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Research Article

Soft Expert Sets

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In 1999, Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty. Many researchers have studied this theory, and they created some models to solve problems in decision making and medical diagnosis, but most of these models deal only with one expert. This causes a problem with the user, especially with those who use questionnaires in their work and studies. In our model, the user can know the opinion of all experts in one model. So, in this paper, we introduce the concept of a soft expert set, which will more effective and useful. We also define its basic operations, namely, complement, union intersection AND, and OR. Finally, we show an application of this concept in decision-making problem.

1. Introduction

Most of the problems in engineering, medical science, economics, environments, and so forth, have various uncertainties. Molodtsov [1] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. After Molodtsov's work, some operations and application of soft sets were studied by Chen et al. [2] and Maji et al. [3, 4]. Alkhazaleh et al. [5] introduced the concept of soft multisets as a generalization of soft set. They also defined in [6, 7] the concepts of possibility fuzzy soft set and fuzzy parameterized interval-valued fuzzy soft set and gave their applications in decision making and medical diagnosis. Many researchers have studied this theory, and they created some models to solve problems in decision making and medical diagnosis, but most of these models deal only with one expert, and if we want to take the opinion of more than one expert, we need to do some operations such as union, intersection, and so forth. This causes a problem with the user, especially with those who use questionnaires in their work and studies. In our model the user can know the opinion of all experts in one model without any operations. Even after any operation on our model the user can know the opinion of all experts. So in this paper we introduce the concept of a soft expert set, which will be more effective and useful. We also define its basic operations, namely, complement, union intersection AND and OR and study their properties. Finally, we give an application of this concept in a decision-making problem.

2. Preliminaries

In this section, we recall some basic notions in soft set theory. Molodtsov [1] defined soft set in the following way. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 (see [1]). A pair (F, A) is called a *soft set* over U, where F is a mapping $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A,\ F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

The following definitions are due to Maji et al. [3].

Definition 2.2. For two soft sets (F, A) and (G, B) over U, (F, A) is called a *soft subset* of (G, B) if

(i) $A \subset B$,

(ii) for all $\varepsilon \in A, F(\varepsilon),$ and $G(\varepsilon)$ are identical approximations.

This relationship is denoted by $(F, A) \mathrel{\tilde{\subset}} (G, B)$. In this case, (G, B) is called a *soft superset* of (F, A).

Definition 2.3. Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4. Let $E = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. The *NOT set* of E denoted by $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, \dots, \rceil e_n\}$ where $\rceil e_i =$ not $e_i,$ for all i.

Definition 2.5. The *complement* of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\rceil \alpha),$ for all $\alpha \in \rceil A$.

Definition 2.6. A soft set (F, A) over U is said to be a *NULL soft* set denoted by $\Phi,$ if for all $\varepsilon \in A, F(\varepsilon) = \emptyset$, (null-set).

Definition 2.7. A soft set (F, A) over U is said to be an *absolute soft set*, denoted by $\widetilde{A},$ if for all $\varepsilon \in A, F(\varepsilon) = U$.

Definition 2.8. If (F, A) and (G, B) are two soft sets then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$, is defined by

$$(F, A) \wedge (G, B) = (H, A \times B), \tag{2.1}$$

where $H(\alpha, \beta) = F(\alpha) \cap G(\beta),$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.9. If (F, A) and (G, B) are two soft sets, then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$, is defined by

$$(F, A) \vee (G, B) = (O, A \times B), \tag{2.2}$$

where $O(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

Definition 2.10. The *union* of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.3}$$

The following definition is due to Ali et al. [8] since they discovered that Maji et al.'s definition of intersection in [3] is not correct.

Definition 2.11. The *extended intersection* of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cap G(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.4}$$

3. Soft Expert Set

In this section, we introduce the concept of a soft expert set, and give definitions of its basic operations, namely, complement, union, intersection, AND, and OR. We give examples for these concepts. Basic properties of the operations are also given.

Let U be a universe, E a set of parameters, and X a set of experts (agents). Let O be a set of opinions, $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 3.1. A pair (F, A) is called a *soft expert set* over U, where F is a mapping given by

$$F : A \longrightarrow P(U), \tag{3.1}$$

where $P(U)$ denotes the power set of U.

Note 3.2. For simplicity we assume in this paper, two-valued opinions only in set O, that is, $O = \{0 = \text{disagree},\ 1 = \text{agree}\}$, but multivalued opinions may be assumed as well.

Example 3.3. Suppose that a company produced new types of its products and wishes to take the opinion of some experts about concerning these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products, $E = \{e_1, e_2, e_3\}$ a set of decision parameters where e_i $(i = 1, 2, 3)$ denotes the decision "easy to use," "quality," and "cheap," respectively, and let $X = \{p, q, r\}$ be a set of experts.

Suppose that the company has distributed a questionnaire to three experts to make decisions on the company's products, and we get the following:

$$
\begin{aligned}
&F(e_1,p,1)=\{u_1,u_2,u_4\}, \quad F(e_1,q,1)=\{u_1,u_4\}, \quad F(e_1,r,1)=\{u_3,u_4\},\\
&F(e_2,p,1)=\{u_4\}, \quad F(e_2,q,1)=\{u_1,u_3\}, \quad F(e_2,r,1)=\{u_1,u_2,u_4\},\\
&F(e_3,p,1)=\{u_3,u_4\}, \quad F(e_3,q,1)=\{u_1,u_2\}, \quad F(e_3,r,1)=\{u_4\},\\
&F(e_1,p,0)=\{u_3\}, \quad F(e_1,q,0)=\{u_2,u_3\}, \quad F(e_1,r,0)=\{u_1,u_2\},\\
&F(e_2,p,0)=\{u_1,u_2,u_3\}, \quad F(e_2,q,0)=\{u_2,u_4\}, \quad F(e_2,r,0)=\{u_3\},\\
&F(e_3,p,0)=\{u_1,u_2\}, \quad F(e_3,q,0)=\{u_3,u_4\}, \quad F(e_3,r,0)=\{u_1,u_2,u_3\}.
\end{aligned}
\tag{3.2}
$$

Then we can view the soft expert set (F, Z) as consisting of the following collection of approximations:

$$
\begin{aligned}
(F,Z)=\{&((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_1,r,1),\{u_3,u_4\}),\\
&((e_2,p,1),\{u_4\}),((e_2,q,1),\{u_1,u_3\}),((e_2,r,1),\{u_1,u_2,u_4\}),\\
&((e_3,p,1),\{u_3,u_4\}),((e_3,q,1),\{u_1,u_2\}),((e_3,r,1),\{u_4\}),\\
&((e_1,p,0),\{u_3\}),((e_1,q,0),\{u_2,u_3\}),((e_1,r,0),\{u_1,u_2\}),\\
&((e_2,p,0),\{u_1,u_2,u_3\}),((e_2,q,0),\{u_2,u_4\}),((e_2,r,0),\{u_3\}),\\
&((e_3,p,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\}),((e_3,r,0),\{u_1,u_2,u_3\})\}.
\end{aligned}
\tag{3.3}
$$

Notice that in this example the first expert, p, "agrees" that the "easy to use" products are u_1, u_2, and u_4. The second expert, q, "agrees" that the "easy to use" products are u_1 and u_4, and the third expert, r, "agrees" that the "easy to use" products are u_3 and u_4. Notice also that all of them "agree" that product u_4 is "easy to use."

Definition 3.4. For two soft expert sets (F, A) and (G, B) over $U, (F, A)$ is called a *soft expert subset* of (G, B) if

(i) $A \subseteq B$,

(ii) for all $\varepsilon \in B, G(\varepsilon) \subseteq F(\varepsilon)$.

This relationship is denoted by $(F, A) \tilde{\subseteq} (G, B)$. In this case (G, B) is called a *soft expert superset* of (F, A).

Definition 3.5. Two soft expert sets (F, A) and (G, B) over U are said to be *equal* if (F, A) is a soft expert subset of (G, B) and (G, B) is a soft expert subset of (F, A).

Example 3.6. Consider Example 3.3. Suppose that the company took the opinion of the experts once again after the products have been in the market for a month.

Suppose

$$A = \{(e_1, p, 1), (e_2, p, 0), (e_3, p, 1), (e_1, q, 1), (e_2, q, 1), (e_3, q, 0), (e_1, r, 0), (e_2, r, 1), (e_3, r, 1)\},$$
$$B = \{(e_1, p, 1), (e_2, p, 0), (e_1, q, 1), (e_2, q, 1), (e_1, r, 0), (e_2, r, 1)\}. \tag{3.4}$$

Clearly $B \subset A$. Let (F, A) and (G, B) be defined as follows:

$$(F, A) = \{((e_1, p, 1), \{u_1, u_2, u_4\}), ((e_1, q, 1), \{u_1, u_4\}), ((e_2, q, 1), \{u_1, u_3\}), ((e_2, r, 1), \{u_1, u_2, u_4\}), ((e_3, p, 1), \{u_3, u_4\}), ((e_3, r, 1), \{u_4\}), ((e_1, r, 0), \{u_1, u_2\}), ((e_2, p, 0), \{u_1, u_2, u_3\}), ((e_3, q, 0), \{u_3, u_4\})\},$$
$$(G, B) = \{((e_1, p, 1), \{u_1, u_4\}), ((e_1, q, 1), \{u_4\}), ((e_2, q, 1), \{u_1, u_3\}), ((e_2, r, 1), \{u_1\}), ((e_1, r, 0), \{u_2\}), ((e_2, p, 0), \{u_1, u_3\})\}. \tag{3.5}$$

Therefore $(G, B) \widetilde{\subseteq} (F, A)$.

Definition 3.7. Let E be a set of parameters and X a set of experts. The NOT *set* of $Z = E \times X \times O$ denoted by $\rceil Z$, is defined by $\rceil Z = \{(\rceil e_i, x_j, o_k),\ \forall\ i, j, k\}$ where $\rceil e_i$ is not e_i.

Definition 3.8. The *complement* of a soft expert set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \rightarrow P(U)$ is a mapping given by $F^c(\alpha) = U - F(\rceil \alpha)$, for all $\alpha \in \rceil A$.

Example 3.9. Consider Example 3.3. Then

$$(F, A)^c = \{((\rceil e_1, p, 1), \{u_3\}), ((\rceil e_1, q, 1), \{u_2, u_3\}), ((\rceil e_1, r, 1), \{u_1, u_2\}), ((\rceil e_2, p, 1), \{u_1, u_2, u_3\}), ((\rceil e_2, q, 1), \{u_2, u_4\}), ((\rceil e_2, r, 1), \{u_3\}), ((\rceil e_3, p, 1), \{u_1, u_2\}), ((\rceil e_3, q, 1), \{u_3, u_4\}), ((\rceil e_3, r, 1), \{u_1, u_2, u_3\}), ((\rceil e_1, p, 0), \{u_1, u_2, u_3\}), ((\rceil e_1, q, 0), \{u_1, u_4\}), ((\rceil e_1, r, 0), \{u_3, u_4\}), ((\rceil e_2, p, 0), \{u_4\}), ((\rceil e_2, q, 0), \{u_1, u_3\}), ((\rceil e_2, r, 0), \{u_1, u_2, u_4\}), ((\rceil e_3, p, 0), \{u_3, u_4\}), ((\rceil e_3, q, 0), \{u_1, u_2\}), ((\rceil e_3, r, 0), \{u_4\})\}. \tag{3.6}$$

Definition 3.10. An *agree-soft expert set* $(F, A)_1$ over U is a soft expert subset of (F, A) defined as follows:

$$(F, A)_1 = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}. \tag{3.7}$$

Example 3.11. Consider Example 3.3. Then the *agree-soft expert set* $(F,A)_1$ over U is

$$
\begin{aligned}
(F,A)_1 = \{&((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,q,1),\{u_1,u_4\}),((e_1,r,1),\{u_3,u_4\}),\\
&((e_2,p,1),\{u_4\}),((e_2,q,1),\{u_1,u_3\}),((e_2,r,1),\{u_1,u_2,u_4\}),\\
&((e_3,p,1),\{u_3,u_4\}),((e_3,q,1),\{u_1,u_2\}),((e_3,r,1),\{u_4\})\}.
\end{aligned}
\tag{3.8}
$$

Definition 3.12. A *disagree-soft expert set* $(F,A)_0$ over U is a soft expert subset of (F,A) defined as follows:

$$
(F,A)_0 = \{F_0(\alpha) : \alpha \in E \times X \times \{0\}\}. \tag{3.9}
$$

Example 3.13. Consider Example 3.3. Then the *disagree-soft expert set* $(F,A)_0$ over U is

$$
\begin{aligned}
(F,A)_0 = \{&((e_1,p,0),\{u_3\}),((e_1,q,0),\{u_2,u_3\}),((e_1,r,0),\{u_1,u_2\}),\\
&((e_2,p,0),\{u_1,u_2,u_3\}),((e_2,q,0),\{u_2,u_4\}),((e_2,r,0),\{u_3\}),\\
&((e_3,p,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\}),((e_3,r,0),\{u_1,u_2,u_3\})\}.
\end{aligned}
\tag{3.10}
$$

Proposition 3.14. *If (F,A) is a soft expert set over U, then*

(i) $((F,A)^c)^c = (F,A)$,

(ii) $(F,A)_1^c = (F,A)_0$,

(iii) $(F,A)_0^c = (F,A)_1$.

Proof. The proof is straightforward. □

Definition 3.15. The *union* of two soft expert sets (F,A) and (G,B) over U denoted by $(F,A)\,\tilde{\cup}\,(G,B)$, is the soft expert set (H,C) where $C = A \cup B$, and for all $\varepsilon \in C$,

$$
H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{3.11}
$$

Example 3.16. Consider Example 3.3. Let

$$
\begin{aligned}
A &= \{(e_1,p,1),(e_2,p,0),(e_3,p,1),(e_1,q,1),(e_2,q,1),(e_3,q,0),(e_1,r,0),(e_2,r,1),(e_3,r,1)\},\\
B &= \{(e_1,p,1),(e_2,p,0),(e_3,p,0),(e_1,q,1),(e_2,q,1),(e_3,q,1),(e_1,r,0),(e_2,r,1)\}.
\end{aligned}
\tag{3.12}
$$

Suppose (F, A) and (G, B) are two soft expert sets over U such that

$$\begin{aligned}(F, A) = \{&((e_1, p, 1), \{u_1, u_2, u_4\}), ((e_1, q, 1), \{u_1, u_4\}), ((e_2, q, 1), \{u_1, u_3\}),\\ &((e_2, r, 1), \{u_1, u_2, u_4\}), ((e_3, p, 1), \{u_3, u_4\}), ((e_3, r, 1), \{u_4\}),\\ &((e_1, r, 0), \{u_1, u_2\}), ((e_2, p, 0), \{u_1, u_2, u_3\}), ((e_3, q, 0), \{u_3, u_4\})\},\\ (G, B) = \{&((e_1, p, 1), \{u_1, u_3, u_4\}), ((e_1, q, 1), \{u_3\}), ((e_2, q, 1), \{u_2, u_3\}),\\ &((e_2, r, 1), \{u_2, u_3\}), ((e_1, r, 0), \{u_2\}), ((e_3, p, 0), \{u_1, u_2, u_4\}),\\ &((e_3, q, 0), \{u_2, u_3\}), ((e_2, p, 0), \{u_2, u_3\})\}.\end{aligned} \tag{3.13}$$

Therefore

$$\begin{aligned}(F, A) \,\tilde{\cup}\, (G, B) &= (H, C)\\ &= \{((e_1, p, 1), U), ((e_1, q, 1), \{u_1, u_3, u_4\}), ((e_2, q, 1), \{u_1, u_2, u_3\}),\\ &\quad ((e_2, r, 1), U), ((e_3, p, 1), \{u_3, u_4\}), ((e_3, r, 1), \{u_4\}),\\ &\quad ((e_1, r, 0), \{u_1, u_2\}), ((e_2, p, 0), \{u_1, u_2, u_3\}), ((e_3, q, 0), \{u_2, u_3, u_4\}),\\ &\quad ((e_3, p, 0), \{u_1, u_2, u_4\})\}.\end{aligned} \tag{3.14}$$

Proposition 3.17. *If (F, A), (G, B), and (H, C) are three soft expert sets over U, then*

(i) $(F, A) \,\tilde{\cup}\, (G, B) = (G, B) \,\tilde{\cup}\, (F, A)$,

(ii) $(F, A) \,\tilde{\cup}\, ((G, B) \,\tilde{\cup}\, (H, C)) = ((F, A) \,\tilde{\cup}\, (G, B)) \,\tilde{\cup}\, (H, C)$.

Proof. The proof is straightforward. □

Definition 3.18. The *intersection* of two soft expert sets (F, A) and (G, B) over U denoted by $(F, A) \,\tilde{\cap}\, (G, B)$ is the soft expert set (H, C) where $C = A \cup B$, for all $\varepsilon \in C$, and

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B,\\ G(\varepsilon) & \text{if } \varepsilon \in B - A,\\ F(\varepsilon) \cap G(\varepsilon) & \text{if } \varepsilon \in A \cap B.\end{cases} \tag{3.15}$$

Example 3.19. Consider Example 3.16. Then

$$\begin{aligned}(F, A) \,\tilde{\cap}\, (G, B) &= (H, C)\\ &= \{((e_1, p, 1), \{u_1, u_4\}), ((e_1, q, 1), \emptyset), ((e_2, q, 1), \{u_3\}),\\ &\quad ((e_2, r, 1), \{u_2\}), ((e_3, p, 1), \{u_3, u_4\}), ((e_3, r, 1), \{u_4\}),\\ &\quad ((e_1, r, 0), \{u_2\}), ((e_2, p, 0), \{u_2, u_3\}),\\ &\quad ((e_3, q, 0), \{u_3\}), ((e_3, p, 0), \{u_1, u_2, u_4\})\}.\end{aligned} \tag{3.16}$$

Proposition 3.20. *If (F,A), (G,B), and (H,C) are three soft expert sets over U, then*

(i) $(F,A)\,\widetilde{\cap}\,(G,B) = (G,B)\,\widetilde{\cap}\,(F,A)$,

(ii) $(F,A)\,\widetilde{\cap}\,((G,B)\,\widetilde{\cap}\,(H,C)) = ((F,A)\,\widetilde{\cap}\,(G,B))\,\widetilde{\cap}\,(H,C)$.

Proof. The proof is straightforward. □

Proposition 3.21. *If (F,A), (G,B), and (H,C) are three soft expert sets over U, then*

(i) $(F,A)\,\widetilde{\cup}\,((G,B)\,\widetilde{\cap}\,(H,C)) = ((F,A)\,\widetilde{\cup}\,(G,B))\,\widetilde{\cap}\,((F,A)\,\widetilde{\cup}\,(H,C))$,

(ii) $(F,A)\,\widetilde{\cap}\,((G,B)\,\widetilde{\cup}\,(H,C)) = ((F,A)\,\widetilde{\cap}\,(G,B))\,\widetilde{\cup}\,((F,A)\,\widetilde{\cap}\,(H,C))$.

Proof. The proof is straightforward. □

Definition 3.22. If (F,A) and (G,B) are two soft expert sets over U then (F,A) AND (G,B) denoted by $(F,A)\wedge(G,B)$, is defined by

$$(F,A)\wedge(G,B) = (H, A\times B), \tag{3.17}$$

where $H(\alpha,\beta) = F(\alpha)\,\widetilde{\cap}\,G(\beta)$, for all $(\alpha,\beta)\in A\times B$.

Example 3.23. Consider Example 3.3. Let

$$\begin{aligned} A &= \{(e_1,p,1),(e_2,p,0),(e_1,r,0),(e_2,r,1)\},\\ B &= \{(e_1,p,1),(e_1,r,0),(e_2,r,1)\}. \end{aligned} \tag{3.18}$$

Suppose (F,A) and (G,B) are two soft expert sets over U such that

$$\begin{aligned} (F,A) = \{&((e_1,p,1),\{u_1,u_2,u_4\}),((e_1,r,0),\{u_1,u_2\}),((e_3,q,0),\{u_3,u_4\}),\\ &((e_2,r,1),\{u_1,u_2,u_4\})\}\\ (G,B) = \{&((e_1,p,1),\{u_1,u_3,u_4\}),((e_1,r,0),\{u_2\}),((e_2,r,1),\{u_2,u_3\})\}. \end{aligned} \tag{3.19}$$

Therefore

$$\begin{aligned} (F,A)\wedge(G,B) &= (H,A\times B)\\ &= \{(((e_1,p,1),(e_1,p,1)),\{u_1,u_4\}),(((e_1,p,1),(e_1,r,0)),\{u_2\}),\\ &\quad (((e_1,p,1),(e_2,r,1)),\{u_2\}),(((e_1,r,0),(e_1,p,1)),\{u_1\}),\\ &\quad (((e_1,r,0),(e_1,r,0)),\{u_2\}),(((e_1,r,0),(e_2,r,1)),\{u_2\}),\\ &\quad (((e_3,q,0),(e_1,p,1)),\{u_3,u_4\}),(((e_3,q,0),(e_1,r,0)),\emptyset),\\ &\quad (((e_3,q,0),(e_2,r,1)),\{u_3\}),(((e_2,r,1),(e_1,p,1)),\{u_1,u_4\}),\\ &\quad (((e_2,r,1),(e_1,r,0)),\{u_2\}),(((e_2,r,1),(e_2,r,1)),\{u_2\})\}. \end{aligned} \tag{3.20}$$

Definition 3.24. If (F, A) and (G, B) are two soft expert sets then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$, is defined by

$$(F, A) \vee (G, B) = (O, A \times B), \tag{3.21}$$

where $O(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

Example 3.25. Consider Example 3.23. Then

$$\begin{aligned}
(F, A) \vee (G, B) &= (H, A \times B) \\
&= \{(((e_1, p, 1), (e_1, p, 1)), U), (((e_1, p, 1), (e_1, r, 0)), \{u_1, u_2, u_4\}), \\
&\quad (((e_1, p, 1), (e_2, r, 1)), U), (((e_1, r, 0), (e_1, p, 1)), U), \\
&\quad (((e_1, r, 0), (e_1, r, 0)), \{u_1, u_2\}), (((e_1, r, 0), (e_2, r, 1)), \{u_1, u_2, u_3\}), \\
&\quad (((e_3, q, 0), (e_1, p, 1)), \{u_1, u_3, u_4\}), \\
&\quad (((e_3, q, 0), (e_1, r, 0)), \{u_2, u_3, u_4\}), \\
&\quad (((e_3, q, 0), (e_2, r, 1)), \{u_2, u_3, u_4\}), (((e_2, r, 1), (e_1, p, 1)), U), \\
&\quad (((e_2, r, 1), (e_1, r, 0)), \{u_1, u_2, u_4\}), \\
&\quad (((e_2, r, 1), (e_2, r, 1)), \{u_1, u_2, u_4\})\}.
\end{aligned} \tag{3.22}$$

Proposition 3.26. *If (F, A) and (G, B) are two soft expert sets over U, then*

(i) $((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$,

(ii) $((F, A) \vee (G, B))^c = (F, A)^c \wedge (G, B)^c$.

Proof. See Maji et al. [3]. □

Proposition 3.27. *If (F, A), (G, B), and (H, C) are three soft expert sets over U, then*

(i) $(F, A) \wedge ((G, B) \wedge (H, C)) = ((F, A) \wedge (G, B)) \wedge (H, C)$,

(ii) $(F, A) \vee ((G, B) \vee (H, C)) = ((F, A) \vee (G, B)) \vee (H, C)$,

(iii) $(F, A) \vee ((G, B) \wedge (H, C)) = ((F, A) \vee (G, B)) \wedge ((F, A) \vee (H, C))$,

(iv) $(F, A) \wedge ((G, B) \vee (H, C)) = ((F, A) \wedge (G, B)) \vee ((F, A) \wedge (H, C))$.

Proof. Straightforward from Definitions 3.22 and 3.24. □

4. An Application of Soft Expert Set

Maji et al. [4] applied the theory of soft sets to solve a decision-making problem using rough mathematics. In this section, we present an application of soft expert set theory in a decision-making problem. The problem we consider is as below.

Assume that a company wants to fill a position. There are eight candidates who form the universe $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. The hiring committee considers a set of parameters, $E = \{e_1, e_2, e_3, e_4, e_5\}$ where the parameters e_i $(i = 1, 2, 3, 4, 5)$ stand for "experience," "computer knowledge," "young age," "good speaking," and "friendly," respectively. Let $X = \{p, q, r\}$ be a set of experts (committee members). Suppose

$$\begin{aligned}
(F, Z) = \{ & ((e_1, p, 1), \{u_1, u_2, u_4, u_7, u_8\}), ((e_1, q, 1), \{u_1, u_4, u_5, u_8\}), \\
& ((e_1, r, 1), \{u_1, u_3, u_4, u_6, u_7, u_8\}), ((e_2, p, 1), \{u_3, u_5, u_8\}), \\
& ((e_2, q, 1), \{u_1, u_3, u_4, u_5, u_6, u_8\}), ((e_2, r, 1), \{u_1, u_2, u_4, u_7, u_8\}), \\
& ((e_3, p, 1), \{u_3, u_4, u_5, u_7\}), ((e_3, q, 1), \{u_1, u_2, u_5, u_8\}), ((e_3, r, 1), \{u_1, u_7, u_8\}), \\
& ((e_4, p, 1), \{u_1, u_7, u_8\}), ((e_4, q, 1), \{u_1, u_4, u_5, u_8\}), ((e_4, r, 1), \{u_1, u_6, u_7, u_8\}), \\
& ((e_5, p, 1), \{u_1, u_2, u_3, u_5, u_8\}), ((e_5, q, 1), \{u_1, u_4, u_5, u_8\}), \\
& ((e_5, r, 1), \{u_1, u_3, u_5, u_7, u_8\}), ((e_1, p, 0), \{u_3, u_5, u_6\}), \\
& ((e_1, q, 0), \{u_2, u_3, u_6, u_7\}), ((e_1, r, 0), \{u_2, u_5\}), \\
& ((e_2, p, 0), \{u_1, u_2, u_4, u_6, u_7\}), ((e_2, q, 0), \{u_2, u_7\}), ((e_2, r, 0), \{u_3, u_5, u_6\}), \\
& ((e_3, p, 0), \{u_1, u_2, u_6, u_8\}), ((e_3, q, 0), \{u_3, u_4, u_6, u_7\}), \\
& ((e_3, r, 0), \{u_2, u_3, u_4, u_5, u_6\}), \\
& ((e_4, p, 0), \{u_2, u_3, u_4, u_5, u_6\}), ((e_4, q, 0), \{u_2, u_3, u_6, u_7\}), \\
& ((e_4, r, 0), \{u_2, u_3, u_4, u_5\}), ((e_5, p, 0), \{u_4, u_6, u_7\}), \\
& ((e_5, q, 0), \{u_2, u_3, u_6, u_7\}), ((e_5, r, 0), \{u_2, u_4, u_6\}) \}.
\end{aligned} \tag{4.1}$$

In Tables 1 and 2 we present the agree-soft expert set and disagree-soft expert set, respectively, such that if $u_i \in F_1(\varepsilon)$ then $u_{ij} = 1$ otherwise $u_{ij} = 0$, and if $u_i \in F_0(\varepsilon)$ then $u_{ij} = 1$ otherwise $u_{ij} = 0$ where u_{ij} are the entries in Tables 1 and 2.

The following *algorithm* may be followed by the company to fill the position.

Algorithm 4.1.

(1) input the soft expert set (F, Z),

(2) find an agree-soft expert set and a disagree-soft expert set,

(3) find $c_j = \sum_i u_{ij}$ for agree-soft expert set,

(4) find $k_j = \sum_i u_{ij}$ for disagree-soft expert set,

(5) find $s_j = c_j - k_j$,

(6) find m, for which $s_m = \max s_j$.

Table 1: Agree-soft expert set.

U	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
(e_1, p)	1	1	0	1	0	0	1	1
(e_2, p)	0	0	1	0	1	0	0	1
(e_3, p)	0	0	1	1	1	0	1	0
(e_4, p)	1	0	0	0	0	0	1	1
(e_5, p)	1	1	1	0	1	0	0	1
(e_1, q)	1	0	0	1	1	0	0	1
(e_2, q)	1	0	1	1	1	1	0	1
(e_3, q)	0	0	1	1	1	0	1	0
(e_4, q)	1	0	0	1	1	0	0	1
(e_5, q)	1	0	0	1	1	0	0	1
(e_1, r)	1	0	1	1	0	1	1	1
(e_2, r)	1	1	0	1	0	0	1	1
(e_3, r)	1	0	0	0	0	0	1	1
(e_4, r)	1	0	0	0	0	1	1	1
(e_5, r)	1	0	1	0	1	0	1	1
$c_j = \sum_i u_{ij}$	$c_1 = 12$	$c_2 = 3$	$c_3 = 7$	$c_4 = 9$	$c_5 = 9$	$c_6 = 3$	$c_7 = 9$	$c_8 = 13$

Table 2: Disagree-soft expert set.

U	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
(e_1, p)	0	0	1	0	0	1	0	0
(e_2, p)	1	1	0	1	0	1	1	0
(e_3, p)	1	1	0	0	0	1	0	1
(e_4, p)	0	1	1	1	1	1	0	0
(e_5, p)	0	0	0	1	0	1	1	0
(e_1, q)	0	1	1	0	0	1	1	0
(e_2, q)	0	1	0	0	0	0	1	0
(e_3, q)	1	1	0	0	0	1	0	1
(e_4, q)	0	1	1	0	0	1	1	0
(e_5, q)	0	1	1	0	0	1	1	0
(e_1, r)	0	1	0	0	1	0	0	0
(e_2, r)	0	0	1	0	1	1	0	0
(e_3, r)	0	1	1	1	1	1	0	0
(e_4, r)	0	1	1	1	1	0	0	0
(e_5, r)	0	1	0	1	0	1	0	0
$k_j = \sum_i u_{ij}$	$k_1 = 3$	$k_2 = 12$	$k_3 = 8$	$k_4 = 6$	$k_5 = 6$	$k_6 = 12$	$k_7 = 6$	$k_8 = 2$

Then s_m is the optimal choice object. If m has more than one value, then any one of them could be chosen by the company using its option.

Now we use this algorithm to find the best choices for the company to fill the position. From Tables 1 and 2 we have Table 3.

Table 3

$c_j = \sum_i u_{ij}$	$k_j = \sum_i u_{ij}$	$s_j = c_j - k_j$
$c_1 = 12$	$k_1 = 3$	$s_1 = 9$
$c_2 = 3$	$k_2 = 12$	$s_2 = -9$
$c_3 = 7$	$k_3 = 8$	$s_3 = -1$
$c_4 = 9$	$k_4 = 6$	$s_4 = 3$
$c_5 = 9$	$k_5 = 6$	$s_5 = 3$
$c_6 = 3$	$k_6 = 12$	$s_6 = -9$
$c_7 = 9$	$k_7 = 6$	$s_7 = 3$
$c_8 = 13$	$k_8 = 2$	$s_8 = 11$

Then max $s_j = s_8$, so the committee will choose candidate 8 for the job.

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