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Research Article

Spectrum of Compact Weighted Composition Operators on the Weighted Hardy Space in the Unit Ball

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Let B_N be the unit ball in the N-dimensional complex space, for ψ, a holomorphic function in B_N, and φ, a holomorphic map from B_N into itself, the weighted composition operator on the weighted Hardy space $H^2(\beta,B_N)$ is given by $(C_{\psi,\varphi})f = \psi(z)f(\varphi(z))$, where $f \in H^2(\beta,B_N)$. This paper discusses the spectrum of $C_{\psi,\varphi}$ when it is compact on a certain class of weighted Hardy spaces and when the composition map φ has only one fixed point inside the unit ball.

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1. Introduction

It is well known that the general principle that the spectrum structure of the composition operator C_φ is closely related to the fixed point behavior of the map φ is well illustrated by compact composition operators. Determining the spectrum of a compact operator is equivalent to finding the eigenvalues of the operator. About the spectrum of a compact operator in a weighted Hardy space defined in the disk or B_N, we refer the reader to see [1], where Cowen and MacCluer proved a theorem of considerable generality, which will show that, essentially, all of the spaces of interest to us these eigenvalues are determined by the derivative of φ at the Denjoy-Wolff fixed point of φ. Weighting a composition operator as a generalization of a multiplication operator and a composition operator, recently, Gunatillake in [2] obtained some results for the spectrum of weighted composition operators on the weighted Hardy spaces of the unit disk. It is, therefore, natural to wonder what results can be obtained for the spectrum of weighted composition operators on the weighted Hardy spaces on B_N. In our paper, we almost completely answer the above question, the fundamental ideas of the proof are those used by Gunatillake in [2] and Cowen and MacCluer in [1], but there are technical difficulties in several variables that we need to consider before we will be ready to give the proof. This statement will also need some clarification in the case of spaces defined on B_N $(N > 1)$ as the Denjoy-Wolff point may not be well defined. In the proof of Lemma 2.1, a technique is inspired by the proof of [3, Theorem 7].

2. The main results

For multiindexes $m = [m_1, m_2, \ldots, m_N]$ and $l = [l_1, l_2, \ldots, l_N]$, we say that $l < m$ for all $|l| < |m|$ or for $l_j < m_j$ if $|l| = |m|$, and $l_n = m_n$ for all $n < j$.

Lemma 2.1. *Suppose $C_{\psi,\varphi}$ is a compact operator on the Hardy space $H^2(\beta, B_N)$. If the composition map φ has only one fixed point a in the unit ball, then $\sigma(C_{\psi,\varphi}) \subset \{0, \psi(a), \psi(a)\mu\}$, where μ denotes all possible products of the eigenvalues of $\varphi'(a)$.*

Proof. Without loss of generality, we suppose $a = 0$. In fact, if $a \neq 0$, let φ_a denote the automorphism commuting 0 and a, then $\varphi_a \circ \varphi_a(z) = z$ for every z in B_N, it is obvious that $C_{\varphi_a} \circ C_{\varphi_a} = I$, C_{φ_a} is invertible, and $\sigma(C_{\psi,\varphi}) = \sigma(C_{\varphi_a} \circ C_{\psi,\varphi} \circ C_{\varphi_a})$.

Let $\psi_0 = \psi \circ \varphi_a$, $\varphi_0 = \varphi_a \circ \varphi \circ \varphi_a$, then $\psi_0(0) = \psi_a$, $\varphi_0(0) = 0$, and $\varphi_0'(0) = \varphi_a'(a) \cdot \varphi'(a) \cdot \varphi_a'(0)$. By $\varphi_a \circ \varphi_a(z) = z$, it follows that $\varphi_a'(a) \cdot \varphi_a'(0) = I$ and $\varphi_0'(0)$ has the same eigenvalue with $\varphi'(a)$. So

$$C_{\psi_0,\varphi_0}(f) = \psi(\varphi_a(z)) f(\varphi_a \circ \varphi \circ \varphi_a(z)) = C_{\varphi_a} \circ C_{\psi,\varphi} \circ C_{\varphi_a}(f), \tag{2.1}$$

$C_{\psi,\varphi}$ and C_{ψ_0,φ_0} are similar and have the same spectrum.

Suppose $C_{\psi,\varphi}$ is compact. For any $\lambda \in \sigma(C_{\psi,\varphi})$, then λ is an eigenvalue, and for the eigenvector g of λ, $C_{\psi,\varphi} g(z) = \lambda g(z)$, that is,

$$\psi(z) g(\varphi(z)) = \lambda g(z). \tag{2.2}$$

If $g(0) \neq 0$, then $\psi(0) g(0) = \lambda g(0)$, $\lambda = \psi(0)$. If $g(0) = 0$ and $\psi(0) = 0$, then

$$\left(\sum_{s\geq 1} \Psi_s(z)\right)\left(\sum_{t\geq 1} G_t(\varphi(z))\right) = \lambda\left(\sum_{t\geq 1} G_t(z)\right), \tag{2.3}$$

where Ψ_s and G_t are the homogeneous expansion of $\psi(z)$ and $g(z)$, and by the assumption $a = 0$ and Schwarz lemma, it follows that $\lim_{|z|\to 0}(|\varphi(z)|/|z|) < +\infty$ (in fact, ≤ 1). Comparing the lowest power terms of two sides, it is easy to know that $\lambda = 0$.

If $g(0) = 0$ and $\psi(0) \neq 0$, differentiating (2.2) with respect to z_j then leads to

$$g(\varphi(z)) \frac{\partial \psi}{\partial z_j} + \psi(z) \varphi_j^s \frac{\partial g}{\partial \varphi_s} = \lambda \frac{\partial g}{\partial z_j}, \tag{2.4}$$

here, $\psi(z)\varphi_j^s(\partial g/\partial \varphi_s)$ stands for $\psi(z)\sum_{s=1}^N((\partial \varphi^s/\partial z_j)(\partial g/\partial z_s))$ by Einstein's convention.

For the higher-order differentiation, we get

$$\sum_{t<m} \alpha_t(z) + \psi(z) \varphi_{j_1 j_2 \cdots s_N}^{s_1 s_2 \cdots s_N} \frac{\partial^m g}{\partial \varphi_1^{s_1} \cdots \partial \varphi_N^{s_n}} = \lambda \frac{\partial^m g}{\partial z_1^{j_1} \cdots \partial z_N^{j_N}}, \tag{2.5}$$

where $\sum_{t<m}\alpha_t(z)$ denotes the sum of all the terms which have the differential order less than m.

Now, let m be the multiindex that $\partial^m g/\partial z^m \neq 0$ and, for any $l<m, \partial^l g/\partial z^l = 0$.

By $g\neq 0$ and $g(0)=0$, $m>0$, it follows that

$$\psi(0)\cdot\underbrace{\varphi'(0)\otimes\varphi'(0)\otimes\cdots\otimes\varphi'(0)}_{|m|\text{ copies}}\partial^m g(\varphi(0)) = \lambda\partial^m g(0). \tag{2.6}$$

Notice that 0 is the fixed point of φ and $\partial^m g/\partial z^m = \partial^m g/\partial\varphi^m \neq 0$, it follows that λ must have the form of eigenvalue of $\varphi'(0)$. The proof is complete. □

If $[l_1,l_2,\dots,l_N]$ is an N-tuple of the integers $1,2,\dots,N$, let $\kappa_a^{[l_1,l_2,\dots,l_N]}$ denote the kernel for evaluation of the corresponding partial derivative at a, that is,

$$\langle f,\kappa_a^{[l_1,l_2,\dots,l_N]}\rangle = \frac{\partial^{|l|}f}{\partial z_1^{l_1}\partial z_2^{l_2}\cdots\partial z_N^{l_N}}(a), \tag{2.7}$$

for all f in $H^2(\beta,B_N)$. For any positive integer m, let $\mathcal{K}_m$ be the subspace spanned by K_a and the derivative evaluation kernel at a for total order up to and including m, that is,

$$\mathcal{K}_m := \operatorname{span}\left\{K_a,\kappa_a^{[1]},\dots,\kappa_a^{[N]},\kappa_a^{[N,N]},\dots,\kappa_a^{[N,N]},\dots,\kappa_a^{\overbrace{[1,1,\dots,1]}^{N\text{ copies}}},\dots,\kappa_a^{\overbrace{[N,N,\dots,N]}^{N\text{ copies}}}\right\}. \tag{2.8}$$

For the details of the space $\mathcal{K}_m$, we also refer the reader to see [1, page 272], in fact, we have the following lemma.

Lemma 2.2. *$\mathcal{K}_m$ is an invariant subspace of $C^*_{\psi,\varphi}$.*

Proof. First, we show that $\mathcal{K}_0$ is invariant as follows:

$$C^*_{\psi,\varphi}K_a = \overline{\psi(a)}K_{\varphi(a)} = \overline{\psi(a)}K_a, \tag{2.9}$$

so $\mathcal{K}_0$ is invariant under $C^*_{\psi,\varphi}$.

For $\mathcal{K}_1$, let f be any function on $H^2(\beta,B_N)$, then

$$\begin{aligned}\left\langle f, C^*_{\psi,\varphi}\kappa_a^{[j]}\right\rangle &= \left\langle \psi\cdot f\circ\varphi,\kappa_a^{[j]}\right\rangle \\ &= f\circ\varphi(a)\frac{\partial\psi}{\partial z_j}(a)+\psi(a)\sum_{k=1}^N(D_kf)(\varphi(a))(D_j\varphi^k)(a) \\ &= f(a)\frac{\partial\psi}{\partial z_j}(a)+\psi(a)\sum_{k=1}^N(D_kf)(a)(D_j\varphi^k)(a) \\ &= \left\langle f,\overline{\frac{\partial\psi}{\partial z_j}(a)}K_a+\overline{\psi(a)\sum_{k=1}^N(D_j\varphi^k)(a)\kappa_a^{[k]}}\right\rangle.\end{aligned} \tag{2.10}$$

That is,

$$C_{\psi,\varphi}^{*}\kappa_a^{[j]} = \overline{\frac{\partial\psi}{\partial z_j(a)}}K_a + \overline{\psi(a)\sum_{k=1}^{N}(D_j\varphi^k)(a)}\kappa_a^{[k]}, \tag{2.11}$$

or we can denote this by Einstein's convention

$$C_{\psi,\varphi}^{*}\kappa_a^{[j]} = \overline{\frac{\partial\psi}{\partial z_j(a)}}K_a + \overline{\psi(a)\varphi_j^k(a)}\kappa_a^{[k]}. \tag{2.12}$$

So $\mathcal{K}_1$ is invariant under $C_{\psi,\varphi}^{*}$.

We can induct this to the higher order and get

$$C_{\psi,\varphi}^{*}\kappa_a^{[j_1,j_2]} = \alpha_1(a) + \overline{\psi(a)\varphi_{j_1,j_2}^{k_1,k_2}(a)}\kappa_a^{[k_1,k_2]}, \tag{2.13}$$

where $\alpha_1(a)$ denotes the lower-order terms which belongs to $\mathcal{K}_1$, as well as

$$C_{\psi,\varphi}^{*}\kappa_a^{[j_1,j_2,\dots,j_m]} = \alpha_{m-1}(a) + \overline{\psi(a)\varphi_{j_1,j_2,\dots,j_m}^{k_1,k_2,\dots,k_m}(a)}\kappa_a^{[k_1,k_2,\dots,k_m]}, \tag{2.14}$$

where $\alpha_{m-1}(a)$ belongs to $\mathcal{K}_{m-1}$.

Thus we have proved that, for any finite positive integer m, $\mathcal{K}_m$ is an invariant subspace of $C_{\psi,\varphi}^{*}$. □

Lemma 2.3. *Suppose $C_{\psi,\varphi}$ is a bounded operator on $H^2(\beta,B_N)$ with only one fixed point of φ in the unit ball. Then $\{\psi(a),\psi(a)\mu\}\subset\sigma(C_{\psi,\varphi})$, where μ denotes the possible product of eigenvalues of $\varphi'(a)$.*

Proof. First, we use (2.9), (2.12), (2.13), and (2.14) to compute the matrix representation of $C_{\psi,\varphi}^{*}$ restricted to the subspace $\mathcal{K}_m$. That is,

$$\begin{pmatrix} \overline{\psi(a)} & * & * & \cdots & * \\ 0 & \overline{\psi(a)\cdot\varphi'(a)} & * & \cdots & * \\ 0 & 0 & \overline{\psi(a)\cdot\varphi'(a)\otimes\varphi'(a)} & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \overline{\psi(a)\cdot\underbrace{\varphi'(a)\otimes\varphi'(a)\otimes\cdots\otimes\varphi'(a)}_{m\text{ copies}}} \end{pmatrix}. \tag{2.15}$$

Let us call this matrix A_m. Then A_m is an $(1+m+m^2+\cdots+m^N)\times(1+m+m^2+\cdots+m^N)$ upper-triangular matrix. The $*'s$ denote $\overline{\alpha_j(a)}'s$.

The subspace $\mathcal{K}_m$ is finite dimensional and, therefore, is closed. The Hardy space $H^2(\beta,B_N)$ can be decomposed as $H^2(\beta,B_N)=\mathcal{K}_m\oplus\mathcal{K}_m^{\perp}$. The block matrix of $C_{\psi,\varphi}^{*}$ with respect to this decomposition is

$$\begin{pmatrix} A_m & B \\ 0 & C_m \end{pmatrix}. \tag{2.16}$$

The fact that $\mathcal{K}_m$ is invariant under $C^*_{\psi,\varphi}$ makes the lower-left corner of this decomposition 0. Since there is a 0 at the lower left and the subspace is finite dimensional, the spectrum of $C^*_{\psi,\varphi}$ is the union of the spectrum of A_m and the spectrum of C_m [1, page 270]. Since A_m is a finite dimensional upper-triangular matrix, its spectrum is the eigenvalue of A_m. By the arguments in [1, pages 274-275], we can conclude that the spectrum of $C^*_{\psi,\varphi}$ contains the set

$$\{\overline{\psi(a)},\overline{\psi(a)\mu}\},\tag{2.17}$$

where μ denotes the product of m eigenvalues of $\varphi'(a)$. So $\{\psi(a),\psi(a)\mu\}$ is contained in $\sigma(C_{\psi,\varphi})$ and this completes the proof. □

Remark 2.4. The set

$$\left\{K_a,\kappa_a^{[1]},\dots,\kappa_a^{[N]},\kappa_a^{[1,1]},\dots,\kappa_a^{[N,1]},\dots,\kappa_a^{[N,N]},\dots,\kappa_a^{\overbrace{[1,1,\dots,1]}^{}_{N\text{copies}}},\dots,\kappa_a^{\overbrace{[N,N,\dots,N]}^{}_{N\text{copies}}}\right\}\tag{2.18}$$

is only the generated element set instead of the basis. So the matrix representation of $C^*_{\psi,\varphi}|_{\mathcal{K}_m}$ is not unique. This matrix is called the *redundant matrix*. It can also be used to prove Lemma 2.1.

By Lemmas 2.1 and 2.3, we can easily get the following theorem, which is the main theorem of this paper.

Theorem 2.5. *Let $C_{\psi,\varphi}$ be a compact operator on the weighted Hardy space $H^2(\beta,B_N)$. If φ has only one fixed point in the unit ball, then the spectrum of $C_{\psi,\varphi}$ is the set*

$$\{0,\psi(a),\psi(a)\mu\},\tag{2.19}$$

where μ is all possible products of $\varphi'(a)$ and a is the only fixed point of φ.

As we will see in the next theorem, compactness of $C_{\psi,\varphi}$ on some $H^2(\beta,B_N)$ for some weight functions ψ implies that φ has only one fixed point in the unit ball.

Theorem 2.6. *Let $C_{\psi,\varphi}$ be a compact operator on $H^2(\beta,B_N)$, where*

$$\sum_{s=0}^{\infty}\frac{(N-1+s)!}{(N-1)!s!}\frac{1}{\beta(s)^2}=\infty.\tag{2.20}$$

If $\liminf_{r\to 1^-}|\psi(r\zeta)|>0$ for ζ is the fixed point of φ, then φ has only one fixed point in the unit ball.

Proof. By contradiction, first, suppose φ has no fixed point, so φ must have its Denjoy-Wollff point denoted by ξ on ∂B_N, and let r belong to the interval $(0,1)$.

Now, we apply the adjoint of $C_{\psi,\varphi}$ to the normalized kernel function $K_{r\xi}/\|K_{r\xi}\|$ as follows:

$$\left\|C^*_{\psi,\varphi}\frac{K_{r\xi}}{\|K_{r\xi}\|}\right\|=|\psi(r\xi)|\frac{\|K_{\varphi(r\xi)}\|}{\|K_{r\xi}\|}.\tag{2.21}$$

Since ξ is the Denjoy-Wollf point on the boundary, there exits a sequence $\{\xi_n\}$ tending to ξ such that $|\varphi(r\xi_n)| \geq |r\xi_n|$. But $\|\mathrm{K}_w\| = \sqrt{\sum_{s=0}^{\infty}(((N-1+s)!/(N-!)!s!)(|w|^{2s}/\beta(s)^2))}$ is an increasing function of $|w|$, $\|K_{\varphi(r\xi_n)}\| \geq \|K_{r\xi_n}\|$, it follows that

$$\left\| C^*_{\psi,\varphi} \frac{K_{r\xi_n}}{\|K_{r\xi_n}\|} \right\| \geq |\psi(r\xi_n)|. \tag{2.22}$$

By [4, Lemma 3.11], it follows that $K_{r\xi_n}/\|K_{r\xi_n}\|$ converges weakly to zero as r tends to 1 and n tends to ∞.

Since $C^*_{\psi,\varphi}$ is compact, the left-hand side of (2.21) tends to 0, but the right-hand side of (2.21) is larger than $\delta_\xi > 0$. That is a contradiction, so φ must have its fixed point in B_N.

Now, we show the singleness of the fixed point of φ. By contradiction, suppose φ has more than one fixed point, then the fixed point set is an affine set, we denote it by E, which must be uncountable if not single. Then $C^*_{\psi,\varphi}K_a = \overline{\psi(a)}K_a$ for all $a \in E$. E is an affine set, it is connected, so $\psi(E)$ is an single point set or an uncountable set.

(i) If $\psi(E)$ is a constant, then $\overline{\psi(a)}$ is the eigenvalue of $C^*_{\psi,\varphi}$, which is infinite multiplicity. This contradicts to the compactness of $C^*_{\psi,\varphi}$.

(ii) If ($\psi(E)$ is not a constant, then it has uncountable elements. That is to say, $C^*_{\psi,\varphi}$ has uncountable eigenvalues. That is impossible.

Hence, it must be the case that φ has only one fixed point in the unit ball and the proof is complete. □

Theorem 1 in [5] gives a method to find ψ so that $C_{\psi,\varphi}$ is compact on the Hardy space $H^2(B_N)$ when φ has fixed points on the boundary, we discuss the spectrum for such operators. First, we quote the theorem as a lemma.

Lemma 2.7. *Suppose φ is a linear-fractional map of B_N with $\varphi(e_1) = e_1$ and for $\zeta \in \partial B_N$, $|\varphi(\zeta)| = 1$ if and only if $\zeta = e_1$. If $b(z)$ is continuous on $\overline{B_N}$ with $b(e_1) = 0$, then the operator T_bC_φ is compact on $H^2(B_N)$.*

If φ has a fixed point inside the ball, Theorem 2.5 gives the spectrum. Therefore, we compute the spectrum when φ has no fixed point inside the unit ball. We will denote the composition of φ with itself n times by φ_n, that is, $\varphi_n = \varphi \circ \varphi \circ \cdots \circ \varphi$ (n times). Now, we give the last theorem of this paper.

Theorem 2.8. *Suppose ψ and φ satisfy the hypothesis in Lemma 2.7, and φ is one-to-one which has no fixed point inside the unit ball. Then $\sigma(C_{\psi,\varphi}) = \{0\}$.*

Proof. We will show that the spectral radius of this operator is 0. Since φ is a nonautomorphism linear fractional map with a fixed point at e_1, it takes the unit sphere to an ellipsoid sphere by [6, Theorem 6] which is tangent to the unit sphere at e_1. e_1 is the only fixed point of φ, so it is the Denjoy-Wollf point.

Let $\epsilon > 0$, there exists $\delta > 0$ such that $|\psi(z)| < \epsilon$ whenever $|z - e_1| < \delta$ and z is in the closed unit ball. Let $W = \{z : |z - e_1| < \delta, |z| \leq 1\}$, clearly, W is open in $\overline{B_N}$. Let $U = \varphi(B_N)$, then $\overline{U}$ is tangent to the unit sphere at e_1. Let $V = \overline{U - W}$, then V is a compact subset of the unit ball. Therefore, the sequence $\{\varphi_n\}$ converges uniformly to e_1 on V.

Considering a point ξ on the unit sphere, then $\varphi(\xi)$ is either in W or V. If $\varphi(\xi)$ is in V, then there is an N_0 that does not depend on ξ such that $\varphi_j(\xi)$ is in W for all $j > N_0$. If $\varphi(\xi)$ is not in V, consider the sequence $\{\varphi_j(\xi)\}_{j=1}^{\infty}$, either $\varphi_j(\xi)$ is in W for all j, or $\varphi_j(\xi)$ will be in V for some j. If $\varphi_j(\xi)$ is in V for some j, take j' to be the smallest integer such that $\varphi_j(\xi)$ is in V. Then $\varphi(\xi)$ is in W for all $j > j' + N_0$. Therefore, for any ξ on the unit sphere, at most N_0 terms of the sequence $\{\varphi_j(\xi)\}_{j=1}^{\infty}$ will be outside W. Hence, at most N_0 terms of the sequence $\{|\psi(\varphi_j(\xi))|\}_{j=1}^{\infty}$ will be larger than ϵ for any ξ. Also ψ is bounded on $\overline{B_N}$, therefore, $|\psi(\varphi_j(\xi))| < M$ for some $M > 0$. Now, if f is in $H^2(B_N)$ and $n > N_0$, then

$$\begin{aligned}
\|C_{\psi,\varphi}^n(f)\|^2 &= \sup_{0<r<1}\int_S |\psi(\zeta)|^2 |\psi(\varphi(\zeta))|^2 \cdots |\psi(\varphi_{n-1}(\zeta))|^2 |f(\varphi_n(\zeta))|^2 d(\zeta) \\
&\le \epsilon^{2(n-N_0-1)} M^{2(N_0+1)} \sup_{0<r<1}\int_S |f(\varphi_n(\zeta))|^2 d(\zeta) \\
&= \epsilon^{2(n-N_0-1)} M^{2(N_0+1)} \|C_{\varphi_n}(f)\|^2 \\
&\le \epsilon^{2(n-N_0-1)} M^{2(N_0+1)} \|C_{\varphi_n}\|^2 \|f\|^2,
\end{aligned} \tag{2.23}$$

but $C_{\varphi_n} = C_\varphi^n$, therefore

$$\|C_{\psi,\varphi}^n\| \le \epsilon^{(n-N_0-1)} M^{(N_0+1)} \|C_{\varphi_n}\|. \tag{2.24}$$

Hence, for all n large enough,

$$\|C_{\psi,\varphi}^n\|^{1/n} \le \epsilon \cdot 2 \|C_\varphi^n\|^{1/n} \le \epsilon \cdot 2 \|C_\varphi\|. \tag{2.25}$$

By [6, Theorem 14], C_φ is bounded.

So we can get that the spectral radius of the operator on $H^2(B_N)$ is 0, therefore, $\sigma(C_{\psi,\varphi}) = \{0\}$. This completes the proof. □

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References

[1] C. C. Cowen and B. D. MacCluer, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, Fla, USA, 1995.

[2] G. Gunatillake, "Spectrum of a compact weighted composition operator," *Proceedings of the American Mathematical Society*, vol. 135, no. 2, pp. 461–467, 2007.

[3] R. Aron and M. Lindström, "Spectra of weighted composition operators on weighted banach spaces of analytic functions," *Israel Journal of Mathematics*, vol. 141, pp. 263–276, 2004.

[4] D. D. Clahane, "Spectra of compact composition operators over bounded symmetric domains," *Integral Equations and Operator Theory*, vol. 51, no. 1, pp. 41–56, 2005.

[5] B. D. MacCluer and R. J. Weir, "Linear-fractional composition operators in several variables," *Integral Equations and Operator Theory*, vol. 53, no. 3, pp. 373–402, 2005.

[6] C. C. Cowen and B. D. MacCluer, "Linear fractional maps of the ball and their composition operators," *Acta Universitatis Szegediensis. Acta Scientiarum Mathematicarum*, vol. 66, no. 1-2, pp. 351–376, 2000.

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