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Research Article

Isometries on Products of Composition and Integral Operators on Bloch Type Space

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We characterize the isometries on the products of composition and integral operators on the Bloch type space in the disk.

1. Introduction

Let $\mathbb{D}$ be the unit disk of the complex plane, and $S(\mathbb{D})$ be the set of analytic self-maps of $\mathbb{D}$. The algebra of all holomorphic functions with domain $\mathbb{D}$ will be denoted by $H(\mathbb{D})$.

We recall that the Bloch type space $\mathcal{B}^\alpha$ $(\alpha > 0)$ consists of all $f \in H(\mathbb{D})$ such that

$$\|f\|_{\mathcal{B}^\alpha} = \sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha |f'(z)| < \infty, \tag{1.1}$$

then $\|\cdot\|_{\mathcal{B}^\alpha}$ is a complete seminorm on $\mathcal{B}^\alpha$, which is Möbius invariant.

It is well known that $\mathcal{B}^\alpha$ is a Banach space under the norm

$$\|f\| = |f(0)| + \|f\|_{\mathcal{B}^\alpha}. \tag{1.2}$$

Let φ be an analytic self-map of $\mathbb{D}$, then the composition operator C_φ induced by φ is defined by

$$(C_\varphi f)(z) = f(\varphi(z)) \tag{1.3}$$

for $z \in \mathbb{D}$ and $f \in H(\mathbb{D})$.

Let $g \in H(\mathbb{D})$, then the integer operator I_g is defined by

$$I_g f(z) = \int_0^z f'(\xi) g(\xi) d\xi, \quad z \in \mathbb{D} \tag{1.4}$$

for $f \in H(\mathbb{D})$.

The products of composition and integral type operators were first introduced and discussed by Li and Stević [1–3], which are defined by

$$\begin{aligned}
(C_\varphi I_g f)(z) &= \int_0^{\varphi(z)} f'(\xi) g(\xi) d\xi, \\
(I_g C_\varphi f)(z) &= \int_0^z (f \circ \varphi)'(\xi) g(\xi) d\xi.
\end{aligned} \tag{1.5}$$

Let X and Y be two Banach spaces, recall that a linear isometry is a linear operator T from X to Y such that $\|Tf\|_Y = \|f\|_X$ for all $f \in X$.

In [4], Banach raised the question concerning the form of an isometry on a specific Banach space. In most cases, the isometries of a space of analytic functions on the disk or the ball have the canonical form of weighted composition operators, which is also true for most symmetric function spaces. For example, the surjective isometries of Hardy and Bergman spaces are certain weighted composition operators (see [5–7]).

The description of all isometric composition operators is known for the Hardy space H^2 (see [8]). An analogous statement for the Bergman space A^2_α with standard radial weights has recently been obtained in [9], and there is a unified proof for all Hardy spaces and also for arbitrary Bergman spaces with reasonable radial weights [10]. For the Dirichlet space and Bloch space, the reader is referred to [11, 12], and for the BMOA, see [13].

The surjective isometries of the Bloch space are characterized in [14]. Trivially, every rotation φ induces an isometry C_φ of $\mathcal{B}$. It has recently been shown in [15] that for composition operators, which induce isometries of $\mathcal{B}$, the conditions $\varphi(0) = 0$ and $\mathbb{D} \subset C(\varphi)$ must hold. Here, $C(\varphi)$ denotes the (global) cluster set of φ, that is, the set of all points $a \in \mathbb{C}$ such that there exists a sequence $\{z_n\}$ in D with the properties $|z_n| \to 1$ and $\varphi(z_n) \to a$, as $n \to \infty$. Plenty of information on cluster sets is contained in [16].

Continued the work, in 2008, Bonet et al. [17] discussed isometric weighted composition operators on weighted Banach spaces of type H^∞. In 2008, Cohen and Colonna [18] discussed the spectrum of an isometric composition operators on the Bloch space of the polydisk. In 2009, Allen and Colonna [19] investigated the isometric composition operators on the Bloch space in $\mathcal{C}^n$. They [20] also discussed the isometries and spectra of multiplication operators on the Bloch space in the disk. Isometries of weighted spaces of holomorphic functions on unbounded domains were discussed by Boyd and Rueda in [21].

Building on those foundations, the present paper continues this line of research, and discusses the isometries on the products of composition and integral operators on the Bloch type space in the disk.

2. Notations and Lemmas

To begin the discussion, let us introduce some notations and state a couple of lemmas.

For $a \in \mathbb{D}$, the involution φ_a which interchanges the origin and point a, is defined by

$$\varphi_a(z) = \frac{a - z}{1 - \overline{a}z}. \tag{2.1}$$

For z, w in $\mathbb{D}$, the pseudohyperbolic distance between z and w is given by

$$\rho(z, w) = |\varphi_z(w)| = \left|\frac{z - w}{1 - \overline{z}w}\right|, \tag{2.2}$$

and the hyperbolic metric is given by

$$\beta(z, w) = \inf_\gamma \int_\gamma \frac{|d\xi|}{1 - |\xi|^2} = \frac{1}{2} \log \frac{1 + \rho(z, w)}{1 - \rho(z, w)}, \tag{2.3}$$

where γ is any piecewise smooth curve in $\mathbb{D}$ from z to w.

The following lemma is well known [22].

Lemma 2.1. *For all $z, w \in \mathbb{D}$, one has*

$$1 - \rho^2(z, w) = \frac{\left(1 - |z|^2\right)\left(1 - |w|^2\right)}{|1 - \overline{z}w|^2}. \tag{2.4}$$

For $\varphi \in S(D)$, the Schwarz-Pick lemma shows that $\rho(\varphi(z), \varphi(w)) \leq \rho(z, w)$, and if the equality holds for some $z \neq w$, then φ is an automorphism of the disk. It is also well known that, for $\varphi \in S(\mathbb{D})$, C_φ is always bounded on $\mathcal{B}$.

A little modification of Lemma 1 in [17] shows the following lemma.

Lemma 2.2. *There exists a constant $C > 0$ such that*

$$\left|\left(1 - |z|^2\right)^\alpha f'(z) - \left(1 - |w|^2\right)^\alpha f'(w)\right| \leq C\|f\|_{\mathcal{B}^\alpha} \cdot \rho(z, w) \tag{2.5}$$

for all $z, w \in \mathbb{D}$ and $f \in \mathcal{B}^\alpha$.

Throughout the remainder of this paper, C will denote a positive constant, the exact value of which will vary from one appearance to the next.

3. Main Theorems

Theorem 3.1. *Let φ be analytic self-maps of the unit disk and $g \in H(\mathbb{D})$ then, the operator $I_g C_\varphi$: $\mathcal{B}^\alpha \to \mathcal{B}^\beta$ is an isometry in the seminorm if and only if the following conditions hold.*

(A) $\sup_{z\in\mathbb{D}}((1 - |z|^2)^\beta |\varphi'(z)|/(1 - |\varphi(z)|^2)^\alpha)|g(z)| \leq 1$;

(B) *For every $a \in \mathbb{D}$, there exists at least a sequence $\{z_n\}$ in $\mathbb{D}$, such that* $\lim_{n\to\infty}\rho(\varphi(z_n), a) = 0$ *and* $\lim_{n\to\infty}((1 - |z_n|^2)^\beta |\varphi'(z_n)|/(1 - |\varphi(z_n)|^2)^\alpha)|g(z_n)| = 1$.

Proof. We prove the sufficiency first.

By condition (A), for every $f \in \mathcal{B}^\alpha$, we have

$$\|I_g C_\varphi f\|_{\mathcal{B}^\beta} = \sup_{z\in D}\left(1-|z|^2\right)^\beta \left|f'(\varphi(z))\right|\left|\varphi'(z)\right|\left|g(z)\right|$$
$$= \sup_{z\in D}\frac{\left(1-|z|^2\right)^\beta \left|\varphi'(z)\right|}{\left(1-\left|\varphi(z)\right|^2\right)^\alpha}\left|g(z)\right|\left(1-\left|\varphi(z)\right|^2\right)^\alpha \left|f'(\varphi(z))\right| \tag{3.1}$$
$$\leq \|f\|_{\mathcal{B}^\alpha}.$$

Next we show that property (B) implies $\|I_g C_\varphi f\|_{\mathcal{B}^\beta} \geq \|f\|_{\mathcal{B}^\alpha}$.

In fact, given any $f \in \mathcal{B}^\alpha$, then $\|f\|_{\mathcal{B}^\alpha} = \lim_{m\to\infty}(1-|a_m|^2)^\alpha |f'(a_m)|$ for some sequence $\{a_m\} \subset \mathbb{D}$. For any fixed m, it follows from (B) that there is a sequence $\{z_k^m\} \subset \mathbb{D}$ such that

$$\rho(\varphi(z_k^m), a_m) \longrightarrow 0, \qquad \frac{\left(1-\left|z_k^m\right|^2\right)^\beta \left|\varphi'(z_k^m)\right|}{\left(1-\left|\varphi(z_k^m)\right|^2\right)^\alpha}\left|g(z_k^m)\right| \longrightarrow 1, \tag{3.2}$$

as $k \to \infty$. By Lemma 2.2, for all m and k,

$$\left|\left(1-\left|\varphi(z_k^m)\right|^2\right)^\alpha f'(\varphi(z_k^m)) - \left(1-|a_m|^2\right)^\alpha f'(a_m)\right| \leq C\|f\|_{\mathcal{B}^\alpha}\cdot\rho(\varphi(z_k^m), a_m). \tag{3.3}$$

Hence,

$$\left(1-\left|\varphi(z_k^m)\right|^2\right)^\alpha \left|f'(\varphi(z_k^m))\right| \geq \left(1-|a_m|^2\right)^\alpha \left|f'(a_m)\right| - C\|f\|_{\mathcal{B}^\alpha}\cdot\rho(\varphi(z_k^m), a_m) \tag{3.4}$$

Therefore,

$$\|I_g C_\varphi f\|_{\mathcal{B}^\beta} = \sup_{z\in\mathbb{D}}\frac{\left(1-|z|^2\right)^\beta \left|\varphi'(z)\right|}{\left(1-\left|\varphi(z)\right|^2\right)^\alpha}\left|g(z)\right|\left(1-\left|\varphi(z)\right|^2\right)^\alpha \left|f'(\varphi(z))\right|$$
$$\geq \limsup_{k\to\infty}\frac{\left(1-\left|z_k^m\right|^2\right)^\beta \left|\varphi'(z_k^m)\right|}{\left(1-\left|\varphi(z_k^m)\right|^2\right)^\alpha}\left|g(z_k^m)\right|\left(1-\left|\varphi(z_k^m)\right|^2\right)^\alpha \left|f'(\varphi(z_k^m))\right| \tag{3.5}$$
$$= \left(1-|a_m|^2\right)^\alpha \left|f'(a_m)\right|.$$

The inequality $\|C_\varphi I_g f\|_{\mathcal{B}^\beta} \geq \|f\|_{\mathcal{B}^\alpha}$ follows by letting $m \to \infty$.

From the above discussions, we have $\|I_g C_\varphi f\|_{\mathcal{B}^\beta} = \|f\|_{\mathcal{B}^\alpha}$, which means that $I_g C_\varphi$ is an isometry operator on the Bloch type space in the seminorm.

Now we turn to the necessity.

For any $a \in \mathbb{D}$, we begin by taking test function

$$f_a(z) = \int_0^z \frac{\left(1-|a|^2\right)^\alpha}{(1-\overline{a}t)^{2\alpha}} dt. \tag{3.6}$$

It is clear that $f_a'(z) = (1-|a|^2)^\alpha/(1-\overline{a}z)^{2\alpha}$. Using Lemma 2.1, we have

$$\left(1-|z|^2\right)^\alpha \left|f_a'(z)\right| = \frac{\left(1-|z|^2\right)^\alpha\left(1-|a|^2\right)^\alpha}{|1-\overline{a}z|^{2\alpha}} = \left(1-\rho^2(a,z)\right)^\alpha. \tag{3.7}$$

So,

$$\|f_a\|_{\mathcal{B}^\alpha} = \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^\alpha \left|f_a'(z)\right| \le 1. \tag{3.8}$$

On the other hand, since $(1-|a|^2)^\alpha |f_a'(a)| = (1-|a|^2)^{2\alpha}/(1-|a|^2)^{2\alpha} = 1$, we have $\|f_a\|_{\mathcal{B}^\alpha} = 1$. By isometry assumption, for any $a \in \mathbb{D}$, we have

$$\begin{aligned} 1 &= \|f_{\varphi(a)}\|_{\mathcal{B}^\alpha} = \|I_g C_\varphi f_{\varphi(a)}\|_{\mathcal{B}^\beta} \\ &= \sup_{z\in D} \frac{\left(1-|z|^2\right)^\beta |\varphi'(z)|}{\left(1-|\varphi(z)|^2\right)^\alpha} |g(z)| \left(1-|\varphi(z)|^2\right)^\alpha \left|f_{\varphi(a)}'(\varphi(z))\right| \\ &\ge \frac{\left(1-|a|^2\right)^\beta |\varphi'(a)|}{\left(1-|\varphi(a)|^2\right)^\alpha} |g(a)|. \end{aligned} \tag{3.9}$$

So (A) follows by noticing a is arbitrary.

Since $\|I_g C_\varphi f_a\|_{\mathcal{B}^\beta} = \|f_a\|_{\mathcal{B}^\alpha} = 1$, there exists a sequence $\{z_m\} \subset \mathbb{D}$ such that

$$\left(1-|z_m|^2\right)^\beta \left|\frac{d(I_g C_\varphi f_a)}{dz}(z_m)\right| = \left(1-|z_m|^2\right)^\beta \left|f_a'(\varphi(z_m))\|\varphi'(z_m)\|g(z_m)\right| \longrightarrow 1, \tag{3.10}$$

as $m \to \infty$.

It follows from (A) that

$$\begin{aligned} &\left(1-z_m|^2\right)^\beta \left|f_a'(\varphi(z_m))\|\varphi'(z_m)\|g(z_m)\right| \\ &\quad = \frac{(1-z_m|^2)^\beta |\varphi'(z_m)|}{\left(1-|\varphi(z_m)|^2\right)^\alpha} |g(z_m)| \left(1-|\varphi(z_m)|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right| \end{aligned} \tag{3.11}$$

$$\le \left(1-|\varphi(z_m)|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right|. \tag{3.12}$$

Combining (3.10) and (3.12), it follows that

$$\begin{aligned}1 &\leq \liminf_{m\to\infty}\left(1-|\varphi(z_m)|^2\right)^{\alpha}\left|f_a'(\varphi(z_m))\right| \\ &\leq \limsup_{m\to\infty}\left(1-|\varphi(z_m)|^2\right)^{\alpha}\left|f_a'(\varphi(z_m))\right| \leq 1.\end{aligned} \tag{3.13}$$

The last inequality follows (3.7) since $\varphi(z_m) \in \mathbb{D}$.

Consequently,

$$\lim_{m\to\infty}\left(1-|\varphi(z_m)|^2\right)^{\alpha}\left|f'_a(\varphi(z_m))\right| = \lim_{m\to\infty}\left(1-\rho^2(\varphi(z_m),a)\right)^{\alpha} = 1. \tag{3.14}$$

That is, $\lim_{n\to\infty}\rho(\varphi(z_m),a) = 0$.

Combining (3.10), (3.11), and (3.14), we know that

$$\lim_{m\to\infty}\frac{\left(1-|z_m|^2\right)^{\beta}|\varphi'(z_m)|}{\left(1-|\varphi(z_m)|^2\right)^{\alpha}}|g(z_m)| = 1. \tag{3.15}$$

This completes the proof of this theorem. □

Theorem 3.2. *Let $g \in H(\mathbb{D})$ and φ be an analytic self-map of the unit disk, such that φ fixes the origin, then the operator $C_\varphi I_g : \mathcal{B}^\alpha \to \mathcal{B}^\beta$ is an isometry in the seminorm if and only if the following conditions hold.*

(C) $\sup_{z\in\mathbb{D}}((1-|z|^2)^{\beta}|\varphi'(z)|/(1-|\varphi(z)|^2)^{\alpha})|g(\varphi(z))| \leq 1$;

(D) *For every $a \in \mathbb{D}$, there exists at least a sequence $\{z_n\}$ in $\mathbb{D}$, such that $\lim_{n\to\infty}\rho(\varphi(z_n),a) = 0$ and $\lim_{n\to\infty}((1-|z_n|^2)^{\beta}|\varphi'(z_n)|/(1-|\varphi(z_n)|^2)^{\alpha})|g(\varphi(z_n))| = 1$.*

Proof. We prove the sufficiency first.

By condition (C), for every $f \in \mathcal{B}$, we have

$$\begin{aligned}\|C_\varphi I_g f\|_{\mathcal{B}^\beta} &= \sup_{z\in\mathbb{D}}\left(1-|z|^2\right)^{\beta}\left|f'(\varphi(z))\right|\left|\varphi'(z)\right|\left|g(\varphi(z))\right| \\ &= \sup_{z\in\mathbb{D}}\frac{\left(1-|z|^2\right)^{\beta}|\varphi'(z)|}{\left(1-|\varphi(z)|^2\right)^{\alpha}}\left|g(\varphi(z))\right|\left(1-|\varphi(z)|^2\right)^{\alpha}\left|f'(\varphi(z))\right| \\ &\leq \|f\|_{\mathcal{B}^\alpha}.\end{aligned} \tag{3.16}$$

Next we show that the property (D) implies $\|C_\varphi I_g f\|_{\mathcal{B}^\beta} \geq \|f\|_{\mathcal{B}^\alpha}$.

In fact, given any $f \in \mathcal{B}^\alpha$, then $\|f\|_{\mathcal{B}^\alpha} = \lim_{n\to\infty}(1-|a_m|^2)^\alpha|f'(a_m)|$ for some sequence $\{a_m\} \subset \mathbb{D}$. For any fixed m, by property (D), there is a sequence $\{z_k^m\} \subset \mathbb{D}$ such that

$$\rho(\varphi(z_k^m), a_m) \longrightarrow 0, \qquad \frac{\left(1-|z_k^m|^2\right)^\beta |\varphi'(z_k^m)|}{\left(1-|\varphi(z_k^m)|^2\right)^\alpha}|g(\varphi(z_k^m))| \longrightarrow 1, \tag{3.17}$$

as $k \to \infty$. By Lemma 2.2, for all m and k,

$$\left|\left(1-|\varphi(z_k^m)|^2\right)^\alpha f'(\varphi(z_k^m)) - \left(1-|a_m|^2\right)^\alpha f'(a_m)\right| \le C\|f\|_{\mathcal{B}^\alpha}\cdot\rho(\varphi(z_k^m), a_m). \tag{3.18}$$

Hence,

$$\left(1-|\varphi(z_k^m)|^2\right)^\alpha |f'(\varphi(z_k^m))| \ge \left(1-|a_m|^2\right)^\alpha |f'(a_m)| - C\|f\|_{\mathcal{B}^\alpha}\cdot\rho(\varphi(z_k^m), a_m). \tag{3.19}$$

Therefore,

$$\begin{aligned}\|C_\varphi I_g f\|_{\mathcal{B}^\beta} &= \sup_{z\in\mathbb{D}} \frac{\left(1-|z|^2\right)^\beta |\varphi'(z)|}{\left(1-|\varphi(z)|^2\right)^\alpha}|g(\varphi(z))|\left(1-|\varphi(z)|^2\right)^\alpha |f'(\varphi(z))| \\ &\ge \limsup_{k\to\infty} \frac{\left(1-|z_k^m|^2\right)^\beta |\varphi'(z_k^m)|}{\left(1-|\varphi(z_k^m)|^2\right)^\alpha}|g(\varphi(z_k^m))|\left(1-|\varphi(z_k^m)|^2\right)^\alpha |f'(\varphi(z_k^m))| \\ &= \left(1-|a_m|^2\right)^\alpha |f'(a_m)|.\end{aligned} \tag{3.20}$$

The inequality $\|C_\varphi I_g f\|_{\mathcal{B}^\beta} \ge \|f\|_{\mathcal{B}^\alpha}$ follows by letting $m \to \infty$.

Now we turn to the necessity.

For any $a \in \mathbb{D}$, we use the same test function f_a defined by (3.6) which satisfies $\|f_a\| = 1$. By isometry assumption, for any $a \in \mathbb{D}$, we have

$$\begin{aligned}1 = \|f_{\varphi(a)}\|_{\mathcal{B}^\alpha} &= \|C_\varphi I_g f_{\varphi(a)}\|_{\mathcal{B}^\beta} \\ &= \sup_{z\in\mathbb{D}} \frac{\left(1-|z|^2\right)^\beta |\varphi'(z)|}{\left(1-|\varphi(z)|^2\right)^\alpha}|g(\varphi(z))|\left(1-|\varphi(z)|^2\right)^\alpha \left|f'_{\varphi(a)}(\varphi(z))\right| \\ &\ge \frac{\left(1-|a|^2\right)^\beta |\varphi'(a)|}{\left(1-|\varphi(a)|^2\right)^\alpha}|g(\varphi(a))|.\end{aligned} \tag{3.21}$$

So, (C) follows by noticing a is arbitrary.

Since $\|C_\varphi I_g f_a\|_{\mathcal{B}^\beta} = \|f_a\|_{\mathcal{B}^\alpha} = 1$, thus there exists a sequence $\{z_m\} \subset \mathbb{D}$ such that

$$\left(1-|z_m|^2\right)^\beta \left|\frac{d(C_\varphi I_g f_a)}{dz}(z_m)\right| = \left(1-|z_m|^2\right)^\beta \left|f_a'(\varphi(z_m))\right|\left|\varphi'(z_m)\right|\left|g(\varphi(z_m))\right| \longrightarrow 1, \tag{3.22}$$

as $m \to \infty$.

It follows from (C) that

$$\begin{aligned}\left(1-|z_m|^2\right)^\beta &\left|f_a'(\varphi(z_m))\right|\left|\varphi'(z_m)\right|\left|g(\varphi(z_m))\right| \\ &= \frac{\left(1-|z_m|^2\right)^\beta \left|\varphi'(z_m)\right|}{\left(1-\left|\varphi(z_m)\right|^2\right)^\alpha}\left|g(\varphi(z_m))\right|\left(1-\left|\varphi(z_m)\right|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right|\end{aligned} \tag{3.23}$$

$$\leq \left(1-\left|\varphi(z_m)\right|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right|. \tag{3.24}$$

Combining (3.22) and (3.24), it follows that

$$\begin{aligned}1 &\leq \liminf_{m\to\infty}\left(1-\left|\varphi(z_m)\right|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right| \\ &\leq \limsup_{m\to\infty}\left(1-\left|\varphi(z_m)\right|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right| \leq 1.\end{aligned} \tag{3.25}$$

The last inequality follows (3.7), since $\varphi(z_m) \in \mathbb{D}$.

Consequently,

$$\lim_{m\to\infty}\left(1-\left|\varphi(z_m)\right|^2\right)^\alpha \left|f_a'(\varphi(z_m))\right| = \lim_{m\to\infty}\left(1-\rho^2(\varphi(z_m),a)\right)^\alpha = 1. \tag{3.26}$$

That is, $\lim_{n\to\infty}\rho(\varphi(z_m),a) = 0$.

Combining (3.22), (3.23), and (3.26), we know that

$$\lim_{m\to\infty}\frac{\left(1-|z_m|^2\right)^\beta \left|\varphi'(z_m)\right|}{\left(1-\left|\varphi(z_m)\right|^2\right)^\alpha}\left|g(\varphi(z_m))\right| = 1; \tag{3.27}$$

the desired results follows. The proof of this theorem is completed. □

Remark 3.3. If $\alpha = \beta = 1$, then $\mathcal{B}^\alpha$, $\mathcal{B}^\beta$ will be Bloch space $\mathcal{B}$, so the similar results on the Bloch space corresponding to Theorems 3.1 and 3.2 also hold.

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