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Research Article

H_∞ Filtering for Discrete-Time Stochastic Systems with Nonlinear Sensor and Time-Varying Delay

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The H_∞ filtering problem for a class of discrete-time stochastic systems with nonlinear sensor and time-varying delay is investigated. By using the Lyapunov stability theory, sufficient conditions are proposed to guarantee the asymptotical stablity with an prescribe H_∞ performance level of the filtering error systems. These conditions are dependent on the lower and upper bounds of the discrete time-varying delays and are obtained in terms of a linear matrix inequality (LMI). Finally, two numerical examples are provided to illustrate the effectiveness of the proposed methods.

1. Introduction

As is well known, time delay exists commonly in many processes due to the after-effect phenomena in their inner dynamics, which has been recognized to be an important source of instability and degraded performance. The presence of time delay must be taken into account in modeling due to the ever-increasing expectations of dynamic performance. Therefore, time-delay systems have drawn much attention in the last few decades, and a great number of important results have been reported in the literature; see, for instance, [1–5] and the references therein. For continuous-time systems, the obtained results can be generally classified into two types: delay-independent and delay-dependent ones. It has been understood that the latter is generally less conservative since the size of delays is considered, especially when time delays are small. Compared with continuous-time systems with time-varying delays, the discrete-time counterpart receives relatively less attention. See, for example, [6–9] and references therein.

In the past few years, considerable attention has been devoted to the topic of H_∞ filtering in the past two decades, and many significant results have been obtained [10–19]. The exponential filtering problem is studied for discrete time-delay stochastic systems with Markovian jump parameters and missing measurements in [20]. The robust fault detection filter problem for fuzzy *Itô* stochastic systems is studied in [21]. The problem of robust H_∞ filtering for uncertain discrete-time stochastic systems with time-varying delays is considered in [22]. Meanwhile, in many industrial processes, the quality and reliability of sensors often influence the performance of the filters. Nonlinearity is present in almost all real sensors in one form or another. So, the filtering problem for a class of nonlinear discrete-time stochastic systems with state delays is considered in [23]. The robust H_∞ filtering problem for a class of nonlinear discrete time-delay stochastic systems is considered in [24]. The H_∞ filtering problem for a general class of nonlinear discrete-time stochastic systems with randomly varying sensor delays is considered in [25]. And the filtering problem for discrete-time fuzzy stochastic systems with sensor nonlinearities is considered in [26]. The problem of H_∞ filtering for discrete-time Takagi-Sugeno (T-S) fuzzy *Itô* stochastic systems with time-varying delay is studied in [27]. Robust H_∞ filter design for systems with sector-bounded nonlinearities is considered in [28, 29]. H_∞ filtering for discrete-time systems with stochastic incomplete measurement and mixed delays is investigated in [30]. Recently, the H_∞ filtering problem of the time-delayed discrete-time deterministic systems with saturation nonlinear sensors, in which process and measurement noise have unknown statistic characteristic but bounded energy, is investigated in [31]. In [24, 26, 28–30], the nonlinearity for filtering problem of systems was assumed to satisfy nonlinear sensor, which may includes actuator saturation and sensor saturation. It is worth mentioning that, although the system in [31] is with nonlinear sensor, the proposed filter design approach only considers the constant time delay, which is not applicable to systems with time-varying delay. To the best of the authors' knowledge, little effort has been made towards the H_∞ filtering of discrete-time stochastic systems with nonlinear sensor and time-varying delay.

Motivated by the works in [20], in this paper, a delay-dependent H_∞ performance analysis result is established for filtering error systems. A new different Lyapunov functional is then employed to deal with systems with nonlinear sensor and time-varying delay. As a result, the H_∞ filter is designed in terms of linear matrix inequalities (LMIs). The resulting filter can ensure that the error system is asymptotically stable and the estimation error is bounded by a prescribed level. Finally, two numerical examples are given to show the effectiveness of the proposed method.

Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. I is the identity matrix. $|\cdot|$ denotes Euclidean norm for vectors, and $\|\cdot\|$ denotes the spectral norm of matrices. N denotes the set of all natural number, that is, $N = \{0, 1, 2, \ldots\}$. $(\Omega, \mathcal{F}, \{\mathcal{F}_k\}_{k\in N}, \mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_k\}_{k\in N}$ satisfying the usual conditions. M^T stands for the transpose of the matrix M. For symmetric matrices X and Y, the notation $X > Y$ (resp., $X \geq Y$) means that the $X - Y$ is positive definite (resp., positive semidefinite). $*$ denotes a block that is readily inferred by symmetry. $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$.

2. Problem Description

Consider a class of discrete-time stochastic systems with nonlinear sensor and time-varying delay as follows:

$$\begin{aligned} x(k+1) = {} & Ax(k) + A_d x(k-\tau(k)) + B_1 v(k) \\ & + \left[Ex(k) + E_d x(k-\tau(k)) + B_2 v(k)\right] w(k), \\ y(k) = {} & f(Cx(k)) + Dv(k), \\ z(k) = {} & Lx(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^q$ is the measurable output vector, $z(k) \in \mathbb{R}^r$ is the state combination to be estimated, and $w(k)$ is a real scalar process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relative to an increasing family $(\mathcal{F}_k)_{k\in N}$ of σ-algebra $\mathcal{F}_k \subset \mathcal{F}$ generated by $(w(k))_{k\in N}$. The stochastic process $\{w(k)\}$ is independent, which is assumed to satisfy

$$\begin{gathered} \mathbf{E}\{w(k)\} = 0, \qquad \mathbf{E}\left\{w^2(k)\right\} = 1, \\ \mathbf{E}\{w(i)w(j)\} = 0 \quad (i \neq j), \end{gathered} \tag{2}$$

where the stochastic variables $w(0), w(1), w(2), \ldots$ are assumed to be mutually independent. The exogenous disturbance signal $v(k) \in \mathbb{R}^p$ is assumed to belong to $L_{e2}([0\ \ \infty), \mathbb{R}^p)$, A, A_d, B_1, E, E_d, B_2, C, D and L are known real constant matrices. And the time-varying delay $\tau(k)$ satisfies

$$\tau_1 \leq \tau(k) \leq \tau_2, \tag{3}$$

where τ_1 and τ_2 are known positive integers representing the minimum and maximum delays, respectively.

In addition, $f_i(\zeta_i)$ $(i = 1, 2, \ldots, p)$ are nonlinear sensor functions. We assume that nonlinear sensor functions are monotonically nondecreasing, bounded, and globally Lipschitz. That is, there exist a set of positive scalars u_i and θ_i such that [31, 32]

$$0 \leq \frac{f_i(\alpha) - f_i(\beta)}{\alpha - \beta} \leq u_i \quad \forall \alpha, \beta \in \mathbb{R},\ i = 1, 2, \ldots, p, \tag{4}$$

$$-\theta_i \leq f_i(\zeta_i) \leq \theta_i, \quad i = 1, 2, \ldots, p, \tag{5}$$

where u_i is the magnification of the sensor, and θ_i is the amplitude of the sensor.

We consider the following linear discrete-time filter for the estimation of $z(k)$:

$$\begin{aligned} \hat{x}(k+1) = {} & A\hat{x}(k) + A_d\hat{x}(k-\tau(k)) \\ & + \left[E\hat{x}(k) + E_d\hat{x}(k-\tau(k))\right] w(k) \\ & + K\left[y(k) - f(C\hat{x}(k))\right], \\ \hat{z}(k) = {} & L\hat{x}(k), \end{aligned} \tag{6}$$

where $\hat{x}(k) \in \mathbb{R}^n$ and $\hat{z}(k) \in \mathbb{R}^r$ denote the estimates of $x(k)$ and $z(k)$, respectively, and the matrix L is constant matrix.

Remark 1. Similar to [26, 31, 32], the nonlinear sensor satisfying (4)-(5) is also considered in this paper. It is noted that in the previous filter, the matrix L is assumed to be constants in order to avoid more verbosely mathematical derivation.

Defining $e(k) = x(k) - \hat{x}(k)$ and augmenting the model (1) to include the states of the filter (6), we obtain the following filtering error systems:

$$\begin{aligned} e(k+1) = {} & Ae(k) + A_d e(k-\tau(k)) + B_1 v(k) \\ & + \left[Ee(k) + E_d e(k-\tau(k)) + B_2 v(k)\right] w(k) \\ & - K\phi(Ce(k)) - KDv(k), \end{aligned} \tag{7}$$

$$\tilde{z}(k) = Le(k), \tag{8}$$

where $\widetilde{z}(k) = z(k) - \widehat{z}(k)$, $\phi(Ce(k)) = f(Cx(k)) - f(C\widehat{x}(k))$, and $\phi_i(C_i e(k))$ $(i = 1, 2, \ldots, p)$ satisfy the following conditions according to (4):

$$0 \leq \frac{\phi_i(C_i e(k))}{C_i e(k)} \leq u_i, \tag{9}$$

where C_i is the ith row of matrix C.

The H_∞ filtering problem to be addressed in this paper can be formulated as follows. Given discrete-time stochastic systems (1), a prescribed level of noise attenuation $\gamma > 0$, and any $f_i(\zeta_i)$ $(i = 1, 2, \ldots, p)$, find a suitable filter in the form of (6) such that the following requirements are satisfied.

(1) The filtering error systems (7)-(8) with $v(k) = 0$ is said to be asymptotically stable if there exists a scalar $c > 0$ such that

$$\mathbf{E}\left\{\sum_{k=0}^{\infty}|x(k)|^2\right\} \leq c\mathbf{E}\left\{|x(0)|^2\right\}, \tag{10}$$

where $x(k)$ denotes the solution of stochastic systems with initial state $x(0)$.

(2) For the given disturbance attenuation level $\gamma > 0$ and under zero initial conditions for all $v(k) \in L_{e2}([0\ \ \infty), \mathbb{R}^p)$, the performance index γ satisfies the following inequality:

$$\|\widetilde{z}(k)\|_{e2} < \gamma\|v(k)\|_{e2}. \tag{11}$$

3. Main Results

3.1. Performance Analysis of H_∞ Filter

Theorem 2. *If there exist symmetric positive definite matrices P, Q, and Υ, diagonal semipositive definite matrices $T = \mathrm{diag}\{t_1, t_2, \ldots, t_p\}$ and $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_p\}$, a nonzero matrix K, and a positive scalar γ, such that the following LMI is satisfied:*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ * & * & \Gamma_{33} & \Gamma_{34} \\ * & * & * & \Gamma_{44} \end{bmatrix} < 0, \tag{12}$$

where

$$\begin{aligned}
\Gamma_{11} &= \tau_{21}Q + \Upsilon - P + A^T RA + E^T RE + L^T L, \\
&\qquad\qquad \tau_{21} = \tau_2 - \tau_1 + 1, \\
\Gamma_{12} &= A^T RA_d + E^T RE_d, \\
\Gamma_{13} &= -A^T RK + C^T UT - C^T \Lambda, \\
\Gamma_{14} &= A^T R(B_1 - KD) + E^T RB_2, \\
\Gamma_{22} &= -\Upsilon - Q + A_d^T RA_d + E_d^T RE_d, \\
\Gamma_{23} &= -A_d^T RK, \\
\Gamma_{24} &= A_d^T R(B_1 - KD) + E_d^T RB_2, \\
\Gamma_{33} &= K^T RK - 2T, \\
\Gamma_{34} &= -K^T R(B_1 - KD), \\
\Gamma_{44} &= (B_1 - KD)^T R(B_1 - KD) + B_2^T RB_2 - \gamma^2 I,
\end{aligned} \tag{13}$$

and $R = P + 2C^T \Lambda UC$, $U = \mathrm{diag}\{u_1, u_2, \ldots, u_p\} \geq 0$, then the filtering error system (7) with $v(k) = 0$ is asymptotically stable, and the optimal H_∞ performance can be obtained by minimizing γ over the variables P, Q, Υ, T, Λ, and K, that is,

$$\begin{aligned}
&\textit{minimize} \quad \gamma \\
&\textit{subject to} \quad P > 0,\ Q > 0,\ \Upsilon > 0,\ T > 0,\ \Lambda > 0.
\end{aligned} \tag{14}$$

Proof. We first establish the condition of asymptotical stability for the filtering error systems (7)-(8). Consider the Lyapunov-Krasovskii functional candidate as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k), \tag{15}$$

where

$$\begin{aligned}
V_1(k) &= e^T(k)Pe(k), \\
V_2(k) &= \sum_{i=k-\tau(k)}^{k-1} e^T(i)Qe(i), \\
V_3(k) &= \sum_{j=-\tau_2+2}^{-\tau_1+1}\ \sum_{l=k+j-1}^{k-1} e^T(l)Qe(l), \\
V_4(k) &= \sum_{i=k-\tau(k)}^{k-1} e^T(i)\Upsilon e(i), \\
V_5(k) &= 2\sum_{i=1}^{p}\lambda_i\phi_i(C_i e(k))C_i e(k).
\end{aligned} \tag{16}$$

First, we consider system (7) with $v(k) = 0$, that is,

$$\begin{aligned}
e(k+1) = {} & Ae(k) + A_d e(k - \tau(k)) - K\phi(Ce(k)) \\
& + [Ee(k) + E_d e(k - \tau(k))]w(k).
\end{aligned} \tag{17}$$

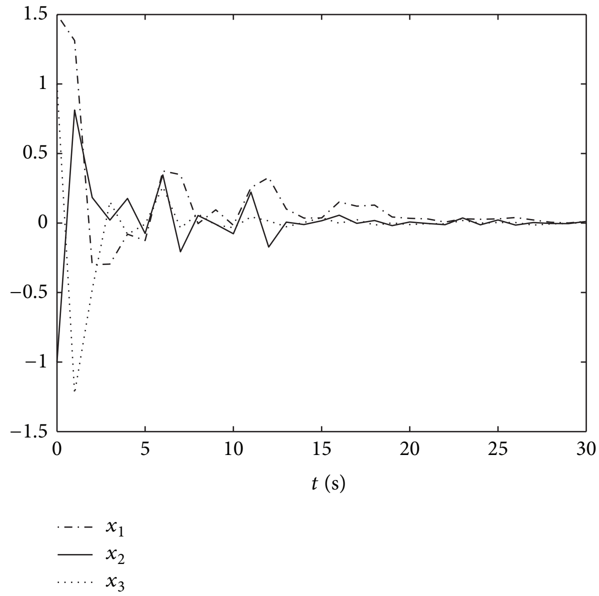

Figure 1: The state response $x(k)$.

Calculating the difference of $V(k)$ along the filtering error system (17), we get

$$\Delta V_1(k) = e^T(k+1)Pe(k+1) - e^T(k)Pe(k), \tag{18}$$

$$\begin{aligned}\Delta V_2(k) = &\sum_{i=k+1-\tau(k+1)}^{k-\tau_1} e^T(i)Qe(i) \\ &- e^T(k-\tau(k))Qe(k-\tau(k)) + e^T(k)Qe(k) \\ &+ \sum_{i=k+1-\tau_1}^{k-1} e^T(i)Qe(i) - \sum_{i=k+1-\tau(k)}^{k-1} e^T(i)Qe(i),\end{aligned} \tag{19}$$

$$\begin{aligned}\Delta V_3(k) = &\sum_{j=-\tau_2+2}^{-\tau_1+1}\left[e^T(k)Qe(k) + \sum_{l=k+j}^{k-1} e^T(l)Qe(l) - \sum_{l=k+j-1}^{k-1} e^T(l)Qe(l)\right] \\ = &\left(\tau_2-\tau_1\right)e^T(k)Qe(k) - \sum_{i=k+1-\tau_2}^{k-\tau_1} e^T(i)Qe(i).\end{aligned} \tag{20}$$

Since $\tau_1 \le \tau(k) \le \tau_2$, we have

$$\begin{aligned}&\sum_{i=k+1-\tau_1}^{k-1} e(i)^T Qe(i) - \sum_{i=k+1-\tau(k)}^{k-1} e(i)^T Qe(i) \le 0, \\ &\sum_{i=k+1-\tau(k+1)}^{k-\tau_1} e(i)^T Qe(i) - \sum_{i=k+1-\tau_2}^{k-\tau_1} e(i)^T Qe(i) \le 0.\end{aligned} \tag{21}$$

Combining (19)–(21), we have

$$\begin{aligned}\Delta V_2(k) + \Delta V_3(k) \le &\left(\tau_2-\tau_1+1\right)e^T(k)Qe(k) \\ &- e^T(k-\tau(k))Qe(k-\tau(k)).\end{aligned} \tag{22}$$

Meanwhile, we have

$$\Delta V_4(k) \le e^T(k)\Upsilon e(k) - e^T(k-\tau(k))\Upsilon e(k-\tau(k)), \tag{23}$$

$$\begin{aligned}\Delta V_5(k) = &2\sum_{i=1}^{p}\lambda_i\phi_i\left(C_ie(k+1)\right)C_ie(k+1) \\ &- 2\phi^T(Ce(k))\Lambda Ce(k).\end{aligned} \tag{24}$$

From condition (9), we have

$$\begin{aligned}&2\sum_{i=1}^{p}\lambda_i\phi_i\left(C_ie(k+1)\right)C_ie(k+1) \\ &\quad\le 2\sum_{i=1}^{p}\lambda_iu_iC_ie(k+1)C_ie(k+1).\end{aligned} \tag{25}$$

From (24)-(25), we obtain

$$\Delta V_5(k) \le 2e^T(k+1)C^T\Lambda UCe(k+1) - 2\phi^T(Ce(k))\Lambda Ce(k). \tag{26}$$

Combining (18), (22), (23), and (26), we have

$$\begin{aligned}\Delta V(k) = &\Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) + \Delta V_5(k) \\ \le &e^T(k+1)\left(P + 2C^T\Lambda UC\right)e(k+1) \\ &+ e^T(k)(\Upsilon - P)e(k) + \left(\tau_2-\tau_1+1\right)e^T(k)Qe(k) \\ &- e^T(k-\tau(k))(\Upsilon + Q)e(k-\tau(k)) \\ &- 2\phi^T(Ce(k))\Lambda Ce(k).\end{aligned} \tag{27}$$

From condition (9), we also have

$$-2\phi_i\left(C_ie(k)\right)t_i\left[\phi_i\left(C_ie(k)\right) - u_iC_ie(k)\right] \ge 0. \tag{28}$$

Then, for $T = \operatorname{diag}\{t_1, t_2, \ldots, t_p\} \geq 0$ and $U = \operatorname{diag}\{u_1, u_2, \ldots, u_p\} \geq 0$, we get

$$-2\phi^T(Ce(k))T\phi^T(Ce(k)) + 2\phi^T(Ce(k))TUCe(k) \geq 0. \tag{29}$$

Adding the left of (29) to (27), we have

$$\begin{aligned}
\Delta V(k) \leq{} & e^T(k+1)\left(P + 2C^T\Lambda UC\right)e(k+1) \\
& + e^T(k)(\Upsilon - P)e(k) + \left(\tau_2 - \tau_1 + 1\right)e^T(k)Qe(k) \\
& - e^T(k-\tau(k))(\Upsilon + Q)e(k-\tau(k)) \\
& - 2\phi^T(Ce(k))\Lambda Ce(k) \\
& - 2\phi^T(Ce(k))T\phi^T(Ce(k)) \\
& + 2\phi^T(Ce(k))TUCe(k) \\
={} & e^T(k+1)Re(k+1) + e^T(k)(\Upsilon - P)e(k) \\
& + \tau_{21}e^T(k)Qe(k) - 2\phi^T(Ce(k))T\phi^T(Ce(k)) \\
& - e^T(k-\tau(k))(\Upsilon + Q)e(k-\tau(k)) \\
& - 2\phi^T(Ce(k))\Lambda Ce(k) + 2\phi^T(Ce(k))TUCe(k).
\end{aligned} \tag{30}$$

Noting (2) and taking the mathematical expectation, we have

$$\begin{aligned}
\mathbf{E}\{\Delta V(k)\} \leq{} & \mathbf{E}\left\{e^T(k+1)Re(k+1) + e^T(k)(\Upsilon - P)e(k)\right. \\
& + \tau_{21}e^T(k)Qe(k) \\
& - e^T(k-\tau(k))(\Upsilon + Q)e(k-\tau(k)) \\
& - 2\phi^T(Ce(k))\Lambda Ce(k) \\
& - 2\phi^T(Ce(k))T\phi^T(Ce(k)) \\
& \left. + 2\phi^T(Ce(k))TUCe(k)\right\} \\
={} & \left[Ae(k) + A_de(k-\tau(k)) - K\phi(Ce(k))\right]^T \\
& \times R\left[Ae(k) + A_de(k-\tau(k)) - K\phi(Ce(k))\right] \\
& + \left[Ee(k) + E_de(k-\tau(k))\right]^T \\
& \times R\left[Ee(k) + E_de(k-\tau(k))\right] \\
& + e^T(k)(\Upsilon - P)e(k) + \tau_{21}e^T(k)Qe(k) \\
& - e^T(k-\tau(k))(\Upsilon + Q)e(k-\tau(k)) \\
& - 2\phi^T(Ce(k))\Lambda Ce(k) \\
& - 2\phi^T(Ce(k))T\phi^T(Ce(k)) \\
& + 2\phi^T(Ce(k))TUCe(k) \\
={} & \zeta^T(k)\widehat{\Gamma}\zeta(k),
\end{aligned} \tag{31}$$

where

$$\begin{gathered}
\zeta^T(k) = \begin{bmatrix} e^T(k) & e^T(k-\tau(k)) & \phi^T(Ce(k)) \end{bmatrix}, \\
\widehat{\Gamma} = \begin{bmatrix} \tau_{21}Q + \Upsilon - P + A^TRA + E^TRE & A^TRA_d + E^TRE_d & -A^TRK + C^TUT - C^T\Lambda \\ A_d^TRA + E_d^TRE & -\Upsilon - Q + A_d^TRA_d + E_d^TRE_d & -A_d^TRK \\ -K^TRA + TUC - \Lambda C & -K^TRA_d & K^TRK - 2T \end{bmatrix}.
\end{gathered} \tag{32}$$

Then, there exists a small scalar $\alpha > 0$ such that

$$\widehat{\Gamma} < \begin{bmatrix} -\alpha I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{33}$$

It can be shown that LMI (12) implies that $\widehat{\Gamma} < 0$; thus, it follows from (33) that

$$\mathbf{E}\{V(k+1)\} - \mathbf{E}\{V(k)\} < -\alpha\mathbf{E}\left\{|e(k)|^2\right\}. \tag{34}$$

Hence, by summing up both sides of (34) from 0 to N for any integer $N > 1$, we have

$$\mathbf{E}\{V(N+1)\} - \mathbf{E}\{V(0)\} < -\alpha\mathbf{E}\left\{\sum_{k=0}^{N}|e(k)|^2\right\} \tag{35}$$

which yields

$$\begin{aligned}
\mathbf{E}\left\{\sum_{k=0}^{N}|e(k)|^2\right\} &< \frac{1}{\alpha}\left[\mathbf{E}\{V(0)\} - \mathbf{E}\{V(N+1)\}\right] \\
&\leq \frac{1}{\alpha}\mathbf{E}\{V(0)\} \\
&\leq c\mathbf{E}\left\{|x(0)|^2\right\},
\end{aligned} \tag{36}$$

where $c = (1/\alpha)\lambda_{\max}(P)$. Taking $N \to \infty$, it is shown from (10) and (36) that the filtering error system (7) is asymptotically stable for $v(k) = 0$.

Next, we will show that the filtering error systems (7)-(8) satisfies

$$\|\tilde{z}(k)\|_{e2} < \gamma\|v(k)\|_{e2} \tag{37}$$

for all nonzero $v(k) \in L_{e2}([0\ \ \infty), \mathbb{R}^p)$. To this end, define

$$J(N) = \mathbf{E}\left\{\sum_{k=1}^{N}\left[|\tilde{z}(k)|^2 < \gamma^2|v(k)|^2\right]\right\} \tag{38}$$

with any integer $N > 0$. Then, for any nonzero $v(k)$, we have

$$\begin{aligned} J(N) &= \mathbf{E}\left\{\sum_{k=1}^{N}\left[|\tilde{z}(k)|^2 - \gamma^2|v(k)|^2 + \mathbf{E}\{\Delta V(k)\}\right]\right\} \\ &\quad - \mathbf{E}\{V(N+1)\} \\ &\le \mathbf{E}\left\{\sum_{k=1}^{N}\left[|\tilde{z}(k)|^2 - \gamma^2|v(k)|^2 + \mathbf{E}\{\Delta V(k)\}\right]\right\} \\ &= \mathbf{E}\left\{\hat{\zeta}^T(k)\Gamma\hat{\zeta}(k)\right\}, \end{aligned} \tag{39}$$

where

$$\hat{\zeta}^T(k) = \left[e^T(k) \quad e^T(k-\tau(k)) \quad \phi^T(Ce(k)) \quad v^T(k)\right]. \tag{40}$$

It can be shown that there exist real matrices $P > 0$, $Q > 0$, and $\Upsilon > 0$, diagonal semipositive definite matrices T and Λ, nonzero matrix K, and scalar $\gamma > 0$ satisfying LMI (12). Since $v(k) \in L_{e2}([0\ \ \infty), \mathbb{R}^p) \neq 0$, it implies that $\Gamma < 0$, and thus $J(N) < 0$. That is, $\|\tilde{z}(k)\|_{e2} < \gamma\|v(k)\|_{e2}$. This completes the proof. □

3.2. Design of H_∞ Filter

Theorem 3. *Consider the discrete-time stochastic systems with nonlinear sensor in (1), a filter of form (6), and constants τ_1 and τ_2. The filtering error systems (7)-(8) is asymptotically stable with performance γ, if there exist positive definite matrices P, Υ, and Q, diagonal semipositive definite matrices T and Λ, and matrix X such that the following LMI is satisfied:*

$$\begin{bmatrix} \tau_{21}Q + \Upsilon - P + L^TL & 0 & C^TUT - C^T\Lambda & 0 & A^TR & E^TR \\ * & -\Upsilon - Q & 0 & 0 & A_d^TR & E_d^TR \\ * & * & -2T & 0 & -X & 0 \\ * & * & * & -\gamma^2 I & B_1^TR - D^TX & B_2^TR \\ * & * & * & * & -R & 0 \\ * & * & * & * & * & -R \end{bmatrix} < 0. \tag{41}$$

Moreover, if the previous condition is satisfied, an acceptable state-space realization of the H_∞ filter is given by

$$K = \left(P + 2C^T\Lambda UC\right)^{-T}X^T. \tag{42}$$

Proof. By the Schur complement, LMI (12) is equivalent to

$$\begin{bmatrix} \tau_{21}Q + \Upsilon - P + L^TL & 0 & C^TUT - C^T\Lambda & 0 & A^TR & E^TR \\ * & -\Upsilon - Q & 0 & 0 & A_d^TR & E_d^TR \\ * & * & -2T & 0 & -K^TR & 0 \\ * & * & * & -\gamma^2 I & (B_1 - KD)^T & B_2^TR \\ * & * & * & * & -R & 0 \\ * & * & * & * & * & -R \end{bmatrix} < 0. \tag{43}$$

By defining

$$K^TR = X, \tag{44}$$

we have LMI (41). The filter parameter K can be deduced from (44). According to Theorem 2, we thus complete the proof. □

Remark 4. When τ_1 and τ_2 are given, matrix inequality (41) is linear matrix inequality in matrix variables $P > 0$, $\Upsilon > 0$, $Q > 0$, $T \ge 0$, $\Lambda \ge 0$, and X, which can be efficiently solved by the developed interior point algorithm [4]. Meanwhile, it is easy to find the minimal attenuation level γ.

In the sequel, special result for the discrete-time deterministic system with nonlinear sensor and time-varying delay, that is to say, there is no stochastic noise:

$$x(k+1) = Ax(k) + A_d x(k - \tau(k)) + B_1 v(k), \quad (45)$$

$$y(k) = f(Cx(k)) + Dv(k), \quad (46)$$

$$z(k) = Lx(k). \quad (47)$$

The following corollary may be obtained from Theorem 3.

Corollary 5. *Consider the discrete-time systems with nonlinear sensor in* (45)–(47)*, a filter of form* (6)*, and constants* τ_1 *and* τ_2*. The corresponding filtering error system is stable with performance* γ*, if there exist positive definite matrices* P*,* Υ*, and* Q*, diagonal semipositive definite matrices* T *and* Λ*, and matrix* X *such that the following LMI is satisfied:*

$$\begin{bmatrix} \tau_{21}Q + \Upsilon - P + L^T L & 0 & C^T UT - C^T \Lambda & 0 & A^T R \\ * & -\Upsilon - Q & 0 & 0 & A_d^T R \\ * & * & -2T & 0 & -X \\ * & * & * & -\gamma^2 I & B_1^T R - D^T X \\ * & * & * & * & -R \end{bmatrix} < 0. \quad (48)$$

Moreover, if the previous condition is satisfied, an acceptable state-space realization of the H_∞ *filter is given by*

$$K = \left(P + 2C^T \Lambda U C\right)^{-T} X^T. \quad (49)$$

Remark 6. When $\tau(k) = \tau$ in system (45), this discrete-time deterministic system model with constant time delay has been considered in [31]. But the proposed filter design approach only considers the constant time delay, which is not applicable to system (45) with time-varying delay.

Remark 7. In many practical industrial processes, the quality and reliability of sensors often influence the performance of the filters. Nonlinearity is present in almost all real sensors in one form or another. Therefore, in order to reduce the effect of the sensor nonlinearity on the filter performance to the lowest level, the nonlinear characteristics of sensors should be taken into account when we design the filters [23, 31].

4. Numerical Example

In this section, two numerical examples are given to illustrate the effectiveness and benefits of the proposed approach.

Example 8. Consider the following discrete-time deterministic system (45)–(47) with nonlinear sensor and time-varying delay as follows:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.5 & 0.15 & 0.5 \\ -0.15 & -0.5 & 0.05 \\ -0.05 & 0.1 & -0.5 \end{bmatrix} x(k) \\ &+ \begin{bmatrix} 0.1 & 0.2 & 0 \\ 0.1 & 0.1 & -0.1 \\ 0.1 & 0 & -0.1 \end{bmatrix} x(k - \tau(k)) \\ &+ \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} v(k), \\ y(k) &= f\left(\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix} x(k)\right) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v(k), \\ z(k) &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x(k), \end{aligned} \quad (50)$$

where the sensor nonlinear functions $f_1(\cdot)$ and $f_2(\cdot)$ satisfy (4) and (5), in which $u_1 = u_2 = 1$, and $\theta_1 = \theta_2 = 1$.

This example has been considered in [31]; however, for system (45), Theorem 2 of [31] is infeasible. Note that different τ_1 and τ_2 yield different $\gamma_{\min}$; if we assume that $0 \le \tau(k) \le 2$, then by Corollary 5, the minimal disturbance attenuation level is $\gamma_{\min} = 1.7495$, and the corresponding filter matrix is

$$K = \begin{bmatrix} 0.2754 & 0.2172 \\ -0.2296 & 0.1602 \\ -0.0453 & 0.0859 \end{bmatrix}. \quad (51)$$

It can be seen from Example 8 that our method is much less conservative than Theorem 2 of [31].

Example 9. Consider the discrete-time stochastic systems (1) with nonlinear sensor and time-varying delay as follows:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.5 & 0.2 & 0.4 \\ -0.2 & -0.5 & 0.1 \\ -0.1 & 0.1 & -0.4 \end{bmatrix} x(k) \\ &+ \begin{bmatrix} 0.2 & 0.3 & 0 \\ 0.2 & 0.1 & -0.1 \\ 0.1 & 0 & -0.2 \end{bmatrix} x(k - \tau(k)) + \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} v(k) \\ &+ \Bigg[\begin{bmatrix} 0.2 & 0.1 & 0.1 \\ -0.1 & -0.3 & 0.1 \\ -0.1 & 0.1 & -0.4 \end{bmatrix} x(k) \\ &+ \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.2 & 0.1 & -0.1 \\ 0.1 & 0.1 & -0.1 \end{bmatrix} x(k - \tau(k)) \\ &+ \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} v(k)\Bigg] w(k), \end{aligned}$$

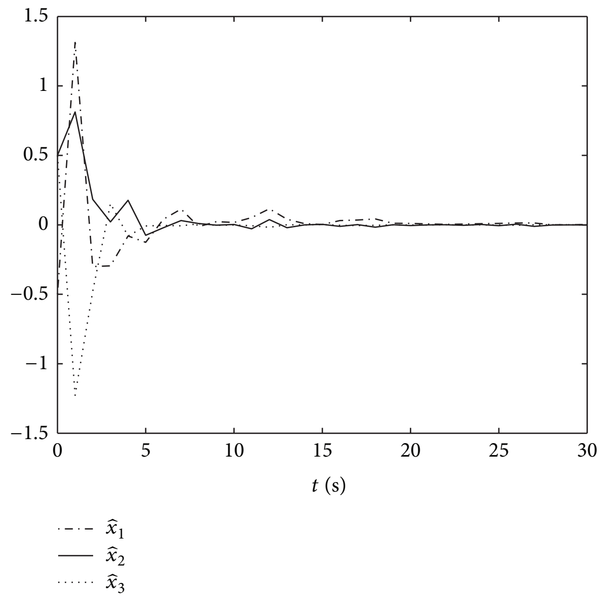

FIGURE 2: The estimation of filter $\hat{x}(k)$.

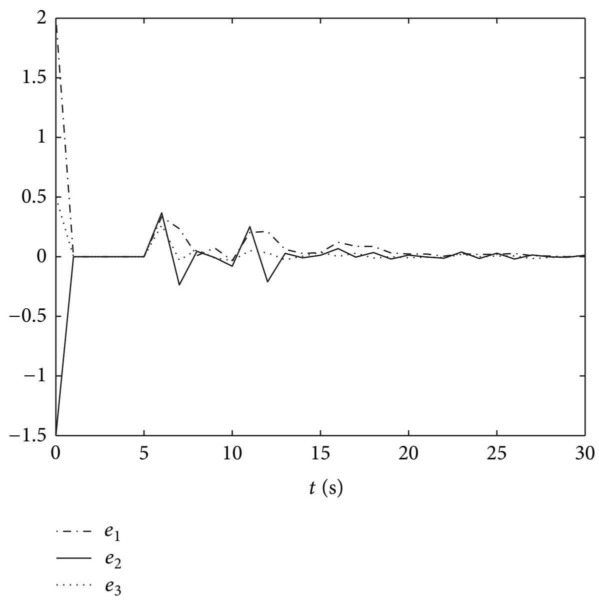

FIGURE 3: The error response $e(k)$.

$$y(k) = f\left(\begin{bmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix} x(k)\right) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v(k),$$
$$z(k) = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix} x(k), \tag{52}$$

where the sensor nonlinear functions $f_1(\cdot)$ and $f_2(\cdot)$ satisfy (4) and (5), in which $u_1 = u_2 = 2$, and $\theta_1 = \theta_2 = 2$.

Note that different τ_1 and τ_2 yield different $\gamma_{\min}$; if we assume that $\tau(k)$ satisfies $1 \le \tau(k) \le 5$, then by Theorem 3, the minimum achievable noise attenuation level is given by $\gamma_{\min} = 0.1392$, and the corresponding filter parameters are as follows:

$$K = \begin{bmatrix} 0.0693 & 0.1005 \\ -0.0350 & 0.0702 \\ -0.0082 & 0.0018 \end{bmatrix}. \tag{53}$$

With the initial conditions, $x(t)$ and $\hat{x}(t)$ are $[1.5 \ \ -1 \ \ 1]^T$ and $[-0.5 \ \ 0.5 \ \ 0.5]^T$, respectively, for an appropriate initial interval. We apply the previous filter parameter K to system (1) and obtain the simulation results as in Figures 1–3. Figure 1 shows the state response $x(k)$ under the initial condition. Figure 2 shows the estimation of filter $\hat{x}(k)$. Figure 3 shows error response $e(k)$. From these simulation results, we can see that the designed H_∞ filter can stabilize the discrete-time stochastic system (1) with nonlinear sensors and time-varying delay.

5. Conclusions

In this paper, the H_∞ filtering problem for a class of discrete-time stochastic systems with nonlinear sensor and time-varying delay has been developed. A new type of Lyapunov-Krasovskii functional has been constructed to derive some sufficient conditions for the filter in terms of LMIs, which guarantees a prescribed H_∞ performance index for the filtering error system. Two numerical examples have shown the usefulness and effectiveness of the proposed filter design method.

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