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Research Article

Parametric Set-Valued Vector Quasi-Equilibrium Problems

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Two kinds of parametric set-valued vector quasi-equilibrium problems are introduced. The existence of solutions to these problems is studied. The upper and lower semicontinuities of their solution maps with respect to the parameters are investigated.

1. Introduction and Preliminaries

Equilibrium problems are a class of general problems that contains many other problems, such as optimization problems, variational inequality problems, saddle point problems, and complementarity problems, as special cases. Up to now, the main efforts for equilibrium problems have been made for the solution existence; see for example [1–6] and the references therein. A few results have been obtained for properties of solution sets, see [7–12].

Motivated and inspired by works in [1, 5, 8–12], in this paper, we will introduce two kinds of parametric set-valued vector quasi-equilibrium problems and study the solution existence of these problems. In addition, we will investigate the upper and lower semicontinuities of their solution maps with respect to the parameters.

Throughout this paper, let X, Y be real Hausdorff topological vector spaces, Λ, M real topological vector spaces, and A a nonempty compact convex subset of X. We denote by $\operatorname{co} A, \operatorname{int} A, \partial A$, and $\operatorname{cl} A$ the convex hull, interior, boundary, and closed hull of A, respectively. Let $K : A \times M \to 2^X$, $T : A \times \Lambda \to 2^Y$, $F : A \times X \times Y \to 2^Y$, and $C : A \to 2^Y$ be set-valued mappings such that $A \cap K(x, \mu) \neq \emptyset$ for all $x \in A$ and $\mu \in M$ and $C(x)$ be a closed convex pointed cone of Y with $\operatorname{int} C(x) \neq \emptyset$ for each $x \in A$.

The mapping F is said to be $Y \setminus -\operatorname{int} C$ quasiconvex of type 2 with respect to T (see [1]) if for any nonempty finite subset $\{y_1, \ldots, y_n\} \subseteq A$ and any $x \in \operatorname{co}\{y_1, \ldots, y_n\}$, there exist

$i \in \{1, \ldots, n\}$ and $z \in T(x)$ such that $F(x, y_i, z) \subseteq Y \setminus -\operatorname{int} C(x)$. F is said to be $Y \setminus -\operatorname{int} C$ quasi convex-like of type 2 with respect to T (see [1]) if for any nonempty finite subset $\{y_1, \ldots, y_n\} \subseteq A$ and any $x \in \operatorname{co}\{y_1, \ldots, y_n\}$, there exist $i \in \{1, \ldots, n\}$ and $z \in T(x)$ such that

$$F(x, y_i, z) \cap (Y \setminus -\operatorname{int} C(x)) \neq \emptyset. \tag{1.1}$$

Let B be a nonempty subset of X. A set-valued mapping $G : B \to 2^Y$ is said to be upper semicontinuous (shortly, u.s.c) at $x_0 \in B$ if for any open set $V \supseteq G(x_0)$, there exists an open neighborhood U of x_0 such that $G(x) \subseteq V$ for each $x \in U \cap B$. G is said to be u.s.c on B if it is u.s.c at each point in B.

The mapping $G : B \to 2^Y$ is said to be lower semicontinuous (shortly, l.s.c) at $x_0 \in B$ if for each $y \in G(x_0)$ and any open neighborhood V of y there exists an open neighborhood U of x_0 such that $G(z) \cap V \neq \emptyset$ for each $z \in U \cap B$, or, equivalently, if for any net $\{x_\alpha\}$ with $x_\alpha \to x_0$ and any $y \in G(x_0)$, there exists a net $\{y_\alpha\}$ with $y_\alpha \in G(x_\alpha)$ for each α such that $y_\alpha \to y$. G is said to be l.s.c on B if it is l.s.c at each point in B.

The mapping $G : B \to 2^Y$ is said to be closed at $x_0 \in B$ if for any net $\{(x_\alpha, y_\alpha)\} : (x_\alpha, y_\alpha) \to (x_0, y_0)$ and $y_\alpha \in G(x_\alpha)$ for each α, one has $y_0 \in G(x_0)$. G is said to be a closed set-valued mapping if its graph, denoted by graph$G$, is a closed set in $X \times Y$, where graph$G = \{(x, y) : x \in B, y \in G(x)\}$. G is said to have closed values if $G(x)$ is a closed set for each $x \in B$.

A set-valued mapping $G : B \to 2^B$ is said to be a KKM mapping if for each nonempty finite subset $\{x_1, \ldots, x_n\}$ of B, one has $\operatorname{co}\{x_1, \ldots, x_n\} \subseteq \bigcup_{i=i}^{n} G(x_i)$.

Lemma 1.1 (Fan-KKM Theorem). *Let B a nonempty convex subset of X and $G : B \to 2^B$ be a KKM mapping. If $G(x)$ is a closed set for every $x \in B$ and there exists $x_0 \in B$ such that $G(x_0)$ is a compact set, then $\bigcap_{x \in B} G(x) \neq \emptyset$.*

Lemma 1.2 (see [13]). *If a set-valued mapping $G : X \to 2^Y$ is u.s.c and has closed values, then it is a closed set-valued mapping.*

Lemma 1.3 (see [14]). *Let the set-valued mapping $G : X \to 2^Y$ have a compact value at x. Then G is u.s.c at $x \in X$ if and only if for any nets $\{x_\alpha\} \subseteq X : x_\alpha \to x$ and $\{y_\alpha\} : y_\alpha \in G(x_\alpha)$ for each α there exist $y \in G(x)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y$.*

For any given parameters $\lambda \in \Lambda$ and $\mu \in M$, in this paper, we consider the following two parametric set-valued vector quasi-equilibrium problems.

PSVQEP 1. Find $x \in A \cap \operatorname{cl} K(x, \mu)$ such that for each $y \in K(x, \mu)$ there exists $z \in T(x, \lambda)$ satisfying

$$F(x, y, z) \subseteq Y \setminus -\operatorname{int} C(x). \tag{1.2}$$

PSVQEP 2. Find $x \in A \cap \operatorname{cl} K(x, \mu)$ such that for each $y \in K(x, \mu)$ there exists $z \in T(x, \lambda)$ satisfying

$$F(x, y, z) \cap (Y \setminus -\operatorname{int} C(x)) \neq \emptyset. \tag{1.3}$$

We denote their solution sets by $S_1(\lambda, \mu)$ and $S_2(\lambda, \mu)$, respectively. Obviously, $S_1(\lambda, \mu) \subseteq S_2(\lambda, \mu)$.

2. Solution Existence

In this section, we will study the existence of solutions for PSVQEP 1 and PSVQEP 2 without any monotonicity. Since parameters play no role in considering solution existence, for the sake of convenience, we state and prove existence results without parameters. We denote the above problems without parameters by SVQEP1 and SVQEP2, and their solution sets by S_1 and S_2, respectively.

Theorem 2.1. *Let*

(i) $\operatorname{co} K(x) \subseteq \operatorname{cl} K(x)$ *for all* $x \in A$,

(ii) $\{x \in A : y \in K(x)\}$ *be an open set,*

(iii) F *be* $Y \setminus -\operatorname{int} C$ *quasi convex of type 2 with respect to* T,

(iv) $\{y \in A : \exists z \in T(x) \ \ s.t. \ \ F(x, y, z) \subseteq Y \setminus -\operatorname{int} C(x)\}$ *be a closed set for each* $x \in A$. *Then (SVQEP1) has at least a solution.*

Proof. Put $E := \{x \in A : x \in \operatorname{cl} K(x)\}$ and define three set-valued mappings $P : A \to 2^A, H : A \to 2^A$, and $Q : A \to 2^A$ by

$$P(x) = \{y \in A : F(x, y, z) \cap -\operatorname{int} C(x) \neq \emptyset, \forall z \in T(x)\}, \quad \forall x \in A,$$
$$H(x) = \begin{cases} K(x) \cap P(x), & x \in E, \\ A \cap K(x), & x \in A \setminus E, \end{cases} \tag{2.1}$$
$$Q(y) = A \setminus \{x \in A : y \in H(x)\}, \quad \forall y \in A.$$

Firstly, we show that Q is a KKM mapping.

Suppose to the contrary that Q is not a KKM mapping. Then there exist a nonempty finite subset $\{y_1, \ldots, y_n\} \subseteq A$ and a point $\widehat{x} = \sum_{j=1}^{n} \alpha_j y_j \in \operatorname{co}\{y_1, \ldots, y_n\}$, where $\alpha_j \geq 0, j = 1, \ldots, n$ and $\sum_{j=1}^{n} \alpha_j = 1$, such that $\widehat{x} \notin \bigcup_{j=1}^{n} Q(y_j)$, which implies that $y_j \in H(\widehat{x}), j = 1, \ldots, n$.

If $\widehat{x} \in E$, then $F(\widehat{x}, y_j, z) \cap -\operatorname{int} C(\widehat{x}) \neq \emptyset$ for all $z \in T(\widehat{x})$ and $j = 1, \ldots, n$, which contradicts (iii).

If $\widehat{x} \notin E$, then $y_j \in K(\widehat{x}), j = 1, \ldots, n$, which indicates that $\widehat{x} = \sum_{j=1}^{n} \alpha_j y_j \in \operatorname{co} K(\widehat{x}) \subseteq \operatorname{cl} K(\widehat{x})$ and then $\widehat{x} \in E$. This is a contradiction.

Thus, Q is a KKM mapping.

Secondly, we show that $\bigcap_{y \in A} Q(y) \neq \emptyset$.

For any given $y \in A$, we can deduce that

$$\begin{aligned} Q(y) &= A \setminus \{x \in A : y \in H(x)\} \\ &= A \setminus (\{x \in E : y \in K(x) \cap P(x)\} \cup \{x \in A \setminus E : y \in A \cap K(x)\}) \\ &= A \setminus (\{x \in A : y \in K(x)\} \cap ((A \setminus E) \cup \{x \in A : y \in P(x)\})) \\ &= (A \setminus (\{x \in A : y \in K(x)\}) \cup (E) \cup \{x \in A : \exists z \in T(x), F(x, y, z) \subseteq Y \setminus -\operatorname{int} C(x)\}). \end{aligned} \tag{2.2}$$

By (ii) and (iv), we can conclude that $Q(y)$ is a closed set. Since X is a Hausdorff topological vector space and A is a compact set, we have that $Q(y)$ is compact for each $y \in A$. By Lemma 1.1, we get $\bigcap_{y\in A} Q(y) \neq \emptyset$.

Finally, we prove that the assertion of the theorem holds.

Taking arbitrarily $\overline{x} \in \bigcap_{y\in A} Q(y)$, we have $\overline{x} \notin \{x \in A : y \in H(x)\}$ for all $y \in A$, which indicates that $H(\overline{x}) = \emptyset$. As $A \cap K(x) \neq \emptyset$ for all x, we know that $\overline{x} \in E$ and then $K(\overline{x}) \cap P(\overline{x}) = \emptyset$. Consequently, for each $\overline{y} \in K(\overline{x})$, there exists $\overline{z} \in T(\overline{x})$ such that $F(\overline{x}, \overline{y}, \overline{z}) \subseteq Y \setminus -\operatorname{int} C(\overline{x})$, which shows that $\overline{x} \in S_1$. □

By a similar proof as for Theorem 2.1, we obtain the following result.

Theorem 2.2. *Let hypotheses (i) and (ii) in Theorem 2.1 hold and let*

(iii) F be $Y \setminus -\operatorname{int} C$ quasi convex-like of type 2 with respect to T,

(iv) $\{y \in A : \exists z \in T(x)\ \ s.t.\ \ F(x, y, z) \cap (Y \setminus -\operatorname{int} C(x)) \neq \emptyset\}$ be a closed set for each $x \in A$. Then (SVQEP2) has at least a solution.

3. Upper Semicontinuity of Solution Sets

In this section, we will study the upper semicontinuity of the solution sets $S_1(\lambda, \mu)$ and $S_2(\lambda, \mu)$ with respect to parameters (λ, μ). For this end, we assume that $S_1(\lambda, \mu)$ and $S_2(\lambda, \mu)$ are nonempty for any $(\lambda, \mu) \in \Lambda \times M$. Let $x_0 \in A, \lambda_0, \lambda \in \Lambda$ and $\mu_0, \mu \in M$.

Theorem 3.1. *Let*

(i) *$E(\cdot) = \{x \in A : x \in \operatorname{cl} K(x, \cdot)\}$ and $W(\cdot)$ be closed set-valued mappings, where $W(x) := Y \setminus -\operatorname{int} C(x)$ for each $x \in A$;*

(ii) *for any nets $\{\lambda_\alpha\} : \lambda_\alpha \to \lambda_0, \{\mu_\alpha\} : \mu_\alpha \to \mu_0, \{x_\alpha\} : x_\alpha \to x_0, \{z_\alpha\} : z_\alpha \in T(x_\alpha, \lambda_\alpha)$ for each α and any $y_0 \in K(x_0, \mu_0)$, there exist nets $\{y_\alpha\} : y_\alpha \in K(x_\alpha, \mu_\alpha)$ for each α, $\{z_\beta\} \subseteq \{z_\alpha\}$ and $z_0 \in T(x_0, \lambda_0)$ such that $y_\alpha \to y_0$ and $z_\beta \to z_0$;*

(iii) *F be l.s.c on $A \times X \times Y$. Then $S_1(\cdot, \cdot)$ is both closed and u.s.c at (λ_0, μ_0).*

Proof. We first show that $S_1(\cdot, \cdot)$ is closed at (λ_0, μ_0).

Suppose to the contrary that $S_1(\cdot, \cdot)$ is not closed at (λ_0, μ_0). Then there exist nets $\{(\lambda_\alpha, \mu_\alpha)\} : (\lambda_\alpha, \mu_\alpha) \to (\lambda_0, \mu_0)$ and $\{x_\alpha\} : x_\alpha \to x_0$ and $x_\alpha \in S_1(\lambda_\alpha, \mu_\alpha)$ for each α such that $x_0 \notin S_1(\lambda_0, \mu_0)$.

$x_\alpha \in S_1(\lambda_\alpha, \mu_\alpha)$ implies that $(\mu_\alpha, x_\alpha) \in \operatorname{graph} E$ for each α. By the closedness of $A \cap \operatorname{cl} K(\cdot, \cdot)$, we get $x_0 \in A \cap \operatorname{cl} K(x_0, \mu_0)$, which together with $x_0 \notin S_1(\lambda_0, \mu_0)$ indicates that there exists $y_0 \in K(x_0, \mu_0)$ such that

$$F(x_0, y_0, z) \cap -\operatorname{int} C(x_0) \neq \emptyset, \quad \forall z \in T(x_0, \lambda_0). \tag{3.1}$$

For $y_0 \in K(x_0, \mu_0)$, by (ii), there exists $y_\alpha \in K(x_\alpha, \mu_\alpha)$ for each α such that $y_\alpha \to y_0$. Due to $x_\alpha \in S_1(\lambda_\alpha, \mu_\alpha)$, for each $y_\alpha \in K(x_\alpha, \mu_\alpha)$, there exists $z_\alpha \in T(x_\alpha, \lambda_\alpha)$ such that $F(x_\alpha, y_\alpha, z_\alpha) \subseteq Y \setminus -\operatorname{int} C(x_\alpha)$. Again by (ii), there exist a subnet $\{z_\beta\} \subseteq \{z_\alpha\}$ and a point $z_0 \in T(x_0, \lambda_0)$ such that $z_\beta \to z_0$ and

$$F(x_\alpha, y_\alpha, z_\beta) \subseteq W(x_\alpha). \tag{3.2}$$

For $z_0 \in T(x_0, \lambda_0)$, by (3.1), there exists $f_0 \in F(x_0, y_0, z_0)$ such that

$$f_0 \in -\operatorname{int} C(x_0). \tag{3.3}$$

By the lower semicontinuity of F, there exists $f_\beta \in F(x_\beta, y_\beta, z_\beta)$ for each β such that $f_\beta \to f_0$, which together with the closedness of $W(\cdot)$ and (3.2) implies that $f_0 \in Y \setminus -\operatorname{int} C(x_0)$. This contradicts (3.3). Hence, $S_1(\cdot,\cdot)$ is closed at (λ_0, μ_0).

Next, we show that $S_1(\cdot,\cdot)$ is u.s.c at (λ_0, μ_0).

By the closedness of $S_1(\cdot,\cdot)$ at (λ_0, μ_0), $S_1(\lambda_0, \mu_0)$ is closed and hence compact as is A.

Suppose to the contrary that $S_1(\cdot,\cdot)$ is not u.s.c at (λ_0, μ_0). By Lemma 1.3, there exist nets $\{(\lambda_\alpha, \mu_\alpha)\} : (\lambda_\alpha, \mu_\alpha) \to (\lambda_0, \mu_0)$ and $\{x_\alpha\} : x_\alpha \in S_1(\lambda_\alpha, \mu_\alpha)$ for each α such that for any $x_0 \in S_1(\lambda_0, \mu_0)$ and any subnet $\{x_\beta\} \subseteq \{x_\alpha\}$ one has

$$x_\beta \nrightarrow x_0. \tag{3.4}$$

$x_\alpha \in S_1(\lambda_\alpha, \mu_\alpha)$ implies that $x_\alpha \in E(\mu_\alpha)$ for each α and $\{x_\alpha\} \subseteq A$. By the compactness of A, there exists a convergent subnet $\{x_\beta\}$ of $\{x_\alpha\}$ such that $x_\beta \to \overline{x} \in A$. By the closedness of $E(\cdot)$, we have $\overline{x} \in E(\mu_0)$. By (3.4), we get $\overline{x} \notin S_1(\lambda_0, \mu_0)$, that is,

$$F(\overline{x}, \overline{y}, z) \cap -\operatorname{int} C(\overline{x}) \neq \emptyset, \quad \forall z \in T(\overline{x}, \lambda_0). \tag{3.5}$$

By using a similar argument as in part one, we can complete the proof. □

Theorem 3.2. *Let hypotheses (i) and (ii) in Theorem 3.1 hold and let*

(iii) F be u.s.c on $A \times X \times Y$.

Then $S_2(\cdot,\cdot)$ is both u.s.c and closed at (λ_0, μ_0).

Proof. We first prove that $S_2(\cdot,\cdot)$ is closed at (λ_0, μ_0).

Suppose to the contrary that $S_2(\cdot,\cdot)$ is not closed at (λ_0, μ_0). Then there exist nets $\{(\lambda_\alpha, \mu_\alpha)\} : (\lambda_\alpha, \mu_\alpha) \to (\lambda_0, \mu_0)$ and $\{x_\alpha\} : x_\alpha \to x_0$ and $x_\alpha \in S_2(\lambda_\alpha, \mu_\alpha)$ for each α such that $x_0 \notin S_2(\lambda_0, \mu_0)$. By using a similar reasoning as in part one of the proof of Theorem 3.1, we can conclude that there exists a net $\{(x_\beta, y_\beta, z_\beta)\}$ such that $(x_\beta, y_\beta, z_\beta) \to (x_0, y_0, z_0)$ and

$$F(x_0, y_0, z_0) \subseteq -\operatorname{int} C(x_0), \tag{3.6}$$

$$F(x_\beta, y_\beta, z_\beta) \cap (Y \setminus -\operatorname{int} C(x_\beta)) \neq \emptyset, \quad \forall \beta, \tag{3.7}$$

where $y_\beta \in K(x_\beta, \mu_\beta)$ with $y_\beta \to y_0 \in K(x_0, \mu_0)$ and $z_\beta \in T(x_\beta, \lambda_\beta)$ with $z_\beta \to z_0 \in T(x_0, \lambda_0)$. By the upper semicontinuity of F and (3.6), we know that there exists β_0 such that

$$F(x_\beta, y_\beta, z_\beta) \subseteq -\operatorname{int} C(x_0), \quad \forall \beta \geq \beta_0, \tag{3.8}$$

which contradicts (3.7). Hence, $S_2(\cdot,\cdot)$ is closed at (λ_0, μ_0).

Next, we prove that $S_2(\cdot,\cdot)$ is u.s.c at (λ_0, μ_0).

By the closedness of $S_2(\cdot,\cdot)$ at (λ_0, μ_0), $S_2(\lambda_0, \mu_0)$ is closed and hence compact as is A.

Suppose to the contrary that $S_2(\cdot,\cdot)$ is not u.s.c at (λ_0,μ_0). By Lemma 1.3, there exist nets $\{(\lambda_\alpha,\mu_\alpha)\}:(\lambda_\alpha,\mu_\alpha)\to(\lambda_0,\mu_0)$ and $\{x_\alpha\}:x_\alpha\in S_2(\lambda_\alpha,\mu_\alpha)$ for each α such that (3.4) holds for any $x_0\in S_2(\lambda_0,\mu_0)$ and any subnet $\{x_\beta\}\subseteq\{x_\alpha\}$.

$x_\alpha\in S_2(\lambda_\alpha,\mu_\alpha)$ implies that $x_\alpha\in E(\mu_\alpha)$ for each α and $\{x_\alpha\}\subseteq A$. By the compactness of A and the closedness of $E(\cdot)$, it follows that there exists a convergent subnet $\{x_\beta\}$ of $\{x_\alpha\}$ such that $x_\beta\to\overline{x}\in E(\mu_0)$. By (3.4), we get $\overline{x}\notin S_2(\lambda_0,\mu_0)$, that is,

$$F(\overline{x},\overline{y},z)\subseteq-\operatorname{int}C(\overline{x}),\quad\forall z\in T(\overline{x},\lambda_0).\tag{3.9}$$

By using a similar argument as in part one, we can complete the proof. □

4. Lower Semicontinuity of Solution Sets

In this section, we will consider the lower semicontinuity of the solution sets $S_1(\cdot,\cdot)$ and $S_2(\cdot,\cdot)$ with respect to parameters (λ,μ).

Theorem 4.1. *Let*

(i) *$E(\cdot):=\{x\in A\mid x\in\operatorname{cl}K(x,\cdot)\}$ be l.s.c on M and $C(\cdot)$u.s.c at x_0;*

(ii) *for any nets $\{\lambda_\alpha\}:\lambda_\alpha\to\lambda_0,\{\mu_\alpha\}:\mu_\alpha\to\mu_0,\{x_\alpha\}:x_\alpha\to x_0,\{y_\alpha\}:y_\alpha\in K(x_\alpha,\mu_\alpha)$ for each α and any $z_0\in T(x_0,\lambda_0)$, there exist nets $\{z_\alpha\}:z_\alpha\in T(x_\alpha,\lambda_\alpha)$ for each α, $\{y_\beta\}\subseteq\{y_\alpha\}$ and a point $y_0\in K(x_0,\mu_0)$ such that $z_\alpha\to z_0$ and $y_\beta\to y_0$;*

(iii) *F be u.s.c and have compact values on $A\times X\times Y$;*

(iv) *$F(x_0,y_0,z_0)\cap-\partial C(x_0)=\emptyset$ for all $x_0\in S_1(\lambda_0,\mu_0),y_0\in K(x_0,\mu_0)$, and $z_0\in T(x_0,\lambda_0)$. Then $S_1(\cdot,\cdot)$ is l.s.c at (λ_0,μ_0).*

Proof. Suppose to the contrary that $S_1(\cdot,\cdot)$ is not l.s.c at (λ_0,μ_0). Then there exist a net $\{(\lambda_\alpha,\mu_\alpha)\}:(\lambda_\alpha,\mu_\alpha)\to(\lambda_0,\mu_0)$ and a point $x_0\in S_1(\lambda_0,\mu_0)$ such that for any net $\{\widetilde{x}_\alpha\}:\widetilde{x}_\alpha\in S_1(\lambda_\alpha,\mu_\alpha)$ for each α one has

$$\widetilde{x}_\alpha\nrightarrow x_0.\tag{4.1}$$

$x_0\in S_1(\lambda_0,\mu_0)$ implies that $x_0\in E(\mu_0)$. By the lower semicontinuity of E, there exists a net $\{x_\alpha\}:x_\alpha\in E(\mu_\alpha)$ for each α such that $x_\alpha\to x_0$, which combining with (4.1) shows that there exists a subnet $\{x_\beta\}$ of $\{x_\alpha\}$ such that $x_\beta\notin S_1(\lambda_\beta,\mu_\beta)$ for all β. Consequently, for each β, there exists $y_\beta\in K(x_\beta,\mu_\beta)$ satisfying

$$F(x_\beta,y_\beta,z_\beta)\cap-\operatorname{int}C(x_\beta)\neq\emptyset,\quad\forall z_\beta\in T(x_\beta,\lambda_\beta).\tag{4.2}$$

By (ii), there exist a subnet $\{\overline{y}_\beta\}\subseteq\{y_\beta\}$ and a point $y_0\in K(x_0,\mu_0)$ such that $\overline{y}_\beta\to y_0$, which together with $x_0\in S_1(\lambda_0,\mu_0)$ and (ii) indicates that there exist $z_0\in T(x_0,\lambda_0)$ and $\overline{z}_\beta\in T(\overline{x}_\beta,\overline{\lambda}_\beta)$ such that $\overline{z}_\beta\to z_0,F(\overline{x}_\beta,\overline{y}_\beta,\overline{z}_\beta)\cap-\operatorname{int}C(\overline{x}_\beta)\neq\emptyset$ for all β and

$$F(x_0,y_0,z_0)\subseteq Y\setminus-\operatorname{int}C(x_0).\tag{4.3}$$

Take arbitrarily $f_\beta \in F(\overline{x}_\beta, \overline{y}_\beta, \overline{z}_\beta) \cap -\operatorname{int} C(\overline{x}_\beta)$ for each β. By Lemma 1.3, there exist $f_0 \in F(x_0, y_0, z_0)$ and a subset $\{f_{\overline{\beta}}\}$ of $\{f_\beta\}$ such that $f_{\overline{\beta}} \to f_0$.

Since $f_{\overline{\beta}} \in -C(\overline{x}_{\overline{\beta}})$ for each $\overline{\beta}$, by the upper semicontinuity of $C(\cdot)$ and Lemma 1.2, we know that $f_0 \in -C(x_0)$, which together with (iv) shows that $f_0 \in -\operatorname{int} C(x_0)$. This contradicts (4.3). Hence, $S_1(\cdot, \cdot)$ is l.s.c at (λ_0, μ_0). □

Theorem 4.2. *Let hypotheses (i) and (ii) in Theorem 4.1 hold and let*

(iii) $F(\cdot, \cdot, \cdot)$ *be l.s.c on* $A \times X \times Y$;

(iv) $F(x_0, y_0, z_0) \cap -\partial C(x_0) = \emptyset$ *for all* $x_0 \in S_2(\lambda_0, \mu_0), y_0 \in K(x_0, \mu_0)$, *and* $z_0 \in T(x_0, \lambda_0)$.

Then $S_2(\cdot, \cdot)$ *is l.s.c at* (λ_0, μ_0).

Proof. By arguments similar to those for Theorem 4.1, we can conclude that there exists a net $\{(\overline{x}_\beta, \overline{y}_\beta, \overline{z}_\beta)\}$ such that $(\overline{x}_\beta, \overline{y}_\beta, \overline{z}_\beta) \to (x_0, y_0, z_0), F(x_0, y_0, z_0) \cap (Y \setminus -\operatorname{int} C(x_0)) \neq \emptyset$, and

$$F\left(\overline{x}_\beta, \overline{y}_\beta, \overline{z}_\beta\right) \subseteq -\operatorname{int} C(\overline{x}_\beta), \quad \forall \beta, \tag{4.4}$$

where $\overline{x}_\beta \in E(\overline{\mu}_\beta), \overline{y}_\beta \in K(\overline{x}_\beta, \overline{\mu}_\beta), \overline{z}_\beta \in T(\overline{x}_\beta, \overline{\lambda}_\beta)$ for all β and $x_0 \in S_2(\lambda_0, \mu_0), y_0 \in K(x_0, \mu_0)$ and $z_0 \in T(x_0, \lambda_0)$.

For any given $f_0 \in F(x_0, y_0, z_0) \cap (Y \setminus -\operatorname{int} C(x_0))$, by the lower semicontinuity of F, there exists $\overline{f}_\beta \in F(\overline{x}_\beta, \overline{y}_\beta, \overline{z}_\beta)$ for each β such that $\overline{f}_\beta \to f_0$. By (4.4), we have $\overline{f}_\beta \in -C(\overline{x}_\beta)$ for each β. By the upper semicontinuity of $C(\cdot)$ and Lemma 1.2, it follows that $\overline{f}_0 \in -C(\overline{x}_0)$, which together with (iv) implies that $f_0 \in -\operatorname{int} C(x_0)$. This is a contradiction. Hence, $S_2(\cdot, \cdot)$ is l.s.c at (λ_0, μ_0). □

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References

[1] N. X. Hai and Ph. Q. Khanh, "The solution existence of general variational inclusion problems," *Journal of Mathematical Analysis and Applications*, vol. 328, no. 2, pp. 1268–1277, 2007.

[2] X. P. Ding, "Existence of solutions for quasi-equilibrium problems in noncompact topological spaces," *Computers & Mathematics with Applications*, vol. 39, no. 3-4, pp. 13–21, 2000.

[3] Z. Lin and J. Yu, "The existence of solutions for the system of generalized vector quasi-equilibrium problems," *Applied Mathematics Letters*, vol. 18, no. 4, pp. 415–422, 2005.

[4] L. G. Huang, "Existence of solutions on weak vector equilibrium problems," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 65, no. 4, pp. 795–801, 2006.

[5] N. X. Hai, Ph. Q. Khanh, and N. H. Quan, "On the existence of solutions to quasivariational inclusion problems," *Journal of Global Optimization*, vol. 45, no. 4, pp. 565–581, 2009.

[6] G. Y. Chen, X. Q. Yang, and H. Yu, "A nonlinear scalarization function and generalized quasi-vector equilibrium problems," *Journal of Global Optimization*, vol. 32, no. 4, pp. 451–466, 2005.

[7] X.-J. Long, N.-J. Huang, and K.-L. Teo, "Existence and stability of solutions for generalized strong vector quasi-equilibrium problem," *Mathematical and Computer Modelling*, vol. 47, no. 3-4, pp. 445–451, 2008.

[8] Ph. Q. Khanh and L. M. Luu, "Lower semicontinuity and upper semicontinuity of the solution sets and approximate solution sets of parametric multivalued quasivariational inequalities," *Journal of Optimization Theory and Applications*, vol. 133, no. 3, pp. 329–339, 2007.

[9] L. Q. Anh and Ph. Q. Khanh, “Semicontinuity of the solution set of parametric multivalued vector quasiequilibrium problems,” *Journal of Mathematical Analysis and Applications*, vol. 294, no. 2, pp. 699–711, 2004.

[10] Ph. Q. Khanh and L. M. Luu, “Upper semicontinuity of the solution set to parametric vector quasivariational inequalities,” *Journal of Global Optimization*, vol. 32, no. 4, pp. 569–580, 2005.

[11] N. J. Huang, J. Li, and H. B. Thompson, “Stability for parametric implicit vector equilibrium problems,” *Mathematical and Computer Modelling*, vol. 43, no. 11-12, pp. 1267–1274, 2006.

[12] L. Q. Anh and Ph. Q. Khanh, “Various kinds of semicontinuity and the solution sets of parametric multivalued symmetric vector quasiequilibrium problems,” *Journal of Global Optimization*, vol. 41, no. 4, pp. 539–558, 2008.

[13] C. J. Zhang, *Set-Valued Analysis with Applications in Economic*, Science Press, Beijing, China, 2004.

[14] F. Ferro, “A minimax theorem for vector-valued functions,” *Journal of Optimization Theory and Applications*, vol. 60, no. 1, pp. 19–31, 1989.