Research Article

On Delay-Range-Dependent Stochastic Stability Conditions of Uncertain Neutral Delay Markovian Jump Systems

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The delay-range-dependent stochastic stability for uncertain neutral Markovian jump systems with interval time-varying delays is studied in this paper. The uncertainties under consideration are assumed to be time varying but norm bounded. To begin with the nominal systems, a novel augmented Lyapunov functional which contains some triple-integral terms is introduced. Then, by employing some integral inequalities and the nature of convex combination, some less conservative stochastic stability conditions are presented in terms of linear matrix inequalities without introducing any free-weighting matrices. Finally, numerical examples are provided to demonstrate the effectiveness and to show that the proposed results significantly improve the allowed upper bounds of the delay size over some existing ones in the literature.

1. Introduction

Time delays are frequently encountered in various engineering systems, such as chemical or process control systems, networked control systems, and manufacturing systems. To sum up, delays can appear in the state, input, or output variables (retarded systems), as well as in the state derivative (neutral systems). In fact, neutral delay systems constitute a more general class than those of the retarded type because such systems can be found in places such as population ecology [1], distributed neural networks [2], heat exchangers, and robots in contact with rigid environments [3]. Since it is shown that the existence of delays in a dynamic system may result in instability, oscillations, or poor performances [3–5], the stability of time-delay systems has been an important problem of recurring interest for many years. Existing results on this topic can be roughly classified into two categories, namely, delay-independent criteria [6] and delay-dependent criteria, and it is generally recognized that the latter cases are less conservative. Actually, the stability of neutral time-delay systems proves to be a more complex issue as well as singular systems [7–9] because the systems involve the derivative of the delayed state. So considerable attention has been devoted to the problem of robust delay-independent stability or delay-dependent stability and stabilization via different approaches for linear neutral systems with delayed state input and parameter uncertainties. Results are mainly presented based on Lyapunov-Krasovskii (L-K) method; see, for example, [10–16] and the references therein. However, there is room for further investigation because the conservativeness of the neutral systems can be further reduced by a better technique.

On the other hand, with the development of science and technology, many practical dynamics, for example, solar thermal central receivers, robotic manipulator systems, aircraft control systems, economic systems, and so on, experience abrupt changes in their structures, whose parameters are caused by phenomena such as component failures or repairs, changes in subsystem interconnections, and sudden environmental changes. This class of systems is more appropriate to be described as Markovian jump systems (MJSs), which can be regarded as a special class of hybrid systems with finite operation modes. The system parameters jump among finite modes, and the mode switching is governed by a Markov process to represent the abrupt variation in their structures and parameters. With so many applications in engineering systems, a great deal of attention has been paid to the stability

analysis and controller synthesis for Markovian jump systems (MJSs) in recent years. Many researchers have made a lot of progress on Markovian jump delay systems and Markovian jump control theory; see, for example, [17–23] and references therein for more details. However, a few of these papers have considered the effect of delay on the stability or stabilization for the corresponding neutral systems. Besides, to the best of the authors' knowledge, it seems that the problem of stochastic stability for neutral Markovian jumping systems with interval time-varying delays has not been fully investigated and it is very challenging. Motivated by the previous description, this paper investigates the stochastic stability of neutral Markovian jumping systems with interval time-varying delays to seek less conservative stochastic stability conditions than some previous ones.

In order to simplify the treatment of the problem, in this paper, we first investigate the nominal systems and construct a new augmented Lyapunov functional containing some triple-integral terms to reduce conservativeness. By some integral inequalities and the nature of convex combination, the delay-range-dependent stochastic stability conditions are derived for the nominal neutral systems with Markovian jump parameters and interval time-varying delays. Then, the results are extended to the corresponding uncertain case on the basis of obtained conditions. In addition, these conditions are expressed in linear matrix inequalities (LMIs), which can be easily checked by utilizing the LMI Toolbox in MATLAB. Numerical examples are given to show the effectiveness and reduced conservativeness over some previous references.

The main contributions of this paper can be summarized as follows: (1) the proposed Lyapunov functional contains some triple-integral terms which is very effective in the reduction of conservativeness, and has not been used in any of the existing literatures in the same context before; (2) the delay-range-dependent stability conditions are obtained in terms of LMIs without introducing any free-weighting matrices besides the Lyapunov matrices, which will reduce the number of variables and decrease the complexity of computation; (3) the proposed results are expressed in a new representation and proved to be less conservative than some existing ones.

The remainder of this paper is organized as follows: Section 2 contains the problem statement and preliminaries; Section 3 presents the main results; Section 4 provides a numerical example to verify the effectiveness of the results; Section 5 draws a brief conclusion.

1.1. Notations. In this paper, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m \times n}$ is for the set of all $m \times n$ matrices. The notation $X < Y$ ($X > Y$), where X and Y are both symmetric matrices, means that $X - Y$ is negative (positive) definite. I denotes the identity matrix with proper dimensions. $\lambda_{\max(\min)}(A)$ is the eigenvalue of matrix A with maximum (minimum) real part. For a symmetric block matrix, we use the sign $*$ to denote the terms introduced by symmetry. $\mathscr{E}$ stands for the mathematical expectation, and $\|v\|$ is the Euclidean norm of vector v, $\|v\| = (v^T v)^{1/2}$, while $\|A\|$ is spectral norm of matrix A, $\|A\| = [\lambda_{\max}(A^T A)]^{1/2}$.

$C([-\rho, 0], \mathbb{R}^n)$ is the space of continuous function from $[-\rho, 0]$ to $\mathbb{R}^n$. In addition, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Problem Statement and Preliminaries

Given a probability space $\{\Omega, \mathscr{F}, \mathbf{P}\}$ where Ω is the sample space, $\mathscr{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathscr{F}$. $\{r_t, t \geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in a finite set $S = \{1, 2, 3, \ldots, N\}$, with the mode transition probability matrix

$$P\left(r_{t+\Delta t} = j \mid r_t = i\right) = \begin{cases} \pi_{ij}\Delta t + o\left(\Delta t\right) & i \neq j, \\ 1 + \pi_{ii}\Delta t + o\left(\Delta t\right) & i = j, \end{cases} \quad (1)$$

where $\Delta t > 0$, $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$, and $\pi_{ij} \geq 0$ ($i, j \in S$, $i \neq j$) denote the transition rate from mode i to j. For any state or mode $i \in S$, we have

$$\pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij}. \quad (2)$$

In this paper, we consider the following uncertain neutral systems with Markovian jump parameters and time-varying delay over the space $\{\Omega, \mathscr{F}, \mathbf{P}\}$ as follows:

$$\dot{x}(t) - C\left(r_t\right)\dot{x}(t - \tau) = \left[A\left(r_t\right) + \Delta A\left(r_t\right)\right]x(t)$$
$$+ \left[B\left(r_t\right) + \Delta B\left(r_t\right)\right]x(t - d(t)), \quad (3)$$

$$x(s) = \varphi(s), \quad r_s = r_0, \quad s \in \left[-\rho, 0\right], \quad (4)$$

where $x(t) \in \mathbb{R}^n$ is the system state and $\tau > 0$ is a constant neutral delay. It is assumed that the time-varying delay $d(t)$ satisfies

$$0 < d_1 \leq d(t) \leq d_2, \qquad \dot{d}(t) \leq \mu, \quad (5)$$

where $d_1 < d_2$ and $\mu \geq 0$ are constant real values. The initial condition $\varphi(s)$ is a continuously differentiable vector-valued function. The continuous norm of $\varphi(s)$ is defined as

$$\|\varphi\|_c = \max_{s \in [-\rho, 0]} |\varphi(s)|, \quad \rho = \max\{\tau, d_2\}; \quad (6)$$

$A(r_t) \in \mathbb{R}^{n \times n}$, $B(r_t) \in \mathbb{R}^{n \times n}$, and $C(r_t) \in \mathbb{R}^{n \times n}$ are known mode-dependent constant matrices, while $\Delta A(r_t) \in \mathbb{R}^{n \times n}$ and $\Delta B(r_t) \in \mathbb{R}^{n \times n}$ are uncertainties. For notational simplicity, when $r_t = i \in S$, $A(r_t)$, $\Delta A(r_t)$, $B(r_t)$, $\Delta B(r_t)$, and $C(r_t)$ are, respectively, denoted as A_i, ΔA_i, B_i, ΔB_i, and C_i. Throughout this paper, the parametric matrix $\|C_i\| < 1$ and the admissible parametric uncertainties are assumed to satisfy the following condition:

$$\left[\Delta A_i(t) \quad \Delta B_i(t)\right] = H_i F_i(t)\left[E_{Ai} \quad E_{Bi}\right], \quad (7)$$

where H_i, E_{Ai}, and E_{Bi} are known mode-dependent constant matrices with appropriate dimensions and $F_i(t)$ is an unknown and time-varying matrix satisfying

$$F_i^T(t) F_i(t) \leq I, \quad \forall t. \quad (8)$$

Particularly, when we consider $F_i(t) = 0$, we get the nominal systems which can be described as

$$\dot{x}(t) - C_i \dot{x}(t - \tau) = A_i x(t) + B_i x(t - d(t)). \quad (9)$$

Before proceeding further, the following assumptions, definitions, and lemmas need to be introduced.

Assumption 1. The system matrix A_i (for all $i \in S$) is Hurwitz matrix with all the eigenvalues having negative real parts for each mode. The matrix H_i (for all $i \in S$) is chosen as a full row rank matrix.

Assumption 2. The Markov process is irreducible, and the system mode r_t is available at time t.

With regard to neutral systems, the operator $\mathfrak{D}$: $C([-\rho, 0], \mathbb{R}^n) \to \mathbb{R}^n$ is defined to be

$$\mathfrak{D}(x_t) = x(t) - Cx(t - \tau). \quad (10)$$

Then, the stability of operator $\mathfrak{D}$ is defined as follows.

Definition 3 (see [4]). The operator $\mathfrak{D}$ is said to be stable if the homogeneous difference equation

$$\mathfrak{D}(x_t) = 0, \quad t \geq 0,$$
$$x_0 = \psi \in \{\phi \in C([-\rho, 0], \mathbb{R}^n) : \mathfrak{D}\phi = 0\} \quad (11)$$

is uniformly asymptotically stable. In order to guarantee the stability of the operator $\mathfrak{D}$, one has assumed that $\|C_i\| < 1$ as previosuly mentioned, which was introduced in [24].

Definition 4 (see [25]). The systems which are described in (3) are said to be stochastically stable if there exists a positive constant Υ such that

$$\mathscr{E}\left\{\int_0^\infty \|x(r_t, t)\|^2 dt \mid \varphi(s), s \in [-\rho, 0], r_0\right\} < \Upsilon. \quad (12)$$

Definition 5 (see [26]). In the Euclidean space $\{\mathbb{R}^n \times S \times R^+\}$, where $x(t) \in \mathbb{R}^n$, $r_t \in S$, and $t \in R^+$, one introduces the stochastic Lyapunov-Krasovskii function of system (3) as $V(x(t), r_t = i, \ t > 0) = V(x_t, i, t)$, the infinitesimal generator satisfying

$$\mathfrak{L}V(x(t), i, t)$$
$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left[\mathscr{E}\left\{V(x(t + \Delta t), r_{t+\Delta t}, t + \Delta t) \mid x(t) = x, r_t = i\right\}\right.$$
$$\left. - V(x(t), i, t)\right]$$
$$= \frac{\partial}{\partial t}V(x(t), i, t) + \frac{\partial}{\partial x}V(x(t), i, t)\dot{x}(t)$$
$$+ \sum_{j=1}^N \pi_{ij}V(x(t), j, t). \quad (13)$$

Lemma 6 (see [27, 28]). *For any constant matrix $H = H^T > 0$ and scalars $\tau_2 > \tau_1 > 0$ such that the following integrations are well defined, then*

(a) $-(\tau_2 - \tau_1)\displaystyle\int_{t-\tau_2}^{t-\tau_1} x^T(s)Hx(s)\,ds$

$$\leq -\left[\int_{t-\tau_2}^{t-\tau_1} x^T(s)\,ds\right] H \left[\int_{t-\tau_2}^{t-\tau_1} x(s)\,ds\right],$$

(b) $-\dfrac{1}{2}(\tau_2^2 - \tau_1^2)\displaystyle\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^t x^T(s)Hx(s)\,dsd\theta$

$$\leq -\left[\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^t x^T(s)\,dsd\theta\right] H \left[\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^t x(s)\,dsd\theta\right]. \quad (14)$$

Lemma 7 (see [19]). *Suppose that $0 \leq \tau_m \leq \tau(t) \leq \tau_M$, Ξ_1, Ξ_2, and Ω are constant matrices of appropriate dimensions, then*

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \quad (15)$$

if and only if $(\tau_M - \tau_m)\Xi_1 + \Omega < 0$ and $(\tau_M - \tau_m)\Xi_2 + \Omega < 0$ hold.

Lemma 8 (see [29]). *For given matrices $Q = Q^T$, M, and N with appropriate dimensions,*

$$Q + MF(t)N + N^T F^T(t)M^T < 0 \quad (16)$$

for all $F(t)$ satisfying $F^T(t)F(t) \leq I$ if and only if there exists a scalar $\delta > 0$ such that

$$Q + \delta^{-1}MM^T + \delta NN^T < 0. \quad (17)$$

Lemma 9 (see [30]). *Given constant matrices Ω_1, Ω_2, and Ω_3, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1}\Omega_3 < 0$ if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3^T \\ * & \Omega_1 \end{bmatrix} < 0. \quad (18)$$

3. Main Results

In this section, we first consider the nominal systems described by (9) and extend to the uncertain case. The following theorems present sufficient conditions to guarantee the stochastic stability for the neutral systems with Markovian jump parameters and time-varying delays.

3.1. Stochastic Stability for the Nominal Systems

Theorem 10. *For the given finite set S of modes with transition rates matrix, scalars d_1, d_2, τ, and μ, the neutral systems with Markovian jump parameters and time-varying delays as described by (9) are stochastically stable if the operator $\mathfrak{D}$ is stable, and there exist symmetric positive matrices $P_i > 0$ ($i \in S$), $Q_j > 0$ ($j = 1, 2, 3, 4, 5$), and $R_k > 0$ ($k = 1, 2, \ldots, 11$) such that the following linear matrix inequalities hold:*

$$\Pi_{i1} + \Gamma^T M\Gamma < 0,$$
$$\Pi_{i2} + \Gamma^T M\Gamma < 0, \quad (19)$$

where

$$\Pi_{i1} = \Pi_{i0} - 2\left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right) - \left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)$$
$$- e_8 R_9 e_8^T - 2e_9 R_9 e_9^T,$$

$$\Pi_{i2} = \Pi_{i0} - \left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right) - 2\left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)$$
$$- 2e_8 R_9 e_8^T - e_9 R_9 e_9^T,$$

$$(20)$$

where

$$\Pi_{i0} = e_1 Y e_1^T - e_2\left[\left(1-\mu\right)R_3\right]e_2^T + e_1 P_i B_i e_2^T + e_2 B_i^T P_i e_1^T$$
$$+ e_1 P_i C_i e_{11}^T + e_{11} C_i^T P_i e_1^T + e_3\left(R_2 + R_3 - R_1\right)e_3^T$$
$$- e_4 R_2 e_4^T + e_5\left(R_5 - R_4\right)e_5^T - e_6 R_5 e_6^T - e_7 R_8 e_7^T$$
$$- e_{10} Q_1 e_{10}^T - e_{11} Q_2 e_{11}^T - e_{12} Q_3 e_{12}^T$$
$$- \left(e_1 - e_{10}\right)Q_4\left(e_1^T - e_{10}^T\right) - \left(e_1 - e_3\right)R_6\left(e_1^T - e_3^T\right)$$
$$- \left(\tau e_1 - e_{12}\right)Q_5\left(\tau e_1^T - e_{12}^T\right)$$
$$- \left(d_1 e_1 - e_7\right)R_{10}\left(d_1 e_1^T - e_7^T\right)$$
$$- \left(d_{12} e_1 - e_8 - e_9\right)R_{11}\left(d_{12} e_1^T - e_8^T - e_9^T\right),$$

$$(21)$$

where $e_i \{i = 1, 2, \ldots, 12\}$ are block entry matrices; for instance,

$$e_2^T = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Y = A_i^T P_i + P_i A_i + \sum_{j=1}^N \pi_{ij} p_j$$
$$+ Q_1 + \tau^2 Q_3 + R_1 + d_1^2 R_8 + d_{12}^2 R_9,$$

$$M = Q_2 + \tau^2 Q_4 + \frac{\tau^4}{4}Q_5 + R_4$$
$$+ d_1^2 R_6 + d_{12}^2 R_7 + \frac{d_1^4}{4}R_{10} + d_m^2 R_{11},$$

$$(22)$$

$$\Gamma = \begin{bmatrix} A_i & B_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_i & 0 \end{bmatrix},$$

$$d_{12} = d_2 - d_1,$$

$$d_m = \frac{1}{2}\left(d_2^2 - d_1^2\right).$$

Proof. Construct the novel Lyapunov functional as follows:

$$V\left(x\left(t\right),i,t\right) = V_r\left(x_t,i\right) + V_\tau\left(x_t,i\right) + V_{d1}\left(x_t,i\right)$$
$$+ V_{d2}\left(x_t,i\right) + V_{d3}\left(x_t,i\right),$$

$$(23)$$

where

$$V_r\left(x_t,i\right) = x^T\left(t\right)P_i x\left(t\right),$$

$$V_\tau\left(x_t,i\right) = \int_{t-\tau}^t x^T\left(s\right)Q_1 x\left(s\right)ds + \int_{t-\tau}^t \dot{x}^T\left(s\right)Q_2 \dot{x}\left(s\right)ds$$
$$+ \int_{-\tau}^0 \int_{t+\theta}^t x^T\left(s\right)\left[\tau Q_3\right]x\left(s\right)dsd\theta$$
$$+ \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T\left(s\right)\left[\tau Q_4\right]\dot{x}\left(s\right)dsd\theta$$
$$+ \int_{-\tau}^0 \int_\theta^0 \int_{t+\lambda}^t \dot{x}^T\left(s\right)\left[\frac{\tau^2}{2}Q_5\right]\dot{x}\left(s\right)dsd\lambda d\theta,$$

$$V_{d1}\left(x_t,i\right) = \int_{t-d_1}^t x^T\left(s\right)R_1 x\left(s\right)ds + \int_{t-d_2}^{t-d_1} x^T\left(s\right)R_2 x\left(s\right)ds$$
$$+ \int_{t-d(t)}^{t-d_1} x^T\left(s\right)R_3 x\left(s\right)ds$$
$$+ \int_{t-d_1}^t \dot{x}^T\left(s\right)R_4 \dot{x}\left(s\right)ds$$
$$+ \int_{t-d_2}^{t-d_1} \dot{x}^T\left(s\right)R_5 \dot{x}\left(s\right)ds,$$

$$V_{d2}\left(x_t,i\right) = \int_{-d_1}^0 \int_{t+\theta}^t \dot{x}^T\left(s\right)\left[d_1 R_6\right]\dot{x}\left(s\right)dsd\theta$$
$$+ \int_{-d_2}^{-d_1} \int_{t+\theta}^t \dot{x}^T\left(s\right)\left[d_{12} R_7\right]\dot{x}\left(s\right)dsd\theta$$
$$+ \int_{-d_1}^0 \int_{t+\theta}^t x^T\left(s\right)\left[d_1 R_8\right]x\left(s\right)dsd\theta$$
$$+ \int_{-d_2}^{-d_1} \int_{t+\theta}^t x^T\left(s\right)\left[d_{12} R_9\right]x\left(s\right)dsd\theta,$$

$$V_{d3}\left(x_t,i\right) = \int_{-d_1}^0 \int_\theta^0 \int_{t+\lambda}^t \dot{x}^T\left(s\right)\left[\frac{d_1^2}{2}R_{10}\right]\dot{x}\left(s\right)dsd\lambda d\theta$$
$$+ \int_{-d_2}^{-d_1} \int_\theta^0 \int_{t+\lambda}^t \dot{x}^T\left(s\right)\left[d_m R_{11}\right]\dot{x}\left(s\right)dsd\lambda d\theta.$$

$$(24)$$

From Definition 5, taking $\mathfrak{L}$ as its infinitesimal generator along the trajectory of system (9), then from (23) and (24) we get the following equalities and inequalities:

$$\mathfrak{L}V\left(x\left(t\right),i,t\right) = \mathfrak{L}V_r\left(x_t,i\right) + \mathfrak{L}V_\tau\left(x_t,i\right) + \mathfrak{L}V_{d1}\left(x_t,i\right)$$
$$+ \mathfrak{L}V_{d2}\left(x_t,i\right) + \mathfrak{L}V_{d3}\left(x_t,i\right),$$

$$(25)$$

$$\mathfrak{L}V_r\left(x_t,i\right) = 2\left[x^T\left(t\right)A_i^T + x^T\left(t - d\left(t\right)\right)B_i^T\right.$$
$$\left. + \dot{x}^T\left(t - \tau\right)C_i^T\right]P_i x\left(t\right)$$
$$+ \sum_{j=1}^N \pi_{ij} x^T\left(t\right)P_j x\left(t\right),$$

$$(26)$$

$$\mathfrak{L}V_\tau\left(x_t, i\right) = x^T(t)\left[Q_1 + \tau^2 Q_3\right] x(t)$$

$$+ \dot{x}^T(t)\left[Q_2 + \tau^2 Q_4 + \frac{\tau^4}{4}Q_5\right]\dot{x}(t)$$

$$- x^T(t-\tau) Q_1 x(t-\tau)$$

$$- \dot{x}^T(t-\tau) Q_2 \dot{x}(t-\tau)$$

$$- \int_{t-\tau}^t x^T(s)\left[\tau Q_3\right] x(s)\, ds$$

$$- \int_{t-\tau}^t \dot{x}^T(s)\left[\tau Q_4\right]\dot{x}(s)\, ds$$

$$- \int_{-\tau}^0\int_{t+\theta}^t \dot{x}(s)\left[\frac{\tau^2}{2}Q_5\right]\dot{x}(s)\, ds d\theta,$$

$$\mathfrak{L}V_{d1}\left(x_t, i\right) = x^T(t) R_1 x(t) + \dot{x}^T(t) R_4 \dot{x}(t)$$

$$+ x^T(t-d_1)\left[R_2 + R_3 - R_1\right] x(t-d_1)$$

$$+ \dot{x}^T(t-d_1)\left[R_5 - R_4\right]\dot{x}(t-d_1)$$

$$- \dot{x}^T(t-d_2) R_5 \dot{x}(t-d_2)$$

$$- x^T(t-d_2) R_2 x(t-d_2)$$

$$- \left(1 - \dot{d}(t)\right) x^T(t-d(t)) R_3 x(t-d(t)),$$

$$\mathfrak{L}V_{d2}\left(x_t, i\right) = x^T(t)\left[d_1^2 R_8 + d_{12}^2 R_9\right] x(t)$$

$$+ \dot{x}^T(t)\left[d_1^2 R_6 + d_{12}^2 R_7\right]\dot{x}(t)$$

$$- \int_{t-d_1}^t \dot{x}^T(s)\left[d_1 R_6\right]\dot{x}(s)\, ds$$

$$- \int_{t-d_2}^{t-d_1} \dot{x}^T(s)\left[d_{12} R_7\right]\dot{x}(s)\, ds$$

$$- \int_{t-d_1}^t x^T(s)\left[d_1 R_8\right] x(s)\, ds$$

$$- \int_{t-d_2}^{t-d_1} x^T(s)\left[d_{12} R_9\right] x(s)\, ds,$$

$$\mathfrak{L}V_{d3}\left(x_t, i\right) = \dot{x}^T(t)\left[\frac{d_1^4}{4} R_{10} + d_m^2 R_{11}\right]\dot{x}(t)$$

$$- \int_{-d_1}^0\int_{t+\theta}^t \dot{x}^T(s)\left[\frac{d_1^2}{2} R_{10}\right]\dot{x}(s)\, ds d\theta$$

$$- \int_{-d_2}^{-d_1}\int_{t+\theta}^t \dot{x}^T(s)\left[d_m R_{11}\right]\dot{x}(s)\, ds d\theta.$$

$$(27)$$

Let us define

$$\xi(t)$$

$$= \text{col}\left\{ x(t)\ \ x(t-d(t))\ \ x(t-d_1)\ \ x(t-d_2)\ \ \dot{x}(t-d_1)\right.$$

$$\dot{x}(t-d_2)\ \int_{t-d_1}^t x(s)\, ds\ \int_{t-d(t)}^{t-d_1} x(s)\, ds$$

$$\left.\int_{t-d_2}^{t-d(t)} x(s)\, ds\ x(t-\tau)\ \dot{x}(t-\tau)\ \int_{t-\tau}^t x(s)\, ds\right\}.$$

$$(28)$$

Applying (a) of Lemma 6, we obtain

$$- \int_{t-\tau}^t x^T(s)\left[\tau Q_3\right] x(s)\, ds$$

$$\leq -\left[\int_{t-\tau}^t x^T(s)\, ds\right] Q_3 \left[\int_{t-\tau}^t x(s)\, ds\right]$$

$$= -\xi^T(t) e_{12} Q_3 e_{12}^T \xi(t).$$

$$(29)$$

Following the same procedure, we also obtain the inequalities as follows:

$$- \int_{t-\tau}^t \dot{x}^T(s)\left[\tau Q_4\right]\dot{x}(s)\, ds$$

$$\leq -\xi^T(t)\left(e_1 - e_{10}\right) Q_4 \left(e_1^T - e_{10}^T\right)\xi(t),$$

$$- \int_{t-d_1}^t \dot{x}^T(s)\left[d_1 R_6\right]\dot{x}(s)\, ds$$

$$\leq -\xi^T(t)\left(e_1 - e_3\right) R_6 \left(e_1^T - e_3^T\right)\xi(t),$$

$$- \int_{t-d_1}^t x^T(s)\left[d_1 R_8\right] x(s)\, ds$$

$$\leq -\left[\int_{t-d_1}^t x^T(s)\, ds\right] R_8 \left[\int_{t-d_1}^t x(s)\, ds\right].$$

$$(30)$$

Applying (b) of Lemma 6, we have

$$- \int_{-\tau}^0\int_{t+\theta}^t \dot{x}^T(s)\left[\frac{\tau^2}{2} Q_5\right]\dot{x}(s)\, ds d\theta$$

$$\leq -\left[\int_{-\tau}^0\int_{t+\theta}^t \dot{x}^T(s)\, ds d\theta\right] Q_5 \left[\int_{-\tau}^0\int_{t+\theta}^t \dot{x}(s)\, ds d\theta\right]$$

$$= -\left[\tau x^T(t) - \int_{t-\tau}^t x^T(s)\, ds\right] Q_5 \left[\tau x(t) - \int_{t-\tau}^t x(s)\, ds\right]$$

$$= -\xi^T(t)\left(\tau e_1 - e_{12}\right) Q_5 \left(\tau e_1^T - e_{12}^T\right)\xi(t).$$

$$(31)$$

Then the following inequalities are obtained by the same technique:

$$-\int_{-d_1}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\left[\frac{d_1^2}{2}R_{10}\right]\dot{x}(s)\,ds\,d\theta$$

$$\leq -\xi^T(t)\left(d_1 e_1 - e_7\right)R_{10}\left(d_1 e_1^T - e_7^T\right)\xi(t),$$

$$-\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{x}^T(s)\left[d_m R_{11}\right]\dot{x}(s)\,ds\,d\theta$$

$$\leq -\xi^T(t)\left(d_{12} e_1 - e_8 - e_9\right)R_{11}\left(d_{12} e_1^T - e_8^T - e_9^T\right)\xi(t).$$

(32)

Let $\lambda(t) = (d(t) - d_1)/d_{12}$, then we have

$$-\int_{t-d_2}^{t-d_1}\dot{x}^T(s)\left[d_{12}R_7\right]\dot{x}(s)\,ds$$

$$= -d_{12}\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)R_7\dot{x}(s)\,ds$$

$$\quad - d_{12}\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)R_7\dot{x}(s)\,ds$$

$$= -\left(d_2 - d(t)\right)\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)R_7\dot{x}(s)\,ds$$

$$\quad - \left(d(t) - d_1\right)\int_{t-d_2}^{t-d(t)}\dot{x}^T(s)R_7\dot{x}(s)\,ds$$

$$\quad - \left(d(t) - d_1\right)\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)R_7\dot{x}(s)\,ds$$

$$\quad - \left(d_2 - d(t)\right)\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)R_7\dot{x}(s)\,ds$$

$$= -\xi^T(t)\left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right)\xi(t)$$

$$\quad - \frac{d(t) - d_1}{d_2 - d(t)}\xi^T(t)\left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right)\xi(t)$$

$$\quad - \xi^T(t)\left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)\xi(t)$$

$$\quad - \frac{d_2 - d(t)}{d(t) - d_1}\xi^T(t)\left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)\xi(t)$$

$$\leq -\xi^T(t)\left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right)\xi(t)$$

$$\quad - \lambda(t)\xi^T(t)\left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right)\xi(t)$$

$$\quad - \xi^T(t)\left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)\xi(t)$$

$$\quad - (1 - \lambda(t))\xi^T(t)\left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)\xi(t)$$

$$= -(1 + \lambda(t))\xi^T(t)\left(e_2 - e_4\right)R_7\left(e_2^T - e_4^T\right)\xi(t)$$

$$\quad - (2 - \lambda(t))\xi^T(t)\left(e_3 - e_2\right)R_7\left(e_3^T - e_2^T\right)\xi(t).$$

(33)

Consistent with the technique of (33), we obtain

$$-\int_{t-d_2}^{t-d_1}x^T(s)\left[d_{12}R_9\right]x(s)\,ds$$

$$\leq -(2 - \lambda(t))\xi^T(t)\left(e_8 R_9 e_8^T\right)\xi(t)$$

$$\quad - (1 + \lambda(t))\xi^T(t)\left(e_9 R_9 e_9^T\right)\xi(t),$$

(34)

considering

$$\dot{x}^T(t)M\dot{x}(t) = \left(A_i x(t) + B_i x(t - d(t)) + C_i \dot{x}(t - \tau)\right)^T$$

$$\quad \times M\left(A_i x(t) + B_i x(t - d(t)) + C_i \dot{x}(t - \tau)\right)$$

$$= \xi^T(t)\Gamma^T M\Gamma\xi(t),$$

(35)

where M, Γ have been defined as before.

We take the previous equalities and inequalities (26)–(35) into (25); thus, we finally get

$$\mathfrak{L}V(x(t), i, t) \leq \xi^T(t)\left[\lambda(t)\Pi_{i1} + (1 - \lambda(t))\Pi_{i2} + \Gamma^T M\Gamma\right]$$

$$\quad \times \xi(t).$$

(36)

Since $0 \leq \lambda(t) \leq 1$, by utilizing Lemma 7, we know that $\lambda(t)\Pi_{i1} + (1 - \lambda(t))\Pi_{i2} + \Gamma^T M\Gamma < 0$ is equivalent to (19). So we choose

$$\beta = \max_{i \in S, \lambda(t) \in [0,1]}\lambda_{\max}\left[\lambda(t)\Pi_{i1} + (1 - \lambda(t))\Pi_{i2} + \Gamma^T M\Gamma\right].$$

(37)

Then $\beta < 0$ and

$$\mathfrak{L}V(x(t), i, t) \leq \beta\|\xi(t)\|^2 \leq \beta\|x(t)\|^2.$$ (38)

According to (38), from Dynkin's formula [31], we obtain

$$\mathscr{E}\{V(x(t), i, t)\} - V(x_0, r_0) \leq \beta\mathscr{E}\left\{\int_0^t \|x(s)\|^2 ds\right\}.$$ (39)

Let $t \to \infty$, then we have

$$\lim_{t \to \infty}\mathscr{E}\left\{\int_0^t \|x(s)\|^2 ds\right\} \leq (-\beta)^{-1}V(x_0, r_0).$$ (40)

From Definition 4, we know that the systems described by (9) are stochastically stable. This completes the proof. □

Remark 11. Theorem 10 provides a delay-range-dependent stochastic stability criterion for nominal neutral systems with interval time-varying delays and Markovian jump parameters as described by (9). By utilizing a new Lyapunov functional, the less conservative criterion is obtained in terms of LMIs and it can be verified in Section 4.

Remark 12. In the same context of the stochastic stability for neutral systems with Markovian jumping parameters

and time-varying delays, the type of augmented Lyapunov functional has not been used in any of the existing literatures. Compared with the existing Lyapunov functional, the proposed one (23) contains some triple-integral terms, which is very effective in the reduction of conservativeness in [28]. Besides, the information on the lower bound of the delay is sufficiently used in the Lyapunov functional by introducing the terms, such as $\int_{t-d_2}^{t-d_1} x^T(s)R_2 x(s)ds$ and $\int_{t-d(t)}^{t-d_1} x^T(s)R_3 x(s)ds$.

In many circumstances, the information on the delay derivative may not be available. That is, μ is usually unknown in the real systems. So we give the following result as a corollary which can be obtained from Theorem 10 by setting $R_3 = 0$.

Corollary 13. *For the given finite set S of modes with transition rates matrix, scalars d_1, d_2, and τ, the neutral systems with Markovian jump parameters and time-varying delays as described by (9) are stochastically stable if the operator $\mathfrak{D}$ is stable, and there exist symmetric positive matrices $P_i > 0$ ($i \in S$), $Q_j > 0$ ($j = 1, 2, 3, 4, 5$), and $R_k > 0$ ($k = 1, 2, 4, 5, \ldots, 11$) such that the following linear matrix inequalities hold:*

$$\widetilde{\Pi}_{i1} + \Gamma^T M \Gamma < 0,$$
$$\widetilde{\Pi}_{i2} + \Gamma^T M \Gamma < 0, \tag{41}$$

where

$$\widetilde{\Pi}_{i1} = \widetilde{\Pi}_{i0} - 2(e_2 - e_4) R_7 \left(e_2^T - e_4^T\right)$$
$$- (e_3 - e_2) R_7 \left(e_3^T - e_2^T\right) - e_8 R_9 e_8^T - 2e_9 R_9 e_9^T, \tag{42}$$
$$\widetilde{\Pi}_{i2} = \widetilde{\Pi}_{i0} - (e_2 - e_4) R_7 \left(e_2^T - e_4^T\right)$$
$$- 2(e_3 - e_2) R_7 \left(e_3^T - e_2^T\right) - 2e_8 R_9 e_8^T - e_9 R_9 e_9^T,$$

where

$$\widetilde{\Pi}_{i0} = e_1 Y e_1^T + e_1 P_i B_i e_2^T + e_2 B_i^T P_i e_1^T + e_1 P_i C_i e_{11}^T + e_{11} C_i^T P_i e_1^T$$
$$+ e_3 (R_2 - R_1) e_3^T - e_4 R_2 e_4^T + e_5 (R_5 - R_4) e_5^T - e_6 R_5 e_6^T$$
$$- e_7 R_8 e_7^T - e_{10} Q_1 e_{10}^T - e_{11} Q_2 e_{11}^T - e_{12} Q_3 e_{12}^T$$
$$- (e_1 - e_{10}) Q_4 \left(e_1^T - e_{10}^T\right) - (e_1 - e_3) R_6 \left(e_1^T - e_3^T\right)$$
$$- (\tau e_1 - e_{12}) Q_5 \left(\tau e_1^T - e_{12}^T\right)$$
$$- (d_1 e_1 - e_7) R_{10} \left(d_1 e_1^T - e_7^T\right)$$
$$- (d_{12} e_1 - e_8 - e_9) R_{11} \left(d_{12} e_1^T - e_8^T - e_9^T\right), \tag{43}$$

and other notations are the same as Theorem 10.

3.2. Stochastic Stability for the Uncertain Neutral Markovian Jump Systems. In this subsection, we consider the uncertain

case which can be described by (3). Based on Theorem 10, we obtain the following theorem to guarantee the stochastic stability for the uncertain neutral systems with interval time-varying delays and Markovian jump parameters.

Theorem 14. *For the given finite set S of modes with transition rates matrix, scalars d_1, d_2, τ, and μ, the uncertain neutral systems with Markovian jump parameters and time-varying delays as described by (3) are stochastically stable if the operator $\mathfrak{D}$ is stable, and there exist scalars $\delta_1 > 0$, $\delta_2 > 0$, symmetric positive matrices $P_i > 0$ ($i \in S$), $Q_j > 0$ ($j = 1, 2, 3, 4, 5$), and $R_k > 0$ ($k = 1, 2, \ldots, 11$) such that the following matrix inequalities hold:*

$$\begin{bmatrix} \Pi_{i1} + \frac{1}{\delta_1} e_1 P_i H_i H_i^T P_i e_1^T + \delta_1 \varepsilon \varepsilon^T & \left(\Gamma^T + \frac{1}{\delta_1} e_1 P_i H_i H_i^T\right) M \\ * & \frac{1}{\delta_1} M H_i H_i^T M - M \end{bmatrix} < 0, \tag{44}$$

$$\begin{bmatrix} \Pi_{i2} + \frac{1}{\delta_2} e_1 P_i H_i H_i^T P_i e_1^T + \delta_2 \varepsilon \varepsilon^T & \left(\Gamma^T + \frac{1}{\delta_2} e_1 P_i H_i H_i^T\right) M \\ * & \frac{1}{\delta_2} M H_i H_i^T M - M \end{bmatrix} < 0, \tag{45}$$

where $\varepsilon^T = E_{Ai} e_1^T + E_{Bi} e_2^T$, Π_{i1}, Π_{i2}, Γ, and M have been defined in Theorem 10.

Proof. On the basis of Theorem 10, we directly replace A_i and B_i with $A_i + \Delta A_i(t)$, $B_i + \Delta B_i(t)$ and obtain

$$\Pi_{i1}(t) + \Gamma^T(t) M \Gamma(t) < 0, \tag{46}$$
$$\Pi_{i2}(t) + \Gamma^T(t) M \Gamma(t) < 0, \tag{47}$$

where

$$\Pi_{i1}(t) = \Pi_{i1} + e_1 \left[\Delta A_i^T(t) P_i + P_i \Delta A_i(t)\right] e_1^T$$
$$+ e_1 P_i \Delta B_i(t) e_2^T + e_2 \Delta B_i^T(t) P_i e_1^T,$$
$$\Pi_{i2}(t) = \Pi_{i2} + e_1 \left[\Delta A_i^T(t) P_i + P_i \Delta A_i(t)\right] e_1^T$$
$$+ e_1 P_i \Delta B_i(t) e_2^T + e_2 \Delta B_i^T(t) P_i e_1^T. \tag{48}$$

Considering (46) and combining the uncertainties condition (7) by Lemma 9, we have

$$\begin{bmatrix} \Pi_{i1} & \Gamma^T M \\ M\Gamma & -M \end{bmatrix} + \begin{bmatrix} e_1 P_i & 0 \\ M & 0 \end{bmatrix} H_i F_i(t) \varepsilon^T$$
$$+ \varepsilon F_i^T(t) H_i^T \begin{bmatrix} P_i e_1^T & M \\ 0 & 0 \end{bmatrix} < 0. \tag{49}$$

With (8), by Lemma 8 from (49), we obtain

$$\begin{bmatrix} \Pi_{i1} & \Gamma^T M \\ M\Gamma & -M \end{bmatrix} + \frac{1}{\delta_1} \begin{bmatrix} e_1 P_i & 0 \\ M & 0 \end{bmatrix} H_i H_i^T \begin{bmatrix} P_i e_1^T & M \\ 0 & 0 \end{bmatrix}$$
$$+ \delta_1 \begin{bmatrix} \varepsilon \varepsilon^T & 0 \\ 0 & 0 \end{bmatrix} < 0. \tag{50}$$

Obviously, (50) is equivalent to (44). Similarly, considering (46) and following the same procedure, we can get (45). Finally, following from the latter proof of Theorem 10, we know that the uncertain neutral systems with Markovian jump parameters and time-varying delay as described by (3) are stochastically stable. This completes the proof. $\square$

Remark 15. It should be noted that (44) and (45) can be viewed as linear matrix inequalities by introducing new variables. That is, define matrices

$$P_i^{(1)} = \frac{1}{\delta_1} P_i H_i H_i^T P_i, \qquad P_i^{(2)} = \frac{1}{\delta_2} P_i H_i H_i^T P_i,$$

$$P_{iM}^{(1)} = \frac{1}{\delta_1} P_i H_i H_i^T M, \qquad P_{iM}^{(2)} = \frac{1}{\delta_2} P_i H_i H_i^T M, \qquad (51)$$

$$M^{(1)} = \frac{1}{\delta_1} M H_i H_i^T M, \qquad M^{(2)} = \frac{1}{\delta_2} M H_i H_i^T M,$$

where H_i, $i \in S$, are known constant matrices and have been defined in (7). Then (44) and (45) can be easily solved by the LMI Toolbox in MATLAB.

Remark 16. It should be mentioned that Theorem 14 is an extension of Theorem 10 to uncertain neutral Markovian jump systems with interval time-varying delays. It provides a stochastic delay-range-dependent stability criterion for (3) and it will be verified to be less conservative than some existing ones in Section 4.

Consistent with the nominal systems, in the uncertain case, we have the following result as a corollary if the information on the delay derivative μ may not be available. The corollary is also obtained by setting $R_3 = 0$ in Theorem 14.

Corollary 17. *For the given finite set S of modes with transition rates matrix, scalars d_1, d_2, and τ, the uncertain neutral systems with Markovian jump parameters and time-varying delays as described by (3) are stochastically stable if the operator $\mathfrak{D}$ is stable, and there exist scalars $\delta_1 > 0$, $\delta_2 > 0$, symmetric positive matrices $P_i > 0$ ($i \in S$), $Q_j > 0$ ($j = 1, 2, 3, 4, 5$), and $R_k > 0$ ($k = 1, 2, 4, 5, \ldots, 11$) such that the following symmetric matrix inequalities hold:*

$$\begin{bmatrix} \widetilde{\Pi}_{i1} + \dfrac{1}{\delta_1} e_1 P_i H_i H_i^T P_i e_1^T + \delta_1 \varepsilon \varepsilon^T & \left(\Gamma^T + \dfrac{1}{\delta_1} e_1 P_i H_i H_i^T\right) M \\ * & \dfrac{1}{\delta_1} M H_i H_i^T M - M \end{bmatrix} < 0,$$

$$\begin{bmatrix} \widetilde{\Pi}_{i2} + \dfrac{1}{\delta_2} e_1 P_i H_i H_i^T P_i e_1^T + \delta_2 \varepsilon \varepsilon^T & \left(\Gamma^T + \dfrac{1}{\delta_2} e_1 P_i H_i H_i^T\right) M \\ * & \dfrac{1}{\delta_2} M H_i H_i^T M - M \end{bmatrix} < 0, \qquad (52)$$

where $\widetilde{\Pi}_{i1}$ and $\widetilde{\Pi}_{i2}$ have been defined in Corollary 13, the remaining notations are the same as Theorem 14.

4. Numerical Examples

In this section, numerical examples are given to show that the proposed theoretical results in this paper are effective and less conservative than some previous ones in the literature.

Example 1. Consider the nominal system in the form of (9) described as follows:

$$\dot{x}(t) - C_i \dot{x}(t - 0.1) = A_i x(t) + B_i x(t - d(t)), \qquad (53)$$

where $i \in S = \{1, 2\}$ and the mode switching is governed by the rate matrix $[\pi_{ij}]_{2 \times 2} = \begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix}$, which is described by Figure 1

$$A_1 = \begin{bmatrix} 2 & 5 \\ -2 & -3 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.3 & 0.5 \\ -0.2 & -0.3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.2 & 0 \\ 0.1 & -0.2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -5 & -1.6 \\ 2 & -4 \end{bmatrix}, \qquad (54)$$

$$B_2 = \begin{bmatrix} -0.3 & 0.5 \\ -0.2 & -0.3 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -0.1 & 0.2 \\ 0 & -0.2 \end{bmatrix}.$$

Given the time-varying delay $d(t) = 0.5(2 + \sin^3 t)$, from the graph of $d(t)$ with $t \in [0, 2\pi]$ in Figure 2, we easily obtain $d_1 = 0.5$ and $d_2 = 1.5$. In addition, we have $\dot{d}(t) = 1.5 \sin^2 t \cos t$ and its maximum $\mu = \sqrt{3}/3$.

By Theorem 10, with the help of LMI toolbox in MAT-LAB, we solve (19) and get a group of matrices for the solution to guarantee the stochastic stability for the system (53) as follows: for simplicity, we only list the matrices for P_i, $i \in S = \{1, 2\}$, Q_j, $j = 1, 2, \ldots, 5$.

$$P_1 = \begin{bmatrix} 1.4857 & -0.3614 \\ * & 0.6329 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 2.0645 & -0.3761 \\ * & 0.5086 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.7342 & -0.1546 \\ * & 0.2978 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 0.4083 & -0.1873 \\ * & 0.3652 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 0.4165 & -0.2137 \\ * & 0.3056 \end{bmatrix}, \qquad Q_4 = \begin{bmatrix} 0.4576 & -0.2539 \\ * & 0.3684 \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} 0.2673 & -0.0845 \\ * & 0.1766 \end{bmatrix}. \qquad (55)$$

Therefore, it can be seen that the system (9) is determined to be stochastically stable by Theorem 10.

Example 2. As said in the literature [32], with the abrupt variation in its structures and parameters, we can present the partial element equivalent circuit (PEEC) model as a stochastic jump one. Then, a general form of PEEC model is given by (3), where we assume that the neutral delay of the PEEC model is constant. Consider the stochastic neutral partial element equivalent circuit (PEEC) model described by the following equation:

$$\dot{x}(t) - C_i \dot{x}(t - 0.3) = (A_i + \Delta A(t)) x(t) \\ + (B_i + \Delta B(t)) x(t - d(t)), \qquad (56)$$
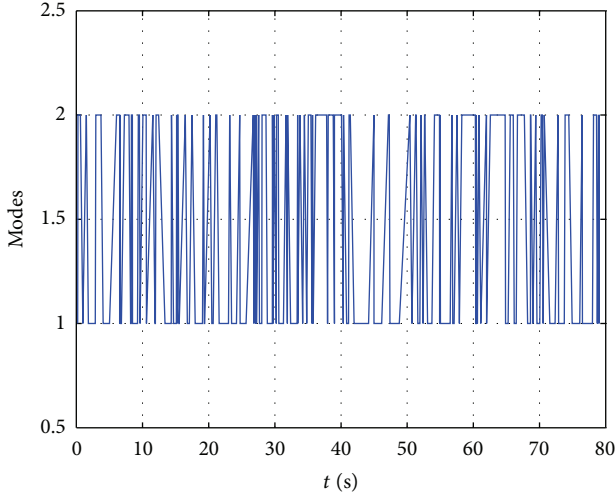
FIGURE 1: Operation modes of Example 1.

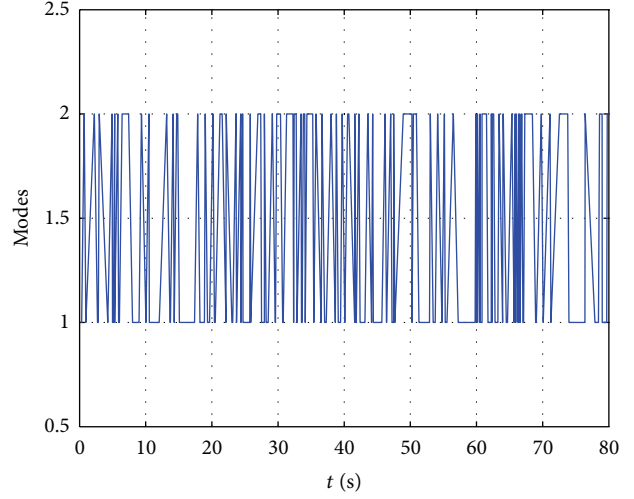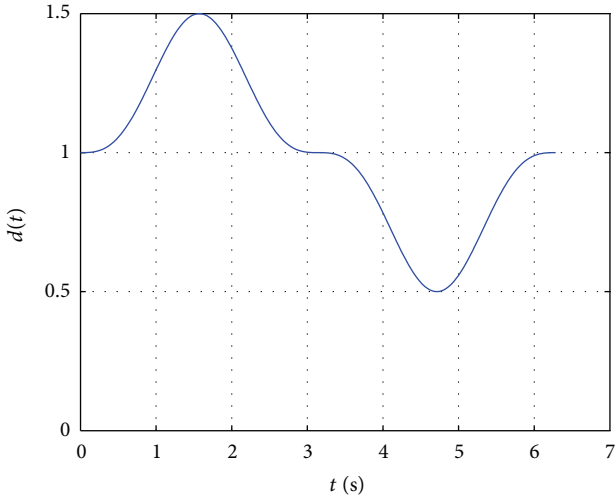FIGURE 3: Operation modes of Example 2.

FIGURE 2: Interval time-varying delay $d(t)$ of Example 1.

where $i \in S = \{1, 2\}$ and the mode switching is governed by the rate matrix $[\pi_{ij}]_{2 \times 2} = \begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}$, which is described by Figure 3

$$A_1 = \begin{bmatrix} -5 & 0 \\ 0 & -6 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -1.6 & 0 \\ -1.8 & -1.5 \end{bmatrix},$$

$$C_1 = 0.5I, \qquad H_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -2 & 0 \\ -0.9 & -1.2 \end{bmatrix}, \qquad (57)$$

$$C_2 = 0.3I, \qquad H_2 = \begin{bmatrix} 0 \\ -0.3 \end{bmatrix},$$

$$E_{A1} = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, \qquad E_{A2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix},$$

$$E_{B1} = \begin{bmatrix} -0.3 & 0.3 \end{bmatrix}, \qquad E_{B2} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}.$$

Given $\|F_i(t)\| < 1$ and the time-varying delay $d(t) = 0.5(2 + \cos^3 t)$, from the graph of $d(t)$ with $t \in [0, 2\pi]$ in Figure 4, we easily obtain $d_1 = 0.5$ and $d_2 = 1.5$. In addition, we have $\dot{d}(t) = -1.5 \cos^2 t \, \sin t$ and its maximum $\mu = \sqrt{3}/3$.

By Theorem 14, with the help of LMI toolbox in MAT-LAB, we solve (44) and (45) and get a group of matrices for the solution to guarantee the stochastic stability for the system (56) as follows: for simplicity, we only list the matrices for P_i, $i \in S = \{1, 2\}$, Q_j, $j = 1, 2, \ldots, 5$.

$$P_1 = \begin{bmatrix} 2.4327 & -0.4713 \\ * & 0.7846 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 2.7685 & -0.4617 \\ * & 0.7432 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.5122 & -0.1558 \\ * & 0.3976 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 0.6083 & -0.1898 \\ * & 0.3976 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 0.3164 & -0.2058 \\ * & 0.4751 \end{bmatrix}, \qquad Q_4 = \begin{bmatrix} 0.2563 & -0.1584 \\ * & 0.3476 \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} 0.1574 & -0.0713 \\ * & 0.1798 \end{bmatrix}. \qquad (58)$$

Therefore, according to Theorem 14, the uncertain neutral PEEC system presented by (3) is stochastically stable.

Example 3. In the study of practical electrical circuit systems, a small test circuit which consists of a partial element equivalent circuit (PEEC) was considered in [33], which can be described as the following form:

$$\dot{x}(t) - C\dot{x}(t - \tau) = Ax(t) + Bx(t - d). \qquad (59)$$

Compared with (9), (59) can be regarded as $i \in S = \{1\}$ and $d(t) = d$. So we have $d_1 = d_2 = d$, $\mu = 0$ and utilize Theorem 10 to compute the maximum discrete delay for system stability.
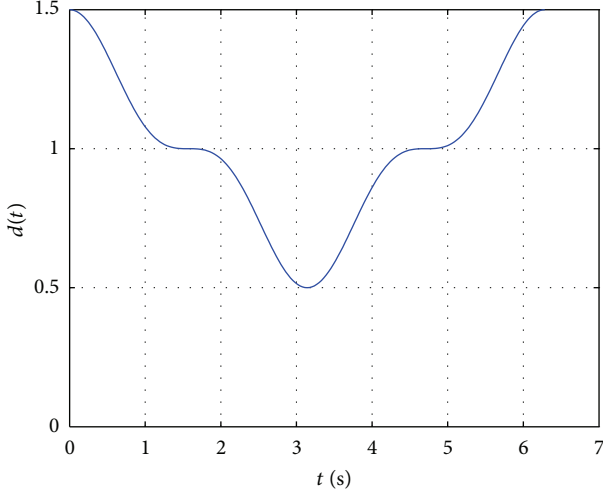
FIGURE 4: Interval time-varying delay $d(t)$ of Example 2.

TABLE 1: Maximum upper bound of d with different neutral delay τ.

Methods	$\tau = 0.1$	$\tau = 0.5$	$\tau = 1$
He et al. [36]	1.7100	1.6718	1.6543
Han [34]	2.1229	2.1229	2.1229
Li et al. [35]	2.2951	2.3471	2.3752
Theorem 10	2.3026	2.3547	2.3835

where A, B, and C are given in Example 3 and the uncertain matrices $\Delta A(t)$ and $\Delta B(t)$ satisfy

$$\|\Delta A(t)\| \leq \kappa, \quad \|\Delta B(t)\| \leq \kappa, \quad \kappa \geq 0. \tag{62}$$

Moreover, in the form of (7) and (8), we assume that

$$H = \kappa I, \quad E_A = E_B = I, \quad 0 \leq \kappa \leq 1. \tag{63}$$

Consider (61), for given τ and κ, by Theorem 14, the maximum upper bound of d, which satisfies the LMIs in (44) and (45), can be calculated by solving a quasiconvex optimization problem. When $\tau = 1.0$, Table 2 gives the comparisons of the maximum allowed delay of d for various parameters κ in different methods.

From Table 2, provided that $\tau = 1.0$, we know that the maximum upper bound of delay $d = 1.5316$ in this paper by setting $\kappa = 0.10$, while the maximum upper bound of delay $d = 1.3864$ for [37], $d = 1.4385$ for [36], and $d = 1.5047$ for [38]. The results are also given by setting $\kappa = 0.15$, $\kappa = 0.20$, and $\kappa = 0.25$, and it is found that the maximum upper bound in this paper is larger than those in [36–38]. So it can be seen that the delay-range-dependent stability condition Theorem 14 in this paper is less conservative than some earlier reported ones in the literature.

Remark 18. It should be pointed out that we require $d_1 \neq d_2$ and $d_1 \neq 0$ in order to conveniently organize this paper. But from the results of the theorems and corollaries in this paper, we know that they are applicable to many special cases, such as $d(t) \equiv d$, $d_1 = d_2$, or $d_1 = 0$, $\tau = 0$. Actually, we just need to delete the corresponding integral terms in the Lyapunov functional and obtain homologous results.

Consider (59) with the following parameters:

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \qquad B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$
$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}. \tag{60}$$

For given τ, by Theorem 10, the maximum d, which satisfies the LMIs in (19), can be calculated by solving a quasiconvex optimization problem. This neutral system was considered in references [34–36]. The results on the maximum upper bound of d are compared in Table 1.

From Table 1, we know that the maximum upper bound of delay $d = 2.3026$ in this paper by setting $\tau = 0.1$, while the maximum upper bound of delay $d = 1.7100$ for [36], $d = 2.1229$ for [34], and $d = 2.2951$ for [35]. The results are also given by setting $\tau = 0.5$ and $\tau = 1$, and it is found that the maximum upper bound in this paper is larger than those in [34–36]. So it can be demonstrated that the stability condition Theorem 10 in this paper yields less conservative results than the previous ones.

Example 4. Further consideration on Example 3, if we take the parameter uncertainties commonly existing in the modeling of a real system into account, a general form of PEEC model is given by

$$\dot{x}(t) - C\dot{x}(t - \tau) = (A + \Delta A(t)) x(t)$$
$$+ (B + \Delta B(t)) x(t - d), \tag{61}$$

Example 5. In this example, to compare the stochastic stability result in Theorem 10 with those in [23, 39, 40], we consider the nominal system (9) with $C_{r_t} = 0$ and $d_1 = 0$. In fact, we consider here that there are no longer neutral delay systems, for given (9) with the following parameters:

$$A_1 = \begin{bmatrix} -3.49 & 0.81 \\ -0.65 & -3.27 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2.49 & 0.29 \\ 1.34 & -0.02 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -0.86 & -1.29 \\ -0.68 & -2.07 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -2.83 & 0.50 \\ -0.84 & -1.01 \end{bmatrix},$$
$$C_1 = C_2 = 0,$$
$$P_{ij} = \left[\pi_{ij}\right]_{2 \times 2}, \quad i, j \in S = \{1, 2\}. \tag{64}$$

As described previously, for given $\pi_{22} = -0.8$, different values of π_{11} and different values of μ, by Theorem 10 and the maximum d_2, which satisfies the LMIs in (19), can be calculated by solving a quasiconvex optimization problem. Tables 3 and 4 give the contrastive results.

From Table 3, provided that $\mu = 0$, we know that the maximum upper bound of delay $d_2 = 0.6853$ in this paper by setting $\pi_{11} = -0.10$, while the maximum upper bound of delay $d_2 = 0.5012$ for [40], $d_2 = 0.5012$ for [39], and

TABLE 2: Maximum upper bound of d with $\tau = 1.0$ and different parameter κ.

Methods	$\kappa = 0.10$	$\kappa = 0.15$	$\kappa = 0.20$	$\kappa = 0.25$
Han [37]	1.3864	1.2705	1.1607	1.0456
He et al. [36]	1.4385	1.3309	1.2396	1.1547
Xu et al. [38]	1.5047	1.4052	1.2998	1.2136
Theorem 14	1.5316	1.4089	1.3028	1.2217

TABLE 3: Maximum upper bound of d_2 with $\mu = 0$ and different parameter π_{11}.

Methods	$\pi_{11} = -0.10$	$\pi_{11} = -0.50$	$\pi_{11} = -0.80$	$\pi_{11} = -1.00$
Cao et al. [40]	0.5012	0.4941	0.4915	0.4903
Chen et al. [39]	0.5012	0.4941	0.4915	0.4903
Xu et al. [23]	0.6797	0.5794	0.5562	0.5465
Theorem 10	0.6853	0.5874	0.5625	0.5574

TABLE 4: Maximum upper bound of d_2 with $\mu = 1.5$ and different parameter π_{11}.

Methods	$\pi_{11} = -0.10$	$\pi_{11} = -0.50$	$\pi_{11} = -0.80$	$\pi_{11} = -1.00$
Cao et al. [40]	—	—	—	—
Chen et al. [39]	—	—	—	—
Xu et al. [23]	0.3860	0.3656	0.3487	0.3378
Theorem 10	0.3953	0.3746	0.3502	0.3449

$d_2 = 0.6797$ for [23]. The results are also given by setting $\pi_{11} = -0.50$, $\pi_{11} = -0.80$, and $\pi_{11} = -1.00$, and it is found that the maximum upper bound of delay d_2 in this paper is larger than those in [23, 39, 40]. So it also can be shown that the stochastic stability result in Theorem 10 is less conservative than those results in [23, 39, 40].

From Table 4, provided that $\mu = 1.5$, we know that the maximum upper bound of delay $d_2 = 0.3953$ in this paper by setting $\pi_{11} = -0.10$, while the methods in [39, 40] cannot be applicable to the case $\mu \geq 1$, and the maximum upper bound of delay $d_2 = 0.3860$ for [23]. So it can be shown that Theorem 10 in this paper is less conservative and can be applied to the time-varying delay without the requirement on $\mu < 1$.

5. Conclusions

In this paper, some new delay-range-dependent conditions have been provided to guarantee the stochastic stability of the neutral systems with Markovian jumping parameters and interval time-varying delays. A novel augmented Lyapunov-Krasovskii functional which contains some triple-integral terms is constructed. By some integral inequalities and the nature of convex combination, some less conservative delay-range-dependent stochastic stability criteria are obtained. Numerical examples are given to demonstrate the effectiveness and less conservativeness of our result.
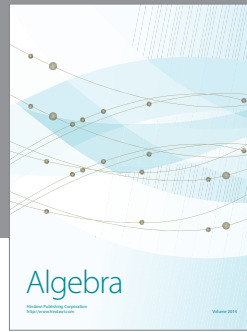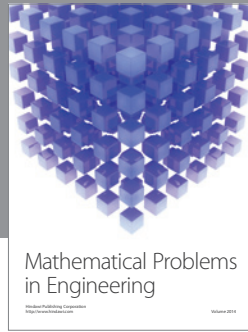
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