MPE — Volume 1, pp. 77-87
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FEEDBACK CONTROL OF TIME-DELAY SYSTEMS WITH BOUNDED CONTROL AND STATE

M. DAMBRINE, J. P. RICHARD and P. BORNE

Ecole Centrale de Lille, LAIL-URA CNRS D 1440, BP 48, 59651 Villeneuve d'Ascq, Cedex, France

(Received 14 October 1994)

This paper is concerned with the problem of stabilizing linear time-delay systems under state and control linear constraints. For this, necessary and sufficient conditions for a given non-symmetrical polyhedral set to be positively invariant are obtained. Then existence conditions of linear state feedback control law respecting the constraints are established, and a procedure is given in order to calculate such a controller. The paper concerns memoryless controlled systems but the results can be applied to cases of delayed controlled systems. An example is given.

AMS Nos.: 34K35, 93D15, 34K20
KEYWORDS: Constrained control, differential–difference equations, polyhedral sets, invariance

1. INTRODUCTION

Practical control engineering must take into account the fact that control input and state variables are constrained to belong to bounded domains. These constraints are consequences of physical limitations such as, for example, limitation of the amplitudes or response velocity of actuators.

A regulator for such constrained systems can be designed by the application of the optimal control theory wherein the constraints are introduced in the Lagrangian formulation. But the main drawback of this method is that it needs the storage of a complicated switching surface to obtain a closed loop solution.

Another approach is based on the notion of Lyapunov functions and on their associated positive invariant sets. This method has been applied for different types of systems (discrete-time systems: see [1]–[3], systems modeled by linear differential equations: [3]–[5], or nonlinear differential equations: [6]–[8]).

In this paper, the design of constrained regulators is studied for systems modeled by time-delay equations or, more precisely, for systems described by differential–difference equations.

*To whom correspondence should be addressed.

2. PROBLEM FORMULATION

Throughout the paper, capital letters generally denote real matrices, lower case letters denote column vectors or scalars, $\mathbb{R}^n$ denotes the Euclidean n-space, and $\mathbb{R}^{n \times m}$ the set of all real $n \times m$ matrices. For vectors $x = [x_1\ x_2 \ldots x_n]^T$ and $y = [y_1\ y_2 \ldots y_n]^T$, the vector inequality $x \le y$ $(x < y)$ is equivalent to $x_i \le y_i$ $(x_i < y_i)$, with $i = 1, 2, \ldots, n$. We associate with a matrix $A = \{a_{ij}\}$ the matrices A^+, A^-, A_+, A_- defined by

$$A^+ = \begin{cases} a_{ij} & \text{for } i = j \\ \max\{0, a_{ij}\} & \text{for } i \ne j \end{cases}, \qquad A^- = A - A^+$$

$$A_+ = \{\max(0, a_{ij})\}, \qquad A_- = A - A_+$$

Systems considered in this paper are described by the vector equation

$$\dot{x}(t) = A\,x(t) + B\,x(t - \tau) + C\,u(t), \qquad \text{for } t > 0 \tag{1}$$

where x (the "instantaneous state vector") belongs to $\mathbb{R}^n$, u (the control vector) is in $\mathbb{R}^m$, and τ (the delay) is a positive number.

It is well known that systems (1) are of infinite dimension: it means that the state of these systems are no longer the vector x as for ordinary differential equations, but the function x_t defined on the interval $[-\tau, 0]$ by

$$x_t(s) = x(t + s),$$

and the state space is the set of all continuous functions defined on $[-\tau, 0]$ with values in $\mathbb{R}^n$. It will be denoted $\mathcal{C}(\mathbb{R}^n)$ in the sequel.

The control vector u(t) is subject to non-symmetrical linear constraints:

$$-d_2 \le u \le d_1, \tag{2}$$

where d_1, d_2 are real m-vectors with non-negative components.

There are also constraints on the current state: for any instant $t \ge -\tau$, the state vector $x(\tau)$ is constrained to belong to the set

$$\mathcal{S} = \{x \in \mathbb{R}^n : -w_2 \le Sx \le w_1\}, \tag{3}$$

where S is a real $q \times n$ matrix of rank n, and w_1, w_2 are real q-vectors with non-negative components.

Moreover, it is assumed that the initial states $x_{t_0}(s)$ belong for all s in $[-\tau, 0]$ to the set

$$\mathcal{S}_0 = \{x \in \mathbb{R}^n : -c_2 \le Dx \le c_1\} \subseteq \mathcal{S}, \tag{4}$$

where D is a real $r \times n$ matrix of rank n, and c_1, c_2 are real r-vectors with non-negative components. Of course, initial states have to satisfy the state constraints (3), that is, $\mathcal{S}_0 \subseteq \mathcal{S}$.

CONSTRAINED CONTROL PROBLEM (CCP) *Find a linear state feedback control law*

$$u(t) = Kx(t) \tag{5}$$

such that for any initial state function x_{t_0} *satisfying constraints (4) the solution of the resulting closed-loop system*

$$\dot{x}(t) = (A + CK)\,x(t) + B\,x(t - \tau), \qquad for\ t > 0 \tag{6}$$

converges towards the origin while the control vector satisfies condition (2) and the state vector remains in the set $\mathcal{S}$.

3. EXISTENCE CONDITIONS OF LINEAR CONSTRAINED CONTROLLERS

Let us associate with the linear state feedback control law $u(t) = Kx(t)$, the subset $\mathcal{R}(K, d_1, d_2)$ of $\mathcal{C}(R^n)$ defined by

$$\mathcal{R}(K, d_1, d_2) = \{\varphi \in \mathcal{C}(R^n) : \forall\ s \in [-\tau, 0], -d_2 \leq K\varphi(s) \leq d_1\}$$

$\mathcal{R}(K, d_1, d_2)$ is obviously the set of the states of the closed-loop system (6) with control satisfying the constraint (2).

In a similar way, the set of the states that satisfy the constraints (3) is

$$\mathcal{R}(S, w_1, w_2) = \{\varphi \in \mathcal{C}(R^n) : \forall\ s \in [-\tau, 0], -w_2 \leq S\varphi(s) \leq w_1\},$$

and the subset of initial state defined by inequalities (4) is expressed as

$$\mathcal{R}(D, c_1, c_2) = \{\varphi \in \mathcal{C}(R^n) : \forall\ s \in [-\tau, 0], -c_2 \leq D\varphi(s) \leq c_1\}$$

The problem CCP can be reformulated as "find a control law $u = Kx$ such that the closed-loop system (6) is asymptotically stable and for any initial state in $\mathcal{R}(D, c_1, c_2)$, the current state x_t belongs to $\mathcal{R}(S, w_1, w_2)$ for any $t \geq 0$, and to $\mathcal{R}(K, d_1, d_2)$ for any $t \geq \tau$". This condition can also be expressed as follows:

THEOREM 1 *The control law $u = Kx$ with $K \in R^{m \times n}$ is a solution of the CCP if and only if*

(i) *the system (6) is asymptotically stable;*
(ii) *there exists a positively invariant set $\Omega \subseteq \mathcal{C}(R^n)$ with respect to (6) such that*

$$\mathcal{R}(D, c_1, c_2) \subseteq \Omega \subseteq \mathcal{R}(S, w_1, w_2),$$

$$x_\tau(\Omega) \subseteq \mathcal{R}(K, d_1, d_2),$$

with $x_\tau(\Omega) = \{x_\tau(\varphi): \varphi \in \Omega\}$ *($x_\tau(\varphi)$ denotes the state function at time t of the solution of (6) starting at $t = 0$ from the initial state φ).*

Proof We just show the necessity of the existence of a such set Ω, the other points being elementary. Let us assume that u is a solution of the CCP, and let Ω_1 be the set defined by:

$$\Omega_1 = \{x_t(\varphi) : \varphi \in \mathcal{R}(D, c_1, c_2)\}$$

Then, obviously Ω_1 is a positive invariant set of (6) satisfying

$$\mathcal{R}(D, c_1, c_2) \subseteq \Omega_1,$$

and moreover, since u is a solution of the CCP, the inequalities

$$\Omega_1 \subseteq \mathcal{R}(S, w_1, w_2),$$

$$x_\tau(\Omega_1) \subseteq \mathcal{R}(K, d_1, d_2)$$

hold.

A direct application of this result is to find a gain matrix K such that u = Kx is a stabilizing control law, ensuring that one of the three subsets $\mathcal{R}(D, c_1, c_2)$, $\mathcal{R}(S, w_1, w_2)$, or $\mathcal{R}(K, d_1, d_2)$ is positively invariant, and inequalities

$$\mathcal{R}(D, c_1, c_2) \subseteq \mathcal{R}(S, w_1, w_2)$$

$$\mathcal{R}(D, c_1, c_2) \subseteq \mathcal{R}(K, d_1, d_2)$$

or

$$\mathcal{R}(D, c_1, c_2) \subseteq \mathcal{R}(S, w_1, w_2) \subseteq \mathcal{R}(K, d_1, d_2)$$

or

$$\mathcal{R}(D, c_1, c_2) \subseteq \mathcal{R}(K, d_1, d_2) \subseteq \mathcal{R}(S, w_1, w_2)$$

respectively hold. In the next section conditions of positive invariance for polyhedral sets will be established for time-delay systems.

4. CONDITIONS OF POSITIVE INVARIANCE FOR POLYHEDRAL SETS

THEOREM 2 *The set*

$$\mathcal{R}(I_n, w_1, w_2) = \{\varphi \in \mathcal{C}(\mathbb{R}^n) : \forall\, s \in [-\tau, 0], -w_2 \le \varphi(s) \le w_1\}$$

is positively invariant for the system

$$\dot{x}(t) = A\,x(t) + B\,x(t-\tau) \tag{7}$$

if and only if:

$$\begin{bmatrix} A^{+}+B_{+} & A^{-}+B_{-} \\ A^{-}+B_{-} & A^{+}+B_{+} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \le 0. \tag{8}$$

Proof. *See* Appendix.

THEOREM 3 ($m \le n$) *Let* $F \in \mathbb{R}^{m\times n}$, *with rank* $F = m$, *then the set*

$$\mathcal{R}(F, w_1, w_2) = \{\varphi \in \mathcal{C}(\mathbb{R}^n) : \forall\, s \in [-\tau, 0], -w_2 \le F\varphi(s) \le w_1\},$$

is positively invariant with respect to the system (7) if and only if there exist two matrices H, and G elements of $\mathbb{R}^{m\times n}$ *such that*

$$FA = GF$$
$$FB = HF$$

$$\begin{bmatrix} G^{+}+H_{+} & G^{-}+H_{-} \\ G^{-}+H_{-} & G^{+}+H_{+} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \le 0 \tag{9}$$

Proof. *See* Appendix.

THEOREM 4 ($m > n$) *Let* $F \in \mathbb{R}^{m\times n}$ *such that rank* $F = n$, *and let G and H be two matrices solutions of*

$$FA = GF$$
$$FB = HF$$

Then (9) is also a sufficient condition for the subset $\mathcal{R}(F, w_1, w_2)$ *to be positively invariant with respect to the system (7).*

5. DESIGN OF A CONSTRAINED CONTROLLER

The following algorithm for the obtention of a constrained controller is based, for sake of simplicity, on the positive invariance of the subset R (S, w_1, w_2).

The control law $u = Kx$ with $K \in R^{m\times n}$ is a solution of the CCP if

(i) the system (5) is asymptotically stable;

(ii) there exist two matrices G and H such that

$$S(A+CK) = GS \tag{10}$$

$$SB = HS \tag{11}$$

$$Mw \leq 0, \tag{12}$$

with

$$M = \begin{bmatrix} G^{+} + H_{+} & G^{-} + H_{-} \\ G^{-} + H_{-} & G^{+} + H_{+} \end{bmatrix}, \qquad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{13}$$

(iii) $\mathcal{R}(S, w_1, w_2) \subseteq \mathcal{R}(K, d_1, d_2)$.

But the following results can be shown:

- If inequality (12) strictly holds then system (5) is asymptotically stable.
- Condition (iii) can be replaced by the following equivalent one ([5]):

$$-d_2 \leq Kx_{(i)} \leq d_1,$$

where $x_{(i)}$ are the vertices of the polyhedral set $\{x \in \mathbb{R}^n: -w_2 \leq Sx \leq w_1\}$.

The previous problem may not possess a unique solution, therefore the control law $u = Kx$ can be chosen so that the rate of convergence of system (5) is improved. This can be done by solving the following optimisation problem:

Find K that maximizes ϵ, under the constraints

$$S(A + CK) = GS \tag{14a}$$

$$SB = HS \tag{14b}$$

$$\begin{bmatrix} G^{+} + e^{\tau\epsilon} H_{+} & G^{-} + e^{\tau\epsilon} H_{-} \\ G^{-} + e^{\tau\epsilon} H_{-} & G^{+} + e^{\tau\epsilon} H_{+} \end{bmatrix} w \leq -\epsilon w \tag{14c}$$

$$-d_2 \leq Kx_{(i)} \leq d_1 \tag{14d}$$

$$\epsilon \geq 0 \tag{14e}$$

This problem is highly nonlinear because of relation (14c), however a quasi-optimal one can be considered by replacing $e^{\tau\epsilon}$ by a positive number β independent of ϵ.

6. EXAMPLE

Let us consider the system

$$\dot{x}(t) = \begin{bmatrix} -3 & 2 \\ 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} x(t-1) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t), \tag{14}$$

with the following limitations: the state vector x(t) has to remains in the set

$$\mathcal{S} = \{x \in \mathbb{R}^2 : -[2\,;2] \le x^T \le [3\,;2]\},$$

the control vector u(t) must satisfy the following constraints

$$-10 \le u(t) \le 9,$$

and it is assumed that there is no more restrictions on initial conditions, that is, $\mathcal{S}_0 = \mathcal{S}$. Here, we have $S = I_2$, so $G = A + CK$ and $H = B$. The set of matrices $K = [k_1, k_2]$ such the feedback law stabilizes the system (15) in respecting the previous constraints is represented in Figure 1. The solution of the optimization problem (14) is given by $K = [-1.12\,; -3.32]$. The obtained value of ϵ is then equal to 0.254.

7. CONCLUSION

In this paper, necessary and sufficient conditions for the control of constrained, time-delay systems have been investigated. The results concern linear models with nonsymmetrical polyhedral domains of constraints. However, the same approach applies to nonlinear systems with symmetrical domains (*see* [9]).

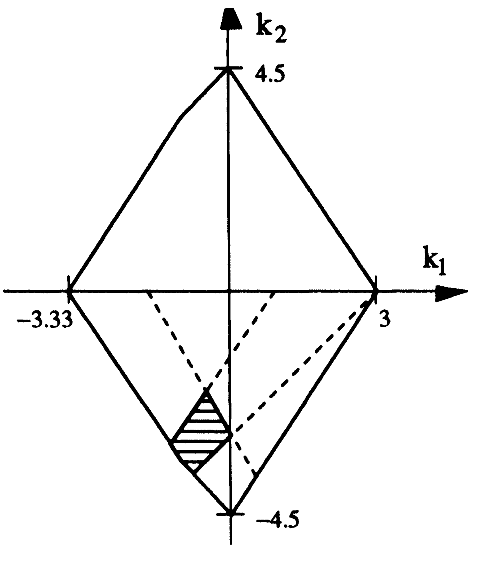

Figure 1.

APPENDIX. PROOFS OF THEOREMS 2–4

We first present two elementary results:

LEMMA 1. *[11]: Let $A \in \mathbb{R}^{n\times n}$ and $F \in \mathbb{R}^{p\times n}$ with rank $F = p$. If*

$$\text{Ker F} \subseteq \text{Ker FA},$$

then there is a matrix $H \in \mathbb{R}^{p\times p}$ solution of the matrix equation

$$FA = HF$$

LEMMA 2. *Let $F \in R^{p\times n}$. If there exist two vectors w_1 and w_2 in R^p with positive components such that*

$$\mathcal{R}(F, w_1, w_2) = \{\varphi \in C(R^m) : \forall\, s \in [-\tau, 0], -w_2 \leq F\,\varphi(s) \leq w_1\}$$

is a positively invariant set for the system (7), then

$$\mathcal{C}(\text{Ker F}) = \{\varphi \in C(R^m) : \forall s \in [-\tau, 0], F\,\varphi(s) = 0\}$$

is also a positively invariant set for (7).

Proof. Because the system (7) is linear, if $\mathcal{R}(F, w_1, w_2)$ is a positively invariant set, then, for any $\alpha > 0$, the set $\mathcal{R}(F, \alpha w_1, \alpha w_2)$ is also positively invariant.

Let $\varphi \in \mathcal{C}$ (Ker F), and assume that $Fx(t_1 ; 0, \varphi) \neq 0$. Then, there is a real $\alpha > 0$ and an index i $(1 \leq i \leq p)$ such that

$$[Fx(t_1 ; 0, \varphi)]_i > \alpha\, w_{1i}, \qquad \text{or } [Fx(t_1 ; 0, \varphi)]_i < -\alpha\, w_{2i}.$$

However, it is obvious that φ belongs to the positively invariant set $\mathcal{R}(F, \alpha w_1, \alpha w_2)$, which is in contradiction with the previous inequalities.

Proof of Theorem 2 We denote w_{ji} the i^{th} component of vector w_j, with j = 1 or 2.

a) Necessity:

Let us assume that there is an index $i \in \{1,\ldots,n\}$ such that the i^{th} component of vector

$$\begin{bmatrix} A^+ + B_+ & A^- + B_- \\ A^- + B_- & A^+ + B_+ \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

is positive. Let $\varphi \in C(R^n)$ be defined by:

$$\varphi(0) : \begin{cases} \varphi_i(0) = w_{1i} \\ \varphi_j(0) = w_{1j} \text{ if } a_{ij} \geq 0 \\ \varphi_j(0) = -w_{2j} \text{ if } a_{ij} < 0 \end{cases}$$

$$\varphi(-\tau) : \begin{cases} (-\tau) = w_{1j} \text{ if } b_{ij} \geq 0 \\ \varphi_j(-\tau) = -w_{2j} \text{ if } b_{ij} < 0' \end{cases}$$

$$\varphi(s) = (s + \tau)\frac{\varphi(0) - \varphi(-\tau)}{\tau} + \varphi(-\tau), \qquad \text{for } s \in (-\tau, 0).$$

The function φ belongs to $\mathcal{R}(I_n, w_1, w_2)$, but the solution x(t) passing through φ satisfies $\dot{x}_i(0) > 0$, therefore x(t) leaves the set $\mathcal{R}(I_n, w_1, w_2)$: this set cannot be positively invariant. If the index i belongs to $\{n + 1, \ldots, 2n\}$, a similar argument leads to the same conclusion.

b) Sufficiency:

Let $v(x) = \max_{1 \leq i \leq n} (\max(\frac{x_i}{w_{1i}}, \frac{-x_i}{w_{2i}}))$.

We shall prove that under condition (8), the positive definite function v is a Lyapunov-Razumikhin function [10]: let x be a solution of (7) such that at time t the following inequality holds:

$$v(x(t - \tau)) \leq v(x(t)) \tag{15}$$

There is an index $i \in \{1, \ldots, n\}$ such that $v(x(t)) = \frac{x_i(t)}{w_{1i}}$, or $v(x(t)) = -\frac{x_j(t)}{w_{2i}}$

Let us assume that $v(x) = \frac{x_i}{w_{1i}}$. Then, along the solutions of (7),

$$\dot{v}(x(t)) = \frac{1}{w_{1i}}[\sum_{j=1}^{n} a_{ij} x_j(t) + \sum_{j=1}^{n} b_{ij} x_j(t - \tau)].$$

Denoting $a^+_{ij} = \max(0, a_{ij})$, and $a^-_{ij} = \max(0, -a_{ij})$ yields:

$$\begin{cases} a_{ij} = a^+_{ij} - a^-_{ij} \\ | a_{ij} | = a^+_{ij} + a^-_{ij} \end{cases}$$

Using the same notation for coefficients b_{ij} yields

$$\dot{v}(x(t)) = \frac{1}{w_{1i}}[a_{ij} x_i + \sum_{j \neq i} a^+_{ij} x_j(t) - \sum_{j \neq i} a^-_{ij} x_j(t) + \sum_{j=1}^{n} b^+_{ij} x_j(t - \tau) - \sum_{j=1}^{n} b^-_{ij} x_j(t - \tau)].$$

According to the definition of v(x):

$$x_j(t) \leq \frac{w_{1j}}{w_{1i}} x_i(t) \text{ and } -x_j(t) \leq \frac{w_{2j}}{w_{1i}} x_i(t),$$

and from inequality (16):

$$x_j(t-\tau) \le \frac{w_{1j}}{w_{1i}} x_i(t) \text{ and } -x_j(t-\tau) \le \frac{w_{2j}}{w_{1i}} x_i(t),$$

So, it follows that

$$\dot{v}(x(t)) \le \frac{1}{w_{1i}} [a_{ii} w_{1i} + \sum_{j \ne i} a_{ij}^{+} w_{1j} + \sum_{j \ne i} a_{ij}^{-} w_{2j} + \sum_{j=1}^{n} b_{ij}^{+} w_{1i} + \sum_{j=1}^{n} b_{ij}^{-} w_{2j}] \, v(x(t)),$$

this is, in a vectorial form:

$$\dot{v}(x(t)) \le \frac{1}{w_{1i}} [A^{+} w_1 + A^{-} w_2 + B_{+} w_1 + B_{-} w_2]_i \, v(x(t)),$$

where $[y]_i$ denotes the i^{th} component of y. Then, from (8), $\dot{v}(x(t)) \le 0$.

If $v(x(t)) = -x_i(t)/w_{2i}$, then a similar argument leads to the same conclusion. So, v is a Lyapunov-Razumikhin function for (7), and the set $\mathcal{R}(I_n, w_1, w_2)) = \{\varphi \in C(R^m)$: $v(\varphi(s)) \le 1, \forall s \in [-\tau, 0]\}$, is positively invariant for (7).

Proof of Theorem 3

a) Necessity:

If the set $\mathcal{R}(F, w_1, w_2)$ is positively invariant for system (7), then Lemma 2 ensures that the set $\mathcal{C}(\mathrm{Ker}\, F)$ is also positively invariant. Let $x \in \mathrm{Ker}\, F$, and consider the function φ_1 defined by

$$\varphi_1(s) = \frac{s+\tau}{\tau} x.$$

It is obvious that $\varphi_1 \in \mathcal{C}(\mathrm{Ker}\, F)$, so $x(t; \varphi_1)$ (the solution of (7) with initial condition φ_1), satisfies $Fx(t; \varphi_1) \equiv 0$, and $F\dot{x}(t; \varphi_1) \equiv 0$. At $t = 0$, $FA\, x = 0$, and according to Lemma 1, there is a matrix G such that $FA = GF$.

Same arguments for $\varphi_2 = -s/\tau\, x$ show that there is a matrix H satisfying $FB = HF$.

Then we define the function $z(t) = Fx(t)$. If $x(t)$ is solution of (7), then $z(t)$ satisfy

$$\dot{z}(t) = G\, z(t) + H\, z(t-\tau) \tag{16}$$

The set $\mathcal{R}(I_p, w_1, w_2)$ is positively invariant for (17) (rank $F = p$), and so according to Theorem 2, G and H satisfy (9).

b) Sufficiency:

Let us assume that there is a function $\varphi \in \mathcal{R}(F, w_1, w_2)$ such that $x(t; \varphi)$ leaves the set $\{x \in \mathbb{R}^n: -w_2 \le Fx \le w_1\}$ at time t_1. Then, $z(t) = Fx(t; \varphi)$ is a solution of (17) with an initial condition belonging to $\mathcal{R}(I_p, w_1, w_2)$, that leaves the set $\{z \in R^p: -w_2 \leq z \leq w_1\}$ at $t = t_1$. But, inequality (4.35) and Theorem 2 imply that $\mathcal{R}(I_p, w_1, w_2)$ is positively invariant for (17). The set $\mathcal{R}(F, w_1, w_2)$ is therefore positively invariant set for system (7).

Proof of Theorem 4 This result can be proved in a similar way than for Theorem 3.

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