Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 767380, 16 pages
http://dx.doi.org/10.1155/2013/767380

Research Article

New Exact Solutions for New Model Nonlinear Partial Differential Equation

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Received 10 February 2013; Accepted 20 April 2013

Academic Editor: Anjan Biswas

In this paper we propose a new form of Padé-II equation, namely, a combined Padé-II and modified Padé-II equation. The mapping method is a promising method to solve nonlinear evaluation equations. Therefore, we apply it, to solve the combined Padé-II and modified Padé-II equation. Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions, rational functions, and elliptic functions.

1. Introduction

In recent years, directly searching for exact solutions of nonlinear partial differential equations (PDEs) has become more and more attractive field in different branches of physics and applied mathematics. These equations appear in condensed matter, solid state physics, fluid mechanics, chemical kinetics, plasma physics, nonlinear optics, propagation of fluxions in Josephson junctions, theory of turbulence, ocean dynamics, biophysics star formation, and many others.

In order to get exact solutions directly, many powerful methods have been introduced such as the (G'/G)-expansion method [1], inverse scattering method [2, 3], Hirota's bilinear method [4, 5], the tanh method [6, 7], the sine-cosine method [8, 9], Bäcklund transformation method [10, 11], the homogeneous balance [12, 13], Darboux transformation [14], and the Jacobi elliptic function expansion method [15].

Recently, Peng [16] introduced a new approach, namely, the mapping method for a reliable treatment of the nonlinear wave equations. The useful mapping method is then widely used by many authors [17, 18].

2. Description of the Method

Consider the general nonlinear partial differential equations (PDEs); say, in two variables,

$$P\left(u, u_x, u_t, u_{xx}, u_{xt}, \ldots\right) = 0. \quad (1)$$

Let $u(x,t) = u(\xi)$, $\xi = \mu(x - ct)$; then (1) reduces to a nonlinear ordinary differential equation (ODE)

$$Q\left(u, u', u'', \ldots\right) = 0. \quad (2)$$

Assume the solution of (2) takes the form

$$u(x,t) = u(\xi) = a_0 + \sum_{i=1}^{m} a_i\left(f(\xi)\right)^i + b_i\left(f(\xi)\right)^{-i}, \quad (3)$$

where the coefficients a_i $(i = 0, 1, 2, \ldots, m)$, μ, and c are constants to be determined, and $f = f(\xi)$ satisfies a nonlinear ordinary differential equation

$$\frac{df(\xi)}{d\xi} = \sqrt{pf^2(\xi) + \frac{1}{2}qf^4(\xi) + r}, \quad p, q, r \in R, \quad (4)$$

where the coefficients a_0, a_i, b_i $(i = 1, 2, \ldots m)$, μ, and c are constants to be determined and $f = f(\xi)$ satisfies (4); the parameter m will be found by balancing the highest-order nonlinear terms with the highest-order partial derivative term in the given equation. Substituting (3) into (2), using (4) repeatedly and setting the coefficients of the each order of $f^i(\xi)$, $f^i(\xi)\sqrt{pf^2(\xi) + (1/2)qf^4(\xi) + r}$ to zero, we obtain a set of nonlinear algebraic equations for a_0, a_i, b_i $(i = 1, 2, \ldots n)$, μ, and c. With the aid of the computer program Maple, we can solve the set of nonlinear algebraic equations and obtain all the constants a_0, a_i, b_i $(i = 1, 2, \ldots n), \mu$, and c. The ODE (4) has the following solutions:

(1) $f(\xi) = \operatorname{sech}(\xi)$, $p = 1$, $q = -2$, $r = 0$,

(2) $f(\xi) = \tanh(\xi)$, $p = -2$, $q = 2$, $r = 1$,

(3) $f(\xi) = (1/2)\tanh(2\xi), (1/2)\coth(2\xi)$, $p = -8$, $q = 32$, $r = 1$,

(4) $f(\xi) = (1/2)\tan(2\xi), -(1/2)\cot(2\xi)$, $p = 8$, $q = 32$, $r = 1$,

(5) $f(\xi) = \operatorname{sn}\xi$, $p = -(k^2+1)$, $q = 2k^2$, $r = 1$,

(6) $f(\xi) = \operatorname{ns}\xi$, $p = -(k^2+1)$, $q = 2$, $r = k^2$,

(7) $f(\xi) = \operatorname{cd}\xi$, $p = -(k^2+1)$, $q = 2k^2$, $r = 1$,

(8) $f(\xi) = \operatorname{dc}\xi$, $p = -(k^2+1)$, $q = 2$, $r = k^2$,

(9) $f(\xi) = \operatorname{cn}\xi$, $p = 2k^2 - 1$, $q = -2k^2$, $r = 1 - k^2$,

(10) $f(\xi) = \operatorname{nc}\xi$, $p = 2k^2 - 1$, $q = 2(1-k^2)$, $r = -k^2$,

(11) $f(\xi) = \operatorname{dn}\xi$, $p = 2 - k^2$, $q = -2$, $r = -(1-k^2)$,

(12) $f(\xi) = \operatorname{nd}\xi$, $p = 2 - k^2$, $q = 2(k^2-1)$, $r = -1$,

(13) $f(\xi) = \operatorname{cs}\xi$, $p = 2 - k^2$, $q = 2$, $r = 1 - k^2$,

(14) $f(\xi) = \operatorname{sc}\xi$, $p = 2 - k^2$, $q = 2(1-k^2)$, $r = 1$,

(15) $f(\xi) = \operatorname{ds}\xi$, $p = -1 + 2k^2$, $q = 2$, $r = -k^2(1-k^2)$,

(16) $f(\xi) = \operatorname{sd}\xi$, $p = -1 + 2k^2$, $q = 2k^2(k^2-1)$, $r = 1$,

(17) $f(\xi) = \operatorname{sc}\xi \pm \operatorname{nc}\xi$, $p = (1+k^2)/2$, $q = (1-k^2)/2$, $r = (1-k^2)/4$,

(18) $f(\xi) = \operatorname{sn}\xi/(1 \pm \operatorname{dn}\xi)$, $p = (k^2-2)/2$, $q = k^2/2$, $r = 1/4$,

(19) $f(\xi) = \operatorname{dn}\xi/(1 \pm k\operatorname{sn}\xi)$, $p = (k^2+1)/2$, $q = (k^2-1)/2$, $r = (1-k^2)/4$,

(20) $f(\xi) = k\operatorname{cn}\xi \pm \operatorname{dn}\xi$, $p = (k^2+1)/2$, $q = -1/2$, $r = -(1-k^2)^2/4$,

(21) $f(\xi) = \operatorname{cn}\xi/(1 \pm \operatorname{sn}\xi)$, $p = (k^2+1)/2$, $q = (1-k^2)/2$, $r = (1-k^2)/4$,

(22) $f(\xi) = k\operatorname{sn}\xi \pm i\operatorname{dn}\xi$, $p = (1-2k^2)/2$, $q = 1/2$, $r = k^2/4$,

(23) $f(\xi) = k\operatorname{sn}\xi \pm i\operatorname{cn}\xi$, $p = (k^2-2)/2$, $q = k^2/2$, $r = k^2/4$,

(24) $f(\xi) = \operatorname{ns}\xi \pm \operatorname{ds}\xi$, $p = (k^2-2)/2$, $q = 1/2$, $r = k^4/4$,

(25) $f(\xi) = \operatorname{ns}\xi - \operatorname{cs}\xi$, $p = (1-2k^2)/2$, $q = 1/2$, $r = 1/4$,

(26) $f(\xi) = \operatorname{cn}\xi/(\sqrt{1-k^2}\operatorname{sn}\xi \pm \operatorname{dn}\xi)$, $p = (1-2k^2)/2$, $q = 1/2$, $r = 1/4$,

(27) $f(\xi) = \operatorname{sn}\xi/(\operatorname{cn}\xi \pm \operatorname{dn}\xi)$, $p = (1+k^2)/2$, $q = (1-k^2)^2/2$, $r = 1/4$,

(28) $f(\xi) = \operatorname{cn}\xi/(\sqrt{1-k^2} \pm \operatorname{dn}\xi)$, $p = (k^2-2)/2$, $q = k^2/2$, $r = 1/4$,

(29) $f(\xi) = -1/\sqrt{c/2}\xi$, $p = 0$, $q = c$, $r = 0$,

(30) $f(\xi) = e^{\xi}$, $p = 1$, $q = 0$, $r = 0$.

3. Application

In this section, we present our proposed equation, namely, a combined Padé-II and modified Padé-II equation, as the form

$$u_t(x,t) + u_x(x,t) + P(u)\,u_x(x,t) + a u_{xxx}(x,t) + b u_{xxt}(x,t) = 0, \tag{5}$$

where $P(u) = u(x,t) + u^2(x,t)$, a, and b are real numbers [19].

Now, we apply the mapping method to solve our equation. Consequently we get the original solutions for our new equation, as the follows.

Substituting $u(x,t) = u(\xi)$, $\xi = \lambda(x - ct)$ in (5) and integrating once yield

$$(1-c)\,u(\xi) + \frac{(u(\xi))^2}{2} + \frac{(u(\xi))^3}{3} + \lambda^2 (a - bc)\,u''(\xi) = 0. \tag{6}$$

Balancing the order of the nonlinear term u^3 with the highest derivative u'' gives $3m = m + 2$ that gives $m = 1$. Thus, the solution of (6) has the form

$$u(\xi) = \sum_{i=0}^{1} a_i f(\xi)^i = a_0 + a_1 f(\xi) + b_1 f(\xi)^{-1}, \tag{7}$$

where

$$\frac{df(\xi)}{d\xi} = \sqrt{p f^2(\xi) + \frac{1}{2} q f^4(\xi) + r}, \quad p, q, r \in R. \tag{8}$$

Substituting (7) in (6) and using (8), collecting the coefficients of each power of f^i, $0 \le i \le 6$, setting each coefficient to zero, and solving the resulting system, we obtain the following sets of solutions:

(1) $a_0 = 0$, $a_1 = b_1 = 0$, $c = c$, $\lambda = \lambda$,

(2) $a_0 = a_0$, $a_1 = b_1 = 0$, $c = c$, $\lambda = \lambda$,

(3) $a_0 = -1/2$, $a_1 = \sqrt{-q/4p}$, $b_1 = 0$, $c = 5/6$, $\lambda = \pm\sqrt{1/(12ap - 10bp)}$,

(4) $a_0 = -1/2$, $a_1 = -\sqrt{-q/4p}$, $b_1 = 0$, $c = 5/6$, $\lambda = \pm\sqrt{1/(12ap - 10bp)}$,

(5) $a_0 = -1/2$, $a_1 = 0$, $b_1 = \sqrt{-r/2p}$, $c = 5/6$, $\lambda = \pm\sqrt{1/(12ap - 10bp)}$,

(6) $a_0 = -1/2$, $a_1 = 0$, $b_1 = -\sqrt{-r/2p}$, $c = 5/6$, $\lambda = \pm\sqrt{1/(12ap - 10bp)}$,

(7) $a_0 = -1/2$, $a_1 = (1/2)\sqrt{(pq + 3q\sqrt{2rq})/(18rq - p^2)}$,
$b_1 = -(1/2)((6rq + p\sqrt{2rq})/$
$\sqrt{(pq + 3q\sqrt{2rq})/(18rq - p^2)}(p^2 - 18rq))$,
$c = 5/6$, $\lambda = \pm(1/\sqrt{2})$
$\times\sqrt{(6p^2a - 108arq - 5p^2b + 90rbq)(p + 3\sqrt{2rq})/}$
$(6p^2a - 108arq - 5p^2b + 90rbq)$,

(8) $a_0 = -1/2$, $a_1 = -(1/2)\sqrt{(pq + 3q\sqrt{2rq})/(18rq - p^2)}$, $b_1 = (1/2)((6rq + p\sqrt{2rq})/\sqrt{(pq + 3q\sqrt{2rq})/(18rq - p^2)}(p^2 - 18rq))$, $c = 5/6$, $\lambda = \pm(1/\sqrt{2}) \times\sqrt{(6p^2a - 108arq - 5p^2b + 90rbq)(p + 3\sqrt{2rq})/(6p^2a - 108arq - 5p^2b + 90rbq)}$,

(9) $a_0 = -1/2$, $a_1 = (1/2)\sqrt{(pq - 3q\sqrt{2rq})/(18rq - p^2)}$, $b_1 = -(1/2)((-6rq + p\sqrt{2rq})/\sqrt{(pq - 3q\sqrt{2rq})/(18rq - p^2)}\,(p^2 - 18rq))$, $c = 5/6$, $\lambda = \pm(1/\sqrt{2}) \times\sqrt{(-10p^2b+180rbq+12p^2a-216arq)(p-3\sqrt{2rq})/(6p^2a - 108arq - 5p^2b + 90rbq)}$,

(10) $a_0 = -1/2$, $a_1 = -(1/2)\sqrt{(pq - 3q\sqrt{2rq})/(18rq - p^2)}$, $b_1 = (1/2)((-6rq + p\sqrt{2rq})/\sqrt{(pq - 3q\sqrt{2rq})/(18rq - p^2)}\,(p^2 - 18rq))$, $c = 5/6$, $\lambda = \pm(1/\sqrt{2}) \times\sqrt{(-10p^2b+180rbq+12p^2a-216arq)(p-3\sqrt{2rq})/(6p^2a - 108arq - 5p^2b + 90rbq)}$.

Using (7), the solution of (8) when $p = 1$, $q = -2$, and $r = 0$, and the sets of solutions (1)–(10), we get

$$u_1(x,t) = 0,$$
$$u_2(x,t) = a_0, \quad \forall a_0 \in R, \tag{9}$$

for $a > (5/6)b$

$$u_{3,4}(x,t) = -\frac{1}{2} \pm \frac{1}{\sqrt{2}}\operatorname{sech}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right), \tag{10}$$

for $a < (5/6)b$

$$u_{5,6}(x,t) = -\frac{1}{2} \pm \frac{1}{\sqrt{2}}\sec\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right). \tag{11}$$

Using (7), the solution of (8) when $p = -2$, $q = 2$, and $r = 1$, and the sets of solutions (3)–(10), we get for $a < (5/6)b$

$$u_{7,8}(x,t) = -\frac{1}{2} \pm \frac{1}{2}\tanh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right),$$

$$u_{9,10}(x,t) = -\frac{1}{2} \pm \frac{1}{2}\coth\left(\frac{1}{2\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right),$$

$$u_{11,12}(x,t) = -\frac{1}{2} \pm \frac{1}{4}\tanh\left(\frac{1}{2\sqrt{2}\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right) \pm \frac{1}{4}\coth\left(\frac{1}{2\sqrt{2}\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right),$$

$$u_{13,14}(x,t) = -\frac{1}{2} \pm \frac{1}{2\sqrt{2}}\tan\left(\frac{1}{2\sqrt{2}\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right) \pm \frac{1}{2\sqrt{2}}\cot\left(\frac{1}{2\sqrt{2}\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right). \tag{12}$$

For $a > (5/6)b$

$$u_{15,16}(x,t) = -\frac{1}{2} \pm \frac{1}{2}i\tan\left(\frac{1}{2\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right),$$

$$u_{17,18}(x,t) = -\frac{1}{2} \pm \frac{1}{2}i\cot\left(\frac{1}{2\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right),$$

$$u_{19,20}(x,t) = -\frac{1}{2} \pm \frac{1}{4}i\tan\left(\frac{1}{2\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm \frac{1}{4}i\cot\left(\frac{1}{2\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right), \tag{13}$$

$$u_{21,22}(x,t) = -\frac{1}{2} \pm \frac{i}{2\sqrt{2}}\tanh\left(\frac{1}{2\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \mp \frac{i}{2\sqrt{2}}\coth\left(\frac{1}{2\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right).$$

Using (7), the solution of (8) when $p = 8$, $q = 32$, and $r = 1$, and the sets of solutions (3)–(10), we get, $[u_{7,8}(x,t), u_{9,10}(x,t), \ldots, u_{21,22}(x,t)]$.

Using (7), the solution of (8) when $p = -8$, $q = 32$, and $r = 1$, and the sets of solutions (3)–(10), we get, $[u_{7,8}(x,t), u_{9,10}(x,t), \ldots, u_{21,22}(x,t)]$.

Using (7), the solution of (8) when $p = -(k^2 + 1)$, $q = 2k^2$, and $r = 1$, and the sets of solutions (3)–(10), we get $u_{23,24,\ldots,30}(x,t) = a_0 + a_1 \operatorname{sn}\xi + b_1 \operatorname{ns}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t),\ldots,u_{21,22}(x,t)]$, and when $k \to 0$, we obtain for $a > (5/6)b$

$$u_{31,32}(x,t) = -\frac{1}{2} \pm \frac{i}{\sqrt{2}}\operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right), \tag{14}$$

for $a < (5/6)b$

$$u_{33,34}(x,t) = -\frac{1}{2} \pm \frac{1}{\sqrt{2}}\csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right). \tag{15}$$

Using (7), the solution of (8) when $p = -(k^2+1)$, $q = 2k^2$, and $r = 1$, and the sets of solutions (3)–(10), we get $u_{35,36,\ldots,42}(x,t) = a_0 + a_1 \operatorname{cd}\xi + b_1 \operatorname{dc}\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$ we obtain constant solutions, when $k \to 0$ we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (7), the solution of (8) when $p = -(k^2+1)$, $q = 2$, and $r = k^2$, and the sets of solutions (3)–(10), we get $u_{43,44,\ldots,50}(x,t) = a_0 + a_1 \operatorname{ns}\xi + b_1 \operatorname{sn}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain, $[u_{7,8}(x,t), u_{9,10}(x,t),\ldots,u_{21,22}(x,t)]$, when $k \to 0$, we obtain $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (7), the solution of (8) when $p = -(k^2+1)$, $q = 2$, and $r = k^2$, and the sets of solutions (3)–(10), we get $u_{51,52,\ldots,57}(x,t) = a_0 + a_1 \operatorname{dc}\xi + b_1 \operatorname{cd}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain constant solution, and when $k \to 0$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (7), the solution of (8) when $p = 2k^2 - 1$, $q = -2k^2$, and $r = 1 - k^2$, and the sets of solutions (3)–(10), we get $u_{58,59,\ldots,65}(x,t) = a_0 + a_1 \operatorname{cn}\xi + b_1 \operatorname{nc}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, when $k \to 0$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (7), the solution of (8) when $p = 2k^2 - 1$, $q = 2(1-k^2)$, and $r = -k^2$, and the sets of solutions (3)–(10), we get $u_{66,67,\ldots,73}(x,t) = a_0 + a_1 \operatorname{nc}\xi + b_1 \operatorname{cn}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, and when $k \to 0$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

Using (7), the solution of (8) when $p = 2 - k^2, q = -2$, and $r = -(1-k^2)$, and the sets of solutions (3)–(10), we get $u_{74,75,\ldots,81}(x,t) = a_0 + a_1 \operatorname{dn}\xi + b_1 \operatorname{nd}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, and when $k \to 0$, we obtain constant solutions.

Using (7), the solution of (8) when $p = 2 - k^2$, $q = 2(k^2-1)$, and $r = -1$, and the sets of solutions (3)–(10), we get $u_{82,83,\ldots,89}(x,t) = a_0 + a_1 \operatorname{nd}\xi + b_1 \operatorname{dn}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$, and when $k \to 0$, we obtain constant solutions.

Using (7), the solution of (8) when $p = 2 - k^2$, $q = 2$, and $r = 1 - k^2$, and the sets of solutions (3)–(10), we get $u_{90,91,\ldots,97}(x,t) = a_0 + a_1 \operatorname{cs}\xi + b_1 \operatorname{sc}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t),\ldots, u_{21,22}(x,t)]$.

Using (7), the solution of (8) when $p = 2 - k^2$, $q = 2(1-k^2)$, and $r = 1$, and the sets of solutions (3)–(10), we get $u_{98,99,\ldots,105}(x,t) = a_0 + a_1 \operatorname{sc}\xi + b_1 \operatorname{cs}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, when $k \to 0$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t),\ldots, u_{21,22}(x,t)]$.

Using (7), the solution of (8), when $p = -1 + 2k^2$, $q = 2$, and $r = -k^2(1-k^2)$, and the sets of solutions (3)–(10), we get $u_{106,107,\ldots,113}(x,t) = a_0 + a_1 \operatorname{ds}\xi + b_1 \operatorname{sd}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, and when $k \to 0$, we obtain also $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (7), the solution of (8), when $p = -1 + 2k^2$, $q = 2k^2(k^2-1)$, and $r = 1$, and the sets of solutions (3)–(10), we get $u_{114,115,\ldots,121}(x,t) = a_0 + a_1 \operatorname{sd}\xi + b_1 \operatorname{ds}\xi$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, and when $k \to 0$, we obtain also $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (7), the solution of (8) when $p = (1+k^2)/2$, $q = (1-k^2)/2$, and $r = (1-k^2)/4$, and the sets of solutions (3)–(10), we get $u_{122,123,\ldots,129}(x,t) = a_0 + a_1(\operatorname{sc}\xi \pm \operatorname{nc}\xi) + b_1(1/(\operatorname{sc}\xi \pm \operatorname{nc}\xi))$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain constant solutions, and when $k \to 0$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t),\ldots, u_{21,22}(x,t)]$, and for $a > (5/6)b$

$$u_{130,131}(x,t) = -\frac{1}{2} + \frac{i}{2} \times \left(\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right),$$

$$u_{132,133}(x,t) = -\frac{1}{2} - \frac{i}{2} \times \left(\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right),$$

$$u_{134,135}(x,t) = \frac{1}{2}\left(-1 + i \times \left(\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)^{-1}\right),$$

$$u_{136,137}(x,t) = \frac{1}{2}\left(-1 - i \times \left(\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)^{-1}\right),$$

$$u_{138,139}(x,t) = -\frac{1}{2} \pm \frac{1}{2\sqrt{2}}\left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right) \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{140,141}(x,t) = -\frac{1}{2} \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right) \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{142,143}(x,t) = -\frac{1}{2} \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right) \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{144,145}(x,t) = -\frac{1}{2} \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right) \pm \frac{1}{2\sqrt{2}} \times \left(i\tanh\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + \operatorname{sech}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{146,147}(x,t) = -\frac{1}{2} + \frac{i}{4} \times \left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right) + i \times \left(4\left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \mp \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{148,149}(x,t) = -\frac{1}{2} - \frac{i}{4} \times \left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right) - i \times \left(4\left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \mp \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)\right)\right)^{-1}. \quad (16)$$

For $a < (5/6)b$

$$u_{150,151}(x,t) = -\frac{1}{2} \pm \frac{1}{2}\tanh\left(\frac{1}{\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right) \pm \frac{i}{2}\operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}}\left(x - \frac{5}{6}t\right)\right),$$

$$u_{152,153}(x,t) = -\frac{1}{2} \pm \frac{1}{2}\tanh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \mp \frac{i}{2}\operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right),$$

$$u_{154,155}(x,t) = \frac{1}{2}\left(-1+1\times\left(\tanh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm i\operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1}\right),$$

$$u_{156,157}(x,t) = \frac{1}{2}\left(-1-1 \times\left(\tanh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm i\operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1}\right),$$

$$u_{158,159}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}}\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \frac{1}{2\sqrt{2}}\times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{160}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) + \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \frac{1}{2\sqrt{2}}\times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) - \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{161}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}}\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) - \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \frac{1}{2\sqrt{2}}\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) + \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{162,163}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \frac{1}{2\sqrt{2}}\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{164}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) + \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \frac{1}{2\sqrt{2}}\times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) - \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{165}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) - \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \frac{1}{2\sqrt{2}}\times\left(\tan\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) + \sec\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)^{-1},$$

$$u_{166,167}(x,t) = -\frac{1}{2} + \frac{1}{4} \times \left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \pm i \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \right) + 1 \times \left(4 \left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \pm i \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \right) \right)^{-1},$$

$$u_{168,169}(x,t) = -\frac{1}{2} - \frac{1}{4} \times \left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \pm i \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \right) - 1 \times \left(4 \left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \pm i \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right) \right) \right)^{-1}. \tag{17}$$

Using (7), the solution of (8) when $p = (k^2 - 2)/2$, $q = k^2/2$, and $r = 1/4$, and the sets of solutions (3)–(10), we get $u_{170,171,\ldots,177}(x,t) = a_0 + a_1(\operatorname{sn}\xi/(1 \pm \operatorname{dn}\xi)) + b_1((1 \pm \operatorname{dn}\xi)/\operatorname{sn}\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain for $a < (5/6)b$

$$u_{178,179}(x,t) = -\frac{1}{2} + \frac{\frac{1}{2}\tanh\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)}{1 \pm \operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{180,181}(x,t) = -\frac{1}{2} - \frac{\frac{1}{2}\left(\tanh \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)}{1 \pm \operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{182,183}(x,t) = -\frac{1}{2} + \frac{\frac{1}{2}\left(1 \pm \operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{\tanh\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{184,185}(x,t) = -\frac{1}{2} - \frac{\frac{1}{2}\left(1 \pm \operatorname{sech}\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{\tanh\left(\frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{186,187}(x,t) = -\frac{1}{2} + \frac{\frac{1}{4}\left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{1 \pm \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)} + \frac{\frac{1}{4}\left(1 \pm \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{188,189}(x,t) = -\frac{1}{2} - \frac{\frac{1}{4}\left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{1 \pm \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)} - \frac{\frac{1}{4}\left(1 \pm \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{190}(x,t) = -\frac{1}{2} + \frac{\frac{1}{4}\left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{1 - \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)} - \frac{\frac{1}{4}\left(1 + \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)},$$

$$u_{191}(x,t) = -\frac{1}{2} - \frac{\frac{1}{4}\left(\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{1 + \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)} + \frac{\frac{1}{4}\left(1 - \operatorname{sech}\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)\right)}{\tanh\left(\frac{1}{2} \frac{1}{\sqrt{-6a+5b}} \left(x - \frac{5}{6}t \right) \right)}. \tag{18}$$

For $a > (5/6)b$

$$u_{192,193}(x,t) = -\frac{1}{2} + i\frac{\frac{1}{2}\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{194,195}(x,t) = -\frac{1}{2} - \frac{\frac{1}{2}\left(\tan\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{196,197}(x,t) = -\frac{1}{2} + \frac{\frac{1}{2}\left(1 \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{i\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{198,199}(x,t) = -\frac{1}{2} - \frac{\frac{1}{2}\left(1 \pm \sec\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{i\tan\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{200,201}(x,t) = -\frac{1}{2} + i\frac{\frac{1}{4}\left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{1 \pm \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\frac{1}{4}\left(1 \pm \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{i\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{202,203}(x,t) = -\frac{1}{2} - i\frac{\frac{1}{4}\left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{1 \pm \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} - \frac{\frac{1}{4}\left(1 \pm \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{i\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{204}(x,t) = -\frac{1}{2} + i\frac{\frac{1}{4}\left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{1 - \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} - \frac{\frac{1}{4}\left(1 + \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{i\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{205}(x,t) = -\frac{1}{2} - i\frac{\frac{1}{4}\left(\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{1 + \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\frac{1}{4}\left(1 - \sec\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)}{i\tan\left(\frac{1}{2}\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}. \quad (19)$$

When $k \to 0$, we obtain [$u_{31,32}(x,t)$ and $u_{33,34}(x,t)$].

Using (7), the solution of (8) when $p = (k^2+1)/2$, $q = (k^2-1)/2$, and $r = (1-k^2)/4$, and the sets of solutions (3)–(10), we get $u_{206,207,\dots,213}(x,t) = a_0 + a_1(\mathrm{dn}\,\xi/(1 \pm k\,\mathrm{sn}\,\xi)) + b_1((1 \pm k\,\mathrm{sn}\,\xi)/\mathrm{dn}\,\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain constant solutions, and when $k \to 0$, we obtain constant solutions.

Using (7), the solution of (8) when $p = (k^2+1)/2$, $q = -1/2$, and $r = -(1-k^2)^2/4$, and the sets of solutions (3)–(10), we get $u_{214,215,\dots,221}(x,t) = a_0 + a_1(k\,\mathrm{cn}\,\xi \pm \mathrm{dn}\,\xi) + b_1/(k\,\mathrm{cn}\,\xi \pm \mathrm{dn}\,\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain [$u_{3,4}(x,t)$ and $u_{5,6}(x,t)$] and when $k \to 0$, we obtain constant solution.

Using (7), the solution of (8) when $p = (k^2+1)/2$, $q = (1-k^2)/2$, and $r = (1-k^2)/4$, and the sets of solutions (3)–(10), we get $u_{222,223,\dots,229}(x,t) = a_0 + a_1(\mathrm{cn}\xi/(1 \pm \mathrm{sn}\xi)) + b_1((1 \pm \mathrm{sn}\xi)/\mathrm{cn}\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain constant solution, and when $k \to 0$, we obtain for $a > (5/6)b$

$$u_{230,231}(x,t) = -\frac{1}{2} + \frac{i}{2}\frac{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{232,233}(x,t) = -\frac{1}{2} - \frac{i}{2}\frac{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{234,235}(x,t) = -\frac{1}{2} + \frac{i}{2}\frac{1 \pm \sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{236,237}(x,t) = -\frac{1}{2} - \frac{i}{2}\frac{1 \pm \sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{238,239}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}}\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm i\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{1}{2\sqrt{2}}\frac{1 \pm i\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{240,241}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}}\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm i\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} - \frac{1}{2\sqrt{2}}\frac{1 \pm i\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{242,243}(x,t) = -\frac{1}{2} \pm \frac{1}{4}\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{i + \sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} \pm \frac{1}{4}\frac{i - \sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{244,245}(x,t) = -\frac{1}{2} \pm \frac{1}{4}\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{i - \sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} \pm \frac{1}{4}\frac{i + \sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}. \quad (20)$$

For $a < (5/6)b$

$$u_{246,247}(x,t) = -\frac{1}{2} + \frac{1}{2}\frac{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{i \pm \sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{248,249}(x,t) = -\frac{1}{2} - \frac{1}{2}\frac{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{i \pm \sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{250,251}(x,t) = -\frac{1}{2} + \frac{1}{2}\frac{i \pm \sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{252,253}(x,t) = -\frac{1}{2} - \frac{1}{2}\frac{i \pm \sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{254,255}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}}\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{1}{2\sqrt{2}}\frac{1 \pm \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{256,257}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}}\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} - \frac{1}{2\sqrt{2}}\frac{1 \pm \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{258,259}(x,t) = -\frac{1}{2} + \frac{i}{4}\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} - \frac{i}{4}\frac{1 \mp \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)},$$

$$u_{260,261}(x,t) = -\frac{1}{2} - \frac{i}{4}\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{1 \pm \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{i}{4}\frac{1 \mp \sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}. \quad (21)$$

Using (7), the solution of (8) when $p = (1-2k^2)/2$, $q = 1/2$, and $r = k^2/4$, and the sets of solutions (3)–(10), we get $u_{262,263,\ldots,269}(x,t) = a_0 + a_1(k\,\mathrm{sn}\,\xi \pm i\,\mathrm{dn}\,\xi) + b_1(1/(k\,\mathrm{sn}\,\xi \pm i\,\mathrm{dn}\,\xi))$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{130,131}(x,t), u_{132,133}(x,t), \ldots, u_{168,169}(x,t)]$, and when $k \to 0$, we obtain constant solutions.

Using (7), the solution of (8) when $p = (k^2-2)/2$, $q = k^2/2$, and $r = k^2/4$, and the sets of solutions (3)–(10), we get $u_{270,271,\ldots,277}(x,t) = a_0 + a_1(k\,\mathrm{sn}\,\xi \pm i\,\mathrm{cn}\,\xi) + b_1(1/(k\,\mathrm{sn}\,\xi \pm i\,\mathrm{cn}\,\xi))$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{130,131}(x,t), u_{132,133}(x,t), \ldots, u_{168,169}(x,t)]$, and when $k \to 0$, we obtain constant solutions.

Using (7), the solution of (8) when $p = (k^2-2)/2$, $q = 1/2$, and $r = k^4/4$, and the sets of solutions (3)–(10), we get $u_{278,279,\ldots,285}(x,t) = a_0 + a_1(\mathrm{ns}\,\xi \pm \mathrm{ds}\,\xi) + b_1(1/(\mathrm{ns}\,\xi \pm \mathrm{ds}\,\xi))$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t), \ldots, u_{21,22}(x,t)]$, and for $a < (5/6)b$

$$u_{286,287}(x,t) = -\frac{1}{2} + \frac{1}{2} \times \left(\coth\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right),$$

$$u_{288,289}(x,t) = -\frac{1}{2} - \frac{1}{2} \times \left(\coth\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right),$$

$$u_{290,291}(x,t) = -\frac{1}{2} + 1 \times \left(2\left(\coth\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{292,293}(x,t) = -\frac{1}{2} - 1 \times \left(2\left(\coth\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{294,295}(x,t) = -\frac{1}{2} + \frac{1}{4} \times \left(\coth\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) + 1 \times \left(4\left(\coth\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{296,297}(x,t) = -\frac{1}{2} - \frac{1}{4} \times \left(\coth\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) - 1 \times \left(4\left(\coth\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{298,299}(x,t) = -\frac{1}{2} + \frac{\sqrt{2}}{4} \times \left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \sqrt{2} \times \left(4\left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{300,301}(x,t) = -\frac{1}{2} - \frac{\sqrt{2}}{4} \times \left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \sqrt{2} \times \left(4\left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{302,303}(x,t) = -\frac{1}{2} + \frac{\sqrt{2}}{4} \times \left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \sqrt{2} \times \left(4\left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \mp \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{304,305}(x,t) = -\frac{1}{2} - \frac{\sqrt{2}}{4} \times \left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \sqrt{2} \times \left(4\left(\cot\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \mp \csc\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1}. \tag{22}$$

For $a > (5/6)b$

$$u_{306,307}(x,t) = -\frac{1}{2} + \frac{i}{2} \times \left(\cot\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right),$$

$$u_{308,309}(x,t) = -\frac{1}{2} - \frac{i}{2} \times \left(\cot\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right),$$

$$u_{310,311}(x,t) = -\frac{1}{2} + i \times \left(2\left(\cot\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{312,313}(x,t) = -\frac{1}{2} - i \times \left(2\left(\cot\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{314,315}(x,t) = -\frac{1}{2} + \frac{i}{4} \times \left(\cot\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right) - i \times \left(4\left(\cot\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{316,317}(x,t) = -\frac{1}{2} - \frac{i}{4} \times \left(\cot\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right) + i \times \left(4\left(\cot\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \csc\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{318,319}(x,t) = -\frac{1}{2} + \frac{\sqrt{2}i}{4} \times \left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \sqrt{2}i \times \left(4\left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{320,321}(x,t) = -\frac{1}{2} - \frac{\sqrt{2}i}{4} \times \left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \sqrt{2}i \times \left(4\left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{322,323}(x,t) = -\frac{1}{2} + \frac{\sqrt{2}i}{4} \times \left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right) - \sqrt{2}i \times \left(4\left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \mp \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1},$$

$$u_{324,325}(x,t) = -\frac{1}{2} - \frac{\sqrt{2}i}{4} \times \left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right) + \sqrt{2}i \times \left(4\left(\coth\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \mp \operatorname{csch}\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)\right)\right)^{-1}. \quad (23)$$

When $k \to 0$, we obtain $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$.

Using (7), the solution of (8) when $p = (1-2k^2)/2$, $q = 1/2$, and $r = 1/4$, and the sets of solutions (3)–(10), we get $u_{326,327,\ldots,333}(x,t) = a_0 + a_1(\operatorname{ns}\xi - \operatorname{cs}\xi) + b_1(1/(\operatorname{ns}\xi - \operatorname{cs}\xi))$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t), \ldots, u_{21,22}(x,t)]$ and $[u_{286,287}(x,t), u_{288,289}(x,t), \ldots, u_{324,325}(x,t)]$, and when $k \to 0$, we obtain $[u_{7,8}(x,t), u_{9,10}(x,t), \ldots, u_{21,22}(x,t)]$ and $[u_{286,287}(x,t), u_{288,289}(x,t), \ldots, u_{324,325}(x,t)]$.

Using (7), the solution of (8) when $p = (1-2k^2)/2$, $q = 1/2$, and $r = 1/4$, and the sets of solutions (3)–(10), we get $u_{334,335,\ldots,341}(x,t) = a_0 + a_1(\operatorname{cn}\xi/(\sqrt{1-k^2}\operatorname{sn}\xi \pm \operatorname{dn}\xi)) + b_1((\sqrt{1-k^2}\operatorname{sn}\xi \pm \operatorname{dn}\xi)/\operatorname{cn}\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$ we obtain constant solutions, when $k \to 0$ we obtain $[u_{230,231}(x,t), u_{232,233}(x,t), \ldots, u_{260,261}(x,t)]$.

Using (7), the solution of (8) when $p = (1+k^2)/2$, $q = (1-k^2)^2/2$, and $r = 1/4$, and the sets of solutions (3)–(10), we get $u_{342,343,\ldots,349}(x,t) = a_0 + a_1(\operatorname{sn}\xi/(\operatorname{cn}\xi \pm \operatorname{dn}\xi)) + b_1((\operatorname{cn}\xi \pm \operatorname{dn}\xi)/\operatorname{sn}\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we get $[u_{31,32}(x,t)$ and $u_{33,34}(x,t)]$, and when $k \to 0$, we obtain for $a > (5/6)b$

$$u_{350,351}(x,t) = -\frac{1}{2} + \frac{i}{2} \times \left(\frac{\sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}\right),$$

$$u_{352,353}(x,t) = -\frac{1}{2} - \frac{i}{2} \times \left(\frac{\sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}\right),$$

$$u_{354,355}(x,t) = -\frac{1}{2} + \frac{i}{2} \times \left(\frac{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}{\sin\left(\frac{1}{\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}\right),$$

$$u_{356,357}(x,t) = -\frac{1}{2} - \frac{i}{2} \times \left(\frac{\cos\left(\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1}{\sin\left(\frac{1}{\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} \right),$$

$$u_{358,359}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1}{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{360,361}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1}{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{362,363}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + 1}{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - 1} \right),$$

$$u_{364,365}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - 1}{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + 1} \right),$$

$$u_{366,367}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + 1}{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - 1} \right),$$

$$u_{368,369}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) - 1}{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{2}\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) + 1} \right),$$

$$u_{370,371}(x,t) = -\frac{1}{2} + \frac{i}{4} \times \left(\frac{\cos\left(\frac{1}{2\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1}{\sin\left(\frac{1}{2\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)} - \frac{\sin\left(\frac{1}{2\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{2\sqrt{6a-5b}}\left(x - \frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{372,373}(x,t) = -\frac{1}{2} - \frac{i}{4} \times \left(\frac{\cos\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}{\sin\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{2\sqrt{6a-5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1} \right). \quad (24)$$

For $a < (5/6)b$

$$u_{374,375}(x,t) = -\frac{1}{2} + \frac{1}{2} \times \left(\frac{\sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{375,377}(x,t) = -\frac{1}{2} - \frac{1}{2} \times \left(\frac{\sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{378,379}(x,t) = -\frac{1}{2} + \frac{1}{2} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}{\sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} \right),$$

$$u_{380,381}(x,t) = -\frac{1}{2} - \frac{1}{2} \times \left(\frac{\cosh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}{\sinh\left(\frac{1}{\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} \right),$$

$$u_{382,383}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times \left(\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{384,385}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times \left(\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1}{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) \pm 1} \right),$$

$$u_{386,387}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times \left(\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) + 1}{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) - 1} \right),$$

$$u_{388,389}(x,t) = -\frac{1}{2} + \frac{1}{2\sqrt{2}} \times \left(\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) - 1}{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right) + 1} \right),$$

$$u_{390,391}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times \left(\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)+1}{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)-1} \right),$$

$$u_{392,393}(x,t) = -\frac{1}{2} - \frac{1}{2\sqrt{2}} \times \left(\frac{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)-1}{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sin\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cos\left(\frac{1}{\sqrt{2}\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)+1} \right),$$

$$u_{394,395}(x,t) = -\frac{1}{2} + \frac{1}{4} \times \left(\frac{\cosh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\pm 1}{\sinh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} + \frac{\sinh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\pm 1} \right),$$

$$u_{396,397}(x,t) = -\frac{1}{2} - \frac{1}{4} \times \left(\frac{\cosh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\pm 1}{\sinh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)} - \frac{\sinh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)}{\cosh\left(\frac{1}{2\sqrt{-6a+5b}}\left(x-\frac{5}{6}t\right)\right)\pm 1} \right). \quad (25)$$

Using (7), the solution of (8) when $p = (k^2 - 2)/2$, $q = k^2/2$, and $r = 1/4$, and the sets of solutions (3)–(10), we get $u_{398,399,\ldots,405}(x,t) = a_0 + a_1(\operatorname{cn}\xi/(\sqrt{1-k^2} \pm \operatorname{dn}\xi)) + b_1((\sqrt{1-k^2} \pm \operatorname{dn}\xi)/\operatorname{cn}\xi)$, where a_0, a_1, and b_1 are defined in the sets of solutions (3)–(10).

Note that, when $k \to 1$, we get constant solutions, and when $k \to 0$, we obtain, $[u_{3,4}(x,t)$ and $u_{5,6}(x,t)]$.

4. Conclusion

In this paper, the mapping method has been successfully implemented to find new traveling wave solutions for our new proposed equation, namely, a combined Padé-II and modified Padé-II equation. The results show that this method is a powerful mathematical tool for obtaining exact solutions for our equation. It is also a promising method to solve other nonlinear partial differential equations.

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