2(\xi)\left(\mp m^2 \pm 1 + \sqrt{1-m^2}\,\mathrm{dn}(\xi)\right)^2}{\delta\mathrm{cn}^2(\xi)\left(\sqrt{1-m^2} \pm \mathrm{dn}(\xi)\right)^2} + C_2, \tag{3.37}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2-2+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 26. Choosing $R = m^2Q^2/(m^2+1)^2P$, $Q < 0$, $P > 0$, and $f(\xi) = \sqrt{-m^2Q/(m^2+1)P}$ $\mathrm{sn}(\sqrt{(-Q/(m^2+1))}\xi)$, we obtain

$$E_{26} = \sqrt{\frac{2\alpha}{k_1\delta}}\sqrt{\frac{-Q}{m^2+1}}\mathrm{cs}\left(\sqrt{\frac{-Q}{m^2+1}}\xi\right)\mathrm{dn}\left(\sqrt{\frac{-Q}{m^2+1}}\xi\right)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{26} = -\frac{2\alpha Q}{\delta(m^2+1)}\mathrm{cs}^2\left(\sqrt{\frac{-Q}{m^2+1}}\xi\right)\mathrm{dn}^2\left(\sqrt{\frac{-Q}{m^2+1}}\xi\right) + C_2, \tag{3.38}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2Q+k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 27. Choosing $R = (1-m^2)Q^2/(m^2-2)^2P$, $Q > 0$, $P < 0$, and $f(\xi) = \sqrt{-Q/(2-m^2)P}\,\mathrm{dn}(\sqrt{(Q/(2-m^2))}\xi)$, we obtain

$$E_{27} = -\sqrt{\frac{2\alpha}{k_1\delta}}\sqrt{\frac{Q}{2-m^2}}m^2\mathrm{cd}\left(\sqrt{\frac{Q}{2-m^2}}\xi\right)\mathrm{sn}\left(\sqrt{\frac{Q}{2-m^2}}\xi\right)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{27} = \frac{2\alpha Q m^4}{\delta(2-m^2)}\mathrm{cd}^2\left(\sqrt{\frac{Q}{2-m^2}}\xi\right)\mathrm{sn}^2\left(\sqrt{\frac{Q}{2-m^2}}\xi\right) + C_2, \tag{3.39}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 28. Choosing $R = m^2(m^2-1)Q^2/(2m^2-1)^2P$, $Q > 0$, $P < 0$, and $f(\xi) = \sqrt{-(m^2Q/(2m^2-1)P)}\mathrm{cn}(\sqrt{(Q/(2m^2-1))}\xi)$, we obtain

$$E_{28} = -\sqrt{\frac{2\alpha}{k_1\delta}}\sqrt{\frac{Q}{2m^2-1}}\mathrm{dc}\left(\sqrt{\frac{Q}{2m^2-1}}\xi\right)\mathrm{sn}\left(\sqrt{\frac{Q}{2m^2-1}}\xi\right)e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{28} = \frac{2\alpha Q}{\delta(2m^2-1)}\mathrm{dc}^2\left(\sqrt{\frac{Q}{2m^2-1}}\xi\right)\mathrm{sn}^2\left(\sqrt{\frac{Q}{2m^2-1}}\xi\right) + C_2, \tag{3.40}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 29. Choosing $P = 1$, $Q = 2 - 4m^2$, $R = 1$, and $f(\xi) = \mathrm{sn}(\xi)\mathrm{dn}(\xi)/\mathrm{cn}(\xi)$, we obtain

$$E_{29} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{(1-2m^2\mathrm{sn}^2(\xi) + m^2\mathrm{sn}^4(\xi))e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{cn}(\xi)\mathrm{sn}(\xi)\mathrm{dn}(\xi)},$$
$$n_{29} = \frac{2\alpha\left(1-2m^2\mathrm{sn}^2(\xi) + m^2\mathrm{sn}^4(\xi)^2\right)^2}{\delta\mathrm{cn}^2(\xi)\mathrm{sn}^2(\xi)\mathrm{dn}^2(\xi)} + C_2, \tag{3.41}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2-4m^2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 30. Choosing $P = m^4$, $Q = 2m^2 - 4$, $R = 1$, and $f(\xi) = \mathrm{sn}(\xi)\mathrm{cn}(\xi)/\mathrm{dn}(\xi)$, we obtain

$$E_{30} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{(1-2\mathrm{sn}^2(\xi) + m^2\mathrm{sn}^4(\xi))e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{dn}(\xi)\mathrm{sn}(\xi)\mathrm{cn}(\xi)},$$
$$n_{30} = \frac{2\alpha(1-2\mathrm{sn}^2(\xi) + m^2\mathrm{sn}^4(\xi))^2}{\delta\mathrm{dn}^2(\xi)\mathrm{sn}^2(\xi)\mathrm{cn}^2(\xi)} + C_2, \tag{3.42}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2-4) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 31. Choosing $P = 1$, $Q = 2m^2 + 2$, $R = 1 - 2m^2 + m^4$, and $f(\xi) = \mathrm{cn}(\xi)\mathrm{dn}(\xi)/\mathrm{sn}(\xi)$, we obtain

$$E_{31} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\left(m^2\mathrm{sn}^4(\xi) - 1\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{sn}(\xi)\mathrm{cn}(\xi)\mathrm{dn}(\xi)},$$
$$n_{31} = \frac{2\alpha\left(m^2\mathrm{sn}^4(\xi) - 1\right)^2}{\delta\mathrm{sn}^2(\xi)\mathrm{cn}^2(\xi)\mathrm{dn}^2(\xi)} + C_2, \tag{3.43}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2 + 2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 32. Choosing $P = A^2(m-1)^2/4$, $Q = (m^2+1)/2 + 3m$, $R = (m-1)^2/4A^2$, and $f(\xi) = \mathrm{dn}(\xi)\mathrm{cn}(\xi)/A(1 + \mathrm{sn}(\xi))(1 + m\mathrm{sn}(\xi))$, we obtain

$$E_{32} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\left(m^2\mathrm{sn}^2(\xi) + m\mathrm{sn}^2(\xi) - 1 - m\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{dn}(\xi)\mathrm{cn}(\xi)},$$
$$n_{32} = \frac{2\alpha\left(m^2\mathrm{sn}^2(\xi) + m\mathrm{sn}^2(\xi) - 1 - m\right)^2}{\delta\mathrm{dn}^2(\xi)\mathrm{cn}^2(\xi)} + C_2, \tag{3.44}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 33. Choosing $P = A^2(m+1)^2/4$, $Q = (m^2+1)/2 - 3m$, $R = (m+1)^2/4A^2$, and $f(\xi) = \mathrm{dn}(\xi)\mathrm{cn}(\xi)/A(1 + \mathrm{sn}(\xi))(1 - m\mathrm{sn}(\xi))$, we obtain

$$E_{33} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\left(m^2\mathrm{sn}^2(\xi) - m\mathrm{sn}^2(\xi) + m - 1\right)e^{[i(kx-\omega t+\xi_0)]}}{\mathrm{dn}(\xi)\mathrm{cn}(\xi)},$$
$$n_{33} = \frac{2\alpha\left(m^2\mathrm{sn}^2(\xi) - m\mathrm{sn}^2(\xi) + m - 1\right)^2}{\delta\mathrm{dn}^2(\xi)\mathrm{cn}^2(\xi)} + C_2, \tag{3.45}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 34. Choosing $P = -4/m$, $Q = 6m - m^2 - 1$, $R = -2m^3 + m^4 + m^2$, and $f(\xi) = m\mathrm{cn}(\xi)\mathrm{dn}(\xi)/(m\mathrm{sn}^2(\xi) + 1)$, we obtain

$$E_{34} = \sqrt{\frac{2\alpha}{k_1\delta}}\frac{\mathrm{sn}(\xi)\left(m\mathrm{sn}^2(\xi) - 1 + 2m^2\mathrm{sn}^2(\xi) + m^3\mathrm{sn}^2(\xi) - m^2 - 2m\right)e^{[i(kx-\omega t+\xi_0)]}}{(m\mathrm{sn}^2(\xi) + 1)\mathrm{cn}(\xi)\mathrm{dn}(\xi)},$$
$$n_{34} = \frac{2\alpha\mathrm{sn}^2(\xi)\left(m\mathrm{sn}^2(\xi) - 1 + 2m^2\mathrm{sn}^2(\xi) + m^3\mathrm{sn}^2(\xi) - m^2 - 2m\right)^2}{\delta(m\mathrm{sn}^2(\xi) + 1)^2\mathrm{cn}^2(\xi)\mathrm{dn}^2(\xi)} + C_2, \tag{3.46}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(6m - m^2 - 1) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 35. Choosing $P = 4/m$, $Q = -6m - m^2 - 1$, $R = 2m^3 + m^4 + m^2$, and $f(\xi) = m\mathrm{cn}(\xi)\mathrm{dn}(\xi)/(m\mathrm{sn}^2(\xi) - 1)$, we obtain

$$E_{35} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{\mathrm{sn}(\xi)\left(m\mathrm{sn}^2(\xi) + 1 - 2m^2\mathrm{sn}^2(\xi) + m^3\mathrm{sn}^2(\xi) + m^2 - 2m\right)e^{[i(kx-\omega t+\xi_0)]}}{(m\mathrm{sn}^2(\xi) - 1)\mathrm{cn}(\xi)\mathrm{dn}(\xi)},$$
$$n_{35} = \frac{2\alpha\mathrm{sn}^2(\xi)\left(m\mathrm{sn}^2(\xi) + 1 - 2m^2\mathrm{sn}^2(\xi) + m^3\mathrm{sn}^2(\xi) + m^2 - 2m\right)^2}{\delta(m\mathrm{sn}^2(\xi) - 1)^2\mathrm{cn}^2(\xi)\mathrm{dn}^2(\xi)} + C_2, \tag{3.47}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(-2(6m + m^2 + 1) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 36. Choosing $P = -(m^2 + 2m + 1)B^2$, $Q = 2m^2 + 2$, $R = (2m - m^2 - 1)/B^2$, and $f(\xi) = (m\mathrm{sn}^2(\xi) - 1)/B(m\mathrm{sn}^2(\xi) + 1)$, we obtain

$$E_{36} = 4\sqrt{\frac{2\alpha}{k_1\delta}} m \frac{\mathrm{sn}(\xi)\mathrm{cn}(\xi)\mathrm{dn}(\xi)}{m^2\mathrm{sn}^4(\xi) - 1} e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{36} = \frac{32\alpha m^2\mathrm{sn}^2(\xi)\mathrm{cn}^2(\xi)\mathrm{dn}^2(\xi)}{\delta\left(m^2\mathrm{sn}^4(\xi) - 1\right)^2} + C_2, \tag{3.48}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2 + 2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 37. Choosing $P = -(m^2 - 2m + 1)B^2$, $Q = 2m^2 + 2$, $R = -(2m + m^2 + 1)/B^2$, and $f(\xi) = (m\mathrm{sn}^2(\xi) + 1)/B(m\mathrm{sn}^2(\xi) - 1)$, we obtain

$$E_{37} = -4\sqrt{\frac{2\alpha}{k_1\delta}} m \frac{\mathrm{sn}(\xi)\mathrm{cn}(\xi)\mathrm{dn}(\xi)}{m^2\mathrm{sn}^4(\xi) - 1} e^{[i(kx-\omega t+\xi_0)]},$$
$$n_{37} = \frac{32\alpha m^2\mathrm{sn}^2(\xi)\mathrm{cn}^2(\xi)\mathrm{dn}^2(\xi)}{\delta\left(m^2\mathrm{sn}^4(\xi) - 1\right)^2} + C_2, \tag{3.49}$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2(2m^2 + 2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

4. Conclusions

In this paper, by using the generalized (G'/G)-expansion method, we have successfully obtained some exact solutions of Jacobi elliptic function form of the Zakharov equations. When the modulus of the Jacobi elliptic function $m \to 0$ or 1, the corresponding solitary wave solutions and trigonometric function solutions are also obtained. This work shows that the generalized (G'/G)-expansion method provides a very effective and powerful tool for solving nonlinear equations in mathematical physics.

Table 1: Relations between the coefficients (P, Q, and R) and corresponding $f(\xi)$ in $f'^2 = Pf^4 + Qf^2 + R$.

Case	P	Q	R	$f(\xi)$
Case 1	m^2	$-(1+m^2)$	1	$\mathrm{sn}\xi$
Case 2	m^2	$-(1+m^2)$	1	$\mathrm{cd}\xi$
Case 3	$-m^2$	$2m^2-1$	$1-m^2$	$\mathrm{cn}\xi$
Case 4	-1	$2-m^2$	m^2-1	$\mathrm{dn}\xi$
Case 5	1	$-(1+m^2)$	m^2	$\mathrm{ns}\xi$
Case 6	1	$-(1+m^2)$	m^2	$\mathrm{dc}\xi$
Case 7	$1-m^2$	$2m^2-1$	$-m^2$	$\mathrm{nc}\xi$
Case 8	m^2-1	$2-m^2$	-1	$\mathrm{nd}\xi$
Case 9	$1-m^2$	$2-m^2$	1	$\mathrm{sc}\xi$
Case 10	$-m^2(1-m^2)$	$2m^2-1$	1	$\mathrm{sd}\,\xi$
Case 11	1	$2-m^2$	$1-m^2$	$\mathrm{cs}\xi$
Case 12	1	$2m^2-1$	$-m^2(1-m^2)$	$\mathrm{ds}\xi$
Case 13	$1/4$	$(1-2m^2)/2$	$1/4$	$\mathrm{ns}\xi \pm \mathrm{cs}\xi$
Case 14	$(1-m^2)/4$	$(1+m^2)/2$	$(1-m^2)/4$	$\mathrm{nc}\xi \pm \mathrm{sc}\xi$
Case 15	$1/4$	$(m^2-2)/2$	$m^2/4$	$\mathrm{ns}\xi \pm \mathrm{ds}\xi$
Case 16	$m^2/4$	$(m^2-2)/2$	$m^2/4$	$\mathrm{sn}\xi \pm i\mathrm{cn}\xi$
Case 17	$m^2/4$	$(m^2-2)/2$	$m^2/4$	$\sqrt{m^2-1}\,\mathrm{sd}\,\xi \pm \mathrm{cd}\xi$
Case 18	$1/4$	$(1-2m^2)/2$	$1/4$	$m\mathrm{cd}\xi \pm i\sqrt{1-m^2}\mathrm{nd}\xi$
Case 19	$1/4$	$(1-2m^2)/2$	$1/4$	$m\mathrm{sn}\xi \pm i\mathrm{dn}\xi$
Case 20	$1/4$	$(1-2m^2)/2$	$1/4$	$\sqrt{m^2-1}\mathrm{sc}\xi \pm i\mathrm{dc}\xi$
Case 21	$(m^2-1)/4$	$(m^2+1)/2$	$(m^2-1)/4$	$m\mathrm{sd}\xi \pm \mathrm{nd}\xi$
Case 22	$m^2/4$	$(m^2-2)/2$	$1/4$	$\mathrm{sn}\xi/(1\pm\mathrm{dn}\xi)$
Case 23	$-1/4$	$(m^2+1)/2$	$(1-m^2)^2/4$	$m\mathrm{cn}\xi \pm \mathrm{dn}\xi$
Case 24	$(1-m^2)^2/4$	$(m^2+1)/2$	$1/4$	$\mathrm{ds}\xi \pm \mathrm{cs}\xi$
Case 25	$1/4$	$(m^2-2)/2$	$m^4/4$	$\mathrm{dc}\xi \pm \sqrt{1-m^2}\mathrm{nc}\xi$
Case 26	$P>0$	$Q<0$	$m^2Q^2/(m^2+1)^2P$	$\sqrt{-m^2Q/(m^2+1)P}\mathrm{sn}(\sqrt{(-Q/(m^2+1))}\xi)$
Case 27	$P<0$	$Q>0$	$(1-m^2)Q^2/(m^2-2)^2P$	$\sqrt{-Q/(2-m^2)P}\mathrm{dn}(\sqrt{(Q/(2-m^2))}\xi)$
Case 28	$P<0$	$Q>0$	$m^2(m^2-1)Q^2/(2m^2-1)^2P$	$\sqrt{-m^2Q/(2m^2-1)P}\mathrm{cn}(\sqrt{(Q/(2m^2-1))}\xi)$
Case 29	1	$2-4m^2$	1	$\mathrm{sn}\xi\mathrm{dn}\xi/\mathrm{cn}\xi$
Case 30	m^4	$2m^2-4$	1	$\mathrm{sn}\xi\mathrm{cn}\xi/\mathrm{dn}\xi$
Case 31	1	$2m^2+2$	$1-2m^2+m^4$	$\mathrm{cn}\xi\mathrm{dn}\xi/\mathrm{sn}\xi$
Case 32	$A^2(m-1)^2/4$	$(m^2+1)/2+3m$	$(m-1)^2/4A^2$	$\mathrm{dn}\xi\mathrm{cn}\xi/A(1+\mathrm{sn}\xi)(1+m\mathrm{sn}\xi)$
Case 33	$A^2(m+1)^2/4$	$(m^2+1)/2-3m$	$(m+1)^2/4A^2$	$\mathrm{dn}\xi\mathrm{cn}\xi/A(1+\mathrm{sn}\xi)(1-m\mathrm{sn}\xi)$
Case 34	$-4/m$	$6m-m^2-1$	$-2m^3+m^4+m^2$	$m\mathrm{cn}\xi\mathrm{dn}\xi/(m\mathrm{sn}^2\xi+1)$
Case 35	$4/m$	$-6m-m^2-1$	$2m^3+m^4+m^2$	$m\mathrm{cn}\xi\mathrm{dn}\xi/(m\mathrm{sn}^2\xi-1)$
Case 36	$-(m^2+2m+1)B^2$	$2m^2+2$	$(2m-m^2-1)/B^2$	$(m\mathrm{sn}^2\xi-1)/B(m\mathrm{sn}^2\xi+1)$
Case 37	$-(m^2-2m+1)B^2$	$2m^2+2$	$-(2m+m^2+1)/B^2$	$(m\mathrm{sn}^2\xi+1)/B(m\mathrm{sn}^2\xi-1)$

Appendix

For more details see Table 1 and [20].

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References

[1] M. L. Wang and Y. B. Zhou, "The periodic wave solutions for the Klein-Gordon-Schrödinger equations," *Physics Letters A*, vol. 318, no. 1-2, pp. 84–92, 2003.

[2] Z. Yan and H. Zhang, "New explicit solitary wave solutions and periodic wave solutions for Whitham-Broer-Kaup equation in shallow water," *Physics Letters A*, vol. 285, no. 5-6, pp. 355–362, 2001.

[3] M. L. Wang, Y. B. Zhou, and Z. B. Li, "Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics," *Physics Letters A*, vol. 216, pp. 67–75, 1996.

[4] S. Liu, Z. Fu, S. Liu, and Q. Zhao, "Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations," *Physics Letters A*, vol. 289, no. 1-2, pp. 69–74, 2001.

[5] Y. L. Ma, B. Q. Li, and C. Wang, "A series of abundant exact travelling wave solutions for a modified generalized Vakhnenko equation using auxiliary equation method," *Applied Mathematics and Computation*, vol. 211, no. 1, pp. 102–107, 2009.

[6] D. Wang and H.-Q. Zhang, "Further improved F-expansion method and new exact solutions of Konopelchenko-Dubrovsky equation," *Chaos, Solitons and Fractals*, vol. 25, no. 3, pp. 601–610, 2005.

[7] H. Bin, M. Qing, L. Yao, and R. Weiguo, "New exact solutions of the double sine-Gordon equation using symbolic computations," *Applied Mathematics and Computation*, vol. 186, no. 2, pp. 1334–1346, 2007.

[8] M. L. Wang, X. Z. Li, and J. L. Zhang, "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics," *Physics Letters A*, vol. 372, no. 4, pp. 417–423, 2008.

[9] E. M. E. Zayed, "New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized (G'/G)-expansion method," *Journal of Physics A*, vol. 42, no. 19, Article ID 195202, p. 13, 2009.

[10] Y.-B. Zhou and C. Li, "Application of modified (G'/G)-expansion method to traveling wave solutions for Whitham-Broer-Kaup-like equations," *Communications in Theoretical Physics*, vol. 51, no. 4, pp. 664–670, 2009.

[11] S. M. Guo and Y. B. Zhou, "The extended (G'/G)-expansion method and its applications to the Whitham-Broer-Kaup-like equations and coupled Hirota-Satsuma KdV equations," *Applied Mathematics and Computation*, vol. 215, no. 9, pp. 3214–3221, 2010.

[12] S. M. Guo, Y. B. Zhou, and C. X. Zhao, "The improved (G'/G)-expansion method and its applications to the Broer-Kaup equations and approximate long water wave equations," *Applied Mathematics and Computation*, vol. 216, no. 7, pp. 1965–1971, 2010.

[13] L. X. Li, E. Q. Li, and M. L. Wang, "The $(G'/G, 1/G)$-expansion method and its application to travelling wave solutions of the Zakharov equations," *Applied Mathematics*, vol. 25, no. 4, pp. 454–462, 2010.

[14] D. J. Huang and H. Q. Zhang, "Extended hyperbolic function method and new exact solitary wave solutions of Zakharov equations," *Acta Physica Sinica*, vol. 53, no. 8, pp. 2434–2438, 2004.

[15] S. D. Liu, Z. T. Fu, S. K. Liu, and Q. Zhang, "The envelope periodic solutions to nonlinearwave equations with Jacobi elliptic function," *Acta Physica Sinica*, vol. 51, pp. 718–722, 2002.

[16] Y. D. Shang, Y. Huang, and W. J. Yuan, "The extended hyperbolic functions method and new exact solutions to the Zakharov equations," *Applied Mathematics and Computation*, vol. 200, no. 1, pp. 110–122, 2008.

[17] M. L. Wang, Y. M. Wang, and J. L. Zhang, "The periodic wave solutions for two systemsof nonlinear wave equations," *Chinese Physics*, vol. 12, pp. 1341–1348, 2003.

[18] C. H. Zhao and Z. M. Sheng, "Explicit travelling wave solutions for Zakharov equations," *Acta Physica Sinica*, vol. 53, no. 6, pp. 1629–1634, 2004.

[19] J. L. Zhang, Y. M. Wang, and M. L. Wang, "Exact solutions to two nonlinear equations," *Acta Physica Sinica*, vol. 52, no. 7, pp. 1574–1578, 2003.

[20] A. Ebaid and E. H. Aly, "Exact solutions for the transformed reduced Ostrovsky equation via the F-expansion method in terms of Weierstrass-elliptic and Jacobian-elliptic functions," *Wave Motion*, vol. 49, no. 2, pp. 296–308, 2012.