Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 896815, 6 pages
http://dx.doi.org/10.1155/2013/896815

Research Article

On Super (a, d)-Edge-Antimagic Total Labeling of Special Types of Crown Graphs

Himayat Ullah,[1] Gohar Ali,[1] Murtaza Ali,[1] and Andrea Semaničová-Feňovčíková[2]

[1] *FAST-National University of Computer and Emerging Sciences, Peshawar 25000, Pakistan*
[2] *Department of Applied Mathematics and Informatics, Technical University, 042 00 Košice, Slovakia*

Correspondence should be addressed to Gohar Ali; gohar.ali@nu.edu.pk

Received 8 April 2013; Accepted 9 June 2013

Academic Editor: Zhihong Guan

For a graph $G = (V, E)$, a bijection f from $V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called (a, d)-edge-antimagic total ((a, d)-EAT) labeling of G if the edge-weights $w(xy) = f(x) + f(y) + f(xy)$, $xy \in E(G)$, form an arithmetic progression starting from a and having a common difference d, where $a > 0$ and $d \geq 0$ are two fixed integers. An (a, d)-EAT labeling is called super (a, d)-EAT labeling if the vertices are labeled with the smallest possible numbers; that is, $f(V) = \{1, 2, \ldots, |V(G)|\}$. In this paper, we study super (a, d)-EAT labeling of cycles with some pendant edges attached to different vertices of the cycle.

1. Introduction

All graphs considered here are finite, undirected, and without loops and multiple edges. Let G be a graph with the vertex set $V = V(G)$ and the edge set $E = E(G)$. For a general reference of the graph theoretic notions, see [1, 2].

A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers, usually to positive or nonnegative integers. In this paper, the domain of the map is the set of all vertices and all edges of a graph. Such type of labeling is called *total labeling*. Some labelings use the vertex-set only, or the edge-set only, and we will call them *vertex labelings* or *edge labelings,* respectively. The most complete recent survey of graph labelings can be seen in [3, 4].

A bijection $g : V(G) \to \{1, 2, \ldots, |V(G)|\}$ is called (a, d)-*edge-antimagic vertex* ((a, d)-EAV) labeling of G if the set of edge-weights of all edges in G is equal to the set $\{a, a + d, a + 2d, \ldots, a + (|E(G)| - 1)d\}$, where $a > 0$ and $d \geq 0$ are two fixed integers. The edge-weight $w_g(xy)$ of an edge $xy \in E(G)$ under the vertex labeling g is defined as the sum of the labels of its end vertices; that is, $w_g(xy) = g(x) + g(y)$. A graph that admits (a, d)-EAV labeling is called an (a, d)-EAV graph.

A bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$ is called (a, d)-*edge-antimagic total* ((a, d)-EAT) labeling of G if the set of edge-weights of all edges in G forms an arithmetic progression starting from a with the difference d, where $a > 0$ and $d \geq 0$ are two fixed integers. The edge-weight $w_f(xy)$ of an edge $xy \in E(G)$ under the total labeling f is defined as the sum of the edge label and the labels of its end vertices. This means that $\{w_f(xy) = f(x) + f(y) + f(xy) : xy \in E(G)\} = \{a, a+d, a+2d, \ldots, a+(|E(G)|-1)d\}$. Moreover, if the vertices are labeled with the smallest possible numbers, that is, $f(V) = \{1, 2, \ldots, |V(G)|\}$, then the labeling is called *super* (a, d)-*edge-antimagic total* (super (a, d)-EAT). A graph that admits an (a, d)-EAT labeling or a super (a, d)-EAT labeling is called an (a, d)-EAT graph or a super (a, d)-EAT graph, respectively.

The super $(a, 0)$-EAT labelings are usually called *super edge-magic*; see [5–8]. Definition of super (a, d)-EAT labeling was introduced by Simanjuntak et al. [9]. This labeling is a natural extension of the notions of edge-magic labeling; see [7, 8]. Many other researchers investigated different types of antimagic graphs. For example, see Bodendiek and Walther [10], Hartsfield and Ringel [11].

In [9], Simanjuntak et al. defined the concept of (a, d)-EAV graphs and studied the properties of (a, d)-EAV labeling and (a, d)-EAT labeling and gave constructions of (a, d)-EAT labelings for cycles and paths. Bača et al. [12] presented some relation between (a, d)-EAT labeling and other labelings, namely, edge-magic vertex labeling and edge-magic total labeling.

In this paper, we study super (a, d)-EAT labeling of the class of graphs that can be obtained from a cycle by attaching some pendant edges to different vertices of the cycle.

2. Basic Properties

Let us first recall the known upper bound for the parameter d of super (a, d)-EAT labeling. Assume that a graph G has a super (a, d)-EAT labeling f, $f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$. The minimum possible edge-weight in the labeling f is at least $1 + 2 + (|V(G)| + 1) = |V(G)| + 4$. Thus, $a \geq |V(G)| + 4$. On the other hand, the maximum possible edge-weight is at most $(|V(G)| - 1) + |V(G)| + (|V(G)| + |E(G)|) = 3|V(G)| + |E(G)| - 1$. So

$$a + (|E(G)| - 1) d \leq 3 |V(G)| + |E(G)| - 1, \tag{1}$$

$$d \leq \frac{2 |V(G)| + |E(G)| - 5}{|E(G)| - 1}. \tag{2}$$

Thus, we have the upper bound for the difference d. In particular, from (2), it follows that, for any connected graph, where $|V(G)| - 1 \leq |E(G)|$, the feasible value d is no more than 3.

The next proposition, proved by Bača et al. [12], gives a method on how to extend an edge-antimagic vertex labeling to a super edge-antimagic total labeling.

Proposition 1 (see [12]). *If a graph G has an (a, d)-EAV labeling, then*

(i) *G has a super $(a + |V(G)| + 1, d + 1)$-EAT labeling, and*

(ii) *G has a super $(a + |V(G)| + |E(G)|, d - 1)$-EAT labeling.*

The following lemma will be useful to obtain a super edge-antimagic total labeling.

Lemma 2 (see [13]). *Let $\mathfrak{A}$ be a sequence $\mathfrak{A} = \{c, c + 1, c + 2, \ldots, c + k\}$, k even. Then, there exists a permutation $\Pi(\mathfrak{A})$ of the elements of $\mathfrak{A}$, such that $\mathfrak{A} + \Pi(\mathfrak{A}) = \{2c + k/2, 2c + k/2 + 1, 2c + k/2 + 2, \ldots, 2c + 3k/2 - 1, 2c + 3k/2\}$.*

Using this lemma, we obtain that, if G is an $(a, 1)$-EAV graph with odd number of edges, then G is also super $(a', 1)$-EAT.

3. Crowns

If G has order n, the *corona* of G with H, denoted by $G \odot H$, is the graph obtained by taking one copy of G and n copies of H and joining the ith vertex of G with an edge to every vertex in the ith copy of H. A cycle of order n with an m pendant edges attached at each vertex, that is, $C_n \odot mK_1$, is called an *m-crown* with cycle of order n. A 1-crown, or only *crown*, is a cycle with exactly one pendant edge attached at each vertex of the cycle. For the sake of brevity, we will refer to the crown with cycle of order n simply as the *crown* if its cycle order is clear from the context. Note that a crown is also known in the literature as a sun graph. In this section, we will deal with the graphs related to 1-crown with cycle of order n.

Silaban and Sugeng [14] showed that, if the m-crown $C_n \odot mK_1$ is (a, d)-EAT, then $d \leq 5$. They also describe (a, d)-EAT, labeling of the m-crown for $d = 2$ and $d = 4$. Note that the $(a, 2)$-EAT labeling of m-crown presented in [14] is super $(a, 2)$-EAT. Moreover, they proved that, if $m \equiv 1 \pmod 4$, n and d are odd; there is no (a, d)-EAT labeling for $C_n \odot mK_1$. They also proposed the following open problem.

Open Problem 1 (see [14]). Find if there is an (a, d)-EAT labeling, $d \in \{1, 3, 5\}$ for m-crown graphs $C_n \odot mK_1$.

Figueroa-Centeno et al. [15] proved that the m-crown graph has a super $(a, 0)$-EAT labeling.

Proposition 3 (see [15]). *For every two integers $n \geq 3$ and $m \geq 1$, the m-crown $C_n \odot mK_1$ is super $(a, 0)$-EAT.*

According to inequality (2), we have that, if the crown $C_n \odot K_1$ is super (a, d)-EAT, then $d \leq 2$. Immediately from Proposition 3 and the results proved in [14], we have that the crown $C_n \odot K_1$ is super $(a, 0)$-EAT and super $(a, 2)$-EAT for every positive integer $n \geq 3$. Moreover, for n odd, the crown is not $(a, 1)$-EAT. In the following theorem, we prove that the crown is super $(a, 1)$-EAT for n even. Thus, we partially give an answer to Open Problem 1.

Theorem 4. *For every even positive integer n, $n \geq 4$, the crown $C_n \odot K_1$ is super $(a, 1)$-EAT.*

Proof. Let $V(C_n \odot K_1) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ be the vertex set and $E(C_n \odot K_1) = \{v_1v_2, v_2v_3 \ldots, v_nv_1, v_1u_1, v_2u_2, \ldots, v_nu_n\}$ be the edge set of $C_n \odot K_1$; see Figure 1.

We define a labeling f, $f : V(C_n \odot K_1) \cup E(C_n \odot K_1) \to \{1, 2, \ldots, 4n\}$ as follows:

$$f(v_i) = \begin{cases} i + n, & \text{for } i = 1, 2, \ldots, \dfrac{n}{2}; \\ i, & \text{for } i = \dfrac{n}{2} + 1, \dfrac{n}{2} + 2, \ldots, n; \end{cases}$$

$$f(u_i) = \begin{cases} i, & \text{for } i = 1, 2, \ldots, \dfrac{n}{2}; \\ i + n, & \text{for } i = \dfrac{n}{2} + 1, \dfrac{n}{2} + 2, \ldots, n; \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} \dfrac{7n}{2} + 1 - i, & \text{for } i = 1, 2, \ldots, \dfrac{n}{2}; \\ \dfrac{9n}{2} + 1 - i, & \text{for } i = \dfrac{n}{2} + 1, \dfrac{n}{2} + 2, \ldots, n - 1; \end{cases}$$

$$f(v_1v_n) = \frac{7n}{2} + 1;$$

$$f(v_iu_i) = 3n + 1 - i, \quad \text{for } i = 1, 2, \ldots, n. \tag{3}$$

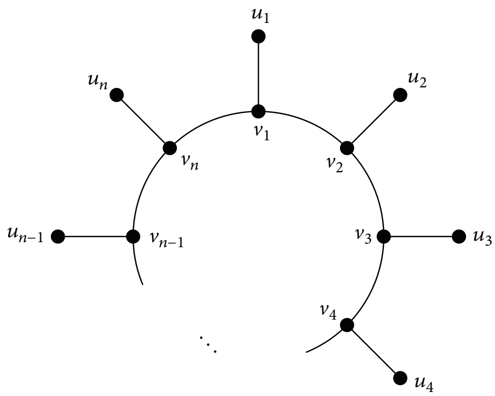

Figure 1: The crown $C_n \odot K_1$.

It is easy to check that f is a bijection. For the edge-weights under the labeling f, we have

$$w_f(v_1v_n) = \frac{11n}{2} + 2;$$
$$w_f(v_iv_{i+1}) = \begin{cases} \frac{11n}{2} + 2 + i, & \text{for } i = 1, 2, \ldots, \frac{n}{2} - 1; \\ \frac{9n}{2} + 2 + i, & \text{for } i = \frac{n}{2}, \frac{n}{2} + 1, \ldots, n - 1; \end{cases}$$
$$w_f(v_iu_i) = 4n + 1 + i, \quad \text{for } i = 1, 2, \ldots, n. \tag{4}$$

Thus, the edge-weights are distinct number from the set $\{4n + 2, 4n + 3, \ldots, 6n + 1\}$. This means that f is a super $(a, 1)$-EAT labeling of the crown $C_n \odot K_1$. □

In [16], was proved the following.

Proposition 5 (see [16]). *Let G be a super $(a, 1)$-EAT graph. Then, the disjoint union of arbitrary number of copies of G, that is, sG, $s \ge 1$, also admits a super $(a', 1)$-EAT labeling.*

In [17], Figueroa-Centeno et al. proved the following.

Proposition 6 (see [17]). *If G is a (super) $(a, 0)$-EAT bipartite or tripartite graph and s is odd, then sG is (super) $(a', 0)$-EAT.*

For (super) $(a, 2)$-EAT labeling for disjoint union of copies of a graph, was shown the following.

Proposition 7 (see [18]). *If G is a (super) $(a, 2)$-EAT bipartite or tripartite graph and s is odd, then sG is (super) $(a', 2)$-EAT.*

Using the above mentioned results, we immediately obtain the following theorem.

Theorem 8. *Let n be an even positive integer, $n \ge 4$. Then, the disjoint union of arbitrary number of copies of the crown $C_n \odot K_1$, that is, $s(C_n \odot K_1)$, $s \ge 1$, admits a super $(a, 1)$-EAT labeling.*

Since the crown $C_n \odot K_1$ is either a bipartite graph, for n even, or a tripartite graph, for n odd, thus, we have the following.

Theorem 9. *Let n be a positive integer, $n \ge 3$. Then, the disjoint union of odd number of copies of the crown $C_n \odot K_1$, that is, $s(C_n \odot K_1)$, $s \ge 1$, admits a super (a, d)-EAT labeling for $d \in \{0, 2\}$.*

According to these results we are able to give partially positive answers to the open problems listed in [19].

Open Problem 2 (see [19]). For the graph $s(C_n \odot mK_1)$, sn even, and $m \ge 1$, determine if there is a super (a, d)-EAT labeling with $d \in \{0, 2\}$.

Open Problem 3 (see [19]). For the graph $s(C_n \odot mK_1)$, s odd, and $n(m + 1)$ even, determine if there is a super $(a, 1)$-EAT labeling.

4. Graphs Related to Crown Graphs

Let us consider the graph obtained from a crown graph $C_n \odot K_1$ by deleting one pendant edge.

Theorem 10. *For n odd, $n \ge 3$, the graph obtained from a crown graph $C_n \odot K_1$ by removing a pendant edge is super (a, d)-EAT for $d \in \{0, 1, 2\}$.*

Proof. Let G be a graph obtained from a crown graph $C_n \odot K_1$ by removing a pendant edge. Without loss of generality, we can assume that the removed edge is v_1u_1. Other edges and vertices we denote in the same manner as that of Theorem 4. Thus $|V(G)| = 2n - 1$ and $|E(G)| = 2n - 1$. It follows from (2) that $d \le 2$.

Define a vertex labeling g as follows:

$$g(v_i) = \begin{cases} i, & \text{for } i = 1, 3, \ldots, n; \\ i + n - 1, & \text{for } i = 2, 4, \ldots, n - 1; \end{cases}$$
$$g(u_i) = \begin{cases} i + n - 1, & \text{for } i = 3, 5, \ldots, n; \\ i, & \text{for } i = 2, 4, \ldots, n - 1. \end{cases} \tag{5}$$

It is easy to see that labeling g is a bijection from the vertex set to the set $\{1, 2, \ldots, 2n - 1\}$. For the edge-weights under the labeling g, we have

$$w_g(v_1v_n) = 1 + n;$$
$$w_g(v_iv_{i+1}) = 2i + n, \quad \text{for } i = 1, 2, \ldots, n - 1; \tag{6}$$
$$w_g(v_iu_i) = 2i + n - 1, \quad \text{for } i = 2, 3, \ldots, n.$$

Thus, the edge-weights are consecutive numbers $n + 1, n + 2, \ldots, 3n - 1$. This means that g is the $(a, 1)$-EAV labeling of G. According to Proposition 1, the labeling g can be extended to the super $(a_0, 0)$-EAT and the super $(a_2, 2)$-EAT labeling of G. Moreover, as the size of G is odd, $|E(G)| = 2n - 1$, and according to Lemma 2, we have that G is also super $(a_1, 1)$-EAT. □

Note that we found some $(a, 1)$-EAV labelings for the graphs obtained from a crown graph $C_n \odot K_1$ by removing a pendant edge only for small size of n, where n is even. In Figure 2, there are depicted $(a, 1)$-EAV labelings of these graphs for $n = 4, 6, 8$. However, we propose the following conjecture.

Conjecture 11. *The graph obtained from a crown graph $C_n \odot K_1$, $n \geq 3$, by removing a pendant edge is super (a, d)-EAT for $d \in \{0, 1, 2\}$.*

Now, we will deal with the graph obtained from a crown graph $C_n \odot K_1$ by removing two pendant edges at distance 1 and at distance 2.

First, consider the case when we remove two pendant edges at distance 1. Let us consider the graph G with the $(a, 1)$-EAV labeling g defined in the proof of Theorem 10. It is easy to see that the vertex u_n is labeled with the maximal vertex label, $g(u_n) = 2n - 1$. Also, the edge-weight of the edge $v_n u_n$ is the maximal possible, $w_g(v_n u_n) = g(v_n) + g(u_n) = (2n - 1) + n = 3n - 1$. Thus, it is possible to remove the edge $v_n u_n$ from the graph G; we denote the graph by G_r, and for the labeling g restricted to the graph G_r, we denote it by g_r. Clearly, r_r is $(a, 1)$-EAV labeling of G_r. According to Proposition 1 from the labeling g_r, we obtain the super $(a_0, 0)$-EAT and the super $(a_2, 2)$-EAT labeling of G_r. Note that, as the size of G_r is even, $|E(G_r)| = 2n - 2$, the labeling g_r can not be extended to a super $(a_1, 1)$-EAT labeling of G_r.

Theorem 12. *For odd n, $n \geq 3$, the graph obtained from a crown graph $C_n \odot K_1$ by removing two pendant edges at distance 1 is super (a, d)-EAT for $d \in \{0, 2\}$.*

Next, we show that, if we remove from a crown graph $C_n \odot K_1, n \geq 5$, two pendant edges at distance 2, the resulting graph is super (a, d)-EAT for $d \in \{0, 2\}$.

Theorem 13. *For odd n, $n \geq 5$, the graph obtained from a crown graph $C_n \odot K_1$ by removing two pendant edges at distance 2 is super (a, d)-EAT for $d \in \{0, 2\}$.*

Proof. Let G be a graph obtained from a crown graph $C_n \odot K_1$ by removing two pendant edges at distance 2. Let $V(G) = \{v_1, v_2, \ldots, v_n, u_2, u_3, \ldots, u_{n-2}, u_n\}$ be the vertex set and $E(G) = \{v_1 v_2, v_2 v_3 \ldots, v_n v_1, v_2 u_2, v_3 u_3, \ldots, v_{n-2} u_{n-2}, v_n u_n\}$ be the edge set of G. Thus, $|V(G)| = 2n - 2$ and $|E(G)| = 2n - 2$. Using (2) gives $d \leq 2$.

Define a vertex labeling g in the following way:

$$
\begin{gathered}
g(v_i) = \begin{cases} \dfrac{i+1}{2}, & \text{for } i = 1, 3, \ldots, n; \\[2ex] \dfrac{n+i+1}{2}, & \text{for } i = 2, 4, \ldots, n-1; \end{cases} \\[3ex]
g(u_i) = \begin{cases} \dfrac{3n-4+i}{2}, & \text{for } i = 3, 5, \ldots, n; \\[2ex] \dfrac{2n+i}{2}, & \text{for } i = 2, 4, \ldots, n-3. \end{cases}
\end{gathered}
\tag{7}
$$

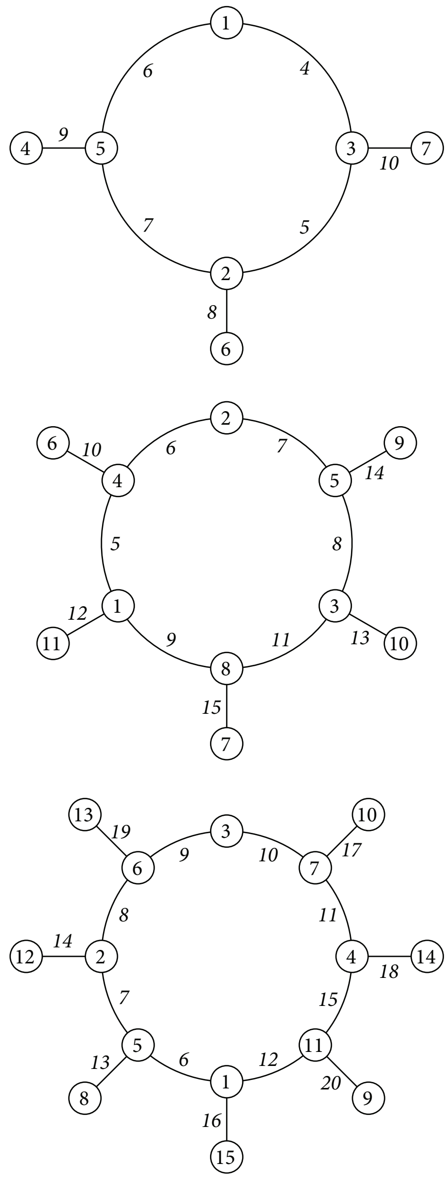

Figure 2: The $(a, 1)$-EAV labelings for the graphs obtained from a crown graph $C_n \odot K_1$ by removing one pendant edge for $n = 4, 6, 8$. The edge-weights under the corresponding labelings are depicted in italic.

It is not difficult to check that the labeling g is a bijection from the vertex set of G to the set $\{1, 2, \ldots, 2n-2\}$ and that the edge-weights under the labeling g are consecutive numbers $(n+3)/2, (n+5)/2, \ldots, (5n-3)/2$. Thus g is the $(a, 1)$-EAV labeling of G. According to Proposition 1, it is possible to extend the labeling g to the super $(a_0, 0)$-EAT and the super $(a_2, 2)$-EAT labelings of G. ☐

Result in the following theorem is based on the Petersen Theorem.

Proposition 14 (Petersen Theorem). *Let G be a $2r$-regular graph. Then, there exists a 2-factor in G.*

Notice that, after removing edges of the 2-factor guaranteed by the Petersen Theorem, we have again an even regular graph. Thus, by induction, an even regular graph has a 2-factorization.

The construction in the following theorem allows to find a super $(a, 1)$-EAT labeling of any graph that arose from an even regular graph by adding even number of pendant edges to different vertices of the original graph. Notice that the construction does not require the graph to be connected.

Theorem 15. *Let G be a graph that arose from a $2r$-regular graph H, $r \geq 0$, by adding k pendant edges to k different vertices of H, $0 \leq k \leq |V(H)|$. If k is an even integer, then the graph G is super $(a, 1)$-EAT.*

Proof. Let G be a graph that arose from a $2r$-regular graph H, $r \geq 0$, by adding k pendant edges to k different vertices of H, $0 \leq k \leq |V(H)|$ and let $m = |V(H)| - k$. Thus, $|V(G)| = 2k + m$, $|E(G)| = k + r(k + m)$, and $|E(H)| = r(k + m)$.

Let k be an even integer. Let us denote the pendant edges of G by symbols $e_1, e_2, \ldots, e_k$. We denote the vertices of G such that

$$e_i = v_i v_{k+m+i}, \quad i = 1, 2, \ldots, k, \tag{8}$$

and moreover

$$\{v_{k/2+1}, v_{k/2+2}, \ldots, v_k, v_{k+m+1}, v_{k+m+2}, \ldots, v_{3k/2+m}\} \subset V(H). \tag{9}$$

We denote the remaining m vertices of $V(H)$ arbitrarily by the symbols $v_{k+1}, v_{k+2}, \ldots, v_{k+m}$.

By the Petersen Theorem, there exists a 2-factorization of H. We denote the 2-factors by F_j, $j = 1, 2, \ldots, r$. Without loss of generality, we can suppose that $V(H) = V(F_j)$ for all j, $j = 1, 2, \ldots, r$ and $E(G) = \cup_{j=1}^{r} E(F_j) \cup \{e_1, e_2, \ldots, e_k\}$. Each factor F_j is a collection of cycles. We order and orient the cycles arbitrarily such that the arcs form oriented cycles. Now, we denote by the symbol $e_j^{\text{out}}(v_t)$ the unique outgoing arc that forms the vertex v_t in the factor F_j, $t = k/2 + 1, k/2 + 2, \ldots, 3k/2 + m$. Note that each edge is denoted by two symbols.

We define a total labeling f of G in the following way:

$$\begin{aligned} f(v_i) &= i, \quad \text{for } i = 1, 2, \ldots, 2k + m; \\ f(e_i) &= 3k + m + 1 - i, \quad \text{for } i = 1, 2, \ldots, k; \\ f\left(e_j^{\text{out}}(v_t)\right) &= \frac{7k}{2} + m + 1 - f(v_t) + j(k + m), \\ &\text{for } t = \frac{k}{2} + 1, \frac{k}{2} + 2, \ldots, \frac{3k}{2} + m, \ j = 1, 2, \ldots, r. \end{aligned} \tag{10}$$

It is easy to see that the vertices are labeled by the first $2k + m$ integers, the edges $e_1, e_2, \ldots, e_k$ by the next k labels, and the edges of H by consecutive integers starting at $3k + m + 1$. Thus, f is a bijection $V(G) \cup E(G) \to \{1, 2, \ldots 3k + m + r(k + m)\}$.

It is not difficult to verify that f is a super $(4k + 2m + 2, 1)$-EAT labeling of G. For the weights of the edges e_i, $i = 1, 2, \ldots, k$, we have

$$\begin{aligned} w_f(e_i) &= f(v_i) + f(v_{k+m+1}) + f(e_i) \\ &= i + (k + m + i) + (3k + m + 1 - i) \\ &= 4k + 2m + 1 + i. \end{aligned} \tag{11}$$

Thus, the edge-weights are the numbers

$$4k + 2m + 2, 4k + 2m + 3, \ldots, 5k + 2m + 1. \tag{12}$$

For convenience, we denote by v_s the unique vertex such that $v_t v_s = e_j^{\text{out}}(v_t)$ in F_j, where $s \in \{k/2+1, k/2+2, \ldots, 3k/2+m\}$. The weights of the edges in F_j, $j = 1, 2, \ldots, r$, are

$$\begin{aligned} w_f\left(e_j^{\text{out}}(v_t)\right) &= f(v_t) + f(v_s) + f\left(e_j^{\text{out}}(v_t)\right) \\ &= f(v_t) + f(v_s) \\ &\quad + \left(\frac{7k}{2} + m + 1 - f(v_t) + j(k + m)\right) \\ &= \frac{7k}{2} + m + 1 + j(k + m) + f(v_s) \end{aligned} \tag{13}$$

for all $t = (k/2)+1, (k/2)+2, \ldots, (3k/2)+m$, and $j = 1, 2, \ldots, r$. Since F_j is a factor in H it holds $\{f(v_s) : v_s \in F_j\} = \{k/2 + 1, k/2 + 2, \ldots, 3k/2 + m\}$. Hence, we have that the set of the edge-weights in the factor F_j is

$$\begin{aligned} \{&4k + m + 2 + j(k + m), 4k + m + 3 + j(k + m), \ldots, \\ &5k + 2m + 1 + j(k + m)\}, \end{aligned} \tag{14}$$

and thus, the set of all edge-weights in G under the labeling f is

$$\{4k + 2m + 2, 4k + 2m + 3, \ldots, 5k + 2m + 1 + r(k + m)\}. \tag{15}$$

□

We conclude the paper with the result that immediately follows from the previous theorem.

Corollary 16. *Let G be a graph obtained from a crown graph $C_n \odot K_1$, $n \geq 3$, by deleting any k pendant edges, $0 \leq k \leq n$. If $n - k$ is even, then G is a super $(a, 1)$-EAT graph.*

Acknowledgments

This research is partially supported by FAST-National University of Computer and Emerging Sciences, Peshawar, and Higher Education Commission of Pakistan. The research for this paper was also supported by Slovak VEGA Grant 1/0130/12.

References

[1] W. D. Wallis, *Magic Graphs*, Birkhäuser, Boston, Mass, USA, 2001.

[2] D. B. West, *An Introduction to Graph Theory*, Prentice Hall, 1996.

[3] M. Bača and M. Miller, *Super Edge-Antimagic Graphs: A Wealth of Problems and Some Solutions*, Brown Walker Press, Boca Raton, Fla, USA, 2008.

[4] J. A. Gallian, "Dynamic survey of graph labeling," *The Electronic Journal of Combinatorics*, vol. 18, 2011.

[5] H. Enomoto, A. S. Lladó, T. Nakamigawa, and G. Ringel, "Super edge-magic graphs," *SUT Journal of Mathematics*, vol. 34, no. 2, pp. 105–109, 1998.

[6] R. M. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, "The place of super edge-magic labelings among other classes of labelings," *Discrete Mathematics*, vol. 231, no. 1–3, pp. 153–168, 2001.

[7] A. Kotzig and A. Rosa, "Magic valuations of finite graphs," *Canadian Mathematical Bulletin*, vol. 13, pp. 451–461, 1970.

[8] A. Kotzig and A. Rosa, *Magic Valuations of Complete Graphs*, CRM Publisher, 1972.

[9] R. Simanjuntak, F. Bertault, and M. Miller, "Two new (a, d)-antimagic graph labelings," in *Proceedings of the 11th Australasian Workshop on Combinatorial Algorithms*, pp. 179–189, 2000.

[10] R. Bodendiek and G. Walther, "Arithmetisch antimagische graphen," in *Graphentheorie III*, K. Wagner and R. Bodendiek, Eds., BI-Wiss, Mannheim, Germany, 1993.

[11] N. Hartsfield and G. Ringel, *Pearls in Graph Theory*, Academic Press, Boston, Mass, USA, 1990.

[12] M. Bača, Y. Lin, M. Miller, and R. Simanjuntak, "New constructions of magic and antimagic graph labelings," *Utilitas Mathematica*, vol. 60, pp. 229–239, 2001.

[13] K. A. Sugeng, M. Miller, Slamin, and M. Bača, "(a, d)-edge-antimagic total labelings of caterpillars," in *Combinatorial Geometry and Graph Theory*, vol. 3330, pp. 169–180, Springer, Berlin, Germany, 2005.

[14] D. R. Silaban and K. A. Sugeng, "Edge antimagic total labeling on paths and unicycles," *Journal of Combinatorial Mathematics and Combinatorial Computing*, vol. 65, pp. 127–132, 2008.

[15] R. M. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, "Magical coronations of graphs," *The Australasian Journal of Combinatorics*, vol. 26, pp. 199–208, 2002.

[16] M. Bača, Y. Lin, and A. Semaničová, "Note on super antimagicness of disconnected graphs," *AKCE International Journal of Graphs and Combinatorics*, vol. 6, no. 1, pp. 47–55, 2009.

[17] R. M. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, "On edge-magic labelings of certain disjoint unions of graphs," *The Australasian Journal of Combinatorics*, vol. 32, pp. 225–242, 2005.

[18] M. Bača, F. A. Muntaner-Batle, A. Semaničová-Fenovčíková, and M. K. Shafiq, "On super $(a, 2)$-edge-antimagic total labeling of disconnectedgraphs," *Ars Combinatoria*. In press.

[19] M. Bača, Dafik, M. Miller, and J. Ryan, "Antimagic labeling of disjoint union of s-crowns," *Utilitas Mathematica*, vol. 79, pp. 193–205, 2009.