Hindawi Publishing Corporation
Journal of Probability and Statistics
Volume 2010, Article ID 196461, 18 pages
doi:10.1155/2010/196461

Research Article

VaR: Exchange Rate Risk and Jump Risk

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Received 7 April 2010; Revised 15 July 2010; Accepted 12 November 2010

Academic Editor: Kelvin K. W. Yau

Incorporating the Poisson jumps and exchange rate risk, this paper provides an analytical VaR to manage market risk of international portfolios over the subprime mortgage crisis. There are some properties in the model. First, different from past studies in portfolios valued only in one currency, this model considers portfolios not only with jumps but also with exchange rate risk, that is vital for investors in highly integrated global financial markets. Second, in general, the analytical VaR solution is more accurate than historical simulations in terms of backtesting and Christoffersen's independence test (1998) for small portfolios and large portfolios. In other words, the proposed model is reliable not only for a portfolio on specific stocks but also for a large portfolio. Third, the model can be regarded as the extension of that of Kupiec (1999) and Chen and Liao (2009).

1. Introduction

With liberalization and globalization of capital markets, foreign currency assets circulate rapidly around the world. In Taiwan, the official monthly statistic reports illustrate that the average percentage of investment in foreign assets relative to domestic assets has been approximately 46% at domestic commercial banks in the recent ten years. In Japan, the ratio is at least 5%, and around 9% in Korea. On average, the weight of foreign assets is around 20% at Asian banks, and the percentage is growing. Thus, controlling the market risk of portfolios composed of domestic assets and foreign assets is an increasing concern for financial institutions.

The VaR approach, which is defined as the maximal loss over a fixed target horizon with a given probability, is widely viewed as a measure of the market risk of portfolios. Portfolios may consist of options and other derivatives. Using the VaR measure, Hofmann and Platen [1] consider the market risk of a large diversified portfolio in which the returns' dynamic is distributed in normal diffusion.[1] Equally, the asset price follows a lognormal distribution. Substantial evidence exists in the empirical financial economic literature of the existence of jumps in equity returns and foreign exchange rates such as the works of Heston [2], Bakshi et al. [3], and Broadie et al. [4]. Therefore, the lognormal assumption is, in actuality, contrary to real life. Daily changes in many variables, especially in exchange rates, illustrate significantly positive kurtosis. This means that the probability distributions of asset returns have fat tails or discontinuity.[2] Gibson [5] demonstrates that event risk poses large jumps to fat tails in market prices. Gibson incorporates event risk into VaR for a portfolio. Differing from the assumption held by Hofmann and Platen [1], Guan et al. [6] consider jump-diffusion asset returns to model large diversified portfolios. As stated above, the literature focuses on portfolios valued only in one currency. However, it is a common phenomenon for institutional investors and individual investors to invest in portfolios comprising a number of domestic-valued assets and foreign-valued assets in highly integrated global financial markets, called international portfolios. Thus, exchange rate risk should be considered in high international investment.

This paper aims to present analytical VaRs of a portfolio including domestic-issued and foreign-issued assets. Using the framework provided by Merton [7], we employ return jumps at the Poisson arrivals to avoid the assumption of normality of asset returns. Also, the Brownian motions of between-jump returns are correlated. An analytical formula of the VaR is then derived. In general, the solution is more accurate than nonparametric techniques often adopted in fat-tail distributions in terms of the system infrastructure and computation time. In addition, this model can be also applied to large portfolios. Compared with that of Hofmann and Platen [1] and Guan et al. [6], it considers not only jumps but also exchange rate risk. This model is more suitable for the global capital markets.

The rest of this paper is organized as follows. The next section outlines the model, and an analytic formula of the value at risk is derived. In the third section, a comparative static analysis on the risk capital measured by the VaR approach is provided. In Section 4, we actually employ two international portfolios including a specific small portfolio and a large portfolio to estimate 1 day VaR (99%). Using in-sample, Section 5 inspects the model accuracy in terms of the usual backtesting criterion and Christoffersen's independence test [8] over the subprime mortgage crisis of August 2007. The samples in this study span from January 1, 2004 to November 27, 2009, or 1367 daily log returns of a line of stock prices and stock indices. From the Taiwanese perspective, a specific small portfolio on a domestic stock traded in Taiwan and a foreign stock traded in USA is used, and a large portfolio is made up of a domestic stock index (Taiwan Stock Exchange Capitalization Weighted Stock Index, TAIEX) and a foreign stock index (S&P 500). The last section provides conclusions.

2. Model Formulation

First, this paper assumes the following: (i) a value of an international portfolio is made up of the value of n kinds of domestic assets with $m_{i,t}$ shares and n classes of foreign assets with $g_{i,t}$ shares for each $i \in \{1, 2, \ldots, n\}$; (ii) the capital market is a complete market with no transaction cost or tax; (iii) there exists a riskless interest rate for lenders and borrowers; (iv) the dynamics of domestic asset returns and exchange rate returns follow Poisson-jump diffusion over the interval of interest; foreign asset returns are distributed in normality, (v) exchange rates are quoted at the price of one unit of the foreign currencies in domestic dollars, and (vi) investment strategies do not vary over an investment horizon. The dynamic processes of asset price and exchange rates are demonstrated as follows, respectively,

$$\frac{dA_{d_i,t}}{A_{d_i,t}} = (\mu_{d_i} - \lambda v)dt + \sigma_{d_i} dW_{1,t} + (\pi - 1)dY_t, \tag{2.1}$$

$$\frac{dA_{f_i,t}}{A_{f_i,t}} = \mu_{f_i} dt + \sigma_{f_i} dW_{2,t}, \tag{2.2}$$

$$\frac{de_{i,t}}{e_{i,t}} = (\mu_{e_i} - \lambda v)dt + \sigma_{e_i} dW_{3,t} + (\pi - 1)dY_t, \tag{2.3}$$

where μ_{d_i}, μ_{f_i}, and μ_{e_i} denote constant drift rates of domestic asset returns, foreign asset returns and exchange rate returns for each $i \in \{1, 2, \ldots, n\}$, respectively; σ_{d_i}, σ_{f_i}, and $\sigma_{e_i,t}$ stand for constant volatilities of domestic asset returns, foreign asset returns and exchange rate returns for each $i \in \{1, 2, \ldots, n\}$, respectively. The $W_{j,t}$ are one-dimensional Brownian motions defined in a filtered probability space (Ω, F, P) under the original probability measure, P for all $j = 1, 2, 3$. Also, the correlation coefficients among the three Brownian motions are defined as $\mathrm{corr}(dW_{1,\,t}, dW_{2,\,t}) = \rho_{1,\,2}$, $\mathrm{corr}(dW_{2,\,t}\,, dW_{3,\,t}) = \rho_{2,3}$, and $\mathrm{corr}(dW_{1,\,t}, dW_{3,\,t}) = \rho_{1,\,3}$.[3] Then, Y_t is the independent Poisson process with the intensity λ at time t; dY_t is independent of $dW_{j,t}$ for all $j = 1, 2, 3$. The v represents $E[\pi - 1]$ where $\pi - 1$ is the random variable percentage in domestic assets or exchange rates resulting from a jump, and $E[\cdots]$ is the symbol of the expectation operator over the random variable Y_t. Assume that the nature logarithm of π, the jump amplitude if the Poisson event occurs, follows normal distributions with the mean u_π and variance σ_π^2, namely, $\ln \pi \sim N(u_\pi, \sigma_\pi^2)$. Therefore, $v = E[\pi - 1] = \exp[u_\pi + (1/2)\sigma_\pi^2] - 1$.

Now, consider the potential daily loss exposure to long trading positions. Typically, the VaR is a specific left-hand critical value of a potential loss distribution. Given conventions, one can define the daily losses valued in domestic dollars relative to the end-of-period expected asset value (relative VaR) and the initial asset value (absolute VaR), denoted by VaR(mean) and VaR(0) as follows, respectively:

$$\begin{aligned} \mathrm{VaR(mean)} &\equiv E_t(V_T) - V_\alpha, \\ \mathrm{VaR}(0) &\equiv V_0 - V_\alpha, \end{aligned} \tag{2.4}$$

in which $E_t(\cdot)$ is the expected value conditional on information at time t, V_α is the value of an international portfolio denominated in domestic dollars given a percentile of α, and V_T is the portfolio value at time T (investment horizon), which consists of n kinds of domestic assets and foreign assets, equally $V_T = \sum_{i=1}^n m_i A_{d_i,T} + \sum_{i=1}^n g_i e_{i,T} A_{f_i,T}$. Which definition of value at risk provides a more suitable measure of risk capital allocation over investment horizon? Kupiec ([9, page 43]) demonstrates that the absolute VaR is more appropriate measure of an asset's risk of posting losses. Thus, we adopt the measure throughout this paper.

Before the derivation of the VaR analytic formula for an international portfolio, it is necessary to employ the following propositions.

Proposition 2.1. *Given the dynamic processes of foreign currency denominated asset price and exchange rate following the Geometric Brownian motion, the dynamic process of* $\sum_{i=1}^{n} g_i e_{i,t} A_{f_i,t}$ *can be expressed as*

$$\frac{dX_{i,t}}{X_{i,t}} = \sum_{i=1}^{n} g_i \left[\left(\mu_{f_i} + \mu_{e_i} - \lambda v + \rho_{2,3}\sigma_{f_i}\sigma_{e_i} \right) dt + \sigma_{f_i} dW_{2,t} + \sigma_{e_i} dW_{3,t} + (\pi - 1) dY_t \right], \tag{2.5}$$

with $X_{i,t} = \sum_{i=1}^{n} g_i e_{i,t} A_{f_i,t}$.

Appendix A provides a detailed proof of Proposition 2.1.

Proposition 2.2. *Given the dynamic processes of asset price and exchange rates, the dynamic process of* V_t *can be expressed as*

$$\begin{aligned} \frac{dV_t}{V_t} = {} & \left[\sum_{i=1}^{n} \gamma_{i,t} \left(\mu_{d_i} - \lambda v \right) + \sum_{i=1}^{n} \beta_{i,t} \left(\mu_{e_i} + \mu_{f_i} - \lambda v + \rho_{2,3}\sigma_{f_i}\sigma_{e_i} \right) \right] dt \\ & + \sum_{i=1}^{n} \gamma_{i,t}\sigma_{d_i} dW_{1,t} + \left[\sum_{i=1}^{n} \beta_{i,t}\sigma_{f_i} \right] dW_{2,t} + \left[\sum_{i=1}^{n} \beta_{i,t}\sigma_{e_i} \right] dW_{3,t} \end{aligned} \tag{2.6}$$

with $V_t = \sum_{i=1}^{n} m_i A_{d_i,t} + \sum_{i=1}^{n} g_i e_{i,t} A_{f_i,t}$, $\gamma_{i,t} = m_i A_{d_i,t}/V_t$, *and* $\beta_{i,t} = g_i\ e_{i,t} A_{f_i,t}/V_t$. $\gamma_{i,t}$ *and* $\beta_{i,t}$ *are also named the weights of the ith kind of domestic asset and foreign asset, respectively.*

Appendix B provides a detailed proof of Proposition 2.2. Conditional on the assumption (vi), the weights can be regarded as constant over the investment horizon.

Using the previous propositions, one can fast obtain the approximation of the absolute VaR by utilizing the analytic formula below:

$$\sum_{k=0}^{\infty} \frac{\exp[-\lambda T][\lambda T]^k}{k!} \Phi\left(\frac{\ln[V_0 + \mathrm{VaR}(0)] - \ln V_0 - \left(\mu_t - (1/2)\sigma_t^2 \right) T - k u_\pi}{\sqrt{\sigma_t^2 T + k\sigma_\pi^2}} \right) = \alpha, \tag{2.7}$$

in which the $Z_{\boldsymbol{\alpha}}$ stands for a critical value with a given probability α and V_0 represents the initial value of an international portfolio, $v = E[\pi - 1] = \exp[u_\pi + (1/2)\sigma_\pi^2] - 1$ and

$$\begin{aligned} \mu_t &= \sum_{i=1}^{n} \gamma_{i,t} \left(\mu_{d_i} - \lambda\ v \right) + \sum_{i=1}^{n} \beta_{i,t} \left(\mu_{f_i} + \mu_{e_i} - \lambda\ v + \rho_{2,3}\sigma_{f_i}\sigma_{e_i} \right), \\ \sigma_t^2 &= \left(\sum_{i=1}^{n} \gamma_{i,t}\sigma_{d_i} \right)^2 + \left(\sum_{i=1}^{n} \beta_{i,t}\sigma_{f_i} \right)^2 + \left(\sum_{i=1}^{n} \beta_{i,t}\sigma_{e_i} \right)^2 + 2\ \rho_{1,2} \left(\sum_{i=1}^{n} \gamma_{i,t}\sigma_{d_i} \right) \left(\sum_{i=1}^{n} \beta_{i,t}\sigma_{f_i} \right) \\ &\quad + 2\ \rho_{2,3} \left(\sum_{i=1}^{n} \beta_{i,t}\sigma_{f_i} \right) \left(\sum_{i=1}^{n} \beta_{i,t}\sigma_{e_i} \right) + 2\ \rho_{1,3} \left(\sum_{i=1}^{n} \gamma_{i,t}\sigma_{d_i} \right) \left(\sum_{i=1}^{n} \beta_{i,t}\sigma_{e_i} \right). \end{aligned} \tag{2.8}$$

Equation (2.7) is provided in Appendix C.

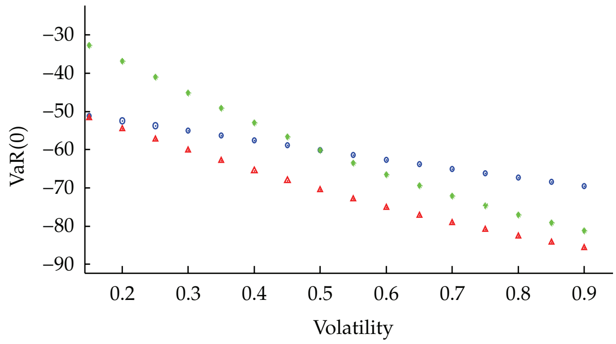

Figure 1: The impact of volatility on VaR. Note that the symbols "∘", "∗", and "△" represent the impact of volatilities of domestic assets, foreign assets, and exchange rates on absolute VaRs, respectively.

By means of (2.7), one can efficiently obtain the approximation of the VaR capital allocation for an international portfolio. The approximated analytical VaR includes some essential elements, such as volatility of underlying assets, volatility of exchange rates, the correlation coefficients, the weights of domestic assets and foreign assets, and the intensity of jumps. Also, (2.7) can be reduced to the analytic solution of Kupiec [9][4] as $\gamma_{i,t} = 1, \beta_{i,t} = 0$, $n = 1$, $\lambda = 0$, and $dY_t = 0$. This case represents that a firm value is only composed of a kind of domestic asset without jumps. Alternatively, (2.7) goes to the closed-form solution of Chen and Liao [10][5] as $\gamma_{i,t} = 0, \beta_{i,t} = 1$, $n = 1$, $\lambda = 0$, and $dY_t = 0$, which means a firm value is made up of only a kind of foreign asset with no jumps. Also, the presented model can be regarded as the extension of Kupiec [9] and Chen and Liao [10].

3. Numerical Analysis

In this section, sensitivity analyses of the impacts of important parameters on VaR capital will be performed in terms of comparative statics. We start by assuming the following: (i) the value of a firm is made up of a line of a domestic asset and a foreign asset, and the exchange rate is the ratio of the domestic currency to the foreign currency; (ii) the initial value of an international portfolio is \$100; (iii) the critical value is −2.33 at a given α of 0.01, and the investment horizon is one year ($T = 1$); (iv) the number of jumps is five; (v) $\lambda = 5$, $u_\pi = 0.5$, $\sigma_\pi^2 = 0.3$, $\mu_{d_1} = 0.1$, $\mu_{f_1} = 0.5$, $\mu_{e_1} = 0.4$, $\rho_{1,2} = 0.5$, $\rho_{1,3} = 0.5$, $\rho_{2,3} = 0.5$, $\gamma_{1,t} = 0.3$, $\beta_{1,t} = 0.7$, $\sigma_{d_1} = 0.5$, $\sigma_{f_1} = 0.5$ and $\sigma_{e_1} = 0.3$.

According to (2.7), the effects of volatilities, correlation coefficients, and the intensities of jumps on the absolute VaR capital allocation are shown in Figures 1, 2, and 3, respectively. There is one common phenomenon exhibited in these figures: the loss amount increases monotonically as volatilities, correlation coefficients and the intensities of jumps rise. As shown in Figures 1 and 2, the sensitivities of the volatility of foreign assets and the correlation coefficient between foreign assets and exchange rates are higher than those of the others. Additionally, Figure 4 illustrates the relationship between the VaR and the weights of domestic assets shapes in hump. Also, the loss amount declines as the weights of foreign assets rise at around 0.5.

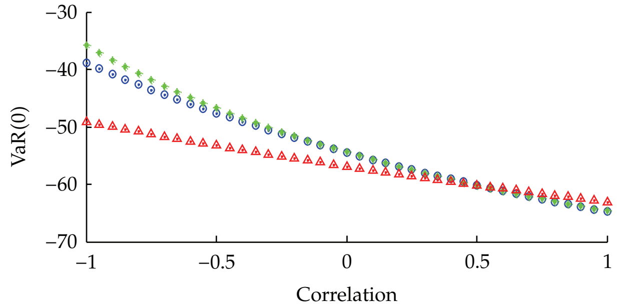

Figure 2: The impact of correlation coefficients on VaR. Note that the symbols "∘", "∗", and "Δ"represent the impact of correlation coefficients, $\rho_{1,2}$, $\rho_{2,3}$, $\rho_{1,3}$ on absolute VaRs, respectively.

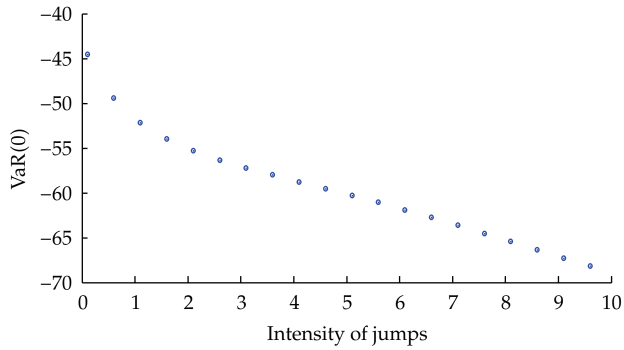

Figure 3: The impact of the intensity of jumps on VaR.

4. Measurement of Value at Risk

For simplicity, this section simply considers the long trading positions of two international portfolios which are a small portfolio and a large portfolio. From the Taiwanese perspective, a small international portfolio includes one domestic-issued stock valued in New Taiwan dollars and one foreign-issued stock valued in U.S. dollars, and a large international portfolio comprises a domestic stock index and a foreign stock index. We then want to know the absolute VaR of the two international portfolios valued in New Taiwan dollars.

4.1. Source of the Data

Assume that a small international portfolio includes two specific domestic and foreign stocks which are TSMC and MSFT, respectively, and a large international portfolio consists of a domestic index, namely, Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and a foreign index of Standard and Poor's Index (S&P 500). The stocks, TSMC, are issued by Taiwan Semiconductor Manufacturing Company Limited and traded in Taiwan; MSFT are issued by Microsoft and traded in USA. We use daily log returns of TAIEX, S&P 500, TSMC, and MSFT in the sample. The S&P 500 has been widely regarded as the best single gauge of the large cap U.S. equities market since the index was first published in 1957. The index is made up of 500 leading companies in leading industries of the U.S. economy, capturing 75% coverage of U.S. equities. The TAIEX is the most widely quoted of

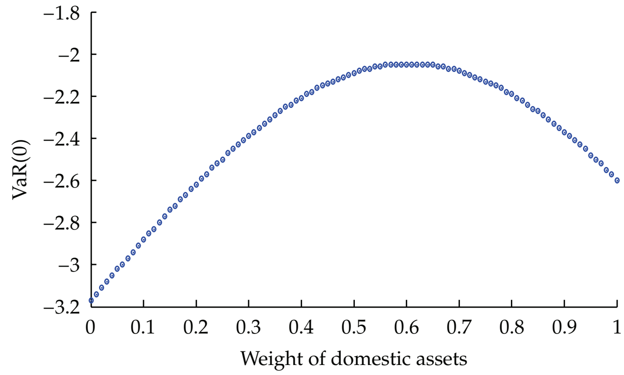

Figure 4: The impact of weights of domestic assets on VaR.

all Taiwan Stock Exchange Corporation indices, which are similar to the Standard & Poor's Index, weighted by the number of outstanding shares. All of these securities and indices are well known to institutional and individual investors in the world. The time window length is the period from January 1, 2004 to November 27, 2009, so that the total of the daily log returns of each asset is 1367. The source of these securities comes from the website of http://www.finance.yahoo.com/. All of the samples span two periods, labelled Period I and Period II. Period I is from January 1, 2004 to July 31, 2007, during which the subprime mortgage crisis had not yet occurred with the total daily log returns being 780. Alternatively, Period II is from August 1, 2007 to November 27, 2009 with a total of 587, which is across the subprime mortgage crisis of August 2007.

4.2. Estimation of Model Parameters

Before evaluation, it is necessary to estimate a set of model parameters for various samples. Assume the number of jumps is ten; $u_\pi = 0.05$, $\sigma_\pi^2 = 0.001$, and $\lambda = 0.03$ for Period I, and $u_\pi = 0.055$, $\sigma_\pi^2 = 0.002$, and $\lambda = 0.035$ for Period II. From the data, the sample means and standard deviations of TSMC, MSFT, TAIEX, S&P 500 and exchange rates in one day are shown in Table 1. Since 1 trading day is equivalent to 1/252 year, one can obtain the sample means and variance of these random variables per annum, which are all multiplied by 252 from Panel A in Table 1, respectively. The results are stated in Panel B in Table 1.

From (2.1), (2.2), and (2.3), the dynamic processes of log returns of random variables in domestic assets, foreign assets and exchange rates can be derived as (4.1), respectively

$$\begin{gathered} d\left(\ln A_{d_1,t}\right) = \left(\mu_{d_1} - \frac{1}{2}\sigma_{d_1}^2 - \lambda v\right)dt + \sigma_{d_1} dW_{1,t} + (\pi - 1)dY_t, \\ d\left(\ln A_{f_1,t}\right) = \left(\mu_{f_1} - \frac{1}{2}\sigma_{f_1}^2\right)dt + \sigma_{d_1} dW_{2,t}, \\ d\left(\ln e_{1,t}\right) = \left(\mu_{e_1} - \frac{1}{2}\sigma_{e_1}^2 - \lambda v\right)dt + \sigma_{e_1} dW_{3,t} + (\pi - 1)dY_t. \end{gathered} \tag{4.1}$$

Table 1: Sample mean and standard deviation of daily log returns of securities and exchange rates in various periods.

Variables	Period I 2004/1/1–2007/7/31 $E[d(\ln H_t)]$	$\sigma(d\ln H_t)$	Period II 2007/8/1–2009/11/27 $E[d(\ln H_t)]$	$\sigma(d\ln H_t)$
Panel A: sample mean and standard deviation of daily log returns				
TSMC	0.000656679	0.015517716	0.000071254	0.010939352
MSFT	0.000064521	0.011705647	0.000034994	0.025837488
TAIEX	0.000251563	0.005154917	−0.000143239	0.008799451
S&P 500	0.000148501	0.003132479	−0.000240238	0.025837488
NTD/USD	−0.000041458	0.002526791	−0.000025263	0.003041025
Panel B: sample mean and variance of log returns per annum				
TSMC	0.165483108	0.060681476	0.017955756	0.030156694
MSFT	0.016259292	0.034529587	0.008818236	0.168229098
TAIEX	0.063393876	0.006696439	−0.036086411	0.019510445
S&P 500	0.037422252	0.002472731	−0.060539976	0.02203105
NTD/USD	−0.010447416	0.001608938	−0.006458256	0.002330454

Note that $E[d(\ln H_t)]$ and $\sigma(d\ln H_t)$ represent the sample means and standard deviations of daily log returns of domestic assets (indices), foreign assets (indices), and exchange rates for all $H_t = A_{d_1,t}, A_{f_1,t}, e_{1,t}$, respectively. Panel B displays the sample means and variances of domestic assets (indices), foreign assets (indices), and exchange rates per annum all multiplied by 252 from Panel A, respectively.

Table 2: Parameter estimation of dynamic processes of asset returns and exchange rate returns in various periods.

Security and exchange rate	Period I 2004/1/1–2007/7/31 μ_i	σ_i	Period II 2007/8/1–2009/11/27 μ_i	σ_i
TSMC	0.1959	0.2463	0.0331	0.1735
MSFT	0.1858	0.1799	0.0929	0.4102
TAIEX	0.0668	0.0816	−0.0263	0.1394
S&P 500	0.0387	0.0497	−0.0495	0.1484
NTD/USD	−0.0096	0.0397	−0.0052	0.0475

Assume the number of jumps is ten. Given $u_\pi = 0.05$, $\sigma_\pi^2 = 0.001$, and $\lambda = 0.03$ for Period I, and $u_\pi = 0.055$, $\sigma_\pi^2 = 0.002$, and $\lambda = 0.035$ for Period II, μ_i and σ_i demonstrate the estimation of drift terms and volatilities of asset (index) returns and foreign exchange returns for $i = d_1$, f_1, and e_1, respectively.

Furthermore, the estimated results of μ_{d_1} and μ_{e_1} can be determined as $E[d(\ln H_t)]+(1/2)\sigma_i^2+\lambda v$ with $v = \exp[u_\pi + (1/2)\sigma_\pi^2] - 1$ for all $H_t = A_{d_1,t}, e_{1,t}$, and $i = d_1, e_1$, respectively. As for the estimation of μ_{f_1}, it is equal to $E[d(\ln A_{f_1,t})] + (1/2)\sigma_{f_1}^2$. Similarly, σ_{d_1}, σ_{f_1}, and σ_{e_1} can be, respectively, estimated through the variances of (4.1) because $\mathrm{Var}[d(\ln A_{d_1,t})] = \sigma_{d_1}^2 dt + \sigma_\pi^2\lambda$, $\mathrm{Var}[d\ (\ln A_{f_1,t})] = \sigma_{f_1}^2 dt$, and $\mathrm{Var}[d\ (\ln A_{e_1,t})] = \sigma_{e_1}^2 dt+\sigma_\pi^2\lambda$. Finally, the estimated results of μ_{d_1}, μ_{f_1}, μ_{e_1}, σ_{d_1}, σ_{f_1} and σ_{e_1} are presented in Table 2.

In addition, Table 3 reports the estimations of the correlation coefficients between each asset (index) and exchange rates in various periods.

Table 3: Estimation of the correlation coefficients in various periods.

Correlation coefficients	Period I 2004/1/1–2007/7/31	Period II 2007/8/1–2009/11/27
Panel A: for a small portfolio on specific domestic and foreign stocks		
$\rho_{1,2}$	0.0162447	0.0458875
$\rho_{2,3}$	0.0023724	0.0333177
$\rho_{1,3}$	−0.0066891	−0.1399387
Panel B: for a large portfolio on a domestic stock index and a foreign stock index		
$\rho_{1,2}$	0.155207	0.206361
$\rho_{2,3}$	0.052375	0.081776
$\rho_{1,3}$	−0.056823	−0.0249387

Note that $\rho_{1,2}$, $\rho_{2,3}$, and $\rho_{1,3}$ denote the correlation coefficients between domestic assets (indices) and foreign assets (indices), foreign assets (indices) and exchange rates (NTD/USD), domestic assets and exchange rates, respectively.

4.3. Calculation of VaR

After the estimation of model parameters, we can fast obtain a one-day VaR at a 0.01 significance level for the two international portfolios consisting of a small portfolio on TSMC and MSFT and a large portfolio on TAIEX and S&P 500 indices through (2.7). These results are summarized in Tables 4 and 5 given that the jump number is 10, $Z_{0.01} = -2.33$ at a 0.01 quantile, $T = 1/252$ and $V_0 = 1$ (initial investment); $u_\pi = 0.05$, $\sigma_\pi^2 = 0.001$, and $\lambda = 0.03$ for Period I, and $u_\pi = 0.055$, $\sigma_\pi^2 = 0.002$, and $\lambda = 0.035$ for Period II. Clearly, there exists a common phenomenon—the maximum losses of initial investment of 1 New Taiwan dollar in Period I are fewer than those in Period II as the weights of foreign assets increase as shown in Tables 4 and 5 for the small portfolio and the large portfolio. This indicates it is necessary for a firm to maintain a sufficient capital amount in order to prevent default risk during the subprime mortgage crisis period. In addition, it can decrease the losses of small and large portfolios for investors to decline weights of foreign assets during the subprime mortgage crisis period.

Alternatively, historical simulation approach is employed to calculate VaRs of the two international portfolios under no specific assumptions about the distribution of risk factors. The approach simply samples from the recent historical data. Given a 0.01 significance level, the one-day VaRs are shown in Tables 4 and 5 in terms of historical simulation approach. Tables 4 and 5 consistently demonstrate that the losses valued by the analytical VaR are higher than those by the historical simulation approach in various domestic weights during both Period I and Period II. If financial managers adopt the historical simulation approach to evaluate financial risk, the firm's financial ratio such as ROE is better, but the default probability of the firm may increase on the account of a shortage of sufficient capital requirement. Hence the conservative policy of the analytical VaR model would be suitable for financial institutions to control market risk.

5. Evaluation of Model Accuracy

Backtesting is a widely used method of evaluating VaR accuracy. However, the criterion neglects conditioning or the important concept of violation clustering in the data. Thus Christoffersen's independence test [8] is provided in order to consider the data conditional on

Table 4: Model accuracy using backtesting for small portfolios in various weights of domestic assets.

	Period I 2004/1/1–2007/7/31				Period II 2007/8/1–2009/11/27			
Domestic Weights	Sample number	VaR	Number of exceptions	LR_{uc}	Sample number	VaR	Number of exceptions	LR_{uc}
Panel A: based on historical simulations								
0	780	−0.0438	11	1.1763	587	−0.0694	6	0.1371
0.1	780	−0.0409	8	0.0051	587	−0.0598	8	0.7012
0.2	780	−0.0396	10	0.5755	587	−0.0513	8	0.7012
0.3	780	−0.0381	8	0.0051	587	−0.0466	5	0.1371
0.4	780	−0.0369	6	0.4558	587	−0.0412	5	0.1371
0.5	780	−0.0356	10	0.5755	587	−0.0398	10	2.4240
0.6	780	−0.0368	10	0.5755	587	−0.0389	10	2.4240
0.7	780	−0.0375	11	1.1763	587	−0.0386	8	0.7012
0.8	780	−0.0378	13	2.9166	587	−0.0385	8	0.7012
0.9	780	−0.0375	13	2.9166	587	−0.0383	8	0.7012
1	780	−0.0371	13	2.9166	587	−0.0380	8	0.7012
Panel B: based on analytical VaR								
0	780	−0.0533	6	0.4558	587	−0.0804	0	11.7990*
0.1	780	−0.0481	6	0.4558	587	−0.0728	1	6.2410*
0.2	780	−0.0436	6	0.4558	587	−0.0659	2	3.4589
0.3	780	−0.0401	6	0.4558	587	−0.0598	2	3.4589
0.4	780	−0.0377	4	2.2760	587	−0.0549	2	3.4589
0.5	780	−0.0366	7	0.0858	587	−0.0516	3	1.7267
0.6	780	−0.0371	7	0.0858	587	−0.0503	2	3.4589
0.7	780	−0.0389	9	0.1777	587	−0.0501	2	3.4589
0.8	780	−0.0419	9	0.1777	587	−0.0502	3	1.7267
0.9	780	−0.0458	9	0.1777	587	−0.0487	4	0.6775
1	780	−0.0504	9	0.1777	587	−0.0547	6	0.0029

Note that this table displays backtesting in terms of in-sample fitting for alternative weights of domestic assets. The critical value is 3.84 at a significant level of 5%. The symbol * denotes the significance at a 5% level. The VaRs are the maximum losses of the initial investment of 1 New Taiwan dollar (NTD) over a one-day horizon.

Table 5: Model accuracy using backtesting for large portfolios in various weights of domestic assets.

	Period I 2004/1/1–2007/7/31				Period II 2007/8/1–2009/11/27			
Domestic weights	Sample number	VaR	Number of exceptions	LR_{uc}	Sample number	VaR	Number of exceptions	LR_{uc}
Panel A: based on historical simulations								
0	780	−0.0374	8	0.0051	587	−0.0438	10	2.4240
0.1	780	−0.0354	8	0.0051	587	−0.0431	10	2.4240
0.2	780	−0.0316	8	0.0051	587	−0.0429	10	2.4240
0.3	780	−0.0305	10	0.5755	587	−0.0426	11	3.6023
0.4	780	−0.0297	12	1.9617	587	−0.0421	11	3.6023
0.5	780	−0.0312	10	0.5755	587	−0.0419	11	3.6023
0.6	780	−0.0301	10	0.5755	587	−0.0413	11	3.6023
0.7	780	−0.0289	11	0.1763	587	−0.0411	11	3.6023
0.8	780	−0.0273	11	0.1763	587	−0.0407	12	4.9661*
0.9	780	−0.0269	11	0.1763	587	−0.0400	12	4.9661*
1	780	−0.0255	11	0.1763	587	−0.0397	12	4.9661*
Panel B: based on analytical VaR								
0	780	−0.0598	6	0.4558	587	−0.0569	8	0.7012
0.1	780	−0.0526	6	0.4558	587	−0.0551	8	0.7012
0.2	780	−0.0466	6	0.4558	587	−0.0523	8	0.7012
0.3	780	−0.0400	8	0.0051	587	−0.0504	10	2.4240
0.4	780	−0.0398	8	0.0051	587	−0.0489	10	2.4240
0.5	780	−0.0415	8	0.0051	587	−0.0480	10	2.4240
0.6	780	−0.0450	7	0.0858	587	−0.0476	10	2.4240
0.7	780	−0.0403	5	0.1633	587	−0.0471	10	2.4240
0.8	780	−0.0399	5	0.1633	587	−0.0462	11	3.6023
0.9	780	−0.0388	5	0.1633	587	−0.0451	11	3.6023
1	780	−0.0374	5	0.1633	587	−0.0449	11	3.6023

Note that this table displays backtesting in terms of in-sample fitting for alternative weights of domestic indices. The critical value is 3.84 at a significant level of 5%. The symbol ∗ denotes the significance at a 5% level. The VaRs are the maximum losses of the initial investment of 1 New Taiwan dollar (NTD) over a one-day horizon.

Table 6: Model accuracy using christoffersen's independence test for a small portfolio.

States	Day Before		
	$I_{t-1} = 0$	$I_{t-1} = 1$	Unconditional
	Historical simulations		
Current day	Panel A: Period I		
$I_t = 0$	410	125	535
$I_t = 1$	110	35	145
Total	520	160	780
	Panel B: Period II		
$I_t = 0$	380	67	447
$I_t = 1$	120	20	140
Total	500	87	587
	Analytical VaR		
Current day	Panel C: Period I		
$I_t = 0$	352	232	584
$I_t = 1$	125	71	196
Total	477	303	780
	Panel D: Period II		
$I_t = 0$	318	115	433
$I_t = 1$	99	55	154
Total	516	170	587

Note that this table displays Christoffersen's independence test [8] for the weights of domestic assets of 0.5. The data samples are split into two parts. The 1-day VaRs are estimated from January 1, 2004 to July 31, 2007 (Period I). The second part is used to forecast VaRs from August 1, 2007 to November 27, 2009 (Period II). $I_t = 0$ represents an indicator function if VaR is not exceeded at time t; otherwise; $I_t = 1$ if VaR is exceeded at time t. The critical value is 3.84 at a significant level of 5 percent.

current conditions. This section will perform backtesting and Christoffersen's independence test [8] to examine the statistical sufficiency of the proposed model for a small international portfolio and a large international portfolio on the basis of the results estimated above. Moreover, we compare the accuracy of the analytical VaR derived from (2.7) with that of the historical simulation in terms of backtesting and the Christoffersen's independence test [8] for the two international portfolios.

5.1. Backtesting Criterion

The usual backtesting techniques consider the number of violations at which the losses are larger than VaR. The proportion of times should be equal to one minus the VaR confidence level; in other words, the model should provide the correct unconditional coverage. In order to test the null hypothesis that the unconditional coverage equals the significant level, Kupiec [11] presents a likelihood ratio statistic. Given a VaR at the 1-percent level left tail over daily horizon for a total of D, one can count how many times the actual loss exceeds one day's VaR. Define d as the number of exceptions and d/D as the exception rate. The null hypothesis is that a given confidence level for losses is the true probability. Kupiec [11] approximates 95% confidence regions,

Table 7: Model accuracy using christoffersen's independence test for a large portfolio.

States	Day before		
	$I_{t-1} = 0$	$I_{t-1} = 1$	Unconditional
	Historical simulations		
Current day	Panel A: Period I		
$I_t = 0$	580	88	668
$I_t = 1$	87	25	112
Total	667	113	780
	Panel B: Period II		
$I_t = 0$	436	66	502
$I_t = 1$	66	19	85
Total	502	85	587
	Analytical VaR		
Current Day	Panel C: Period I		
$I_t = 0$	484	112	596
$I_t = 1$	153	31	184
Total	637	143	780
	Panel D: Period II		
$I_t = 0$	436	56	492
$I_t = 1$	80	15	95
Total	516	71	587

Note that this table displays Christoffersen's independence test [8] for the weights of domestic assets of 0.5. The data samples are split into two parts. The 1-day VaRs are estimated from January 1, 2004 to July 31, 2007 (Period I). The second part is used to forecast VaRs from August 1, 2007 to November 27, 2009 (Period II). $I_t = 0$ represents an indicator function if VaR is not exceeded at time t; otherwise; $I_t = 1$ if VaR is exceeded at time t. The critical value is 3.84 at a significant level of 5 percent.

denoted by q for the test. The unconditional coverage is defined by the log-likelihood ratio:

$$\mathrm{LR}_{\mathrm{uc}} = -2\ln\left[(1-q)^{D-d}q^{d}\right] + 2\ln\left\{\left[1-\frac{d}{D}\right]^{D-d}\left(\frac{d}{D}\right)^{d}\right\}. \tag{5.1}$$

The $\mathrm{LR}_{\mathrm{uc}}$ statistic has a chi-square distribution with one degree of freedom. One would reject the null hypothesis if $\mathrm{LR}_{\mathrm{uc}} > 3.84$ at a 95% confidence level. The test procedure described above is called backtesting.

Assume that the jump number is 10, $Z_{0.01} = -2.33$ at a 0.01 quantile, $T = 1/252$ and $V_0 = 1$ (initial investment); $u_\pi = 0.05$, $\sigma_\pi^2 = 0.001$, and $\lambda = 0.03$ for Period I, and $u_\pi = 0.055$, $\sigma_\pi^2 = 0.002$, and $\lambda = 0.035$ for Period II. In in-sample fitting, the time window length is the period from January 1, 2004 to November 27, 2009, which is broken into two periods, labelled Period I and Period II. Period I is from January 1, 2004 to July 31, 2007, during which the subprime mortgage crisis had not yet occurred; Period II is from August 1, 2007 to November 27, 2009. Tables 4 and 5 demonstrate that using the historical approach, the null hypothesis for a small portfolio is accepted at a significance level of 5% in various weights during Period I, while it is not accepted for a large portfolio during Period II. The accuracy of the analytical VaR is higher than that of the historical approach during Period II as shown in Table 5.

To summarize, the VaR model presented by this paper can be almost used to accurately calculate VaRs with various domestic weights for the two international portfolios based on the backtesting criterion.

5.2. Christoffersen's Independence Test

In 1998, Christoffersen extended the $\mathrm{LR}_{\mathrm{uc}}$ statistic to present interval forecast test for testing conditional coverage hypothesis. It can be done in a likelihood ratio, symbolized by $\mathrm{LR}_{\mathrm{ind}}$. The relevant test statistic is

$$\mathrm{LR}_{\mathrm{ind}} = -2\ln\left[\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right], \tag{5.2}$$

in which π_i stands for the probability of observing an exception conditional on state i the previous day; n_{ij} is defined as the number of days in which state j occurred in one day while it was at i the previous day and $\pi = (n_{01} + n_{11})/(n_{00} + n_{01} + n_{10} + n_{11})$. Each day we set a deviation indicator to 0 if VaR is not exceeded and to 1 otherwise. The critical value is 3.84 at the 95% percentile of the χ^2 distribution with one degree of freedom. One would reject the null hypothesis if $\mathrm{LR}_{\mathrm{ind}} > 3.84$.

The combined test statistic for conditional coverage is then

$$\mathrm{LR}_{\mathrm{cc}} = \mathrm{LR}_{\mathrm{uc}} + \mathrm{LR}_{\mathrm{ind}}. \tag{5.3}$$

The sum is distributed as $\chi^2(2)$. Thus one would reject the null hypothesis at the 95 percent test confidence level if $\mathrm{LR}_{\mathrm{cc}} > 5.991$. Assume the domestic weights are 0.5 for the small portfolio and the large portfolio.

Suppose the weights of domestic assets are 50% for a small portfolio and a large portfolio. As illustrated in Table 6, of 780 days, Panel A illustrates $n_{00} = 410$, $n_{01} = 110$, $n_{10} = 125$, and $n_{11} = 35$, which are the fractions of $\pi_0 = 0.2115$, $\pi_1 = 0.2188$ and $\pi = 0.2132$ in Period I, and Panel B exhibits, of 587 days, $n_{00} = 380$, $n_{01} = 120$, $n_{10} = 67$, and $n_{11} = 20$, which are the fractions of $\pi_0 = 0.24$, $\pi_1 = 0.2299$ and $\pi = 0.2385$ in Period II for a small portfolio through the historical simulation approach. Consequently, the statistics of $\mathrm{LR}_{\mathrm{ind}} = 0.0378$ and $\mathrm{LR}_{\mathrm{ind}} = 0.042$ can be, respectively, obtained in terms of the historical simulation in Period I and Period II. Similarly, from Panels C and D in Table 6, the statistics are $\mathrm{LR}_{\mathrm{ind}} = 0.7619$ and $\mathrm{LR}_{\mathrm{ind}} = 4.5129$ based on the analytical VaR for a small international portfolio in Period I and Period II, respectively. As for a large portfolio, $\mathrm{LR}_{\mathrm{ind}} = 5.8499$, $\mathrm{LR}_{\mathrm{ind}} = 4.4902$, $\mathrm{LR}_{\mathrm{ind}} = 0.3603$ and $\mathrm{LR}_{\mathrm{ind}} = 1.3659$ are found from Panels A, B, C and D in Table 7, respectively. As a result, the null hypothesis is almost accepted except Panel D in Table 6 for a small portfolio through the analytical VaR during the subprime mortgage crisis. However, as for a large portfolio, the null hypothesis cannot be accepted under the historical simulation method during in Period I and Period II.

On the other hand, as the domestic weights are 0.5, all the $\mathrm{LR}_{\mathrm{cc}}$-statistics based on the analytical VaR approach are consistently less than the cutoff value of 5.991 in any scenario, whereas the sum of $\mathrm{LR}_{\mathrm{uc}}$ and $\mathrm{LR}_{\mathrm{ind}}$ from the historical simulation criterion is higher than the critical value of 5.991 for a large portfolio as shown in Tables 5 and 7. Generally speaking, these results disclose that the accuracy of the analytical VaR model is more reliable than that of the historical simulation in terms of Christoffersen's Independence Test. Hence, the analytical VaR model can efficiently evaluate the market risk of a small portfolio and a large international portfolio distributed in nonnormality in terms of backtesting and Christoffersen's independence test [8] over the subprime mortgage crisis.

6. Conclusion

One advantage of VaR is that it is an intuitively appealing measure of risk that can be easily conveyed to a firm's senior manager. The measure most commonly used assumes that the probability distribution of daily asset returns is normal. However, this assumption is far from conditions in the actual world. This paper provides a mixed Poisson-jump model for an international portfolio to manage market risk, in particular the subprime mortgage crisis of August 2007. Differing from past studies whose portfolios were valued only in one currency, this model considers portfolios not only with jumps but also with exchange rate risk. It is vital for investors to consider exchange rate risk in highly integrated global financial markets.

Additionally, in terms of backtesting and Christoffersen's independence test [8], the finding is that the model is more capable of accurately reflecting the loss probability of 1% than the historical simulation approach. Especially for a large portfolio, the proposed method in this paper is a more efficient way in the presence of asymmetric and fat-tail portfolio returns during periods of financial turbulence. In other words, the proposed model is reliable not only for a small portfolio on specific stocks but also for a large portfolio.

Appendices

A. The Derivation of Proposition 2.1

Let $X_{i,t} = \sum_{i=1}^{n} g_i e_{i,t} A_{f_i,t}$ with $g_{i,t}$ being foreign asset shares. Conditional on self-financing strategy and by means of Ito's lemma, one can obtain

$$\frac{dX_{i,t}}{X_{i,t}} = \sum_{i=1}^{n} g_i \left(\frac{dA_{f_i,t}}{A_{f_i,t}} + \frac{de_{i,t}}{e_{i,t}} + \frac{dA_{f_i,t}}{A_{f_i,t}} \cdot \frac{de_{i,t}}{e_{i,t}} \right). \tag{A.1}$$

Substituting the dynamic processes of foreign asset returns and exchange rates shown in (2.2) and (2.3) into (A.1), and (A.1) can be expressed as

$$\frac{dX_{i,t}}{X_{i,t}} = \sum_{i=1}^{n} g_i \left[\left(\mu_{f_i} + \mu_{e_i} - \lambda v + \rho_{2,3} \sigma_{f_i} \sigma_{e_i} \right) dt + \sigma_{f_i} dW_{2,t} + \sigma_{e_i} dW_{3,\,t} + (\pi - 1) dY_t \right]. \tag{A.2}$$

B. The Derivation of Proposition 2.2

Suppose that $V_t = \sum_{i=1}^{n} m_i A_{d_i,t} + \sum_{i=1}^{n} g_i e_{i,t} A_{f_i,t}$. Conditional on the assumption (vi) and using Ito's lemma, one can obtain

$$\frac{dV_t}{V_t} = \sum_{i=1}^{n} \gamma_{i,t} \frac{dA_{d_i,\,t}}{A_{A_{d_i,t}}} + \sum_{i=1}^{n} \beta_{i,t} \frac{dX_{i,t}}{X_{i,t}}, \tag{B.1}$$

with $\gamma_{i,t} = m_i A_{d_i,t}/V_t$ and $\beta_{i,t} = g_i e_{i,t} A_{f_i,t}/V_t$. Substituting Proposition 2.1 and (2.1) into (B.1), the result is:

$$\begin{aligned}\frac{dV_t}{V_t} &= \left[\sum_{i=1}^{n}\gamma_{i,t}(\mu_{d_i} - \lambda v) + \sum_{i=1}^{n}\beta_{i,t}(\mu_{e_i} + \mu_{f_i} - \lambda v + \rho_{2,3}\sigma_{f_i}\sigma_{e_i})\right]dt \\ &\quad + \sum_{i=1}^{n}\gamma_{i,t}\sigma_{d_i}dW_{1,t} + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i}\right]dW_{2,t} + \left[\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i}\right]dW_{3,t}.\end{aligned} \tag{B.2}$$

C. The Derivation of Equation (2.7)

Given a confidence level of α, VaR can be expressed as

$$P_r(V_T \le V_\alpha) = \alpha. \tag{C.1}$$

Based on the absolute VaR being denoted by $\mathrm{VaR}(0) \equiv V_0 - V_\alpha$, (C.1) can be transformed into

$$P_r(V_T \le V_0 + \mathrm{VaR}(0)) = \alpha. \tag{C.2}$$

Let $\sigma_t dW_t \equiv \sum_{i=1}^{n}\gamma_{i,t}\sigma_{d_i}dW_{1,t} + [\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i}]dW_{2,t} + [\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i}]\ dW_{3,t}$. From Proposition 2.2, one can obtain

$$\ln V_T \mid Y_T = k \mid N\left(\ln V_0 + \left(\mu_t - \frac{1}{2}\sigma_t^2\right)T + ku_\pi, \sigma_t^2 T + k\sigma_\pi^2\right), \tag{C.3}$$

in which k stands for the number of jumps and satisfies $k = 0, 1, \ldots, \infty$ and $N(\cdots)$ represents a normal distribution

$$\begin{aligned}\mu_t &= \sum_{i=1}^{n}\gamma_{i,t}(\mu_{d_i} - \lambda v) + \sum_{i=1}^{n}\beta_{i,t}(\mu_{f_i} + \mu_{e_i} - \lambda v + \rho_{2,3}\sigma_{f_i}\sigma_{e_i}); \\ \sigma_t^2 &= \left(\sum_{i=1}^{n}\gamma_{i,t}\sigma_{d_i}\right)^2 + \left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i}\right)^2 + \left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i}\right)^2 + 2\rho_{1,2}\left(\sum_{i=1}^{n}\gamma_{i,t}\sigma_{d_i}\right)\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i}\right) \\ &\quad + 2\rho_{2,3}\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{f_i}\right)\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i}\right) + 2\rho_{1,3}\left(\sum_{i=1}^{n}\gamma_{i,t}\sigma_{d_i}\right)\left(\sum_{i=1}^{n}\beta_{i,t}\sigma_{e_i}\right).\end{aligned} \tag{C.4}$$

Assume that $\sigma_B dB_t \equiv \sigma_t dW_t + \ln \pi dk$. Thus, (C.2) becomes

$$\begin{aligned}\sum_{k=0}^{\infty}P_r(Y_T = k)P_r\Bigg(&\sigma_t W_T + k\ln\pi \le \ln[V_0 + \mathrm{VaR}(0)] - \ln V_0 \\ &-\left(\mu_t - \frac{1}{2}\sigma_t^2\right)\ T \mid Y_T = k\Bigg) = \alpha.\end{aligned} \tag{C.5}$$

Because jumps follow Poisson distribution, (C.5) can be easily written through (C.3) as follows

$$\sum_{k=0}^{\infty} \frac{\exp[-\lambda T][\lambda T]^k}{k!} P_r \left(\frac{\sigma_t W_T + k \ln \pi - k u_\pi}{\sqrt{\sigma_t^2 \ T + k\sigma_\pi^2}} \right. \\ \left. \leq \frac{\ln[V_0 + \mathrm{VaR}(0)] - \ln V_0 - \left(\mu_t - (1/2)\sigma_t^2\right)T - ku_\pi}{\sqrt{\sigma_t^2 T + k\sigma_\pi^2}} \right) = \alpha. \tag{C.6}$$

Consequently, (2.7) can be proved as follows:

$$\sum_{k=0}^{\infty} \frac{\exp[-\lambda T][\lambda T]^k}{k!} \Phi \left(\frac{\ln[V_0 + \mathrm{VaR}(0)] - \ln V_0 - \left(\mu_t - (1/2)\sigma_t^2\right)T - ku_\pi}{\sqrt{\sigma_t^2 T + k\sigma_\pi^2}} \right) = \alpha. \tag{C.7}$$

Acknowledgments

The author is indebted to the anonymous referees and the editor, Kelvin K. W. Yau, for helpful suggestions that improved this paper substantially. The views expressed, as well as any errors, are those of the author.

Endnotes

1. Research related to these ideas was introduced by Jorion [12], Simons [13], Duffie and Pan [14], Kupiec [9], Brooks and Persand [15], and Chen and Liao [10].

2. Literature related to these studies has been presented by Stock and Watson [16], Hull and White [17], Hansen [18], and Consigli [19].

3. For simplicity, we assume that the dependence structure between exchange rates and equity returns is linear. However, there are some drawbacks. First, it is not invariant to transformations of the original variables. Second, conditional correlations are not accounted for. Third, the proposed method cannot be used in the case of portfolios that include assets with nonlinear payoffs.

4. Kupiec [9] shows the absolute VaR as follows:

$$\mathrm{VaR}_k(0) = A_{d_i,t_0}\left[\exp\left(\left(\mu_{d_i} - \frac{1}{2}\sigma_{d_i}^2\right)T + Z_\alpha\sqrt{\sigma_{d_i}^2 \ T}\right) - 1\right].$$

5. Chen and Liao [10] derives the absolute VaR of foreign-issued assets as below:

$$\mathrm{VaR}_c(0) = A_{f_{i,t_0}} e_{i,t_0} \left\{ \exp\left[\left(\mu_{f_i} + \mu_{e_i} - \frac{1}{2}\sigma_{f_i}^2 - \frac{1}{2}\sigma_{e_i}^2 \right) T + Z_\alpha \sqrt{\left(\sigma_{f_i}^2 + \sigma_{e_i}^2 + 2\rho_{2,3}\sigma_{f_i}\sigma_{e_i} \right) T} \right] - 1 \right\}.$$

References

[1] N. Hofmann and E. Platen, "Approximating large diversified portfolios," *Mathematical Finance*, vol. 10, no. 1, pp. 77–88, 2000.

[2] S. Heston, "A closed form solution for options with stochastic volatility with applications to bond and currency options," *Review of Financial Studies*, vol. 6, pp. 327–343, 1993.

[3] G. Bakshi, C. Charles, and Z. Chen, "Empirical performance of alternative option pricing models," *Journal of Finance*, vol. 52, no. 5, pp. 2003–2049, 1997.

[4] M. Broadie, M. Chernov, and M. Johannes, "Model specification and risk premia: evidence from futures options," *Journal of Finance*, vol. 62, no. 3, pp. 1453–1490, 2007.

[5] M. Gibson, "Incorporating event risk into value-at-risk," FEDS Papers 2001-17, 2001.

[6] L. K. Guan, L. Xiaoqing, and T. K. Chong, "Asymptotic dynamics and value-at-risk of large diversified portfolios in a jump-diffusion market," *Quantitative Finance*, vol. 4, no. 2, pp. 129–139, 2004.

[7] R. C. Merton, "Option pricing when underlying stock returns are discontinuous," *Journal of Financial Economics*, vol. 3, no. 1-2, pp. 125–144, 1976.

[8] P. F. Christoffersen, "Evaluating interval forecasts," *International Economic Review*, vol. 39, no. 4, pp. 841–862, 1998.

[9] P. Kupiec, "Risk capital and VaR," *Journal of Derivatives*, vol. 7, pp. 41–52, 1999.

[10] F. Y. Chen and S. L. Liao, "Modelling VaR for foreign-asset portfolios in continuous time," *Economic Modelling*, vol. 26, no. 1, pp. 234–240, 2009.

[11] P. Kupiec, "Techniques for verifying the accuracy of risk measurement models," *Journal of Derivatives*, vol. 3, pp. 73–84, 1995.

[12] P. Jorion, "Risk: measuring the risk in value at risk," *Financial Analysts Journal*, vol. 52, no. 6, pp. 47–56, 1996.

[13] K. Simons, "Model error," *New England Economic Review*, vol. 4, no. 6, pp. 17–28, 1997.

[14] D. Duffie and J. Pan, "An overview of value at risk," *Journal of Derivatives*, vol. 4, no. 3, pp. 7–49, 1997.

[15] C. Brooks and G. Persand, "Model choice and value-at-risk performance," *Financial Analysts Journal*, vol. 58, no. 5, pp. 87–97, 2002.

[16] J. H. Stock and M. W. Watson, "Evidence on structural instability in macroeconomic time series relations," *Journal of Business and Economic Statistics*, vol. 14, no. 1, pp. 11–30, 1996.

[17] J. Hull and A. White, "Value at risk when daily changes in market variables are not normally distributed," *Journal of Derivatives*, vol. 5, pp. 9–19, 1998.

[18] B. E. Hansen, "The new econometrics of structural change: dating breaks in U.S. labor productivity," *Journal of Economic Perspectives*, vol. 15, no. 4, pp. 117–128, 2001.

[19] G. Consigli, "Tail estimation and mean-VaR portfolio selection in markets subject to financial instability," *Journal of Banking and Finance*, vol. 26, no. 7, pp. 1355–1382, 2002.