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Research Article

Resolvent Iterative Methods for Solving System of Extended General Variational Inclusions

Muhammad Aslam Noor,[1,2] Khalida Inayat Noor,[1] and Eisa Al-Said[2]

[1] *Mathematics Department, COMSATS Institute of Information Technology, Islamabad 44000, Pakistan*
[2] *Mathematics Department, College of Science, King Saud University, Riyadh 11451, Saudi Arabia*

Correspondence should be addressed to Muhammad Aslam Noor, noormaslam@hotmail.com

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We introduce and consider some new systems of extended general variational inclusions involving six different operators. We establish the equivalence between this system of extended general variational inclusions and the fixed points using the resolvent operators technique. This equivalent formulation is used to suggest and analyze some new iterative methods for this system of extended general variational inclusions. We also study the convergence analysis of the new iterative method under certain mild conditions. Several special cases are also discussed.

1. Introduction

In the recent years, much attention has been given to study the system of variational inclusions/inequalities, which occupies a central and significant role in the interdisciplinary research between analysis, geometry, biology, elasticity, optimization, imaging processing, biomedical sciences, and mathematical physics. One can see an immense breadth of mathematics and its simplicity in the works of this research. A number of problems leading to the system of variational inclusions/inequalities arise in applications to variational problems and engineering, see; for example, [1–31]. Variational inclusions/inequalities can be viewed as innovative and novel extension of the variational principles.

Inspired and motivated by research going on in this area, we introduce and consider a new system of extended general variational inclusions involving six different nonlinear operators. This new class of system of extended general variational inclusions includes the system of variational inclusions/inequalities involving five, four, three, and two operators and quasi variational inclusions/inequalities as special cases. Using the resolvent operator technique, we establish the equivalence between the new system of general variational inclusions and the fixed point problem. This alternative equivalent formulation is used to suggest and analyze some iterative methods for solving this system of extended general variational inclusions. Several special cases of these iterative algorithms are also discussed. We also prove the convergence of the proposed iterative methods under weaker conditions. Since the new system of extended general variational inclusions/inequalities includes the system of variational inclusions/inequalities and related optimization problems as special cases, results proved in this paper continue to hold for these problems. Our result can be viewed as refinement and improvement of the previous results in this field. The interested readers are advised to explore this field further and discover some new and novel applications of these system of extended general variational inclusions/inequalities in various branches of pure and applied sciences. This field of study is not much developed and offers several opportunities for future research. For example, see [5, 6] and the references therein, for the applications of recurrent neural network regarding the extended general variational inequalities.

2. Preliminaries

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$, respectively, Let K be a closed and convex set in H. Let $T_1, T_2, A, g, h, g_1 : H \to H$ be nonlinear different operators, and let $\varphi : H \to R \cup \{+\infty\}$ be a continuous function.

We now consider the problem of finding $x^*, y^* \in H$ such that

$$\begin{aligned} &0 \in \rho T_1(y^*) + \rho A(g_1(x^*)) - g(y^*) + g_1(x^*), \quad \rho > 0, \\ &0 \in \eta T_2(x^*) + \eta A(h_1(y^*)) + g_1(y^*) - h(x^*), \quad \eta > 0, \end{aligned} \tag{2.1}$$

which is called the system of general variational inclusions involving seven different operators.

We now discuss some special cases of the system of general variational inclusions (2.1).

(i) If $T_1 = T_2 = T$ and $g = h = g_1,\ \rho = \eta,\ x = x^* = y^*$, then (2.1) is equivalent to finding $x \in H$, such that

$$0 \in \rho T(x) + \rho A(g(x)), \tag{2.2}$$

which is known as the variational inclusion problem or finding the zero of the sum of two (more) monotone operators [8–12]. It is well known that a wide class of linear and nonlinear problems can be studied via variational inclusion problems.

(ii) We note that, if $A(\cdot) = \partial\varphi(\cdot)$, the subdifferential of a proper, convex, and lower-semicontinuous function, then (2.1) is equivalent to finding $x^*, y^* \in H$, such that

$$\begin{aligned} &\langle \rho T_1(y^*) + g_1(x^*) - g(y^*), g(x) - g_1(x^*) \rangle \geq \rho\varphi(g_1(x^*)) - \rho\varphi(g(x)), \quad \forall x \in H,\ \rho > 0, \\ &\langle \eta T_2(x^*) + h_1(y^*) - h(x^*), h(x) - g_1(y^*) \rangle \geq \eta\varphi(g_1(y^*)) - \eta\varphi(h(x)), \quad \forall x \in H,\ \eta > 0, \end{aligned} \tag{2.3}$$

which is called the system of mixed general variational inequalities involving five different nonlinear operators and appears to be a new one.

(iii) If $T_1 = T_2 = T$, then (2.3) reduces to the following system of mixed general variational inequalities of finding $x^*, y^* \in H$, such that

$$\begin{aligned} &\langle \rho T(y^*) + g_1(x^*) - g(y^*), g(x) - g_1(x^*) \rangle \geq \rho\varphi(g_1(x^*)) - \rho\varphi(g(x)), \quad \forall x \in H,\ \rho > 0, \\ &\langle \eta T(x^*) + h_1(y^*) - h(x^*), h(x) - g_1(y^*) \rangle \geq \eta\varphi(g_1(y^*)) - \eta\varphi(h(x)), \quad \forall x \in H,\ \eta > 0. \end{aligned} \tag{2.4}$$

(iv) If φ is an indicator function of a closed and convex set K in H, then (2.4) is equivalent to finding $x^*, y^* \in K$, such that

$$\begin{aligned} &\langle \rho T(y^*) + g_1(x^*) - g(y^*), g(x) - g_1(x^*) \rangle \geq 0, \quad \forall x \in H : g(x) \in K,\ \rho > 0, \\ &\langle \eta T(x^*) + g_1(y^*) - h(x^*), h(x) - g_1(y^*) \rangle \geq 0, \quad \forall x \in H : h(x) \in K,\ \eta > 0, \end{aligned} \tag{2.5}$$

is called the system of extended general variational inequalities involving five different operators, which has been studied by Noor [23].

(v) If $T_1 = T_2 = T, h = g_1$, then (2.5) is equivalent to finding $x^* \in K$ such that

$$\langle Tx^*, g(x) - h(x^*) \rangle \geq 0, \quad \forall x \in H : g(x) \in K, \tag{2.6}$$

which is known as the extended general variational inequality introduced and studied by Noor [16] in 2009. It has been shown [16] that the minimum of a differentiable nonconvex function on the nonconvex set can be characterized by the extended general variational inequality (2.6). For the neural network technique for solving (2.6), see [5, 6]. In particular, for suitable and appropriate choice of the operators, one can obtain the various classes of variational inclusions and variational inequalities. This shows that the system of extended general variational inclusions involving seven different operators (2.1) is more general and includes several classes of variational inclusions/inequalities and related optimization problems as special cases. For the recent applications, numerical methods, and formulations of variational inequalities and variational inclusions, see [1–31] and the references therein.

3. Iterative Algorithms

In this section, we suggest some explicit iterative algorithms for solving the system of general variational inclusion (2.1). First of all, we establish the equivalence between the system of variational inclusions and fixed point problems. For this purpose, we recall the following well-known result.

Definition 3.1 (see [1]). For any maximal operator T, the resolvent operator associated with T, for any $\rho > 0$, is defined as

$$J_T(u) = (I + \rho T)^{-1}(u), \quad \forall u \in H. \tag{3.1}$$

It is well known that an operator T is maximal monotone if and only if its resolvent operator J_T is defined everywhere. It is single valued and nonexpansive, that is,

$$\|J_A u - J_A v\| \leq \|u - v\|, \quad \forall u, v \in H. \tag{3.2}$$

We now show that the system of extended general variational inclusions (2.1) is equivalent to the fixed point problem and this is the motivation of our next result.

Lemma 3.2. *If the operator A is maximal monotone, then $(x^*, y^*) \in H$ is a solution of* (2.1), *if and only if, $x^*, y^* \in H$ satisfies*

$$\begin{aligned} g_1(x^*) &= J_A\left[g(y^*) - \rho T_1(y^*)\right], \\ g_1(y^*) &= J_A\left[h(x^*) - \eta T_2(x^*)\right]. \end{aligned} \tag{3.3}$$

Proof. Let $(x^*, y^*) \in H$ be a solution of (2.1). Then

$$\begin{aligned} g(y^*) - \rho T_1(y^*) &\in (I + \rho A)(g_1(x^*)), \\ h(x^*) - \eta T_2(x^*) &\in (I + \eta A)(g_1(y^*)), \end{aligned} \tag{3.4}$$

which implies that

$$\begin{aligned} g_1(x^*) &= J_A\left[g(y^*) - \rho T_1(y^*)\right], \\ g_1(y^*) &= J_A\left[h(x^*) - \eta T_2(x^*)\right], \end{aligned} \tag{3.5}$$

the required result. □

This equivalent formulation is used to suggest and analyze an iterative method for solving (2.1). To do so, one rewrite (3.3) in the following form:

$$x^* = (1 - a_n)x^* + a_n(x^* - g_1(x^*)) + a_n J_A\left[g(y^*) - \rho T_1(y^*)\right], \tag{3.6}$$

$$y^* = y^* - g_1(y^*) + J_A\left[h(x^*) - \eta T_2(x^*)\right], \tag{3.7}$$

where $a_n \in [0, 1]$ for all $n \geq 0$ satisfies some suitable conditions.

This alternative equivalence formulation enables us to suggest the following explicit iterative method for solving (2.1).

Algorithm 1. For arbitrarily chosen initial points $x_0, y_0 \in K$ compute the sequence $\{x_n\}$ and $\{y_n\}$ by

$$x_{n+1} = (1-a_n)x_n + a_n(x_{n+1} - g_1(x_{n+1})) + a_n J_A[g(y_n) - \rho T_1(y_n)],$$
$$y_{n+1} = y_{n+1} - g_1(y_{n+1}) + J_A[h(x_{n+1}) - \eta T_2(x_{n+1})], \tag{3.8}$$

where $a_n \in [0,1]$ for all $n \geq 0$ satisfies some suitable conditions.

For $g_1 = g$ and $g_1 = h$, Algorithm 1 reduces to the following algorithm for solving (2.1).

Algorithm 2. For arbitrarily chosen initial points $x_0, y_0 \in K$ compute the sequence $\{x_n\}$ and $\{y_n\}$ by

$$x_{n+1} = (1-a_n)x_n + a_n(x_{n+1} - g(x_{n+1})) + a_n J_A[g(y_n) - \rho T_1(y_n)], \tag{3.9}$$

$$y_{n+1} = y_{n+1} - h(y_{n+1}) + J_A[h(x_{n+1}) - \eta T_2(x_{n+1})], \tag{3.10}$$

where $a_n \in [0,1]$ for all $n \geq 0$ satisfies some suitable conditions.

For suitable and appropriate choice of the operators T_1, T_2, A, g, h, g_1 and spaces, one can obtain a wide class of iterative methods for solving different classes of variational inclusions and related optimization problems. This shows that Algorithm 1 is quite flexible and general and includes various known and new algorithms for solving variational inequalities and related optimization problems as special cases.

Definition 3.3. A mapping $T : H \to H$ is called r-strongly monotone, if and only if, there exists a constant $r > 0$, such that

$$\langle Tx - Ty, x - y\rangle \geq r\|x-y\|^2, \quad \forall x, y \in H. \tag{3.11}$$

Definition 3.4. A mapping $T : H \to H$ is called relaxed γ-cocoercive, if and only if, there exists a constant $\gamma > 0$, such that

$$\langle Tx - Ty, x - y\rangle \geq -\gamma\|Tx-Ty\|^2, \quad \forall x, y \in H. \tag{3.12}$$

Definition 3.5. A mapping $T : H \to H$ is called relaxed (γ, r)-cocoercive, if and only if, there exists constants $\gamma > 0, r > 0$, such that

$$\langle Tx - Ty, x - y\rangle \geq -\gamma\|Tx-Ty\|^2 + r\|x-y\|^2, \quad \forall x, y \in H. \tag{3.13}$$

The class of relaxed (γ, r)-cocoercive mappings is more general than the class of strongly monotone mappings. It is known that the relaxed (γ, r)-cocoercivity implies strongly monotonicity, but the converse is not true.

Definition 3.6. A mapping $T : H \to H$ is called μ-Lipschitzian, if and only if, there exists a constant $\mu > 0$, such that

$$\|Tx - Ty\| \leq \mu\|x - y\|, \quad \forall x, y \in H. \tag{3.14}$$

4. Main Results

In this section, we consider the convergence criteria of Algorithm 2 under some suitable mild conditions and this is the main motivation of this paper. In a similar way, one can consider the convergence analysis of Algorithm 1.

Theorem 4.1. *Let x^*, y^* be a solution of* (2.1)*. If $T_1 : H \to H$ is relaxed (γ_1, r_1)-cocoercive and μ_1-Lipschitzian and $T_2 : H \times H \to H$ is relaxed (γ_2, r_2)-cocoercive and μ_3-Lipschitzian, Let g be a relaxed (γ_3, r_3)-cocoercive and μ_3-Lipschitzian. Let the operator h be relaxed (γ_4, r_4)-cocoercive and μ_4-Lipschitzian. If the operator g_1 is relaxed (γ_5, r_5)-cocoercive and μ_5-Lipschitzian, then*

$$\left|\rho - \frac{r_1 - \gamma_1\mu_1^2}{\mu_1^2}\right| < \frac{\sqrt{\left(r_1 - \gamma_1\mu_1^2\right)^2 - \mu_1^2\mu(2-\mu)}}{\mu_1^2}, \quad r_1 > \gamma_1\mu_1^2 + \mu_1\sqrt{\mu(2-\mu)}, \ \mu = k + k_3 < 1, \tag{4.1}$$

$$\left|\eta - \frac{r_2 - \gamma_2\mu_2^2}{\mu_2^2}\right| < \frac{\sqrt{\left(r_2 - \gamma_2\mu_2^2\right)^2 - \mu_2^2\nu(2-\nu)}}{\mu_2^2}, \quad r_2 > \gamma_2\mu_2^2 + \mu_2\sqrt{\nu(2-\nu)}, \ \nu = k_1 + k_3 < 1, \tag{4.2}$$

where

$$k = \sqrt{1 - 2\left(r_3 - \gamma_3\mu_3^2\right) + \mu_3^2}, \qquad k_1 = \sqrt{1 - 2\left(r_4 - \gamma_4\mu_4^2\right) + \mu_4^2}, \qquad k_3 = \sqrt{1 - 2\left(r_5 - \gamma_5\mu_5^2\right) + \mu_5^2}, \tag{4.3}$$

and $a_n \in [0,1]$, $\sum_{n=0}^{\infty} a_n = \infty$, then for arbitrarily chosen initial points $x_0, y_0 \in H$, x_n and y_n obtained from Algorithm 1 converge strongly to x^* and y^*, respectively.

Proof. From (3.6), (3.9), and the nonexpansive property of the resolvent operator J_A, we have

$$\begin{aligned}
&\|x_{n+1} - x^*\| \\
&\quad = \|x_{n+1} - g_1(x_{n+1}) + J_\varphi[g(y_n) - \rho T_1(y_n)] - (x^* - g_1(x^*)) - J_\varphi[g(y^*) - \rho T_1(y^*)]\| \\
&\quad \leq \|x_{n+1} - x^* - (g_1(x_{n+1}) - g_1(x^*))\| + \|J_\varphi[g(y_n) - \rho T_1(y_n)] - J_\varphi[g(y^*) - \rho T_1(y^*)]\| \\
&\quad \leq \|x_{n+1} - x^* - (g_1(x_{n+1}) - g_1(x^*))\| + \|[g(y_n) - \rho T_1(y_n)] - [g(y^*) - \rho T_1(y^*)]\| \\
&\quad = \|x_{n+1} - x^* - (g_1(x_{n+1}) - g_1(x^*))\| + \|y_n - y^* - \rho[T_1(y_n) - T_1(y^*)]\| \\
&\qquad + \|y_n - y^* - (g(y_n) - g(y^*))\|.
\end{aligned} \tag{4.4}$$

From the relaxed (γ_1, r_1)-cocoercive and μ_1-Lipschitzian of T_1, we have

$$\begin{aligned}
&\left\|y_n - y^* - \rho\left[T_1(y_n) - T_1(y^*)\right]\right\|^2 \\
&\quad = \left\|y_n - y^*\right\|^2 - 2\rho\langle T_1(y_n) - T_1(y^*), y_n - y^*\rangle + \rho^2\left\|T_1(y_n) - T_1(y^*)\right\|^2 \\
&\quad \le \left\|y_n - y^*\right\|^2 - 2\rho\left[-\gamma_1\left\|T_1(y_n) - T_1(y^*)\right\|^2 + r_1\left\|y_n - y^*\right\|^2\right] \\
&\qquad + \rho^2\left\|T_1(y_n) - T_1(y^*)\right\|^2 \\
&\quad \le \left\|y_n - y^*\right\|^2 + 2\rho\gamma_1\mu_1^2\left\|y_n - y^*\right\|^2 - 2\rho r_1\left\|y_n - y^*\right\|^2 + \rho^2\mu_1^2\left\|y_n - y^*\right\|^2 \\
&\quad = \left[1 + 2\rho\gamma_1\mu_1^2 - 2\rho r_1 + \rho^2\mu_1^2\right]\left\|y_n - y^*\right\|^2.
\end{aligned} \tag{4.5}$$

In a similar way, using the (γ_3, r_3)-cocoercivity and μ_3-Lipschitz continuity of the operator g and (γ_5, r_5)-cocoercivity and μ_5-Lipschitz continuity of the operator g_1, we have

$$\left\|y_n - y^* - (g(y_n) - g(y^*))\right\| \le k\left\|y_n - y^*\right\|, \tag{4.6}$$

$$\left\|y_n - y^* - (g_1(y_n) - g_1(y^*))\right\| \le k_3\left\|y_n - y^*\right\|, \tag{4.7}$$

where k and k_3 are defined by (4.3). Set

$$\theta_1 = \frac{k + \left[1 + 2\rho\gamma_1\mu_1^2 - 2\rho r_1 + \rho^2\mu_1^2\right]^{1/2}}{1 - k_3}. \tag{4.8}$$

It is clear from condition (4.1) that $0 \le \theta_1 < 1$. Hence from (4.5),(4.6), and (4.7), it follows that

$$\|x_{n+1} - x^*\| \le \theta_1\left\|y_n - y^*\right\|. \tag{4.9}$$

Similarly, from the relaxed (γ_2, r_2)-cocoercive and μ_2-Lipschitzian of T_2, we obtain

$$\begin{aligned}
&\left\|x_{n+1} - x^* - \eta[T_2(x_{n+1}) - T_2(x^*)]\right\|^2 \\
&\quad = \|x_{n+1} - x^*\|^2 - 2\eta\langle T_2(x_{n+1}) - T_2(x^*), x_{n+1} - x^*\rangle + \eta^2\|T_2(x_{n+1}) - T_2(x^*)\|^2 \\
&\quad \le \|x_{n+1} - x^*\|^2 - 2\eta\left[-\gamma_2\|T_2(x_{n+1}) - T_2(x^*)\|^2 + r_2\|x_{n+1} - x^*\|^2\right] \\
&\qquad + \eta^2\|T_2(x_{n+1}) - T_2(x^*)\|^2 \\
&\quad = \|x_{n+1} - x^*\|^2 + 2\eta\gamma_2\|T_2(x_{n+1}) - T_2(x^*)\|^2 - 2\eta r_2\|x_{n+1} - x^*\|^2 \\
&\qquad + \eta^2\|T_2(x_{n+1}) - T_2(x^*)\|^2 \\
&\quad \le \|x_{n+1} - x^*\|^2 + 2\eta\gamma_2\mu_2^2\|x_{n+1} - x^*\|^2 - 2\eta r_2\|x_{n+1} - x^*\|^2 + \eta^2\mu_2^2\|x_{n+1} - x^*\|^2 \\
&\quad = \left[1 + 2\eta\gamma_2\mu_2^2 - 2\eta r_2 + \eta^2\mu_2^2\right]\|x_{n+1} - x^*\|^2.
\end{aligned} \tag{4.10}$$

Also, using the (γ_4, r_4)-cocoercivity and μ_4-Lipschitz continuity of the operator h, we have

$$\left\| y_n - y^* - \left(h(y_n) - h(y^*)\right)\right\| \leq k_1 \left\| y_n - y^* \right\|, \tag{4.11}$$

where k_1 is defined by (4.3).

Hence from (3.7), (3.10), (4.7), (3.7), and (4.11), we have

$$\begin{aligned}
\left\| y_{n+1} - y^* \right\| &= \left\| y_{n+1} - y^* - \left(g_1(y_{n+1}) - g_1(y^*)\right)\right\| \\
&\quad + \left\| J_\varphi\left[h(x_{n+1}) - \eta T_2(x_{n+1})\right] - J_\varphi\left[h(x^*) - \eta T_2(x^*)\right]\right\| \\
&\leq \left\| y_{n+1} - y^* - \left(g_1(y_{n+1}) - g_1(y^*)\right)\right\| + \left\| x_{n+1} - x^* - \eta(T_2(x_{n+1}) - T_2(x_n))\right\| \\
&\quad + \| x_{n+1} - x^* - (h(x_{n+1}) - h(x^*))\|,
\end{aligned} \tag{4.12}$$

which implies that

$$\left\| y_{n+1} - y^* \right\| \leq \theta_2 \| x_{n+1} - x^* \|, \tag{4.13}$$

where

$$\theta_2 = \frac{k_1 + \left[1 + 2\rho\gamma_1\mu_1^2 - 2\rho r_1 + \rho^2\mu_1^2\right]^{1/2}}{1 - k_3}. \tag{4.14}$$

From (4.2), it follows that $\theta_2 < 1$.

From (4.9) and (4.13), we obtain that

$$\| x_{n+1} - x^* \| \leq \theta_1\theta_2 \| x_n - x^* \|. \tag{4.15}$$

Since $\theta_1\theta_2 < 1$, it follows that $\lim_{n\to\infty}\{\|x_n - x^*\|\} = 0$. Hence the result $\lim_{n\to\infty}\{\|y_n - y^*\|\} = 0$ is from (4.11). This completes the proof. □

Remarks 4.2. It is well known [5, 6] that the traditional algorithms may not be efficient due to the structure of the problems. To overcome this drawback, one usually uses the artificial neural network based on the circuit implementation. It has been shown [5, 6] that the neural network models are efficient in solving variational inequalities and related optimization problems. The recurrent neural network methods have applications in kinematics control, support vector machine learning, and related branches of engineering. Using the technique and ideas of Liu and Cao [5] and Liu and Yang [6], one can consider the recurrent neural network based on the resolvent operator for solving the system of extended general variational inclusions (2.1) and its special cases. This is an interesting problem for future research. Such type of systems of extended general variational inclusions may have important and significant applications in engineering and applied sciences. For more general systems of general variational inequalities/inclusions, see the work of Noor and Noor [27, 28] and the references therein.

5. Conclusion

In this paper, we have introduced and considered a new system of extended general variational inclusions involving six different operators. We have established the equivalent between the system of variational inclusions and the fixed point problem using the resolvent operator. This equivalence is used to suggest and analyze some iterative methods for solving the extended general system of variational inclusion. Several special cases are also discussed.

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