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WELL POSEDNESS FOR EVOLUTION INCLUSIONS

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ABSTRACT

We prove the existence of a continuous selection of the multivalued map $\varphi \rightarrow \Phi(\varphi)$ which is the set of all mild solutions of the evolution inclusion

$$\dot{x}(t) \in Ax(t) + F(t, x(t)) + \int_0^t h(t-s)g(x(s))ds$$

$$x(0) = \varphi.$$

Here F is a multivalued map, Lipschitzian with respect to x, and A is the infinitesimal generator of a C_0-semigroup.

Key words: Well-posedness, Evolution Inclusion, Mild Solution.

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1. Introduction

According to a generally accepted terminology in the theory of differential equations, an initial value problem, whose solutions exist, are unique and depend continuously on the initial data, is called *well posed.* For problems lacking uniqueness, (i.e., those for which any solution through a point, can be embedded in a continuous, single valued family of solutions depending on the initial point) can be considered as the natural extension of the well posedness.

The existence of a continuous map $\varphi \rightarrow x_\varphi$, such that x_φ is a solution of the Cauchy problem

$$\dot{x} \in F(t, x), \; x(0) = \varphi$$

where F is a nonempty set-valued function, Lipschitzian with respect to x, was proved first by Cellina in [4]. Then, the same problem for a differential inclusion with Lipschitzian right-hand side defined on an open set, was studied by several authors [3, 4, 6, 7]. Well posedness for a differential inclusion on closed sets was proved in [5]. A continuous function $f: X \rightarrow Y$ is said to be a *continuous selection* of a multivalued map $F: X \rightarrow 2^Y$ if $f(x) \in F(X)$ for all $x \in X$. Continuous selections exist due to continuous selection theorems. A detailed study of continuous selection theorems is given in [2]. The existence of a continuous selection of the set-valued

$$\dot{x}(t) \in Ax(t) + F(t, x(t)), \quad x(0) = \varphi,$$

function $\varphi \to \Phi(\varphi)$, where $\Phi(\varphi)$ is the set of all mild solutions of the Cauchy problem is established in [7, 9]. Here F is Lipschitzian with respect to x and A is the infinitesimal generator of a C_0-semigroup.

In this paper, we consider the evolution inclusion of the form

$$\dot{x}(x) \in Ax(t) + F(t, x(t)) + \int_0^t h(t-s)g(x(s))ds,$$

$$x(0) = \varphi, \tag{1}$$

where F is a set-valued function and $\varphi \in X$. We prove the existence of a continuous selection of the set-valued function $\varphi \to \Phi(\varphi)$, where $\Phi(\varphi)$ is the set of all mild solutions of the Cauchy problem (1), assuming that F is Lipschitzian with resect to x and A is the infinitesimal generator of a C_0-semigroup. This work was motivated by the existence of unique mild solution of the evolution integrodifferential equation studied by Ahmed [1].

2. Preliminaries

Let $T > 0$, $I = [0, T]$ and denote $\mathcal{L}$ the σ-algebra of all Lebesgue measurable subsets of I. Let X be a real separable Banach space with the norm $\|\cdot\|$. Denote by $\mathfrak{B}(X)$ the family of all Borel subsets of X. For any subset $A \subset X$ and $x \in X$, we set $d(x, A) = inf\{\|x - y\| : y \in A\}$. Furthermore, for two closed bounded nonempty subsets A and B of X, we denote by $h(A, B)$ the Hausdorff distance from A to B, that is, $h(A, B) = max\{\sup_{x \in A} d(x, B), \sup_{y \in B} d(y, A)\}$.

Denote by $C(I, X)$ the Banach space of all continuous functions $x: I \to X$ endowed with the norm $\|x\|_\infty = sup\{\|x(t)\| : t \in I\}$ and by $\mathcal{L}^1(I, X)$, the Banach space of all Bochner integrable functions $x: I \to X$ with norm $\|x\|_1 = \int_0^T \|x(t)\| dt$. A subset K of $\mathcal{L}^1(I, X)$ is called *decomposable* if for every u and v in K and $A \in L$ we have $u\aleph_A + v\aleph_{I-A} \in K$, where $\aleph_A$ stands for the characteristic function of A. Denote by $\mathfrak{D}$ the family of all closed nonempty decomposable subsets of $\mathcal{L}^1(I, X)$.

Let X be a separable Banach space and $\{G(t): t \geq 0\} \subset \mathcal{L}(X, X)$ be a strongly continuous semigroup of bounded linear operators from X to X having infinitesimal generator A. Consider the Cauchy problem (1) where $\varphi \in X$ and $F: I \times X \to 2^X$ is a set-valued function satisfying the following hypotheses:

- (i) F is the $\mathcal{L} \otimes \mathfrak{B}(X)$ measurable,
- (ii) there exists $k \in \mathcal{L}^1(I, R)$ such that $h(F(t,x), F(t,y)) \leq k(t)\|x - y\|$, for all $x, y \in X$ *a.e.* in I,
- (iii) there exists $\mu \in \mathcal{L}^1(I, R)$ such that $d(0, F(t, 0)) \leq \mu(t)$, $t \in I$ *a.e.*,
- (iv) $g: X \to X$ be a continuous function and there exists a constant $C > 0$ such that $\|g(x)\| \leq C(1 + \|x\|)$ and $\|g(x) - g(y)\| \leq C\|x - y\|$, for all $x, y \in X$,
- (v) $h \in \mathcal{L}^1(I, R)$ and there exists a constant $H \in \mathcal{L}^1(I, R)$ such that for each pair $s, t \in I$ with $s < t$, $\int_s^t |h(\tau)| d\tau \leq \int_0^T |h(\tau)| d\tau = H$.

Definition 1: A function $x(\cdot, \varphi): I \to X$ is called a *mild solution* of the Cauchy problem (1) if there exists $f(\cdot, \varphi) \in \mathcal{L}^1(I, X)$ such that

- (i) $f(t, \varphi) \in F(t, x(t, \varphi))$ for almost all $t \in I$; and

(ii) $x(t,\varphi) = G(t)\varphi + \int_0^t G(t-\tau)f(\tau,\varphi)d\tau + \int_0^t G(t-\tau)(\int_0^\tau h(\tau - s)g(x(s,\varphi))ds)d\tau)$

for each $t \in I$.

We denote by $\Phi(\varphi)$ the set of all mild solutions of (1).

Let S be a separable metric space. A set-valued function $G: S \to 2^X$ is called *lower semicontinuous* if the set $\{s \in S: G(s) \subset C\}$ is closed in S for any closed $C \subset X$. The following two lemmas are used in the sequel.

Lemma 1:[7] *Let $F: I \times S \to 2^X$ be $L \otimes \mathfrak{B}(S)$ measurable and lower semicontinuous in s. Then, the function $s \to G_F(s)$ given by*

$$G_F(s) = \{v \in \mathcal{L}^1(I,X): v(t) \in F(t,s) \text{ a.e. in } I\}$$

is lower semicontinuous from S into $\mathfrak{D}$ if and only if there exists a continuous function $\mu: S \to \mathcal{L}^1(I,R)$ such that for every $s \in S$, $\mu(s)(t) \le d(0, F(t,s))$ a.e. in I.

Lemma 2: [7] *Consider a lower semicontinuous function $G: S \to \mathfrak{D}$ and assume that $p: S \to \mathcal{L}^1(I,X)$ and $q: S \to \mathcal{L}^1(I,R)$ are continuous functions and for every $s \in S$ the set*

$$H(s) = cl\{u \in G(s): \| u(t) - p(s)(t) \| \le q(s)(t) \text{ a.e. in } I\}$$

is nonempty. Then the function $H: S \to \mathfrak{D}$ is lower semicontinuous, so it has a continuous selection.

3. Well Posedness

Theorem 1: *Let A be the infinitesimal generator of a C_0-semigroup $\{G(t): t \ge 0\}$ and let the hypotheses (i)-(v) be satisfied. Then, there exists a function $x(\cdot,\cdot): I \times X \to X$ such that*

(i) *$x(\cdot,\varphi) \in \Phi(\varphi)$ for every X; and*

(ii) *$\varphi \to x(\cdot,\varphi)$ is continuous from X into $C(I,X)$.*

Proof: Let $M = sup\{\| G(t) \| : t \in I\}$, and for $\varphi \in X$ defined $x_0(\cdot,\varphi): I \to X$ by $x_0(t,\varphi) = G(t)\varphi$. Clearly $\varphi \to x_0(\cdot,\varphi)$ is continuous from X to $C(I,X)$. For each $\varphi \in X$, let $\alpha(\varphi): I \to R$ be given by

$$\alpha(\varphi)(t) = \mu(t) + k(t) \| x_0(t,\varphi) \| .$$

Clearly, $\alpha(\cdot)$ is continuous from X to $\mathcal{L}^1(I,R)$. Moreover, for each $\varphi \in X$,

$$d(0, F(t, x_0(t,\varphi)) \le \beta(t) + k(t) \| x_0(t,\varphi) \| = \alpha(\varphi)(t).$$

Let $\epsilon > 0$ be fixed, and for $n \in N$, set $\epsilon_n = \epsilon / 2^{n+1}$. Now define

$$G_0: X \to 2^{\mathcal{L}(I,X)} \text{ and } H_0: X \to 2^{\mathcal{L}(I,X)} \text{ by}$$

$$G_0(\varphi) = \{v \in \mathcal{L}^1(I,X): v(t) \in F(t, x_0(t,\varphi)) \text{ a.e. } t \in I\} \text{ and}$$

$$H_0(\varphi) = cl\{v \in G_0(\varphi): \| v(t) \| \le \alpha(\varphi)(t) + \epsilon_0 \text{ a.e. } t \in I\}$$

Clearly by Lemma 1, $G_0(\cdot)$ is lower semicontinuous from X into $\mathfrak{D}$ and $H_0(\varphi) \ne \emptyset$ for each $\varphi \in X$, and, hence by Lemma 2, there exists a continuous function $h_0: X \to \mathcal{L}^1(I,X)$, which is a continuous selection of $H_0(\cdot)$.

For each $\varphi \in X$, let $\beta(\varphi): I \to R$ be given by $\beta(\varphi) = HC(1 + \| x_0(t,\varphi) \|)$. Clearly, $\beta(\cdot)$ is continuous from X to $\mathcal{L}^1(I,R)$. Let $K(t) = k(t) + CH$ and set $m(t) = \int_0^t K(\tau)d\tau$. For $n \geq 1$, define $\Gamma_n: X \to \mathcal{L}^1(I,R)$ by

$$\Gamma_n(\varphi)(t) = M^n \int_0^t \alpha(\varphi(s))\frac{[m(t)-m(s)]^{n-1}}{(n-1)!}ds$$

$$+ M^n \int_0^t \beta(\varphi)(s)\frac{[m(t)-m(s)]^{n-1}}{(n-1)!}ds$$

$$+ M^n t(\sum_{i=0}^{n} \epsilon_i)\frac{[m(t)]^{n-1}}{(n-1)!}, \qquad t \in I.$$

Set $f_0(t,\varphi) = h_0(\varphi)(t)$ and $g_0(x(t,\varphi)) = g(x_0(t,\varphi))$. Since $x_0(\cdot,\varphi)$ is continuous, $g_0(x(\cdot,\varphi))$ is also continuous. Define

$$x_1(t,\varphi) = G(t)\varphi + \int_0^t G(t-\tau)f_0(\tau,\varphi)d\tau + \int_0^t G(t-\tau)(\int_0^\tau h(\tau-s)g_0(x(s,\varphi))ds)d\tau, \quad t \in I.$$

Then, $f_0(t,\varphi) \in F(t, x_0(t,\varphi))$, $\| f_0(t,s) \| \leq \alpha(s)(t) + \epsilon_0$, and $\| g_0(x(t,\varphi)) \| = \| g(x_0(t,\varphi) \| \leq C(1 + \| x_0(t,\varphi) \|)$. For $t \in I \backslash \{0\}$ and by Fubini's theorem,

$$\| x_1(t,\varphi) - x_0(T,\varphi) \| \leq \int_0^t \| G(t-\tau) \| \, \| f_0(\tau,\varphi) \| d\tau$$

$$+ \int_0^t (\int_0^t \| G(t-\tau) \| \, | h(\tau - s) | d\tau) \| g_0(x(s,\varphi)) \| ds$$

$$\leq M \int_0^t \| f_0(\tau,\varphi) \| d\tau + M \int_0^t H \| g(x_0(\tau,\varphi)) \| d\tau$$

$$\leq M \int_0^t (\alpha(\varphi)(\tau) + \epsilon_0)d\tau + M \int_0^t HC(1 + \| x_0(\tau,\varphi) \|)d\tau$$

$$\leq M \int_0^t \alpha(\varphi)(\tau)d\tau + MT\epsilon_0 + M \int_0^t \beta(\varphi)(\tau)d\tau < \Gamma_1(\varphi)(t).$$

We claim that there exist three sequence $\{f_n(\cdot,\varphi)\}_{n \in N}$, $\{x_n(\cdot,\varphi)\}_{n \in N}$ and $\{g_n(x(\cdot,\varphi))\}_{n \in N}$ such that for each $n \geq 1$ the following conditions are satisfied.

(*a*) $\varphi \to f_n(\cdot,\varphi)$ is continuous from X into $\mathcal{L}^1(I,X)$,

(*b*) $f_n(t,\varphi) \in F(t, x_n(t,\varphi))$ for each $\varphi \in X$ and *a.e.* $t \in I$,

(*c*) $\| f_n(t,\varphi) - f_{n-1}(t,\varphi) \| \leq K(t)\Gamma_n(\varphi)(t)$,

(*d*) $g_i(x(\cdot,\varphi)) = g(x_i(\cdot,\varphi))$ and $g_n(x(\cdot,\varphi))$ is continuous from X into $C(X,X)$ for $i = 1, 2, \ldots,$

(*e*) $\| g_n(x(t,\varphi)) - g_{n-1}(x(t,\varphi)) \| \leq K(t)\Gamma_n(\varphi)(t)$,

(*f*) $x_{n+1}(t,\varphi) = G(t)\varphi + \int_0^t G(t-\tau)\delta f_n(\tau,\varphi)d\tau + \int_0^t G(t-\tau)(\int_0^\tau h(\tau-s)\cdot$
$g_n(x(s,\varphi))ds)d\tau$.

Suppose that we already constructed $f_1, f_2, \ldots, f_n$, $x_1, x_2, \ldots, x_n$ and $g_1, g_2, \ldots, g_n$ satisfying (*a*)-(*e*). Define $x_{n+1}(\cdot,\varphi): I \to X$ by

$$x_{n+1}(t,\varphi) = G(t)\varphi + \int_0^t G(t-\tau)f_n(\tau,\varphi)d\tau + \int_0^t G(t-\tau)(\int_0^\tau h(\tau-s)g_n(x(s,\varphi))ds)d\tau.$$

Then, for $t \in I\backslash\{0\}$, we have that

$$\| x_{n+1}(t,\varphi) - x_n(t,\varphi) \| \leq \int_0^t \| G(t-u) \| \; \| f_n(u,\varphi) - f_{n-1}(u,\varphi) \| du$$

$$+ \int_0^t \| G(t-u) \| (\int_0^u | h(u-s) | \; \| g_n(x(s,\varphi)) - g_{n-1}(x(s,\varphi)) \| ds)du$$

$$\leq M \int_0^t k(u) \| x_n(u,\varphi) - x_{n-1}(u,\varphi) \| du$$

$$+ M \int_0^t CH \| x_n(u,\varphi) - x_{n-1}(u,\varphi) \| du$$

(by Fubini's theorem and making use of (*iv*) and (*v*))

$$\leq M \int_0^t (k(u) + CH) \| x_n(u,\varphi) - x_{n-1}(u,\varphi) \| du$$

$$\leq M \int_0^t K(u)\Gamma_n(\varphi)(u)du.$$

By making use of calculations provided in [2] we get

$$\| x_{n+1}(t,\varphi) - x_n(t,\varphi) \| \leq M^{n+1} \int_0^t \alpha(s)(\tau)\frac{[m(t)-m(\tau)]^n}{n!}d\tau$$

$$+ M^{n+1}T(\sum_{i=0}^n \epsilon_i)\frac{[m(t)]^n}{n!} + M^{n+1} \int_0^t \beta(s)(\tau)\frac{[m(t)-m(\tau)]^n}{n!}d\tau$$

$$\leq \Gamma_{n+1}(\varphi)(t)$$

and $d(f_n(t,\varphi), F(t, x_{n+1}(t,\varphi)) \leq k(t) \| x_{n+1}(t,\varphi) - x_n(t,\varphi) \|$

$$\leq K(t) \| x_{n+1}(t,\varphi) - x_n(t,\varphi) \|$$

$$\leq K(t)\Gamma_{n+1}(\varphi)(t).$$

Define $G_{n+1}: X \to 2^{\mathfrak{L}(I,X)}$ and $H_{n+1}: X \to 2^{\mathfrak{L}(I,X)}$ by

$$G_{n+1} = \{v \in \mathfrak{L}^1(I,X): v(t) \in F(t, x_{n+1}(t,\varphi)) \text{ a.e. in } I\} \text{ and}$$

$$H_{n+1} = cl\{v \in G_{n+1}(\varphi): \| v(t) - f_n(t,\varphi) \| \leq K(t)\Gamma_{n+1}(\varphi)(t) \text{ a.e. in } I\}.$$

Again, by Lemma 1, $G_{n+1}(\cdot)$ is lower semicontinuous from S into $\mathfrak{D}$ and $H_{n+1}(\varphi)$ is not empty for each $\varphi \in X$. Hence, by Lemma 2 there exists $h_{n+1}: X \to \mathfrak{L}^1(I,X)$ as a continuous selection of $H_{n+1}(\cdot)$. Then, $f_{n+1}(t,\varphi) = h_{n+1}(\varphi)(t)$ satisfies the conditions (a)-(c). Also

$$\begin{aligned} \| g_{n+1}(x(t,\varphi)) - g_n(x(t,\varphi)) \| &= \| g(x_{n+1}(t,\varphi)) - g(x_n(t,\varphi) \| \\ &\leq C \| x_{n+1}(t,\varphi) - x_n(t,\varphi) \| \\ &\leq K(t) \| x_{n+1}(t,\varphi) - x_n(t,\varphi) \| \\ &\leq K(t)\Gamma_{n+1}(\varphi)(t). \end{aligned}$$

Therefore, g_{n+1} satisfies (e) and (f) and since $x_n(\cdot,\varphi)$ is continuous, $g_{n+1}(t,\varphi)$ is continuous.

Now,

$$\begin{aligned} \| f_n(\cdot,\varphi) - f_{n-1}(\cdot,\varphi) \|_1 &= \int_0^T \| f_n(u,\varphi) - f_{n-1}(u,\varphi) \| du \\ &\leq \int_0^T k(u) \| x_n(u,\varphi) - x_{n-1}(u,\varphi) \| du \\ &\leq \int_0^T K(u)\Gamma_n(\varphi)(u) du \\ &\leq M^n \int_0^T \alpha(\varphi)(u) \frac{[m(T) - m(u)]^n}{n!} du \\ &\quad + M^n T\Big(\sum_{i=0}^n \epsilon_i\Big) \frac{[m(t)]^n}{n!} \\ &\quad + M^n \int_0^T \beta(\varphi)(u) \frac{[m(T) - m(u)]^n}{n!} du \\ &\leq \frac{[M \| K_1 \|]^n}{n!} [\| \alpha(\varphi) \|_1 + \| \beta(\varphi) \|_1 + T\epsilon]. \end{aligned}$$

Since $\varphi \to \| \alpha(\varphi) \|_1$ and $\varphi \to \| \beta(\varphi) \|_1$ are continuous it is locally bounded. Therefore, $\{f_n(\cdot,\varphi)\}_{n \in N}$ is a Cauchy sequence in $\mathfrak{L}^1(I,X)$. If $f(\cdot,\varphi) \in \mathfrak{L}^1(I,X)$ is the limit of $\{f_n(\cdot,\varphi)\}_{n \in N}$, then $\varphi \to f(\cdot,\varphi)$ is continuous from X into $\mathfrak{L}^1(I,X)$.

Similarly,

$$\begin{aligned} \| g_n(x(\cdot,\varphi)) - g_{n-1}(x(\cdot,\varphi)) \|_1 &\leq \int_0^T C \| x_n(u,\varphi) - x_{n-1}(u,\varphi) \| du \\ &\leq \int_0^T K(u)\Gamma_n(\varphi)(u) du \end{aligned}$$

$$\leq \frac{[M \| K_1 \|]^n}{n!} (\| \alpha(\varphi) \|_1 + \| \beta(\varphi) \|_1 + T\epsilon)$$

and so, as previously, $\{g_n(x(\cdot,\varphi))\}_{n \in N}$ is a Cauchy sequence in $C(X,X)$. If $g(x(\cdot,\varphi)) \in C(X,X)$ is its limit, then $x(\cdot,\varphi) \rightarrow g(x(\cdot,\varphi))$ is continuous from X into $C(X,X)$.

On the other hand,

$$\| x_{n+1}(\cdot,\varphi) - x_n(\cdot,\varphi) \|_\infty \leq \int_0^T M \| f_n(u,\varphi) - f_{n-1}(u,\varphi) \| du$$

$$+ MH \int_0^T \| g_n(x(u,\varphi)) - g_{n-1}(x(u,\varphi)) \| du$$

$$\leq M \| f_n(u,\varphi) - f_{n-1}(u,\varphi) \|_1 + MH \| g_n(x(u,\varphi)) - g_{n-1}(x(u,\varphi)) \|_1$$

$$\leq M \int_0^T [K(u) + CH] \| x_n(u,\varphi) - x_{n-1}(u,\varphi) \| du$$

$$\leq \frac{[M \| K_1 \|]^n}{n!} [M \| \alpha(\varphi) \|_1 + M \| \beta(\varphi) \|_1 + MT\epsilon].$$

Hence, $\{x_n(\cdot,\varphi)\}_{n \in N}$ is a Cauchy sequence in $C(I,X)$. If $x(\cdot,\varphi) \in C(I,X)$ is its limit then it follows that $\varphi \rightarrow x(\cdot,\varphi)$ is continuous from X into $C(I,X)$.

Since $x_n(\cdot,\varphi)$ converges to $x(\cdot,\varphi)$ uniformly, and $d(F_n(t,\varphi), F(t,x(t,\varphi))) \leq K(t) \| x_n(t,\varphi) - x(t,\varphi) \|$, the limit of a subsequence $\{f_{n_k}\}_{k \in N}$ of $\{f_n\}_{n \in N}$ converges pointwise to f, so we obtain $f(t,\varphi) \in F(t,x(t,\varphi))$ for $\varphi \in X$ and $t \in I$ *a.e.*

Furthermore, $g_n(x(\cdot,\varphi))$ converges to $g(x(\cdot,\varphi))$ uniformly, passing the limit in the condition (f) we obtain

$$x(t,\varphi) = G(t)\varphi + \int_0^t G(t-\tau) f(\tau,\varphi) d\tau$$

$$+ \int_0^t G(t-\tau) \left(\int_0^\tau h(\tau - s) g(x(s,\varphi)) ds \right) d\tau \text{ for each } t \in I.$$

Therefore, $x(\cdot,\varphi) \in \Phi(\varphi)$ for every $\varphi \in X$ and the proof is complete.

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