Hindawi Publishing Corporation
Journal of Applied Mathematics and Decision Sciences
Volume 2009, Article ID 897024, 20 pages
doi:10.1155/2009/897024

Research Article

A New Decision-Making Method for Stock Portfolio Selection Based on Computing with Linguistic Assessment

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Received 30 November 2008; Revised 18 March 2009; Accepted 13 May 2009

Recommended by Lean Yu

The purpose of stock portfolio selection is how to allocate the capital to a large number of stocks in order to bring a most profitable return for investors. In most of past literatures, experts considered the portfolio of selection problem only based on past crisp or quantitative data. However, many qualitative and quantitative factors will influence the stock portfolio selection in real investment situation. It is very important for experts or decision-makers to use their experience or knowledge to predict the performance of each stock and make a stock portfolio. Because of the knowledge, experience, and background of each expert are different and vague, different types of 2-tuple linguistic variable are suitable used to express experts' opinions for the performance evaluation of each stock with respect to criteria. According to the linguistic evaluations of experts, the linguistic TOPSIS and linguistic ELECTRE methods are combined to present a new decision-making method for dealing with stock selection problems in this paper. Once the investment set has been determined, the risk preferences of investor are considered to calculate the investment ratio of each stock in the investment set. Finally, an example is implemented to demonstrate the practicability of the proposed method.

1. Introduction

The purpose of stock portfolio selection is how to allocate the capital to a large number of stocks in order to bring a most profitable return for investors [1]. For this point of view, stock portfolio decision problem can be divided into two questions.

(1) Which stock do you choose?

(2) Which investment ratio do you allocate your capital to this stock?

There are some literatures to handle the stock portfolio decision problem. Markowitz proposed the mean-variance method for the stock portfolio decision problem in 1952 [2].

In his method, an expected return rate of a bond is treated as a random variable. Stochastic programming is applied to solve the problem. The basic concept of his method can be expressed as follows.

(1) When the risk of stock portfolio is constant, we should pursue to maximize the return rate of stock portfolio.

(2) When the return rate of stock portfolio is constant, we should pursue to minimize the risk of stock portfolio.

The capital asset pricing model (CAPM), Sharpe-Lintner model, Black model, and two-factor model are derived from the mean-variance method [3, 4]. The capital asset pricing model (CAPM) was developed in 1960s. The concept of the CAPM is that the excepted return rate of the capital with risk is equal to the interest rate of the capital without risk and market risk premium [4]. The methods and theory of the financial decision making can be found in [5–7]. In 1980, Saaty proposed Analytic Hierarchy Process (AHP) to deal with the stock portfolio decision problem by evaluating the performance of each company in different level of criteria [8]. Edirisinghe and Zhang [9] selected the securities by using Data Envelopment Analysis (DEA). Huang [1] defined a new definition of risk and use genetic algorithm to cope with stock portfolio decision problem. Generally, in the portfolio selection problem the decision maker considers simultaneously conflicting objectives such as rate of return, liquidity, and risk. Multiobjective programming techniques such as goal programming (GP) and compromise programming (CP) are used to choose the portfolio [10–12]. Considering the uncertainty of investment environment, Tiryaki transferred experts' linguistic value into triangle fuzzy number and used a new fuzzy ranking and weighting algorithm to obtain the investment ratio of each stock [4]. In fact, the stock portfolio decision problem can be described as multiple criteria decision making (MCDM) problem.

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method is developed by Hwang and Yoon [13], which is one of the well-known MCDM methods. The basic principle of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). It is an effective method to determine the total ranking order of decision alternatives.

The Elimination et choice in Translating to Reality (ELECTRE) method is a highly developed multicriteria analysis model which takes into account the uncertainty and vagueness in the decision process [14]. It is based on the axiom of partial comparability and it can simplify the evaluation procedure of alternative selection. The ELECTRE method can easily compare the degree of difference among all of alternatives.

In MCDM method, experts can express their opinions by using crisp value, triangle fuzzy numbers, trapezoidal fuzzy numbers, interval numbers, and linguistic variables. Due to imprecise information and experts' subjective opinion that often appear in stock portfolio decision process, crisp values are inadequate for solving the problems. A more realistic approach may be to use linguistic assessments instead of numerical values [15, 16]. The 2-tuple linguistic representation model is based on the concept of symbolic translation [17, 18]. Experts can apply 2-tuple linguistic variables to express their opinions and obtain the final evaluation result with appropriate linguistic variable. It is an effective method to reduce the mistakes of information translation and avoid information loss through computing with words [19]. In general, decision makers would use the different 2-tuple linguistic variables based on their knowledge or experiences to express their opinions [20]. In this paper, we use different type of 2-tuple linguistic variable to express experts' opinions and combine

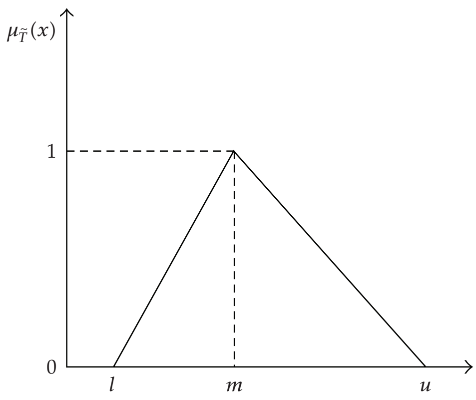

Figure 1: Triangular fuzzy number $\tilde{T}$.

linguistic ELECTRE method with TOPSIS method to obtain the final investment ratio which is reasonable in real decision environment.

This paper is organized as follows. In Section 2, we present the context of fuzzy set and the definition and operation of 2-tuple linguistic variable. In Section 3, we describe the detail of the proposed method. In Section 4, an example is implemented to demonstrate the procedure for the proposed method. Finally, the conclusion is discussed at the end of this paper.

2. The 2-Tuple Linguistic Representation

2.1. Fuzzy Set and Triangular Fuzzy Number

Fuzzy set theory is first introduced by Zadeh in 1965 [21]. Fuzzy set theory is a very feasible method to handle the imprecise and uncertain information in a real world [22]. Especially, it is more suitable for subjective judgment and qualitative assessment in the evaluation processes of decision making than other classical evaluation methods applying crisp values [23, 24].

A positive triangular fuzzy number (PTFN) $\tilde{T}$ can be defined as $\tilde{T} = (l, m, u)$, where $l \leq m \leq u$ and $l > 0$, shown in Figure 1. The membership function $\mu_{\tilde{T}}(x)$ of positive triangular fuzzy number (PTFN) $\tilde{T}$ is defined as [15]

$$\mu_{\tilde{T}}(x) = \begin{cases} \dfrac{x-l}{m-l}, & l < x < m, \\ \dfrac{u-x}{u-m}, & m < x < u, \\ 0, & \text{otherwise}. \end{cases} \tag{2.1}$$

A linguistic variable is a variable whose values are expressed in linguistic terms. In other words, variable whose values are not numbers but words or sentences in a nature or artificial language [25–27]. For example, "weight" is a linguistic variable whose values are very low, low, medium, high, very high, and so forth. These linguistic values can also be represented by fuzzy numbers. There are two advantages for using triangular fuzzy number to express linguistic variable [28]. First, it is a rational and simple method to use triangular fuzzy number to express experts' opinions. Second, it is easy to do fuzzy arithmetic when using triangular fuzzy number to express the linguistic variable. It is suitable to represent the degree of subjective judgment in qualitative aspect than crisp value.

2.2. The 2-Tuple Linguistic Variable

Let $S = \{s_0, s_1, s_2, \ldots, s_g\}$ be a finite and totally ordered linguistic term set. The number of linguistic term is $g+1$ in set S. A 2-tuple linguistic variable can be expressed as (s_i, α_i), where s_i is the central value of ith linguistic term in S and α_i is a numerical value representing the difference between calculated linguistic term and the closest index label in the initial linguistic term set. The symbolic translation function Δ is presented in [29] to translate crisp value β into a 2-tuple linguistic variable. Then, the symbolic translation process is applied to translate $\beta\,(\beta \in [0,1])$ into a 2-tuple linguistic variable. The generalized translation function can be represented as [30]:

$$\begin{gathered} \Delta : [0,1] \longrightarrow S \times \left[-\frac{1}{2g}, \frac{1}{2g}\right) \\ \Delta(\beta) = (s_i, \alpha_i), \end{gathered} \tag{2.2}$$

where $i = \text{round}(\beta \times g)$, $\alpha_i = \beta(-i/g)$ and $\alpha_i \in [-1/2g, 1/2g)$.

A reverse function Δ^{-1} is defined to return an equivalent numerical value β from 2-tuple linguistic information (s_i, α_i). According to the symbolic translation, an equivalent numerical value β is obtained as follow [30]

$$\Delta^{-1}(s_i, \alpha_i) = \frac{i}{g} + \alpha_i = \beta. \tag{2.3}$$

Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a 2-tuple linguistic variable set. The arithmetic mean $\overline{\text{X}}$ is computed as [31]

$$\overline{\text{X}} = \Delta\left(\frac{1}{n}\sum_{i=1}^{n} \Delta^{-1}(r_i, \alpha_i)\right) = (s_m, \alpha_m), \tag{2.4}$$

where n is the amount of 2-tuple linguistic variable. The (s_m, α_m) is a 2-tuple linguistic variable which is represented as the arithmetic mean.

In general, decision makers would use the different 2-tuple linguistic variables based on their knowledge or experiences to express their opinions [20]. For example, the different types of linguistic variables show as Table 1. Each 2-tuple linguistic variable can be represented as a triangle fuzzy number. A transformation function is needed to transfer these 2-tuple linguistic variables from different linguistic sets to a standard linguistic set at unique domain. In the method of Herrera and Martinez [29], the domain of the linguistic variables will increase as the number of linguistic variable is increased. To overcome this drawback, a new translation function is applied to transfer a crisp number or 2-tuple linguistic variable to a standard linguistic term at the unique domain [30]. Suppose that the interval $[0,1]$ is the unique domain. The linguistic variable sets with different semantics (or types) will be defined by partitioning the interval $[0,1]$. Transforming a crisp number β ($\beta \in [0,1]$) into ith linguistic term $(s_i^{n(t)}, \alpha_i^{n(t)})$ of type t as

$$\Delta_t(\beta) = \left(s_i^{n(t)}, \alpha_i^{n(t)}\right), \tag{2.5}$$

where $i = \text{round}(\beta \times g_t)$, $\alpha_i^{n(t)} = \beta(-i/g_t)$, $g_t = n(t) - 1$, and $n(t)$ is the number of linguistic variable of type t.

Transforming ith linguistic term of type t into a crisp number β ($\beta \in [0,1]$) as

$$\Delta_t^{-1}\left(s_i^{n(t)}, \alpha_i^{n(t)}\right) = \frac{i}{g_t} + \alpha_i^{n(t)} = \beta, \tag{2.6}$$

where $g_t = n(t) - 1$ and $\alpha_i^{n(t)} \in [-1/2g_t, 1/2g_t)$.

Therefore, the transformation from ith linguistic term $(s_i^{n(t)}, \alpha_i^{n(t)})$ of type t to kth linguistic term $(s_k^{n(t+1)}, \alpha_k^{n(t+1)})$ of type $t+1$ at interval $[0,1]$ can be expressed as

$$\Delta_{t+1}\left(\Delta_t^{-1}\left(s_i^{n(t)}, \alpha_i^{n(t)}\right)\right) = \left(s_k^{n(t+1)}, \alpha_k^{n(t+1)}\right), \tag{2.7}$$

where $g_{t+1} = n(t+1) - 1$ and $\alpha_k^{n(t+1)} \in [-1/2g_{t+1}, 1/2g_{t+1})$.

3. Proposed Method

Because of the knowledge, experience and background of each expert is different and experts' opinions are usually uncertain and imprecise, it is difficult to use crisp value to express experts' opinions in the process of evaluating the performance of stock. Instead of crisp value, the 2-Tuple linguistic valuable which is an effective method to reduce the mistakes of information translation and avoid information loss through computing with words to express experts' opinions [19]. In this paper, different types of 2-tuple linguistic variables are used to express experts' opinions.

The TOPSIS method is one of the well-known MCDM methods. It is an effective method to determine the ranking order of decision alternatives. However, this method cannot distinguish the difference degree between two decision alternatives easily. Based on the axiom of partial comparability, the ELECTRE method can easily compare the degree of difference among of all alternatives. This method always cannot provide the total ordering of all decision alternatives. Therefore, the ELECTRE and TOPSIS methods are combined to determine the final investment ratio.

In the proposed model, the subjective opinions of experts can be expressed by different 2-tuple linguistic variables in accordance with their habitual knowledge and experience. After aggregating opinions of all experts, the linguistic TOPSIS and linguistic ELECTRE methods are applied to obtain the investment portfolio sets Ω_t and Ω_e, respectively. The strict stock portfolio set Ω_{ip} is determined by intersection Ω_t with Ω_e. In general, the risk preference of investor can be divided into three types such as risk-averter, risk-neutral, and risk-loving. Considering the risk preference of investor, we can calculate the investment ratio of each

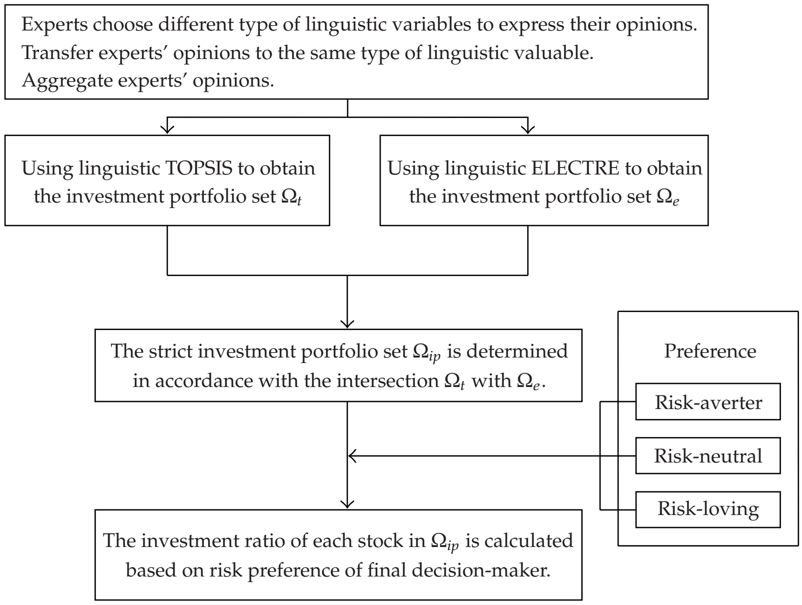

Figure 2: The decision-making process of the proposed method.

stock in strict stock portfolio set Ω_{ip}. The decision process of the proposed method is shown as in Figure 2.

In general, a stock portfolio decision may be described by means of the following sets:

(i) a set of experts or decision-makers called $E = \{E_1, E_2, \ldots, E_K\}$;

(ii) a set of stocks called $S = \{S_1, S_2, \ldots, S_m\}$;

(iii) a set of criteria $C = \{C_1, C_2, \ldots, C_n\}$ with which stock performances are measured;

(iv) a weight vector of each criterion $W = (W_1, W_2, \ldots, W_n)$;

(v) a set of performance ratings of each stock with respect to each criterion called $\tilde{S}_{ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

According to the aforementioned description, there are K experts, m stocks and n criteria in the decision process of stock portfolio. Experts can express their opinions by different 2-tuple linguistic variables. The kth expert's opinion about the performance rating of ith stock with respect to jth criterion can be represented as $\tilde{S}_{ij}^k = (S_{ij}^k, \alpha_{ij}^k)$. The kth expert's opinion about the importance of jth criterion can be represented as $\widetilde{W}_{jk} = (S_{jk}^w, \alpha_{jk}^w)$.

The aggregated linguistic rating $\tilde{S}_{ij}$ of each stock with respect to each criterion can be calculated as

$$\tilde{S}_{ij} = \Delta\left(\frac{1}{K}\sum_{k=1}^{K}\Delta^{-1}\left(S_{ij}^k, \alpha_{ij}^k\right)\right) = \left(S_{ij}, \alpha_{ij}\right). \tag{3.1}$$

The aggregated linguistic weight $\tilde{w}_j$ of each criterion can be calculated as

$$\widetilde{W}_j = \Delta\left(\frac{1}{K}\sum_{k=1}^{K}\Delta^{-1}\left(S_{jk}^{w}, \alpha_{jk}^{w}\right)\right) = \left(S_j^{w}, \alpha_j^{w}\right). \tag{3.2}$$

3.1. Linguistic TOPSIS Method

Considering the different importance of each criterion, the weighted linguistic decision matrix is constructed as

$$\widetilde{V}[\tilde{v}_{ij}]_{m\times n}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n, \tag{3.3}$$

where $\tilde{v}_{ij} = \tilde{x}_{ij}(\cdot)\tilde{w}_j = \Delta(\Delta^{-1}(S_{ij}, \alpha_{ij}) * \Delta^{-1}(S_j^{w}, \alpha_j^{w})) = (S_{ij}^{v}, \alpha_{ij}^{v})$.

According to the weighted linguistic decision matrix, the linguistic positive-ideal solution (LPIS, S^*) and linguistic negative-ideal solution (LNIS, S^-) can be defined as

$$\begin{aligned} S^* &= \left(\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*\right), \\ S^- &= \left(\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\right), \end{aligned} \tag{3.4}$$

where $\tilde{v}_j^* = \max_i\{(S_{ij}^{v}, \alpha_{ij}^{v})\}$ and $\tilde{v}_j^- = \min_i\{(S_{ij}^{v}, \alpha_{ij}^{v})\}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

The distance of each stock S_i $(i = 1, 2, \ldots, m)$ from S^* and S^-can be currently calculated as

$$\begin{aligned} d_i^* = d(S_i, S^*) &= \sqrt{\sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j)} = \sqrt{\sum_{j=1}^{n}\left(\Delta^{-1}\left(\max_i\left\{\left(S_{ij}^{v}, \alpha_{ij}^{v}\right)\right\}\right) - \Delta^{-1}\left(S_{ij}^{v}, \alpha_{ij}^{v}\right)\right)^2}, \\ d_i^- = d(S_i, S^-) &= \sqrt{\sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j)} = \sqrt{\sum_{j=1}^{n}\left(\Delta^{-1}\left(S_{ij}^{v}, \alpha_{ij}^{v}\right) - \Delta^{-1}\left(\min_i\left\{\left(S_{ij}^{v}, \alpha_{ij}^{v}\right)\right\}\right)\right)^2}. \end{aligned} \tag{3.5}$$

A closeness coefficient is defined to determine the ranking order of all stocks once d_i^* and d_i^- of each stock S_i $(i = 1, 2, \ldots, m)$ have been calculated. The closeness coefficient represents the distances to the linguistic positive-ideal solution (S^*) and the linguistic negative-ideal solution (S^-) simultaneously by taking the relative closeness to the linguistic positive-ideal solution. The closeness coefficient (CC_i) of each stock is calculated as

$$\mathrm{CC}_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1, 2, \ldots, m. \tag{3.6}$$

The higher CC_i means that stock S_i relatively close to positive ideal solution, the stock S_i has more ability to compete with each others. If the closeness coefficient of stock S_i is greater than the predetermined threshold value β_t, we consider stock S_i is good enough to choose in the investment portfolio set. According to closeness coefficient of each stock, the investment portfolio set Ω_t can be determined based on investment threshold value β_t as $\Omega_t = \{S_i \mid \mathrm{CC}_i \geq \beta_t\}$. Finally, the investment ratio of each stock in Ω_t can be calculated as

$$P_t(S_i) = \begin{cases} \dfrac{\mathrm{CC}(S_i)}{\sum_{S_i \in \Omega_t} \mathrm{CC}(S_i)}, & S_i \in \Omega_t, \\ 0, & S_i \notin \Omega_t, \end{cases} \tag{3.7}$$

where $P_t(S_i)$ is the investment ratio of each stock by linguistic TOPSIS method.

3.2. Linguistic ELECTRE Method

According to the ELECTRE method, the concordance index $C_j(S_i, S_l)$ is calculated for S_i and S_l ($i \neq l$, $i, l = 1, 2, \ldots, m$) with respect to each criterion as

$$C_j(S_i, S_l) = \begin{cases} 1, & \Delta^{-1}(\tilde{s}_{ij}) \geq \Delta^{-1}(\tilde{s}_{lj}) - q_j, \\ \dfrac{\Delta^{-1}(\tilde{s}_{ij}) - \Delta^{-1}(\tilde{s}_{lj}) + p_j}{p_j - q_j}, & \Delta^{-1}(\tilde{s}_{lj}) - q_j \geq \Delta^{-1}(\tilde{s}_{ij}) \geq \Delta^{-1}(\tilde{s}_{lj}) - p_j, \\ 0, & \Delta^{-1}(\tilde{s}_{ij}) \leq \Delta^{-1}(\tilde{s}_{lj}) - p_j, \end{cases} \tag{3.8}$$

where q_j and p_j are indifference and preference threshold values for criterion C_j, $p_j > q_j$.

The discordance index $D_j(S_i, S_l)$ is calculated for each pair of stocks with respect to each criterion as

$$D_j(S_i, S_l) = \begin{cases} 1, & \Delta^{-1}(\tilde{s}_{ij}) \leq \Delta^{-1}(\tilde{s}_{lj}) - v_j, \\ \dfrac{\Delta^{-1}(\tilde{s}_{lj}) - p_j - \Delta^{-1}(\tilde{s}_{ij})}{v_j - p_j}, & \Delta^{-1}(\tilde{s}_{lj}) - p_j \geq \Delta^{-1}(\tilde{s}_{ij}) \geq \Delta^{-1}(\tilde{s}_{lj}) - v_j, \\ 0, & \Delta^{-1}(\tilde{s}_{ij}) \geq \Delta^{-1}(\tilde{s}_{lj}) - p_j, \end{cases} \tag{3.9}$$

where v_j is the veto threshold for criterion C_j, $v_j > p_j$.

Calculate the overall concordance index $C(S_i, S_l)$ as

$$C(S_i, S_l) = \sum_{j=1}^{n} \Delta^{-1}(\tilde{w}_j) * C_j(S_i, S_l). \tag{3.10}$$

The credibility matrix $S(S_i, S_l)$ of each pair of the stocks is calculated as

$$S(S_i, S_l) = \begin{cases} C(S_i, S_l), & \text{if } D_j(S_i, S_l) \leq C(S_i, S_l)\ \forall j, \\ C(S_i, S_l) \displaystyle\prod_{j \in J(S_i, S_l)} \dfrac{1 - D_j(S_i, S_l)}{1 - C(S_i, S_l)}, & \text{otherwise}, \end{cases} \tag{3.11}$$

where $J(S_i, S_l)$ is the set of criteria for which $D_j(S_i, S_l) > C(S_i, S_l)$, $i \neq l$, $i, l = 1, 2, \ldots, m$.

The concordance credibility and discordance credibility degrees are defined as [32]

$$\phi^{+}(S_i) = \sum_{i \neq l} S(S_i, S_l),$$
$$\phi^{-}(S_i) = \sum_{i \neq l} S(S_l, S_i). \tag{3.12}$$

The concordance credibility degree represents that the degree of stock S_i is at least as good as all the other stocks. The discordance credibility degree represents that the degree of all the other stocks is at least as good as stock S_i.

Then, the net credibility degree is defined as $\phi(S_i) = \phi^{+}(S_i) - \phi^{-}(S_i)$. If the net credibility degree of stock S_i is higher, then it represents a higher attractiveness of stock S_i. In order to determine the investment ratio, the outranking index of stock S_i can be defined as

$$\mathrm{OTI}(S_i) = \frac{\phi(S_i)/(m-1)+1}{2}. \tag{3.13}$$

Property 3.1. According to the definition of $\mathrm{OTI}(S_i)$, we can find $0 \leq \mathrm{OTI}(S_i) \leq 1$.

Proof. Because $\phi(S_i) = \phi^{+}(S_i) - \phi^{-}(S_i) = \sum_{i \neq l} S(S_i, S_l) - \sum_{i \neq l}(S_l \cdot S_i)$, $i \neq l$, $i, l = 1, 2, \ldots, m$.

If the stock S_i is better than S_l with respect to each criterion, the best case is

$$\sum_{i \neq l} S(S_i, S_l) - \sum_{i \neq l} (S_l, S_i) = m - 1. \tag{3.14}$$

If the stock S_i is worse than S_l with respect to each criterion, the worst case is

$$\sum_{i \neq l} S(S_i, S_l) - \sum_{i \neq l} (S_l, S_i) = -(m - 1). \tag{3.15}$$

Therefore, $-(m-1) \leq \phi(S_i) \leq m-1$.

Then, $-1 \leq \phi(S_i)/(m-1) \leq 1$. Finally, we can prove $0 \leq (\phi(S_i)/(m-1)+1)/2 = \mathrm{OTI}(S_i) \leq 1$.

The $\mathrm{OTI}(S_i)$ denotes the standardization result of the net credibility degree. According to the definition, it is easy to understand and transform the net credibility degree into interval $[0, 1]$.

If the outranking index of stock S_i is greater than the predetermined threshold value β_e, we consider stock S_i is good enough to choose in the investment portfolio set. According to the outranking index of each stock, the investment portfolio set Ω_e can be determined based on investment threshold value β_e as $\Omega_e = \{S_i \mid \mathrm{OTI}(S_i) \geq \beta_e\}$. Finally, the investment ratio of each stock in Ω_e can be calculated as

$$P_e(S_i) = \begin{cases} \dfrac{\mathrm{OTI}(S_i)}{\sum_{S_i \in \Omega_e} \mathrm{OTI}(S_i)}, & S_i \in \Omega_e, \\ 0, & S_i \notin \Omega_e, \end{cases} \tag{3.16}$$

where $P_e(S_i)$ is the investment ratio of each stock by using linguistic ELECTRE method. □

3.3. Stock Portfolio Decision

We can consider Linguistic TOPSIS and Linguistic ELECTRE methods as two financial experts to provide investment ratio of each stock, respectively. Smart investor will make a stock portfolio decision by considering the suggestions of investment ratio of each stock simultaneously. Therefore, the portfolio set Ω_{ip} is defined as strict stock portfolio set $\Omega_{ip} = \Omega_t \cap \Omega_e$.

According to the closeness coefficient, the investment ratio of each stock in strict stock portfolio set Ω_{ip} can be calculated as

$$P_{t\_ip}(S_i) = \begin{cases} \dfrac{\mathrm{CC}(S_i)}{\sum_{S_i\in\Omega_{ip}} \mathrm{CC}(S_i)}, & S_i \in \Omega_{ip}, \\ 0, & S_i \notin \Omega_{ip}. \end{cases} \tag{3.17}$$

According to the outranking index, the investment ratio of each stock in strict stock portfolio set Ω_{ip} can be calculated as

$$P_{e\_ip}(S_i) = \begin{cases} \dfrac{\mathrm{OTI}(S_i)}{\sum_{S_i\in\Omega_{ip}} \mathrm{OTI}(S_i)}, & S_i \in \Omega_{ip}, \\ 0, & S_i \notin \Omega_{ip}. \end{cases} \tag{3.18}$$

In general, the investment preference of investors can be divided into three types such as risk-averter (RA), risk-neutral (RN), and risk-loving (RL). If a person is risk-averter, he/she will consider the smaller investment rates between $P_{t\_ip}(S_i)$ and $P_{e\_ip}(S_i)$. Therefore, the final ratio of each stock in strict portfolio set can be calculated as

$$P_{\mathrm{RA}}(S_i) = \frac{\min\left(P_{t\_ip}(S_i), P_{e\_ip}(S_i)\right)}{\sum_{S_i\in\Omega_{ip}} \min\left(P_{t\_ip}(S_i), P_{e\_ip}(S_i)\right)}. \tag{3.19}$$

If a person is risk-neutral, he/she will consider the average investment rates between $P_{t\_ip}(S_i)$ and $P_{e\_ip}(S_i)$. Therefore, the final ratio of each stock in strict portfolio set can be calculated as

$$P_{\mathrm{RN}}(S_i) = \frac{\left(P_{t\_ip}(S_i) + P_{e\_ip}(S_i)\right)/2}{\sum_{S_i\in\Omega_{ip}} \left(\left(P_{t\_ip}(S_i) + P_{e\_ip}(S_i)\right)/2\right)}. \tag{3.20}$$

If a person is risk-loving, he/she will consider the bigger investment rates between $P_{t\_ip}(S_i)$ and $P_{e\_ip}(S_i)$. Therefore, the final ratio of each stock in portfolio set can be calculated as

$$P_{\mathrm{RL}}(S_i) = \frac{\max\left(P_{t\_ip}(S_i), P_{e\_ip}(S_i)\right)}{\sum_{S_i\in\Omega_{ip}} \max\left(P_{t\_ip}(S_i), P_{e\_ip}(S_i)\right)}. \tag{3.21}$$

Table 1: Ten stocks of semiconduct industry in Taiwan.

S_1	Taiwan Semiconductor Manufacturing Co. Ltd.	S_2	United Microelectronics Corp.
S_3	Advanced Semiconductor Engineering, Inc.	S_4	Via Technologies, Inc.
S_5	MediaTek Inc.	S_6	King Yuan Electronics Co. Ltd.
S_7	Taiwan Mask Corp.	S_8	Winbond Electronics Corp.
S_9	SunPlus Technology Co. Ltd.	S_{10}	Nanya Technology Corporation

4. Numerical Example

An example with ten stocks of semiconduct industry in placecountry-region, Taiwan, will be considered to determine the investment ratio of each stock in this paper. Ten stocks are shown as Table 1. A committee of three financial experts $E = \{E_1, E_2, E_3\}$ has been formed to evaluate the performance of each stock. They are famous professors of a department of finance at well-known university in country-regionplace, Taiwan. Their knowledge and experiences are enough to evaluate the stock performance of each company for this example. In the process of criteria selection, they considered the quantitative and qualitative factors to deal with the portfolio selection. After the serious discussion and selection by three financial experts, six criteria are considered to determined the investment ratio of each stock such as profitability (C_1), asset utilization (C_2), liquidity (C_3), leverage (C_4), valuation (C_5), growth (C_6).

Profitability (C_1)

The goal of enterprise is tomakeaprofit. There are some indexes to evaluate the profitability of a company such as earnings per share (EPS), net profit margin, return on assets (ROA), and return on equity (ROE). The profitability of a company will influence the performance of each stock.

Asset Utilization (C_2)

Asset utilization means the efficiency of using company's resource in a period. A good company will promote the resource using efficiency as more as possible. Experts evaluate the asset utilization of the company based on receivables turnover, inventory turnover, and asset turnover.

Liquidity (C_3)

Liquidity will focus on cash flow generation and a company's ability to meet its financial obligations. When company's transfer assets (1 and, factory buildings, equipment, patent, goodwill) to currency in a short period, there will have some loss because the company's manager do not have enough time to find out the buyer who provide the highest price. An appropriate liquidity ratio (debt to equity ratio, current ratio, quick ratio) will both prevent liquidity risk and minimize the working capital.

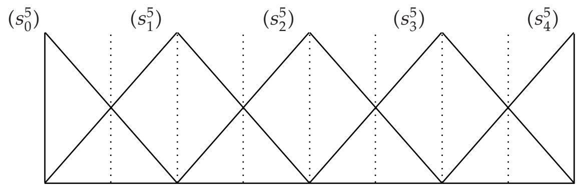

Figure 3: Membership functions of linguistic variables at type 1 ($t = 1$).

Table 2: Different types of linguistic variables.

Type		Linguistic variable	Figure
1	Performance	Extremely Poor (s_0^5), Poor (s_1^5), Fair (s_2^5), Good (s_3^5), Extremely Good (s_4^5)	Figure 3
	Weight	Extremely Low (s_0^5), Low (s_1^5), Fair (s_2^5), High (s_3^5), Extremely High (s_4^5)	
2	Performance	Extremely Poor (s_0^7), Poor (s_1^7), Medium Poor (s_2^7), Fair (s_3^7), Medium Good (s_4^7), Good (s_5^7), Extremely Good (s_6^7)	Figure 4
	Weight	Extremely Low (s_0^7), Low (s_1^7), Medium Low (s_2^7), Fair (s_3^7), Medium High (s_4^7), High (s_5^7), Extremely High (s_6^7)	
3	Performance	Extremely Poor (s_0^9) , Very Poor (s_1^9), Poor (s_2^9), Medium Poor (s_3^9), Fair (s_4^9), Medium Good (s_5^9), Good (s_6^9), Very Good (s_7^9) , Extremely Good (s_8^9)	Figure 5
	Weight	Extremely Low (s_0^9) , Very Low (s_1^9), Low (s_2^9), Medium Low (s_3^9), Fair (s_4^9), Medium High (s_5^9), High (s_6^9), Very High (s_7^9) , Extremely High (s_8^9)	

Leverage (C_4)

When the return on assets is greater than lending rate, it is time for a company to lend money to operate. But increasing the company's debt will increase risk if the company does not earn enough money to pay the debt in the future. A suitable leverage ratio is one of the criteria to evaluate the performance of each stock.

Valuation (C_5)

Book value means the currency which all of the company's assets transfer to, stock value means the price if you want to buy now, earnings before amortization, interest and taxes ratio (EBAIT) means the company earns in this year, expert must consider the best time point to buy the stock by Technical Analysis (TA) and Time Series Analysis (TSA). So, valuation is also one of the criteria to evaluate the performance of each stock.

Growth (C_6)

If the scale of a company was expanded year by year, EBAIT will increase which is like "compound interest." Because of economies of scale, the growth of the company will promote asset utilization and then raise the EBAIT and EPS.

According to the proposed method, the computational procedures of the problem are summarized as follows.

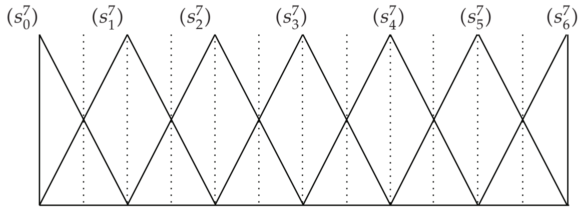

Figure 4: Membership functions of linguistic variables at type 2 ($t = 2$).

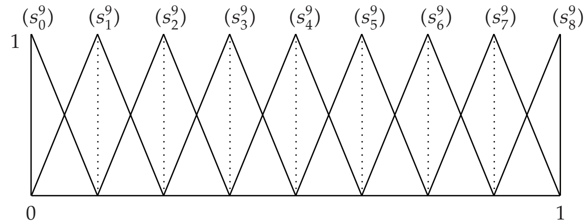

Figure 5: Membership functions of linguistic variables at type 3 ($t = 3$).

Table 3: Evaluation decisions (the ratings of the all stocks under all criteria) by three experts.

	C_1			C_2			C_3			C_4			C_5			C_6		
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3
S_1	F	F	G	EG	MG	VG	G	MG	VG	P	F	EG	F	G	VG	P	MG	VG
S_2	P	F	MG	F	MG	G	F	F	G	EP	F	MG	P	MG	VG	P	MG	G
S_3	F	F	G	G	F	MG	F	MG	G	F	F	MG	P	MG	G	F	MG	MG
S_4	F	G	MG	G	G	G	F	MG	MG	F	G	G	F	MG	MG	P	EG	MG
S_5	F	MG	EG	G	MG	VG	F	G	G	G	MG	VG	G	MG	VG	P	G	G
S_6	P	F	G	G	F	VG	F	F	VG	P	MG	VG	F	F	G	F	F	G
S_7	G	F	G	P	MG	VG	F	F	G	F	F	VG	P	MG	VG	F	MG	VG
S_8	EP	MG	G	F	F	VG	EP	F	VG	EP	MG	EG	EP	MG	VG	P	MG	VG
S_9	G	MG	VG	F	MG	G	F	F	VG	F	MG	VG	F	MG	VG	F	G	G
S_{10}	EP	G	G	F	G	G	F	MG	MG	EP	MG	G	EP	F	MG	EP	MG	MG

Step 1. Each expert selects the suitable 2-tuple linguistic variables to express their opinions. Expert 1 uses linguistic variables with 5 scale of linguistic term set to express his opinion, expert 2 uses linguistic variables with 7 scale of linguistic term set and expert 3 uses linguistic variables with 9 scale of linguistic term set, respectively (see Table 2).

Step 2. Each expert expresses his opinion about the performance of each stock with respect to each criterion as shown in Table 3.

Step 3. Each expert expresses his opinion about the importance of each criterion as shown in Table 4.

Step 4. Transform the linguistic ratings into the linguistic variables of type 2 and aggregate the linguistic ratings of each stock with respect to criteria as Table 5.

Table 4: Evaluation decisions (the weightings of all criteria) by three experts.

	C_1	C_2	C_3	C_4	C_5	C_6
E_1	EH	H	H	H	EH	F
E_2	EH	H	H	MH	H	H
E_3	EH	EH	VH	EH	VH	H

Table 5: Transfer to the linguistic variable of type 2.

Stock	Criterion	E_1	E_1	E_1	Average
C_1	S_1	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, −0.0833	S_4^7, −0.0833
	S_2	S_2^7, −0.0833	S_3^7, 0.0000	S_4^7, −0.0417	S_3^7, −0.0417
	S_3	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, −0.0833	S_4^7, −0.0833
	S_4	S_3^7, 0.0000	S_5^7, 0.0000	S_4^7, −0.0417	S_4^7, −0.0139
	S_5	S_3^7, 0.0000	S_4^7, 0.0000	S_6^7, 0.0000	S_4^7, 0.0556
	S_6	S_2^7, −0.0833	S_3^7, 0.0000	S_5^7, −0.0833	S_3^7, 0.0000
	S_7	S_5^7, −0.0833	S_3^7, 0.0000	S_5^7, −0.0833	S_4^7, 0.0000
	S_8	S_0^7, 0.0000	S_4^7, 0.0000	S_5^7, −0.0833	S_3^7, −0.0278
	S_9	S_5^7, −0.0833	S_4^7, 0.0000	S_5^7, 0.0417	S_5^7, −0.0694
	S_{10}	S_0^7, 0.0000	S_5^7, 0.0000	S_5^7, −0.0833	S_3^7, 0.0278
C_2	S_1	S_6^7, 0.0000	S_4^7, 0.0000	S_5^7, 0.0417	S_5^7, 0.0139
	S_2	S_3^7, 0.0000	S_4^7, 0.0000	S_5^7, −0.0833	S_4^7, −0.0278
	S_3	S_5^7, −0.0833	S_3^7, 0.0000	S_4^7, −0.0417	S_4^7, −0.0417
	S_4	S_5^7, −0.0833	S_5^7, 0.0000	S_5^7, −0.0833	S_5^7, −0.0556
	S_5	S_5^7, −0.0833	S_4, 0.0000	S_5^7, 0.0417	S_5^7, −0.0694
	S_6	S_5^7, −0.0833	S_3^7, 0.0000	S_5^7, 0.0417	S_4^7, 0.0417
	S_7	S_2^7, −0.0833	S_4^7, 0.0000	S_5^7, 0.0417	S_4^7, −0.0694
	S_8	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, 0.0417	S_4^7, −0.0417
	S_9	S_3^7, 0.0000	S_4, 0.0000	S_5^7, −0.0833	S_4^7, −0.0278
	S_{10}	S_3^7, 0.0000	S_5^7, 0.0000	S_5^7, −0.0833	S_4^7, 0.0278
C_3	S_1	S_5^7, −0.0833	S_4^7, 0.0000	S_5^7, 0.0417	S_5^7, −0.0694
	S_2	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, −0.0833	S_4^7, −0.0833
	S_3	S_3^7, 0.0000	S_4^7, 0.0000	S_5^7, −0.0833	S_4^7, −0.0278
	S_4	S_3^7, 0.0000	S_4^7, 0.0000	S_4^7, −0.0417	S_4^7, −0.0694
	S_5	S_3^7, 0.0000	S_5^7, 0.0000	S_5^7, −0.0833	S_4^7, 0.0278
	S_6	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, 0.0417	S_4^7, −0.0417
	S_7	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, −0.0833	S_4^7, −0.0833
	S_8	S_0^7, 0.0000	S_3^7, 0.0000	S_5^7, 0.0417	S_3^7, −0.0417
	S_9	S_3^7, 0.0000	S_3^7, 0.0000	S_5^7, 0.0417	S_4^7, −0.0417
	S_{10}	S_3^7, 0.0000	S_4^7, 0.0000	S_4^7, −0.0417	S_4^7, −0.0694

Table 5: Continued.

Stock	Criterion	E_1	E_1	E_1	Average
C_4	S_1	$S_2^7, -0.0833$	$S_3^7, 0.0000$	$S_6^7, 0.0000$	$S_4^7, -0.0833$
	S_2	$S_0^7, 0.0000$	$S_3^7, 0.0000$	$S_4^7, -0.0417$	$S_2^7, 0.0417$
	S_3	$S_3^7, 0.0000$	$S_3^7, 0.0000$	$S_4^7, -0.0417$	$S_3^7, 0.0417$
	S_4	$S_3^7, 0.0000$	$S_5^7, 0.0000$	$S_5^7, -0.0833$	$S_4^7, 0.0278$
	S_5	$S_5^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_5^7, -0.0694$
	S_6	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, -0.0694$
	S_7	$S_3^7, 0.0000$	$S_3^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, -0.0417$
	S_8	$S_0^7, 0.0000$	$S_4^7, 0.0000$	$S_6^7, 0.0000$	$S_3^7, 0.0556$
	S_9	$S_3^7, 0.0000$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, 0.0139$
	S_{10}	$S_0^7, 0.0000$	$S_4^7, 0.0000$	$S_5^7, -0.0833$	$S_3^7, -0.0278$
C_5	S_1	$S_3^7, 0.0000$	$S_5^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, 0.0694$
	S_2	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, -0.0694$
	S_3	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, -0.0833$	$S_3^7, 0.0556$
	S_4	$S_3^7, 0.0000$	$S_4^7, 0.0000$	$S_4^7, -0.0417$	$S_4^7, -0.0694$
	S_5	$S_5^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_5^7, -0.0694$
	S_6	$S_3^7, 0.0000$	$S_3^7, 0.0000$	$S_5^7, -0.0833$	$S_4^7, -0.0833$
	S_7	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, -0.0694$
	S_8	$S_0^7, 0.0000$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_3^7, 0.0139$
	S_9	$S_3^7, 0.0000$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, 0.0139$
	S_{10}	$S_0^7, 0.0000$	$S_3^7, 0.0000$	$S_4^7, -0.0417$	$S_2^7, 0.0417$
C_6	S_1	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, -0.0694$
	S_2	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, -0.0833$	$S_3^7, 0.0556$
	S_3	$S_3^7, 0.0000$	$S_4^7, 0.0000$	$S_4^7, -0.0417$	$S_4^7, -0.0694$
	S_4	$S_2^7, -0.0833$	$S_6^7, 0.0000$	$S_4^7, -0.0417$	$S_4^7, -0.0417$
	S_5	$S_2^7, -0.0833$	$S_5^7, 0.0000$	$S_5^7, -0.0833$	$S_4^7, -0.0556$
	S_6	$S_3^7, 0.0000$	$S_3^7, 0.0000$	$S_5^7, -0.0833$	$S_4^7, -0.0833$
	S_7	$S_3^7, 0.0000$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, 0.0139$
	S_8	$S_2^7, -0.0833$	$S_4^7, 0.0000$	$S_5^7, 0.0417$	$S_4^7, -0.0694$
	S_9	$S_3^7, 0.0000$	$S_5^7, 0.0000$	$S_5^7, -0.0833$	$S_4^7, 0.0278$
	S_{10}	$S_0^7, 0.0000$	$S_4^7, 0.0000$	$S_4^7, -0.0417$	$S_3^7, -0.0694$

Table 6: Transfer to the linguistic variable of type 2.

Criterion	E_1	E_2	E_3	Average
C_1	$S_6^7, 0.0000$	$S_6^7, 0.0000$	$S_6^7, 0.0000$	$S_6^7, 0.0000$
C_2	$S_5^7, -0.0833$	$S_5^7, 0.0000$	$S_6^7, 0.0000$	$S_5^7, 0.0278$
C_3	$S_5^7, -0.0833$	$S_5^7, 0.0000$	$S_5^7, 0.0417$	$S_5^7, -0.0139$
C_4	$S_5^7, -0.0833$	$S_4^7, 0.0000$	$S_6^7, 0.0000$	$S_5^7, -0.0278$
C_5	$S_6^7, 0.0000$	$S_5^7, 0.0000$	$S_5^7, 0.0417$	$S_5^7, 0.0694$
C_6	$S_3^7, 0.0000$	$S_5^7, 0.0000$	$S_5^7, -0.0833$	$S_4^7, 0.0278$

Table 7: The weighted linguistic decision matrix.

	C_1	C_2	C_3	C_4	C_5	C_6
S_1	0.1148	0.1435	0.1231	0.0924	0.1307	0.0816
S_2	0.0902	0.1082	0.0940	0.0594	0.1061	0.0759
S_3	0.1148	0.1059	0.1030	0.0858	0.0987	0.0816
S_4	0.1284	0.1318	0.0963	0.1100	0.1061	0.0854
S_5	0.1421	0.1294	0.1119	0.1211	0.1357	0.0835
S_6	0.0984	0.1200	0.1008	0.0946	0.1036	0.0797
S_7	0.1311	0.1012	0.0940	0.0990	0.1061	0.0930
S_8	0.0929	0.1059	0.0739	0.0880	0.0913	0.0816
S_9	0.1503	0.1082	0.1008	0.1078	0.1209	0.0949
S_{10}	0.1038	0.1176	0.0963	0.0748	0.0666	0.0588

Table 8: Linguistic positive-ideal solution (LPIS, S^*) and linguistic negative-ideal solution (LNIS, S^-).

	C_1	C_2	C_3	C_4	C_5	C_6
S^*	0.1503	0.1435	0.1231	0.1211	0.1357	0.0949
S^-	0.0902	0.1012	0.0739	0.0594	0.0666	0.0588

Table 9: Calculate the distance from S^* and the distance from S^-, the closeness coefficient of each stock.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
d^*	0.0478	0.1036	0.0766	0.0492	0.0228	0.0755	0.0661	0.1018	0.0463	0.1084
d^-	0.1027	0.0480	0.0610	0.0879	0.1188	0.0647	0.0799	0.0444	0.1048	0.0346
CC	0.6826	0.3166	0.4432	0.6409	0.8388	0.4614	0.5471	0.3038	0.6937	0.2419

Table 10: The overall concordance matrix.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1.0000	1.0000	1.0000	1.0000	0.9868	1.0000	1.0000	1.0000	0.9836	1.0000
S_2	0.9046	1.0000	1.0000	0.8219	0.7268	0.9472	0.8716	0.9868	0.7040	1.0000
S_3	0.9287	1.0000	1.0000	1.0000	0.9028	1.0000	1.0000	1.0000	0.9836	1.0000
S_4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_6	1.0000	1.0000	1.0000	1.0000	0.9196	1.0000	1.0000	1.0000	0.8852	1.0000
S_7	0.9019	1.0000	1.0000	0.9859	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_8	0.7500	1.0000	0.9866	0.9836	0.7061	1.0000	0.9672	1.0000	0.8525	1.0000
S_9	0.9577	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_{10}	0.6257	0.7441	0.7885	0.6685	0.4954	0.7589	0.6758	0.8033	0.5360	0.8033

Step 5. Transform the linguistic evaluations of weight of each criterion into the linguistic variables of type 2 and aggregate the linguistic weight of each criterion as Table 6.

Step 6. Calculate the weighted linguistic decision matrix $V = [v_{ij}]_{m*n}$ as Table 7.

Step 7. Calculate the linguistic positive-ideal solution (LPIS, S^*) and linguistic negative-ideal solution (LNIS, S^-) as Table 8.

Table 11: The credibility matrix.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_1	1.0000	1.0000	1.0000	1.0000	0.9868	1.0000	1.0000	1.0000	0.9836	1.0000
S_2	0.9046	1.0000	1.0000	0.8219	0.4845	0.9472	0.8716	0.9868	0.7040	1.0000
S_3	0.9287	1.0000	1.0000	1.0000	0.9028	1.0000	1.0000	1.0000	0.9836	1.0000
S_4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_6	1.0000	1.0000	1.0000	1.0000	0.9196	1.0000	1.0000	1.0000	0.8852	1.0000
S_7	0.9019	1.0000	1.0000	0.9859	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_8	0.7500	1.0000	0.9866	0.9836	0.7061	1.0000	0.9672	1.0000	0.8525	1.0000
S_9	0.9577	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
S_{10}	0.5213	0.7440	0.7884	0.6684	0.3302	0.7588	0.6757	0.8032	0.5359	0.8032

Table 12: The concordance credibility degree, the discordance credibility degree, the net credibility degree, and the outranking index.

Stock	$\phi^+(S_i)$	$\phi^-(S_i)$	$\phi(S_i)$	OTI
S_1	9.9704	8.9641	1.0063	0.5559
S_2	8.7206	9.7440	−1.0234	0.4431
S_3	9.8151	9.7750	0.0402	0.5022
S_4	10.0000	9.4599	0.5401	0.5300
S_5	10.0000	8.3300	1.6700	0.5928
S_6	9.8049	9.7060	0.0989	0.5055
S_7	9.8878	9.5145	0.3732	0.5207
S_8	9.2459	9.7900	−0.5441	0.4698
S_9	9.9577	8.9448	1.0128	0.5563
S_{10}	6.6292	9.8032	−3.1740	0.3237

Table 13: Compute the ratio of investment in accordance with the risk preference.

Rank	$P_t(S_i)$	$P_e(S_i)$	$P_{\mathrm{RA}}(S_i)$	$P_{\mathrm{RN}}(S_i)$	$P_{\mathrm{RL}}(s_i)$
1	S_5, 0.2465	S_5, 0.1575	S_5, 0.2225	S_5, 0.2308	S_5, 0.2385
2	S_9, 0.2038	S_9, 0.1478	S_9, 0.2088	S_9, 0.2028	S_9, 0.1973
3	S_1, 0.2006	S_1, 0.1477	S_1, 0.2075	S_1, 0.2012	S_1, 0.1952
4	S_4, 0.1883	S_4, 0.1408	S_4, 0.1948	S_4, 0.1903	S_4, 0.1861
5	S_7, 0.1608	S_7, 0.1384	S_7, 0.1663	S_7, 0.1749	S_7, 0.1829
6		S_6, 0.1343			
7		S_3, 0.1335			

Step 8. Calculate the distance of each stock from S^* and the distance from S^-, and the closeness coefficient of each stock as Table 9.

Step 9. Define investment threshold value as the average of the closeness coefficient $\beta_t = \sum_{i=1}^{n} \mathrm{CC}(S_i)/n$, so the investment portfolio set is $\Omega_t = \{S_1, S_4, S_5, S_7, S_9\}$ in accordance with TOPSIS. The ratio of investment based on TOPSISmethod is shown as Table 13.

Step 10. The indifference threshold, preference threshold, and veto threshold values of each criterion can be determined in accordance with the linguistic variables of type 2 as

$$q_j = \Delta^{-1}\left(S_1^7\right) - \Delta^{-1}\left(S_0^7\right) = \frac{1}{6}, \qquad p_j = \Delta^{-1}\left(S_2^7\right) - \Delta^{-1}\left(S_0^7\right) = \frac{2}{6},$$
$$v_j = \Delta^{-1}\left(S_3^7\right) - \Delta^{-1}\left(S_0^7\right) = \frac{3}{6}, \quad j = 1, \ldots, 6. \tag{4.1}$$

Step 11. Calculate the concordance matrix and the discordance matrix of each pair stock with respect to each criterion.Then, calculate the overall concordance matrix as Table 10 and the credibility matrix as Table 11.

Step 12. Calculate the concordance credibility degree, the discordance credibility degree, the net credibility degree, and the outranking index as Table 12.

Step 13. Define investment threshold value as the average of the outranking index $\beta_e = \sum_{i=1}^{n} \mathrm{OTI}(S_i)/n$, so the investment portfolio set is $\Omega_e = \{S_1, S_3, S_4, S_5, S_6, S_7, S_9\}$ in accordance with ELECTRE method. The ratio of investment based on ELECTRE method is shown as Table 13.

Step 14. Compute strict stock portfolio set as $\Omega_{ip} = \Omega_t \cap \Omega_e = \{S_1, S_4, S_5, S_7, S_9\}$.

Step 15. According to the investment preference of investor, the result of the ratio of investment based on combining linguistic ELECTRE with TOPSIS can be calculated as Table 13.

According to the result of numerical example, experts considered that the proposed method is useful to help investor determine the stock portfolio.

5. Conclusion

In general, the stock portfolio decision problem adheres to uncertain and imprecise data, and fuzzy set theory is adequate to deal with it. In this proposed model, different types of 2-tuple linguistic variables are applied to express the subjective judgment of each expert. Expert can easily express his opinion by different types of 2-tuple linguistic variables. The generalized translation method of different types of 2-tuple linguistic variables is applied to aggregate the subjective judgment of each expert. It is a flexible way to aggregate the opinions of all experts. Then, a new decision-making method has been presented in this paper by combining the advantages of ELECTRE with TOPSIS methods. According to the experts' opinions, the linguistic ELECTRE method and linguistic TOPSIS method are used to derive the closeness coefficient and the outranking index of each stock, respectively. Based on the closeness coefficient, the outranking index, and selection threshold, we can easily obtain three type of the investment ratio in accordance with different investment preference of final decision-maker. It is a reasonable way in real decision environment. In other words, the proposed method provides a flexible way to determine the stock portfolio under the uncertain environment. In the future, the concept of combing different decision methods for deciding stock portfolio will be applied to different fields such as R&D projects investment, bonus distribution in a company. A decision support system will be developed based on the proposed method for dealing with the stock selection problems in the future.

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