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Upper Bounds on the Solution of Coupled Algebraic Riccati Equation

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Upper bounds for eigenvalues of a solution to continuous time coupled algebraic Riccati equation (CCARE) and discrete time coupled algebraic Riccati equation (DCARE) are developed as special cases of bounds for the unified coupled algebraic Riccati equation (UCARE). They include bounds of the maximal eigenvalues, the sums of the eigenvalues and the trace.

Keywords: Coupled Riccati equation; Jump linear systems; JLQ problem; Eigenvalues

AMS Subject Classifications: 93E03, 93C05, 15A42, 15A24, 49N10

I. INTRODUCTION

It is well known that algebraic Riccati and Lyapunov equations are widely applied to various engineering areas such as signal processing and, especially, control theory. In the area of control systems analysis and design, these equations play important roles in system stability analysis, optimal controllers and filters design, the transient behavior estimates, *etc*. There are many numerical algorithms of computation of their solution. Despite that the problem to find bounds to the solution of these equation has been intensively studied in past two decades. The surveys of such results can be found in [14, 9, 10]. Majority of papers in this area deal separately with continuous or

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discrete type Riccati equation. Recently, a unified approach for continuous and discrete Riccati equation has been proposed in [15]. Based on this treatment in [16, 6, 7] the bounds for the unified Riccati and Lyapunov equations have been obtained. The reasons that problem to estimate the upper and lower bounds of these equation has become an attractive topic are that the bounds are also applied to solve many control problems such as stability analysis [12, 18], time-delay system controller design [13], estimation of the minimal cost and the suboptimal controller design [11], convergence of numerical algorithms [3], robust stabilization problem [4] and so on. The last example is connected with linear dynamical systems with Markovian jumps in parameter values, which have recently attracted a great deal of interest. Considering the linear-quadratic problem for such a system instead of one equation a set of coupled algebraic equations arises. And all the reasons mentioned above could be repeated to show in what way the bounds for coupled algebraic Riccati and Lyapunov equations can be used. There is one more reason why it is important to have an estimation for the solution of the coupled Riccati equation. On the contrary to the standard Riccati equation the problem of numerical solving of the coupled Riccati equation is not well studied. There are only few numerical algorithms ([1, 2, 19]) and all of them are recurrent so the efficiency of evaluating depends on how close to the final solution the algorithm's starting value is.

The objective of this paper is to present bounds for the sums of the eigenvalues of the solution of the unified coupled algebraic Riccati equation. In the limiting cases we obtain bounds for the discrete coupled algebraic Riccati equation and continuous coupled algebraic Riccati equation. This paper seems to be the first where the upper bounds for such a type of Riccati equation are proposed.

The eigenvalues $\lambda_i(X)$, $i = 1, \ldots, n$, of a symmetric matrix $X \in R^{n \times n}$ are assumed to be arranged such that

$$\lambda_1(X) \geq \lambda_2(X) \geq \cdots \geq \lambda_n(X).$$

All our results will be expressed concisely by using the following scalar functions

$$f(a, b, c) = \frac{-a + \sqrt{a^2 + bc}}{b}, \quad b \neq 0.$$

When we consider the discrete time jump linear-quadratic control problem the following coupled Riccati equation (DCARE) arises [5]:

$$Q_i + \tilde{A}_i'\tilde{F}_i\tilde{A}_i - \tilde{A}_i'\tilde{F}_i\tilde{B}_i(R_i + \tilde{B}_i'\tilde{F}_i\tilde{B}_i)^{-1}\tilde{B}_i'\tilde{F}_i\tilde{A}_i - P_i = 0, \tag{1}$$

where

$$\tilde{F}_i = \sum_{j \in S} \tilde{p}_{ij} P_j$$

and $\tilde{A}_i$, Q_i, $P_i \in R^{n \times n}$, $\tilde{B}_i \in R^{n \times m}$, $R_i \in R^{m \times m}$, $\tilde{p}_{ij} \in [0, 1]$, $\sum_{j \in S} \tilde{p}_{ij} = 1$, $i \in S$, S is a finite set. We assume that $\tilde{p}_{ii} > 0$. Using the following notation

$$A_i = \sqrt{\tilde{p}_{ii}}\tilde{A}_i, \quad B_i = \sqrt{\tilde{p}_{ii}}\tilde{B}_i R_i^{-1/2}, \quad p_{ij} = \frac{\tilde{p}_{ij}}{\tilde{p}_{ii}}, \quad F_i = \sum_{j \in S} p_{ij} P_j,$$

we can write (1) as

$$Q_i + A_i'F_iA_i - A_i'F_iB_i(I + B_i'F_iB_i)^{-1}B_i'F_iA_i - P_i = 0 \tag{2}$$

where

$$F_i = \sum_{j \in S} p_{ij} P_j = P_i + \sum_{j \neq i} p_{ij} P_j.$$

For the continuous time jump linear-quadratic control problem the following coupled Riccati equation (CCARE) arises [20]:

$$Q_i + P_i\tilde{A}_i + \tilde{A}_i'P_i - P_i\tilde{B}_iR_i^{-1}\tilde{B}_i'P_i + \sum_{j \in S} q_{ij}P_j = 0. \tag{3}$$

$\tilde{A}_i$, Q_i, $P_i \in R^{n \times n}$, $\tilde{B}_i \in R^{n \times m}$, $R_i \in R^{m \times m}$, $q_{ij} \in R$, $\sum_{j \in S} q_{ij} = 0$, $q_{ij} \geq 0$, $i \in S$, S is a finite set. In this case we also introduce a new notations

$$A_i = \tilde{A}_i + \frac{1}{2}q_{ii}I, \quad B_i = \tilde{B}_iR_i^{-1/2}.$$

Using this notation we can rewrite (3) as

$$Q_i + P_iA_i + A_i'P_i - P_iB_iB_i'P_i + \sum_{j \neq i} q_{ij}P_j = 0. \tag{4}$$

The first important observation is that both DCARE (3) and CCARE (4) can be obtained as special cases of the following unified coupled algebraic Riccati equation (UCARE)

$$\begin{aligned} &Q_i + F_iA_i + A_i'F_i + \Delta A_i'F_iA_i \\ &\quad - (I+\Delta A_i)'F_iB_i(I+\Delta B_i'F_iB_i)^{-1}B_i'F_i(I+\Delta A_i) + \sum_{j\neq i}\pi_{ij}P_j = 0. \end{aligned} \tag{5}$$

where

$$F_i = \Delta\sum_{j\neq i}\pi_{ij}P_j + P_i. \tag{6}$$

and A_i, Q_i, $P_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, π_{ij}, $\Delta \in [0,\infty)$, $i\in S$, S is a finite set with s elements. In our future consideration we assume that there exists positive definite solution of (5).

Remark 1 For $\Delta = 0$ and $\pi_{ij} = q_{ij}$ UCARE (5) reduces to CCARE (4) and for $\Delta = 1$ and $\pi_{ij} = p_{ij}$ UCARE (5) reduces to DCARE (3) by substituting A_i by $A_i - I$.

We need the following lemmas.

LEMMA 1 ([17]) *Let X, $Y \in R^{n\times n}$ with $X = X'$, $Y = Y'$, X, $Y \geq 0$. Then the following inequalities hold*

$$\sum_{k=1}^{l}\lambda_k(XY) \leq \sum_{k=1}^{l}\lambda_k(X)\lambda_k(Y), \tag{7}$$

$$\sum_{k=1}^{l}\lambda_k(X+Y) \leq \sum_{k=1}^{l}\lambda_k(X) + \sum_{k=1}^{l}\lambda_k(Y), \tag{8}$$

for any $l = 1,\ldots,n$.

LEMMA 2 ([17]) *For $l = 1,\ldots,n$, let*

$$\sum_{k=1}^{l}x_k \leq \sum_{k=1}^{l}y_k, \tag{9}$$

for real numbers arranged in nonincreasing order. Then

$$\sum_{k=1}^{l}u_kx_k \leq \sum_{k=1}^{l}u_ky_k, \tag{10}$$

where real numbers u_k arranged in nonincreasing order, are nonnegative.

LEMMA 3 ([8]) *Let matrix $P>0$, and matrix $R\geq 0$. For any $l=1,\ldots,n$*

$$\sum_{k=1}^{l}\lambda_k((P^{-1}+R)^{-1})\leq\sum_{k=1}^{l}\frac{\lambda_k(P)}{1+\lambda_k(P)\lambda_{n-k+1}(R)}. \tag{11}$$

II. MAIN RESULTS

Using the matrix identity

$$(I+ST)^{-1}=I-S(I+TS)^{-1}T,$$

where $S,\ T\in R^{n\times n}$, (5) can be transformed to

$$F_i=(I+\Delta A_i)'(F_i^{-1}+\Delta B_iB_i')^{-1}(I+\Delta A_i)+\Delta\sum_{j\neq i}\pi_{ij}P_j+\Delta Q_i. \tag{12}$$

and using (6) we have

$$P_i=(I+\Delta A_i)'(F_i^{-1}+\Delta B_iB_i')^{-1}(I+\Delta A_i)+\Delta Q_i. \tag{13}$$

LEMMA 4 *Let positive definite matrices P_i, $i\in S$ satisfy the UCARE (5) and assume that there exists a positive constant α such that*

$$\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)\leq\alpha, \tag{14}$$

for $l=1,\ldots,n$ and $i\in S$. Then

$$\sum_{k=1}^{l}\lambda_k(P_i)\leq f(W_i,2U_i,2V_i) \tag{15}$$

for $l=1,\ldots,n$ and $i\in S$, where

$$a_i=1+\Delta\lambda_1(A_i+A_i'+\Delta A_i'A_i),\quad q_i=\sum_{k=1}^{l}\lambda_k(Q_i),$$

$$T_i=\sum_{k=1}^{l}\frac{1+\Delta\lambda_k(A_i+A_i'+\Delta A_i'A_i)}{\Delta\lambda_{n-k+1}(B_iB_i')},\quad U_i=a_i(1-\Delta\max_{j\in S,j\neq i}\pi_{ij}),$$

$$W_i = T_i + \Delta\alpha a_i \max_{j\in S, j\neq i} \pi_{ij} - U_i T_i - \Delta U_i q_i,$$
$$V_i = \Delta\alpha a_i T_i \max_{j\in S, j\neq i} \pi_{ij} + \Delta q_i T_i + \Delta^2 \alpha a_i q_i \max_{j\in S, j\neq i} \pi_{ij}.$$

Proof From (13) it follows, by using (7) and (8), that

$$\begin{aligned}\sum_{k=1}^{l} \lambda_k(P_i) &\leq \sum_{k=1}^{l} \lambda_k((I+\Delta A_i)'(F_i^{-1} + \Delta B_i B_i')^{-1}(I+\Delta A_i)) \\ &\quad + \Delta \sum_{k=1}^{l} \lambda_k(Q_i) \\ &\leq \sum_{k=1}^{l} \lambda_k((I+\Delta A_i)'(I+\Delta A_i))\lambda_k((F_i^{-1} + \Delta B_i B_i')^{-1}) \\ &\quad + \Delta \sum_{k=1}^{l} \lambda_k(Q_i).\end{aligned} \tag{16}$$

Now (11) with $P = F_i$ and $R = \Delta B_i B_i'$, produces

$$\sum_{k=1}^{l} \lambda_k((F_i^{-1} + \Delta B_i B_i')^{-1}) \leq \sum_{k=1}^{l} \frac{\lambda_k(F_i)}{1 + \lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')}, \tag{17}$$

then an application of (10) with

$$u_k = \lambda_k((I+\Delta A_i)'(I+\Delta A_i)), \quad x_k = \lambda_k((F_i^{-1} + \Delta B_i B_i')^{-1})$$

and

$$y_k = \frac{\lambda_k(F_i)}{1 + \lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')},$$

gives

$$\begin{aligned}&\sum_{k=1}^{l} \lambda_k((I+\Delta A_i)'(I+\Delta A_i))\lambda_k((F_i^{-1} + \Delta B_i B_i')^{-1}) \\ &\quad \leq \sum_{k=1}^{l} \frac{\lambda_k((I+\Delta A_i)'(I+\Delta A_i))\lambda_k(F_i)}{1 + \lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')} \\ &\quad = \sum_{k=1}^{l} \frac{\lambda_k((I+\Delta A_i)'(I+\Delta A_i))}{\lambda_{n-k+1}(\Delta B_i B_i')} \frac{\lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')}{1 + \lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')}.\end{aligned} \tag{18}$$

The function $\tilde{f}:[0,\infty) \to R$, $\tilde{f}(x) = x/(1+x)$ is concave it means that for any $x_k \in [0,\infty)$, $\alpha_k \in (0,\infty)$, $k = 1,\ldots,l$ the following inequality holds

$$\sum_{k=1}^{l} \alpha_k \tilde{f}(x_k) \leq T\tilde{f}\left(\frac{\sum_{k=1}^{l} \alpha_k x_k}{T}\right), \tag{19}$$

where $T = \sum_{k=1}^{l} \alpha_k$. By (19) with

$$x_k = \lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i'), \quad \alpha_k = \frac{\lambda_k((I+\Delta A_i)'(I+\Delta A_i))}{\lambda_{n-k+1}(\Delta B_i B_i')}$$

we have

$$\begin{aligned}
&\sum_{k=1}^{l} \frac{\lambda_k((I+\Delta A_i)'(I+\Delta A_i))}{\lambda_{n-k+1}(\Delta B_i B_i')} \frac{\lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')}{1+\lambda_k(F_i)\lambda_{n-k+1}(\Delta B_i B_i')} \\
&\quad \leq T_i \frac{\sum_{k=1}^{l} \lambda_k((I+\Delta A_i)'(I+\Delta A_i))\lambda_k(F_i)}{T_i + \sum_{k=1}^{l} \lambda_k((I+\Delta A_i)'(I+\Delta A_i))\lambda_k(F_i)} \\
&\quad \leq \frac{T_i \lambda_1((I+\Delta A_i)'(I+\Delta A_i)) \sum_{k=1}^{l} \lambda_k(F_i)}{T_i + \lambda_1((I+\Delta A_i)'(I+\Delta A_i)) \sum_{k=1}^{l} \lambda_k(F_i)}
\end{aligned} \tag{20}$$

and to obtain (20), the monotonicity of $\tilde{f}$ was used. Combine (20) with (18) and (16) gives

$$\begin{aligned}
\sum_{k=1}^{l} \lambda_k(P_i) \leq &\frac{T_i \lambda_1((I+\Delta A_i)'(I+\Delta A_i)) \sum_{k=1}^{l} \lambda_k(F_i)}{T_i + \lambda_1((I+\Delta A_i)'(I+\Delta A_i)) \sum_{k=1}^{l} \lambda_k(F_i)} \\
&+ \Delta \sum_{k=1}^{l} \lambda_k(Q_i).
\end{aligned} \tag{21}$$

Notice that by (8)

$$\begin{aligned}
\sum_{k=1}^{l} \lambda_k(F_i) &= \sum_{k=1}^{l} \lambda_k\left(\Delta \sum_{j \in S, j \neq i} \pi_{ij} P_j + P_i\right) \\
&\leq \sum_{k=1}^{l} \sum_{j \in S, j \neq i} \Delta \pi_{ij} \lambda_k(P_j) + \sum_{k=1}^{l} \lambda_k(P_i) \\
&\leq \Delta \max_{j \in S, j \neq i} \pi_{ij} \sum_{j \in S, j \neq i} \sum_{k=1}^{l} \lambda_k(P_j) + \sum_{k=1}^{l} \lambda_k(P_i).
\end{aligned} \tag{22}$$

We can rewrite (14) as

$$\sum_{j\in S, j\neq i}\sum_{k=1}^{l}\lambda_k(P_j)\leq\alpha-\sum_{k=1}^{l}\lambda_k(P_i). \tag{23}$$

Then (22) and (23) imply

$$\sum_{k=1}^{l}\lambda_k(F_i)\leq\Delta\max_{j\in S,j\neq 1}\pi_{ij}\alpha+(1-\Delta\max_{j\in S,j\neq i}\pi_{ij})\sum_{k=1}^{l}\lambda_k(P_i). \tag{24}$$

Since the function $g:[0,\infty)\to R$, $g(x)=(ax/(b+cx))$, $a,b,c>0$ is non-decreasing we can use the bound (24) in (21) and obtain

$$\sum_{k=1}^{l}\lambda_k(P_i)\leq\Delta\sum_{k=1}^{l}\lambda_k(Q_i)$$
$$+\frac{T_i\lambda_1((I+\Delta A_i)'(I+\Delta A_i))(\Delta\max_{j\in S,j\neq i}\pi_{ij}\alpha+(1-\Delta\max_{j\in S,j\neq i}\pi_{ij})\sum_{k=1}^{l}\lambda_k(P_i))}{T_i+\lambda_1((I+\Delta A_i)'(I+\Delta A_i))(\Delta\max_{j\in S,j\neq i}\pi_{ij}\alpha+(1-\Delta\max_{j\in S,j\neq i}\pi_{ij})\sum_{k=1}^{l}\lambda_k(P_i))}.$$

Solving this inequality with respect to $\sum_{k=1}^{l}\lambda_k(P_i)$ and rearranging imply result (15). ■

LEMMA 5 *Let the positive definite matrices P_i, $i\in S$ satisfy the UCARE (5). The*

$$\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)\leq f(W,2U,2V)=\alpha, \tag{25}$$

for $l=1,\dots,n$ and $i\in S$, where

$$a=1+\Delta\,\max_{i\in S}\lambda_1(A_i+A_i'+\Delta A_i'A_i),\quad q=\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(Q_i),$$
$$T=\max_{i\in S}\sum_{k=1}^{l}\frac{1+\Delta\lambda_k(A_i+A_i'+\Delta A_i'A_i)}{\Delta\lambda_{n-k+1}(B_iB_i')},$$
$$U=\left(\Delta\left(\max_{i,j\in S,i\neq j}\pi_{ij}\right)+1\right)a,\quad W=Ts-TUs-\Delta Uq,\quad V=\Delta Tsq.$$

Proof Summing (21) over $i\in S$ we have

$$\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)\leq\sum_{i\in S}\frac{T_i\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i)}{T_i+\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i)}$$
$$+\Delta\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(Q_i)$$

$$\leq \sum_{i\in S} \frac{T\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i)}{T+\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i)} + \Delta\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(Q_i), \tag{26}$$

where $T = \max_{i\in S} T_i$. To estimate the first term on the right hand side we again use the inequality (19) with $\tilde{f}:[0,\infty)\to R, \tilde{f}(x) = (Tx/(T+x))$,

$$\alpha_i = \frac{1}{s},\quad x_i = \lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i),\qquad i\in S.$$

On this way we have

$$\begin{aligned}
&\sum_{i\in S} \frac{T\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i)}{T+\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i)} \\
&\quad\leq \frac{T_s\sum_{i\in S}(\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i))}{Ts+\sum_{i\in S}(\lambda_1((I+\Delta A_i)'(I+\Delta A_i))\sum_{k=1}^{l}\lambda_k(F_i))} \\
&\quad\leq \frac{Ts(\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(F_i))\max_{i\in S}(\lambda_1((I+\Delta A_i)'(I+\Delta A_i)))}{Ts+(\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(F_i))\max_{i\in S}(\lambda_1((I+\Delta A_i)'(I+\Delta A_i)))}.
\end{aligned} \tag{27}$$

Notice that by (8)

$$\begin{aligned}
\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(F_i) &= \sum_{i\in S}\sum_{k=1}^{l}\lambda_k\Big(\Delta\sum_{j\neq i}\pi_{ij}P_j + P_i\Big) \\
&\leq \sum_{i\in S}\sum_{k=1}^{l}\sum_{j\in S, j\neq i}\Delta\pi_{ij}\lambda_k(P_j) + \sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i) \\
&= \Delta\sum_{i\in S}\Big(\sum_{j\in S, j\neq i}\pi_{ji}\sum_{k=1}^{l}\lambda_k(P_i)\Big) + \sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i) \\
&\leq \Delta\Big(\max_{i,j\in S, i\neq j}\pi_{ij}\Big)\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i) + \sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i) \\
&= \Big(\Delta\Big(\max_{i,j\in S, i\neq j}\pi_{ij}\Big)+1\Big)\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i).
\end{aligned} \tag{28}$$

From (26), (27) and (28) we can conclude that

$$\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)\leq\Delta\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(Q_i)$$
$$+\frac{Ts\left(\Delta\left(\max\limits_{i,j\in S,i\neq j}\pi_{ij}\right)+1\right)\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)\max_{i\in S}(\lambda_1((I+\Delta A_i)'(I+\Delta A_i)))}{Ts+\left(\Delta\left(\max\limits_{i,j\in S,i\neq j}\pi_{ij}\right)+1\right)\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)\max_{i\in S}(\lambda_1((I+\Delta A_i)'(I+\Delta A_i)))}. \quad (29)$$

Solving this inequality with respect to $\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(P_i)$ and rearranging imply result (25). ∎

Specializing the result of Lemmas 4 and 5 to the DCARE and CCARE according to Remark 1, we obtain the following two theorems.

THEOREM 1 *Let the positive definite matrices P_i, $i\in S$ satisfy the CCARE* (4). *Then*

$$\sum_{k=1}^{l}\lambda_k(P_i)\leq f(W_{ci},2,2V_{ci}), \quad (30)$$

where

$$\alpha_c=f(W_c,2,2V_c),$$

$$W_c=s\left(\max_{j\in S,j\neq i}q_{ij}-\max_{j\in S}\lambda_1(A_i+A_i')\right)\max_{j\in S}\sum_{k=1}^{l}\frac{1}{\lambda_{n-k+1}(B_jB_j')},$$

$$V_c=s\max_{j\in S}\sum_{k=1}^{l}\frac{1}{\lambda_{n-k+1}(B_jB_j')}\sum_{i\in S}\sum_{k=1}^{l}\lambda_k(Q_i)$$

$$W_{ci}=\left(\max_{j\in S,j\neq i}q_{ij}-\lambda_1(A_i+A_i')\right)\sum_{k=1}^{l}\frac{1}{\lambda_{n-k+1}(B_iB_i')},$$

$$V_{ci}=\alpha_c\sum_{k=1}^{l}\frac{1}{\lambda_{n-k+1}(B_iB_i')}\max_{j\in S,j\neq i}q_{ij}$$
$$+\sum_{k=1}^{l}\lambda_k(Q_i)\sum_{k=1}^{l}\frac{1}{\lambda_{n-k+1}(B_iB_i')}.$$

THEOREM 2 *Let the positive definite matrices P_i, $i \in S$ satisfy the DCARE* (2). *Then*

$$\sum_{k=1}^{l} \lambda_k(P_i) \leq f(W_{di}, 2U_{di}, 2V_{di}) \tag{31}$$

for $l = 1, \ldots, n$ and $i \in S$, where

$$\alpha_d = f(W_d, 2U_d, 2V_d),$$

$$W_d = s \max_{i \in S} \sum_{k=1}^{l} \frac{\lambda_k(A_i A_i')}{\lambda_{n-k+1}(B^i B_i')}$$

$$- sU_d \max_{i \in S} \sum_{k=1}^{l} \frac{\lambda_k(A_i A_i')}{\lambda_{n-k+1}(B_i B_i')} - U_d \sum_{i \in S} \sum_{k=1}^{l} \lambda_k(Q_i),$$

$$U_d = \left(\max_{i,j \in S, i \neq j} p_{ij} + 1 \right) \max_{i \in S} \lambda_1(A_i' A_i),$$

$$V_d = s \max_{i \in S} \sum_{k=1}^{l} \frac{\lambda_k(A_i A_i')}{\lambda_{n-k+1}(B_i B_i')} \max_{i \in S} \sum_{k=1}^{l} \frac{\lambda_k(A_i A_i')}{\lambda_{n-k+1}(B_i B_i')},$$

$$T_{di} = \sum_{k=1}^{l} \frac{\lambda_k(A_i' A_i)}{\lambda_{n-k+1}(B_i B_i')}, \quad q_{di} = \sum_{k=1}^{l} \lambda_k(Q_i),$$

$${}_{id}W = T_{di} + \alpha_d \lambda_1(A_i' A_i) \max_{j \in S, j \neq i} p_{ij} - \lambda_1(A_i' A_i) \left(1 - \max_{j \in S, j \neq i} p_{ij} \right) T_{di}$$

$$- \lambda_1(A_i' A_i) \left(1 - \max_{j \in S, j \neq i} p_{ij} \right) q_{di}$$

$${}_{id}V = \alpha_d \lambda_1(A_i' A_i) T_{di} \max_{j \in S, j \neq i} p_{ij} + q_{di} T_{di} + \alpha_d \lambda_1(A_i' A_i) q_{di} \max_{j \in S, j \neq i} \pi_{ij}.$$

III. CONCLUSIONS

The upper bounds for the sums of eigenvalues of the solution to unified-type coupled algebraic Riccati equation are presented in this paper. In the special cases the results reduce to bounds for the continuous and discrete coupled algebraic equations.

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