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Research Article

Oscillatory Singularity Behaviors Near Interface Crack Tip for Mode II of Orthotropic Bimaterial

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The fracture behaviors near the interface crack tip for mode II of orthotropic bimaterial are discussed. The oscillatory singularity fields are researched. The stress functions are chosen which contain twelve undetermined coefficients and an unknown singularity exponent. Based on the boundary conditions and linear independence, the system of twelve nonhomogeneous linear equations is derived. According to the condition for the system of nonhomogeneous linear equations which has a solution, the singularity exponent is determined. Total coefficients are found by means of successive elimination of the unknowns. The theoretical formulae of stress intensity factors and analytic solutions of stress field near the interface crack tip are obtained. The crack tip field is shown by figures.

1. Introduction

Many researchers have studied the singularity behavior near interface crack tip for isotropic, orthotropic, or anisotropic bimaterial. The method of eigenfunction expansions is used by Williams [1]. The complex function method is developed by Rice and Sih [2]. Erdogan [3] and England [4], present and research various interface crack problems. These academic authorities play a leading role in fracture mechanics for interface crack, and they have made the tremendous contribution. The subsequent papers are published one after another over several decades [5–16]. In this paper, the solution method is proposed to research the singularity behavior near interface crack tip for mode II of orthotropic bimaterial.

It can be seen that the following differences exist between this method and previous methods by comparison.

(1) The stress function contains twelve undetermined coefficients, rather than eight.

(2) The system of twelve nonhomogeneous linear equations is deduced based on the boundary conditions, rather than the system of eight homogeneous linear equations.

(3) The characteristic equation is found by using the condition for the system of nonhomogeneous linear equations that possess a solution, rather than being based on the condition for the system of homogeneous linear equations which has a nontrivial solution.

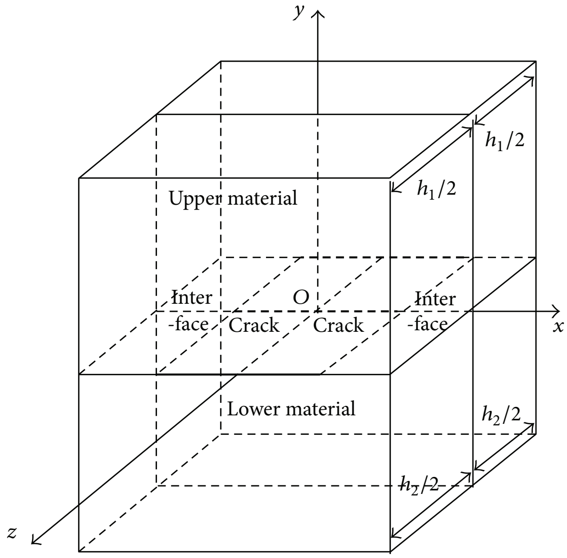
(4) In order to determine total coefficients, we only need to solve the system of nonhomogeneous linear equations, rather than to solve the system of homogeneous linear equations first, and then use the load conditions at infinity.

(5) The stress intensity factors are defined by right-hand limit and left-hand limit rather than by limit.

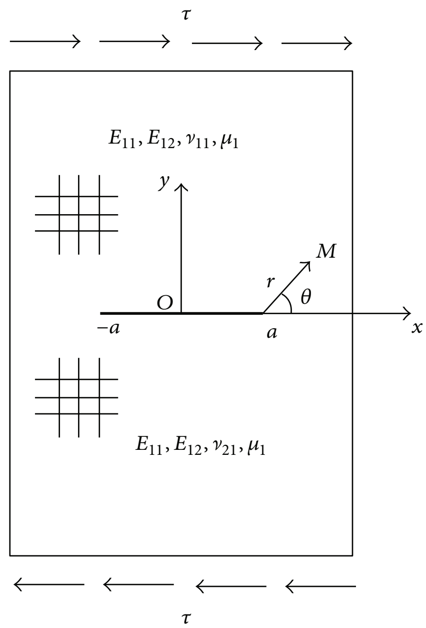
(6) The oscillatory singularity fields near interface crack tip of three orthotropic bimaterial are illustrated by the help of two tables and seven figures.

2. Mechanical Model

The plane $y > 0$ is the upper orthotropic material ($j = 1$), and its elastic constants are E_{11}, E_{12}, ν_{11} and μ_1. The plane $y < 0$ is the lower orthotropic material ($j = 2$), and its elastic constants are E_{21}, E_{22}, ν_{21}, and μ_2. $y = 0, |x| < a$ is the crack surfaces, and $y = 0, |x| > a$ is the bonded interface of two dissimilar materials.



(a) Geometric state



(b) Shear load

FIGURE 1: Interface crack for mode II of orthotropic bimaterial.

The stress functions $U_j(x, y)$, $(j = 1, 2)$ satisfy the governing equations [16–20]:

$$(b_{22})_j \frac{\partial^4 U_j}{\partial x^4} + \left[2(b_{12})_j + (b_{66})_j\right] \frac{\partial^4 U_j}{\partial x^2 \partial y^2} + (b_{11})_j \frac{\partial^4 U_j}{\partial y^4} = 0, \quad (j = 1, 2). \tag{1}$$

The boundary conditions of the interface crack for mode II are as follows:

$$y = 0, \quad |x| < a : \left(\sigma_y\right)_1 = \left(\sigma_y\right)_2 = 0, \quad \left(\tau_{xy}\right)_1 = \left(\tau_{xy}\right)_2 = 0, \tag{2}$$

$$y = 0, \quad |x| > a : \left(\sigma_y\right)_1 = \left(\sigma_y\right)_2, \quad \left(\tau_{xy}\right)_1 = \left(\tau_{xy}\right)_2, \quad (u)_1 = (u)_2, \quad (v)_1 = (v)_2, \tag{3}$$

$$|y| \longrightarrow +\infty : \left(\sigma_y\right)_1 = \left(\sigma_y\right)_2 = 0, \quad \left(\tau_{xy}\right)_1 = \left(\tau_{xy}\right)_2 = \tau. \tag{4}$$

The geometric and load conditions for the investigated problem are given as shown in Figure 1.

3. Stress Function

The relationships between the stresses and the stress functions can be obtained as

$$\left(\sigma_x\right)_j = \frac{\partial^2 U_j}{\partial y^2} = \sum_{k=1}^{2} \operatorname{Re}\left\{\left[\left(A_{jk,\lambda} + iC_{jk,\lambda}\right) - i\left(B_{jk,\lambda} + iC_{jk,\lambda}\right)\right] \times \left(-\beta_{jk}^2\right) U_{jk,\lambda}\left(z_{jk}\right)\right\}, \tag{5a}$$

$$\left(\sigma_y\right)_j = \frac{\partial^2 U_j}{\partial x^2} = \sum_{k=1}^{2} \operatorname{Re}\left\{\left[\left(A_{jk,\lambda} + iC_{jk,\lambda}\right) - i\left(B_{jk,\lambda} + iC_{jk,\lambda}\right)\right] U_{jk,\lambda}\left(z_{jk}\right)\right\}, \tag{5b}$$

$$\left(\tau_{xy}\right)_j = -\frac{\partial^2 U_j}{\partial x \partial y} = -\sum_{k=1}^{2} \operatorname{Re}\left\{\left[\left(A_{jk,\lambda} + iC_{jk,\lambda}\right) - i\left(B_{jk,\lambda} + iC_{jk,\lambda}\right)\right] \times \left(i\beta_{jk}\right) U_{jk,\lambda}\left(z_{jk}\right)\right\}, \quad (j = 1, 2) \tag{5c}$$

in which the stress functions U_j of right side of (5a)–(5c) contain twelve undetermined coefficients $A_{jk,\lambda}$, $B_{jk,\lambda}$, $C_{jk,\lambda}$ $(j,k=1,2)$ and an unknown singularity exponent λ.

Considering the boundary conditions (2), (3), and (4), the functions $U_{jk,\lambda}(z_{jk})$ of right side of (5a)–(5c) can be chosen as

$$U_{jk,\lambda}(z_{jk}) = \tau\left(1-\frac{a^2}{z_{jk}^2}\right)^{\lambda} = \tau\frac{z_{jk}^{-2\lambda}}{\left(z_{jk}^2-a^2\right)^{-\lambda}}, \quad (j,k=1,2). \tag{6}$$

in which

$$z_{jk} = x + s_{jk}y = x + i\beta_{jk}y, \quad (j,k=1,2) \tag{7}$$

$$z_{jk} - a = x - a + i\beta_{jk}y = r\left(\cos\theta + i\beta_{jk}\sin\theta\right), \quad (j,k=1,2). \tag{8}$$

From (5a)–(5c) and (6), an unknown number λ is called singularity exponent, and sometimes it is also called eigenvalue.

4. Singularity Exponent

If the singularity exponent λ is the complex number, the stress near the interface crack tip shows oscillatory singularity. We let [1–16]

$$\lambda = -\frac{1}{2} + i\varepsilon. \tag{9}$$

Substituting (9) into (6), it can be found as

$$\begin{aligned} U_{jk,\varepsilon}(z_{jk}) &= \tau\left(1-\frac{a^2}{z_{jk}^2}\right)^{-(1/2)+i\varepsilon} \\ &= \tau\frac{z_{jk}^{1-i2\varepsilon}}{\left(z_{jk}^2-a^2\right)^{(1/2)-i\varepsilon}}, \quad (j,k=1,2). \end{aligned} \tag{10}$$

Equations (7) and (8) are substituted into (10), and we have

$$\begin{aligned} U_{jk,\varepsilon}(z_{jk})\Big|_{y=0,|x|<a} &= e^{(-1)^j\varepsilon\pi}\frac{\tau x}{\sqrt{a^2-x^2}}(-1)^{j-1} \\ &\times\left[\sin\left(2\varepsilon\ln\frac{\sqrt{a^2-x^2}}{x}\right)\right. \\ &\left.-i\cos\left(2\varepsilon\ln\frac{\sqrt{a^2-x^2}}{x}\right)\right]. \end{aligned} \tag{11}$$

at the crack surfaces.

Substituting (11), (5b), and (5c) into the boundary condition (2), by the help of the functions $\cos(2\varepsilon\ln((\sqrt{a^2-x^2})/x))$ and $\sin(2\varepsilon\ln((\sqrt{a^2-x^2})/x))$ which are linearly independent, the four homogeneous linear equations can be deduced as

$$A_{11,\varepsilon} + A_{12,\varepsilon} - B_{11,\varepsilon} - B_{12,\varepsilon} + 2C_{11,\varepsilon} + 2C_{12,\varepsilon} = 0, \tag{12a}$$

$$-A_{21,\varepsilon} - A_{22,\varepsilon} + B_{21,\varepsilon} + B_{22,\varepsilon} - 2C_{21,\varepsilon} - 2C_{22,\varepsilon} = 0, \tag{12b}$$

$$-\beta_{11}A_{11,\varepsilon} - \beta_{12}A_{12,\varepsilon} - \beta_{11}B_{11,\varepsilon} - \beta_{12}B_{12,\varepsilon} = 0, \tag{12c}$$

$$\beta_{21}A_{21,\varepsilon} + \beta_{22}A_{22,\varepsilon} + \beta_{21}B_{21,\varepsilon} + \beta_{22}B_{22,\varepsilon} = 0. \tag{12d}$$

Similarly, we can find other eight linear equations (12e) to (12l) which are omitted here. By $(\tau_{xy})_1 = (\tau_{xy})_2 = \tau$ in condition (4), two nonhomogeneous linear equations are contained in the eight linear equations.

The system of nonhomogeneous linear equations (12a) to (12l) is solved by eliminating the unknowns. To let the rank of the coefficient matrix A_λ for the system (12a)–(12l) and the rank of the augmented matrix $\overline{A}_\lambda = (A_\lambda, b_{\mathrm{II}})$ be equal, the rank must be 8; that is, $\operatorname{rank}(A_\lambda) = \operatorname{rank}(\overline{A}_\lambda) = 8$. For this reason [21], the number ε must satisfy the following characteristic equation:

$$\left(e_{12} - f_{12}\right)f_{12} - 4g_{12}h_{12}\varepsilon^2 = 0 \tag{13}$$

in which bimaterial parameters are given as

$$e_{12} = \frac{1}{\mu_1} - \frac{1}{\mu_2}, \qquad f_{12} = \frac{\beta_{11}\beta_{12}-\nu_{11}}{E_{11}} - \frac{\beta_{21}\beta_{22}-\nu_{21}}{E_{21}}, \tag{14a}$$

$$\begin{aligned} g_{12} &= \frac{\beta_{11}+\beta_{12}}{E_{11}} + \frac{\beta_{21}+\beta_{22}}{E_{21}}, \\ h_{12} &= \frac{\beta_{11}+\beta_{12}}{E_{11}}\beta_{11}\beta_{12} + \frac{\beta_{21}+\beta_{22}}{E_{21}}\beta_{21}\beta_{22}. \end{aligned} \tag{14b}$$

The singularity exponent λ of (9) is a complex number, so $\varepsilon \neq 0$ in (13), and $e_{12} \neq 0, f_{12} \neq 0, g_{12} > 0, h_{12} > 0$ in (14a) and (14b) for determinate bimaterial. To the equation (13) has a solution, the discrimination rule can be deduced as

(∗) D_{12}: *if* e_{12} and f_{12} in (14a) satisfy $e_{12} > 0$ and $f_{12} > 0$, *then* the negative root of (13) is the bielastic constant ε.

Substituting ε solved by (13) into (9), the complex singularity exponent λ can be obtained.

The system of eight equations which remained through sequence elimination is solved by means of the inverse sequence backsubstitution. We found all coefficients as follows:

$$A_{jk,\varepsilon} + C_{jk,\varepsilon} = (-1)^k\frac{f_{12} + 2\beta_{jk^*}g_{12}\varepsilon}{\left(\beta_{j2}-\beta_{j1}\right)f_{12}}, \tag{15a}$$

$$B_{jk,\varepsilon} - C_{jk,\varepsilon} = (-1)^{k-1}\frac{f_{12} - 2\beta_{jk^*}g_{12}\varepsilon}{\left(\beta_{j2}-\beta_{j1}\right)f_{12}}, \quad (j,k=1,2). \tag{15b}$$

in which $k=1$, $k^*=2$; $k=2$, $k^*=1$. By (15a) and (15b), the stress functions U_j of the right side of (5a)–(5c) contain practically eight coefficients.

5. Stress Intensity Factors

Considering the stress expressions (5b) and (5c) and also the load condition in (4), the stress intensity factors are defined as

$$\begin{aligned}(k_2^+)_j = -\sum_{k=1}^{2}\lim_{z_{jk}\to a^+}\operatorname{Re}\Big\{&\left(2\pi\left|z_{jk}-a\right|\right)^{(1/2)-i\varepsilon}\\&\times\left[\left(A_{jk,\varepsilon}+C_{jk,\varepsilon}\right)\right.\\&\left.-i\left(B_{jk,\varepsilon}-C_{jk,\varepsilon}\right)\right]\\&\times\left(i\beta_{jk}\right)U_{jk,\varepsilon}\left(z_{jk}\right)\Big\},\end{aligned} \tag{16a}$$

$$\begin{aligned}(k_2^-)_j = -\sum_{k=1}^{2}\lim_{z_{jk}\to a^-}\operatorname{Re}\Big\{&\left(2\pi\left|z_{jk}-a\right|\right)^{(1/2)-i\varepsilon}\\&\times\left[\left(A_{jk,\varepsilon}+C_{jk,\varepsilon}\right)\right.\\&\left.-i\left(B_{jk,\varepsilon}-C_{jk,\varepsilon}\right)\right]\\&\times(-1)^{j-1}(2\varepsilon-i)\left(i\beta_{jk}\right)\\&\times U_{jk,\varepsilon}\left(z_{jk}\right)\Big\},\quad (j=1,2),\end{aligned} \tag{16b}$$

$$k_2 = \frac{k_2^+ - k_2^-}{2} = \frac{1}{2}\sum_{j=1}^{2}\frac{(k_2^+)_j}{2} - \frac{1}{2}\sum_{j=1}^{2}\frac{(k_2^-)_j}{2}, \tag{16c}$$

$$\begin{aligned}(k_1^+)_j = \sum_{k=1}^{2}\lim_{z_{jk}\to a^+}\operatorname{Re}\Big\{&\left(2\pi\left|z_{jk}-a\right|\right)^{(1/2)-i\varepsilon}\\&\times\left[\left(A_{jk,\varepsilon}+C_{jk,\varepsilon}\right)-i\left(B_{jk,\varepsilon}-C_{jk,\varepsilon}\right)\right]\\&\times U_{jk,\varepsilon}\left(z_{jk}\right)\Big\},\end{aligned} \tag{17a}$$

$$\begin{aligned}(k_1^-)_j = \sum_{k=1}^{2}\lim_{z_{jk}\to a^-}\operatorname{Re}\Big\{&\left(2\pi\left|z_{jk}-a\right|\right)^{(1/2)-i\varepsilon}\\&\times\left[\left(A_{jk,\varepsilon}+C_{jk,\varepsilon}\right)-i\left(B_{jk,\varepsilon}-C_{jk,\varepsilon}\right)\right]\\&\times(-1)^{j-1}(2\varepsilon-i)U_{jk,\varepsilon}\left(z_{jk}\right)\Big\},\\&(j=1,2),\end{aligned} \tag{17b}$$

$$k_1 = \frac{k_1^+ + k_1^-}{2} = \frac{1}{2}\sum_{j=1}^{2}\frac{(k_1^+)_j}{2} + \frac{1}{2}\sum_{j=1}^{2}\frac{(k_1^-)_j}{2}, \tag{17c}$$

$$k = k_2 + ik_1. \tag{18}$$

In order to express the change process of $z_{jk} \to a^-$ completely, it is necessary that the factor of (11): $e^{(-1)^j\varepsilon\pi}(-1)^{j-1}[\sin(2\varepsilon\ln((\sqrt{a^2-x^2})/x)) - i\cos(2\varepsilon\ln((\sqrt{a^2-x^2})/x))] = (-1)^{j-1}(2\varepsilon - i)$, $(\varepsilon \to 0)$ is substituted into the expressions (16b) and (17b). At the same time, the minus is used in (16c) because the load condition is given in (4).

Substituting (10), (7), (8), (15a), and (15b) into (16a)–(16c), (17a)–(17c), and (18), the stress intensity factors of interface crack for mode II can be deduced as

$$k_2 = \tau(\pi a)^{1/2}p(\varepsilon), \qquad k_1 = -\tau(\pi a)^{1/2}\frac{2g_{12}\varepsilon}{f_{12}}q(\varepsilon), \qquad k = k_2 + ik_1 \tag{19}$$

in which

$$\begin{aligned}p(\varepsilon) = \frac{1}{2}\{&[1+(1+2\varepsilon)\,ch\varepsilon\pi]\cos(\varepsilon\ln\pi a)\\&-[1+(1-2\varepsilon)\,ch\varepsilon\pi]\sin(\varepsilon\ln\pi a)\},\end{aligned} \tag{20a}$$

$$\begin{aligned}q(\varepsilon) = \frac{1}{2}\{&[1-(1+2\varepsilon)\,ch\varepsilon\pi]\cos(\varepsilon\ln\pi a)\\&-[1-(1-2\varepsilon)\,ch\varepsilon\pi]\sin(\varepsilon\ln\pi a)\}.\end{aligned} \tag{20b}$$

6. Stress

From (10), (8), and (7), it can be found as follows that

$$\lim_{z_{jk}\to a}\left\{\left[2\pi\left(z_{jk}-a\right)\right]^{(1/2)-i\varepsilon}U_{jk,\varepsilon}\left(z_{jk}\right)\right\} = \tau(\pi a)^{(1/2)-i\varepsilon}, \quad (j,k=1,2) \tag{21}$$

in the vicinity of the crack tip.

By (21), it can be known that

$$U_{jk,\varepsilon}\left(z_{jk}\right) = \frac{\tau(\pi a)^{(1/2)-i\varepsilon}}{(2\pi)^{(1/2)-i\varepsilon}}\frac{1}{\left(z_{jk}-a\right)^{(1/2)-i\varepsilon}}, \quad \left(z_{jk}\longrightarrow a;\ j,k=1,2\right). \tag{22}$$

Substituting (22), (15a), (15b), (19), and (8) into (5a)–(5c), the stresses in the vicinity of the interface crack tip ($z_{jk} \to a, r \to 0;\ j,k = 1,2$) for mode II of orthotropic bimaterial can be expressed as follows:

$$\begin{aligned}(\sigma_x)_j = \frac{k_2}{(2\pi r)^{1/2}}&\frac{\sqrt{2}}{\left(\beta_{j2}-\beta_{j1}\right)p(\varepsilon)}\\&\times\Big\{\Big[\cos\left(\frac{\pi}{4}-\varepsilon\ln\pi a\right)\cdot\cos(\varepsilon\ln 2\pi r)\\&-\sin\left(\frac{\pi}{4}-\varepsilon\ln\pi a\right)\sin(\varepsilon\ln 2\pi r)\Big]\end{aligned}$$

$$\cdot \left[\operatorname{Re} \frac{\beta_{j1}^{2}}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Re} \frac{\beta_{j2}^{2}}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} \right]$$

$$- \left[\cos\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \cos\left(\varepsilon \ln 2\pi r\right) + \sin\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right]$$

$$\left. \cdot \left[\operatorname{Im} \frac{\beta_{j1}^{2}}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Im} \frac{\beta_{j2}^{2}}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} \right] \right\}$$

$$+ \frac{k_1}{(2\pi r)^{1/2}} \frac{\sqrt{2}\beta_{j1}\beta_{j2}}{\left(\beta_{j2} - \beta_{j1}\right) q(\varepsilon)}$$

$$\times \left\{ \left[\cos\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \cdot \cos\left(\varepsilon \ln 2\pi r\right) + \sin\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right] \right.$$

$$\cdot \left[\operatorname{Re} \frac{\beta_{j2}}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Re} \frac{\beta_{j1}}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} \right]$$

$$+ \left[\cos\left(\frac{\pi}{4} - \varepsilon \ln \pi a\right) \cos\left(\varepsilon \ln 2\pi r\right) - \sin\left(\frac{\pi}{4} - \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right]$$

$$\left. \cdot \left[\operatorname{Im} \frac{\beta_{j2}}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Im} \frac{\beta_{j1}}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} \right] \right\}, \tag{23a}$$

$$\left(\sigma_y\right)_j = \frac{k_2}{(2\pi r)^{1/2}} \frac{\sqrt{2}}{\left(\beta_{j2} - \beta_{j1}\right) p(\varepsilon)}$$

$$\times \left\{ \left[\cos\left(\frac{\pi}{4} - \varepsilon \ln \pi a\right) \cdot \cos\left(\varepsilon \ln 2\pi r\right) - \sin\left(\frac{\pi}{4} - \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right] \right.$$

$$\cdot \left[\operatorname{Re} \frac{1}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Re} \frac{1}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} \right]$$

$$- \left[\cos\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \cos\left(\varepsilon \ln 2\pi r\right) + \sin\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right]$$

$$\left. \cdot \left[\operatorname{Im} \frac{1}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Im} \frac{1}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} \right] \right\}$$

$$+ \frac{k_1}{(2\pi r)^{1/2}} \frac{\sqrt{2}}{\left(\beta_{j2} - \beta_{j1}\right) q(\varepsilon)}$$

$$\times \left\{ \left[\cos\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \cdot \cos\left(\varepsilon \ln 2\pi r\right) + \sin\left(\frac{\pi}{4} + \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right] \right.$$

$$\cdot \left[\operatorname{Re} \frac{\beta_{j2}}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Re} \frac{\beta_{j1}}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} \right]$$

$$+ \left[\cos\left(\frac{\pi}{4} - \varepsilon \ln \pi a\right) \cos\left(\varepsilon \ln 2\pi r\right) - \sin\left(\frac{\pi}{4} - \varepsilon \ln \pi a\right) \sin\left(\varepsilon \ln 2\pi r\right) \right]$$

$$\left. \cdot \left[\operatorname{Im} \frac{\beta_{j2}}{\left(\cos\theta + i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}} - \operatorname{Im} \frac{\beta_{j1}}{\left(\cos\theta + i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}} \right] \right\}, \tag{23b}$$

$$\left(\tau_{xy}\right)_j = \frac{k_2}{(2\pi r)^{1/2}} \frac{\sqrt{2}}{\left(\beta_{j2} - \beta_{j1}\right) p(\varepsilon)}$$

$$\times\left\{\left[\cos\left(\frac{\pi}{4}+\varepsilon\ln\pi a\right)\cdot\cos\left(\varepsilon\ln 2\pi r\right)\right.\right.$$
$$\left.+\sin\left(\frac{\pi}{4}+\varepsilon\ln\pi a\right)\sin\left(\varepsilon\ln 2\pi r\right)\right]$$
$$\cdot\left[\operatorname{Re}\frac{\beta_{j2}}{\left(\cos\theta+i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}}\right.$$
$$\left.-\operatorname{Re}\frac{\beta_{j1}}{\left(\cos\theta+i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}}\right]$$
$$+\left[\cos\left(\frac{\pi}{4}-\varepsilon\ln\pi a\right)\cos\left(\varepsilon\ln 2\pi r\right)\right.$$
$$\left.-\sin\left(\frac{\pi}{4}-\varepsilon\ln\pi a\right)\sin\left(\varepsilon\ln 2\pi r\right)\right]$$
$$\cdot\left[\operatorname{Im}\frac{\beta_{j2}}{\left(\cos\theta+i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}}\right.$$
$$\left.\left.-\operatorname{Im}\frac{\beta_{j1}}{\left(\cos\theta+i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}}\right]\right\}$$
$$+\frac{k_1}{(2\pi r)^{1/2}}\frac{\sqrt{2}\beta_{j1}\beta_{j2}}{\left(\beta_{j2}-\beta_{j1}\right)q(\varepsilon)}$$
$$\times\left\{\left[\cos\left(\frac{\pi}{4}-\varepsilon\ln\pi a\right)\cdot\cos\left(\varepsilon\ln 2\pi r\right)\right.\right.$$
$$\left.-\sin\left(\frac{\pi}{4}-\varepsilon\ln\pi a\right)\sin\left(\varepsilon\ln 2\pi r\right)\right]$$
$$\cdot\left[\operatorname{Re}\frac{1}{\left(\cos\theta+i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}}\right.$$
$$\left.-\operatorname{Re}\frac{1}{\left(\cos\theta+i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}}\right]$$
$$-\left[\cos\left(\frac{\pi}{4}+\varepsilon\ln\pi a\right)\cos\left(\varepsilon\ln 2\pi r\right)\right.$$
$$\left.+\sin\left(\frac{\pi}{4}+\varepsilon\ln\pi a\right)\sin\left(\varepsilon\ln 2\pi r\right)\right]$$
$$\cdot\left[\operatorname{Im}\frac{1}{\left(\cos\theta+i\beta_{j2}\sin\theta\right)^{(1/2)-i\varepsilon}}\right.$$
$$\left.\left.-\operatorname{Im}\frac{1}{\left(\cos\theta+i\beta_{j1}\sin\theta\right)^{(1/2)-i\varepsilon}}\right]\right\} \tag{23c}$$

in which $p(\varepsilon)$ and $q(\varepsilon)$ are given by (20a) and (20b).

TABLE 1: Mechanical properties of each orthotropic material.

Materials		E_{j1}/GPa	E_{j2}/GPa	ν_j	μ_j/GPa	Δ_j	β_{j1}	β_{j2}
A	$j=1$	12.45	11.56	0.33	4.35	0.54	0.85	1.21
	$j=2$	201	12.5	0.21	6.69	813.31	0.74	5.39
B	$j=1$	181	10.3	0.28	7.17	539.01	0.86	4.89
	$j=2$	40.06	38.26	0.26	11.24	5.07	0.63	1.63
C	$j=1$	18.3	15.51	0.32	5.87	1.42	0.80	1.35
	$j=2$	135	10.3	0.21	6.6	348.96	0.82	4.40

TABLE 2: Mechanical properties of three orthotropic bimaterials.

Bimaterial	e_{12}	f_{12}	g_{12}	h_{12}	ε	λ
A	0.0804	0.0379	0.1966	0.2948	−0.0834	−0.5 −0.0834i
B	0.0505	0.0026	0.0881	0.1908	−0.0427	−0.5 −0.0427i
C	0.0188	0.0166	0.1565	0.2681	−0.0149	−0.5 −0.0149i

Note that (23a)–(23c) of the stress field contain the following terms:

$$\begin{pmatrix}(\sigma_x)_j\\(\sigma_y)_j\\(\tau_{xy})_j\end{pmatrix}\supset\frac{1}{(2\pi r)^{1/2}}\begin{pmatrix}\cos(\varepsilon\ln 2\pi r)\\\sin(\varepsilon\ln 2\pi r)\end{pmatrix}. \tag{24}$$

The right side of (24) tends to change the sign rapidly at the crack tip ($r\to 0$), and therefore the stresses possess an oscillatory character.

7. Oscillatory Field

The test and calculus results [16, 22–24] for the mechanicals properties of three orthotropic bimaterial are shown in Tables 1 and 2.

From (19), the normalized stress intensity factors k_2/τ and k_1/τ depend on the length of crack a and the bielastic constant ε. The variations of the stress intensity factors versus crack length are plotted in Figure 2. The variations of the stress intensity factors with bielastic constant are illustrated in Figure 3. The minute variations of factor k_1/τ can be observed by the inner small figure of Figure 3. The factors k_2/τ and k_1/τ increase almost linearly when the length a or the constant ε increases as shown in Figures 2 and 3.

By (23a)–(23c), the normalized stresses σ_x/τ, σ_y/τ, and τ_{xy}/τ depend on the polar angle θ and the polar radius r. The variations of the normalized stresses for the above three bimaterials with respect to polar angle θ are plotted in Figure 4. Figure 4 shows that three stresses have alternately the increase and decrease with θ increase, their maximum values can be always reached at both sides of the crack.

Figures 5 and 6 show the variations of the normalized stresses for the above three bimaterials with respect to polar radius r for $\theta=\pm45^\circ$ and $\theta=\pm60^\circ$. Two figures mean that the stresses σ_x/τ, σ_y/τ, and τ_{xy}/τ are the monotonic decreasing functions of r/a on the two half-planes.

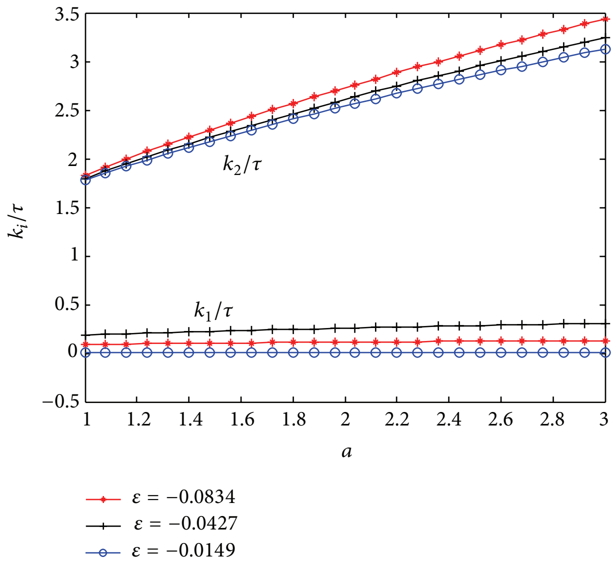


FIGURE 2: Normalized stress intensity factors as a function of a.

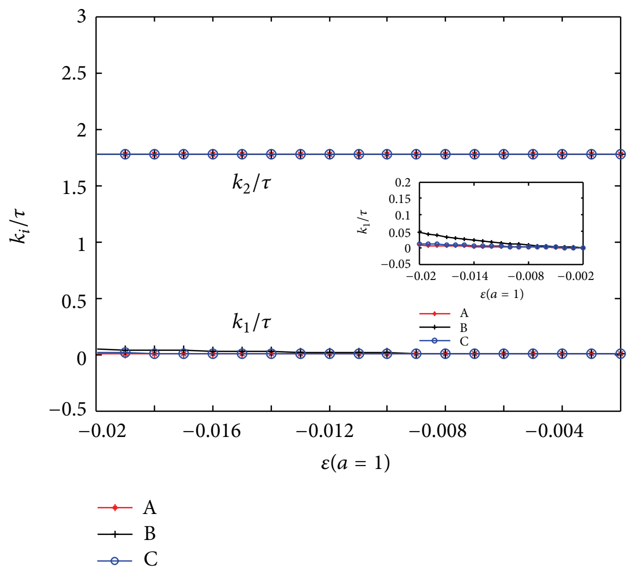


FIGURE 3: Normalized stress intensity factors as a function of ε when $a = 1$.

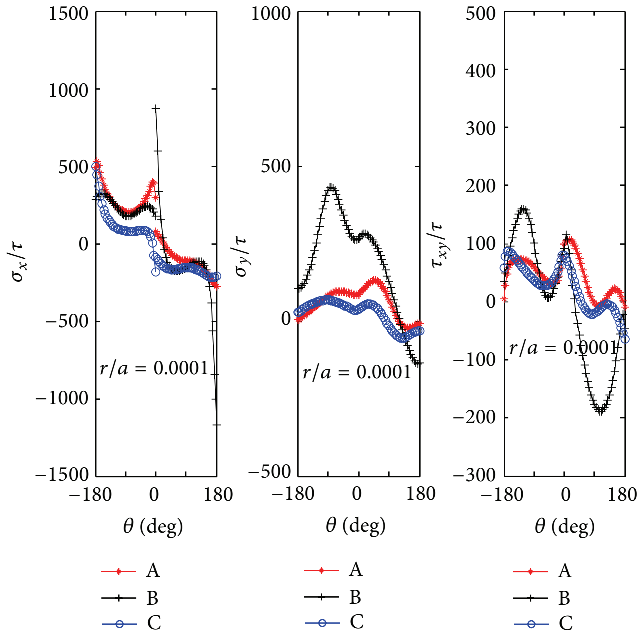


FIGURE 4: Normalized stresses versus θ for $r/a = 0.0001$.

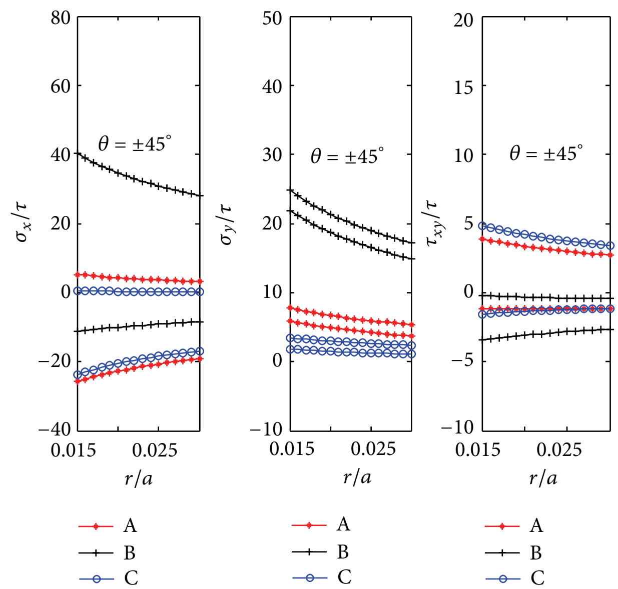


FIGURE 5: Normalized stresses versus r for $\theta = \pm 45^\circ$.

Figures 7 and 8 show the variations of the normalized logarithmic stresses with respect to the polar radius r for $\theta = \pm 45^\circ$ and $\theta = \pm 60^\circ$. It can be found that the stress distribution in Figures 7 and 8 is not straight line and does not parallel to each other. Such distributions mean oscillatory singularity state.

8. Conclusion

From the above derivation, the following results are very significant.

(1) New stress functions are chosen.

(2) The system of twelve nonhomogeneous linear equations is derived.

(3) The characteristic equation can be given.

(4) By the help of the distinction rule $(*)$, the complex singularity exponent λ can be found.

(5) Total coefficients are determined.

(6) For the oscillatory field, the theoretical formulae of stress intensity factors and the analytic solutions for stresses near interface crack tip of mode II are

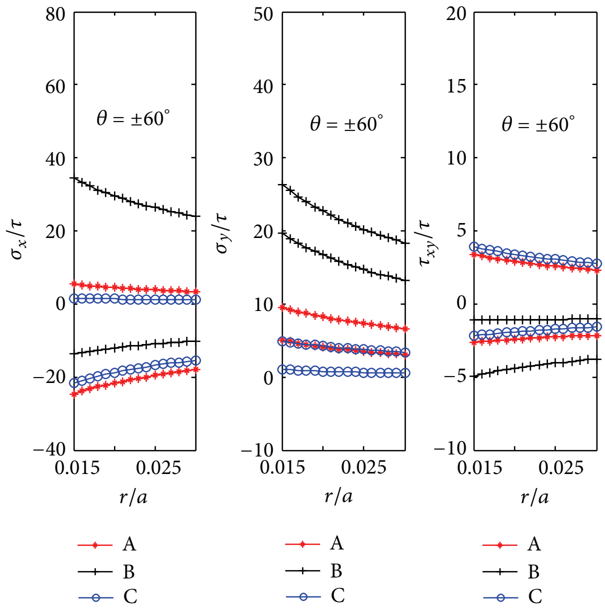


Figure 6: Normalized stresses versus r for $\theta = \pm 60^\circ$.

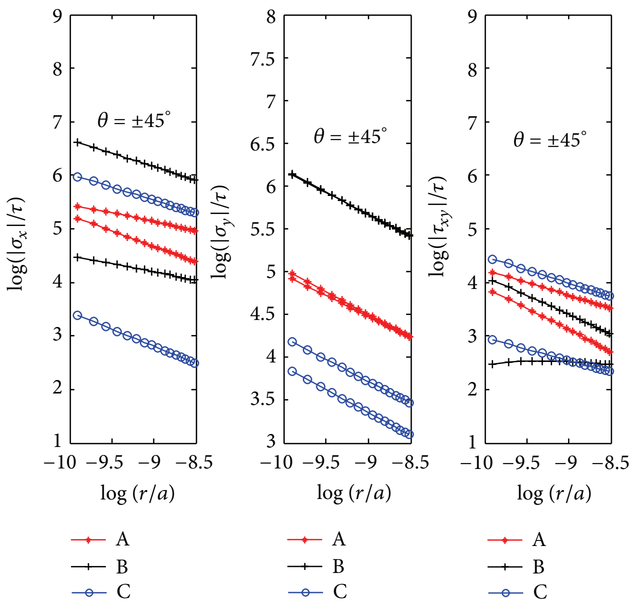


Figure 7: Normalized stresses versus r for $\theta = \pm 45^\circ$.

yielded, respectively. The stress intensity factors and the stresses show mixed crack characteristics.

(7) The crack tip fields for oscillatory singularities are illustrated in Figures 2 to 8.

Acknowledgment

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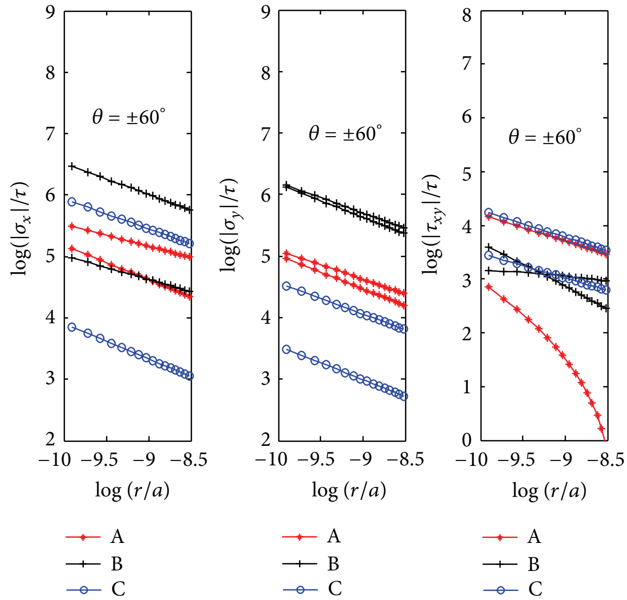

Figure 8: Normalized stresses versus r for $\theta = \pm 60^\circ$.

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