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Research Article

Some New Parallel Flows in Weakly Conducting Fluids with an Exponentially Decaying Lorentz Force

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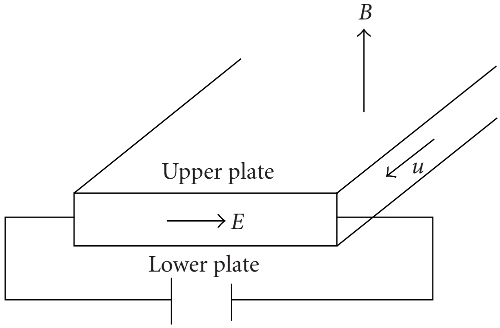
We investigate the fully developed flow between two parallel plates and the film flow over a plate in an electrically conducting fluid under the action of a parallel Lorentz force. The Lorentz force varies exponentially in the vertical direction due to low-fluid electrical conductivity and the special arrangement of the magnetic and electric fields at the lower plate. Exact analytical solutions are derived for velocity, flow rate, and wall shear stress at the plates. The velocity results are presented in figures. All these flows are new and are presented for the first time in the literature.

1. Introduction

Magnetohydrodynamics (MHD) is the study of the interaction between magnetic fields and moving conducting fluids. The simplest MHD flow is the flow between two infinite, horizontal, parallel plates under the action of an external vertical magnetic field and a horizontal electric field (Figure 1.1). This type of flow was first investigated by Hartmann and Lazarus [1] and is called the Hartmann flow (Davidson [2, page 153]). In 1961 Gailitis and Lielausis [3] introduced the idea of using a Lorentz force to control the flow of an electrically conducting fluid over a flat plate. This is achieved by applying an external electromagnetic field (see Figure 1.2) by a stripwise arrangement of flush mounted electrodes and permanent magnets of alternating polarity and magnetization. A similar arrangement was proposed by Rice [4]. The Lorentz force, which acts parallel to the plate, can either assist or oppose the flow. This idea was later abandoned and only recently attracted new attention (Henoch and Stace [5], Crawford and Karniadakis [6], O'Sullivan and Biringen [7], Berger et al. [8], Kim and Lee [9], Du and Karniadakis [10], Du et al. [11], Breuer et al. [12], Lee and Sung [13], Spong et al. [14]). In addition, in the last



E = horizontal electric field

B = vertical magnetic field

FIGURE 1.1. The flow configuration in the classical Hartmann flow.

years much investigation on flow control using the Lorentz force is being conducted at the Rossendorf Institute and at the Institute for Aerospace Engineering in Dresden, Germany (Weier et al. [15], Posdziech and Grundmann [16], Weier et al. [17], Weier and Gerbeth [18], Weier [19], Mutschke et al. [20], Albrecht and Grundmann [21], Shatrov and Gerbeth [22]). The work of [22] is concerned with turbulent flow in a channel with a Lorentz force. As a special case, the laminar Poiseuille flow is treated with a Lorentz force only for a flow rate equal to 4/3.

The width of both electrodes and magnets is assumed to be equal to a. Due to the crossing electric and magnetic field lines, the Lorentz force acts in the streamwise direction. It is known that when a conducting fluid moves inside a magnetic field, an electric current is produced and this electric current influences the Lorentz force. In the present work, the fluid electrical conductivity is assumed to be small and therefore the induced electric current is negligible. This means that the fluid motion has no influence on the Lorentz force. The Lorentz force depends only on the external electric and magnetic field. Assuming hard ferromagnetic properties, the magnetic field of the chain of magnets can be easily calculated analytically. The current distribution of the electrode array can be found in closed form. As a result, apart from inhomogeneities near the magnet corners, the Lorentz force decays exponentially in the y direction. After averaging over the spanwise direction z, the Lorentz force can be calculated [19].

Although the Lorentz force has been applied in many flow configurations, some flows under the influence of this force have not been investigated until now. In the present paper, we will investigate the classical flow between two parallel, infinite plates with a Lorentz force created at the lower plate according to the arrangement shown in Figure 1.2. The present problem is equivalent to the classical Hartmann flow with a different arrangement of the magnetic and electric fields, except that we will investigate the problem of a film flowing over a plate with a Lorentz force created at the plate according to the arrangement shown in Figure 1.2.

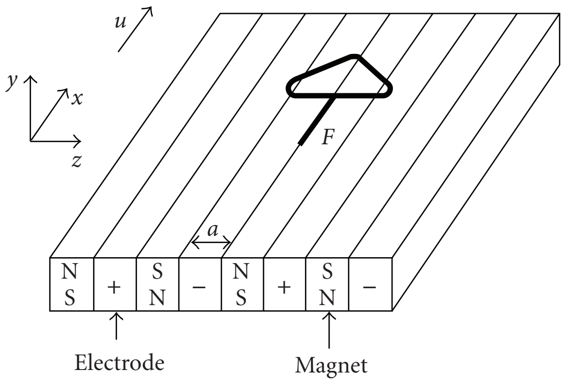


FIGURE 1.2. Arrangement of electrodes and magnets for the creation of an EMHD Lorentz force F in the flow along a flat plate (Weier [19]).

2. The mathematical model

Consider the flow between two horizontal, infinite, parallel plates with u and v denoting, respectively, the velocity components in the x and y directions, where x is the coordinate along the plates and y is the coordinate perpendicular to x. It is assumed that an electromagnetic field exists at the lower plate and therefore a Lorentz force, parallel to the plates, is produced. The fluid is forced to move due to the action of the Lorentz force. For steady, two-dimensional flow, the boundary layer equations with constant fluid properties are [19, 23]

$$\begin{aligned} &\text{continuity equation:} && \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \\ &\text{momentum equation:} && u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+\frac{\pi j_0 M_0}{8\rho}\exp\left(-\frac{\pi}{a}y\right), \end{aligned} \tag{2.1}$$

where p is the pressure, v is the fluid kinematic viscosity, j_0 (A/m^2) is the applied current density in the electrodes, M_0 (Tesla) is the magnetization of the permanent magnets, a is the width of magnets and electrodes, and ρ is the fluid density. The last term in the momentum equation is the Lorentz force which decreases exponentially with y and is independent of the flow. For fully developed conditions, the flow is parallel, the transverse velocity is zero, and the flow is described only by the following momentum equation:

$$\text{momentum equation:} \qquad -\frac{1}{\rho}\frac{dp}{dx}+v\frac{\partial^2 u}{\partial y^2}+\frac{\pi j_0 M_0}{8\rho}\exp\left(-\frac{\pi}{a}y\right)=0. \tag{2.2}$$

3. Results and discussion

3.1. The classical Couette flow with Lorentz force. The first viscous fluid flow treated in the classical book by White [24] is the steady flow between a fixed and a moving plate (Couette flow) and this happens in almost all fluid mechanics books because this flow is the simplest in fluid mechanics (Liggett [25, page 156], Kleinstreuer [26, page 121], Panton [27, page 132]). This flow is called Couette flow in honour of the french Couette [28] who performed experiments on the flow between a fixed and moving concentric cylinder. The boundary conditions for this case are

$$\begin{aligned} &\text{at } y = 0: \qquad u = 0, \\ &\text{as } y = h: \qquad u = u_2, \end{aligned} \tag{3.1}$$

where h is the distance between the plates and u_2 is the velocity of the upper plate.

Here we will investigate this flow under the action of a Lorentz force produced at the lower plate. The momentum (2.2), without the pressure gradient, with boundary conditions (3.1) has the following exact analytical solution:

$$\frac{u}{u_2} = \frac{y}{h} + Z\left[1 - \exp\left(-\frac{\pi}{a}y\right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right]. \tag{3.2}$$

The first term on the right is due to the motion of the upper plate (classical Couette flow) and the second term due to the action of the Lorentz force. The parameter Z is defined as

$$Z = \frac{j_0 M_0 a^2}{8\pi\mu u_2}, \tag{3.3}$$

where μ is the fluid dynamic viscosity. The parameter Z is dimensionless, expresses the balance between the electromagnetic forces to viscous forces, and is equivalent to square of the classical Hartmann number or to Chandrasekhar number (Burr et al. [29, page 23], Aurnou and Olson [30, page 284]). This number is used in the analysis of the boundary layer flow over a flat plate situated in a free stream and there is characteristic velocity used as the free stream velocity [19]. We see that this number appears also in the Couette flow with characteristic velocity of the moving plate. The dimensionless velocity given by (3.2) depends on both Z and the ratio h/a. The above combination of the classical Couette flow with Lorentz forces is presented here for the first time in the literature.

The dimensionless flow rate M between the plates is

$$M = \frac{1}{u_2 h}\int_0^h u\,dy \tag{3.4}$$

and is obtained by integrating the velocity function. Thus, we have

$$M = \frac{1}{2} + \frac{Z}{2\pi}\left[\left(\pi + 2\frac{a}{h}\right)\exp\left(-\frac{\pi}{a}h\right) + \pi - 2\frac{a}{h}\right]. \tag{3.5}$$

The wall shear stresses at the two plates are

$$\begin{aligned} \tau_1 &= \frac{\mu u_2}{h} + \mu u_2 Z\left[\frac{\pi}{a} - \frac{1}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right], \\ \tau_2 &= \frac{\mu u_2}{h} + \mu u_2 Z\left[-\frac{1}{h} + \left(\frac{\pi}{a} + \frac{1}{h}\right)\exp\left(-\frac{\pi}{a}h\right)\right]. \end{aligned} \tag{3.6}$$

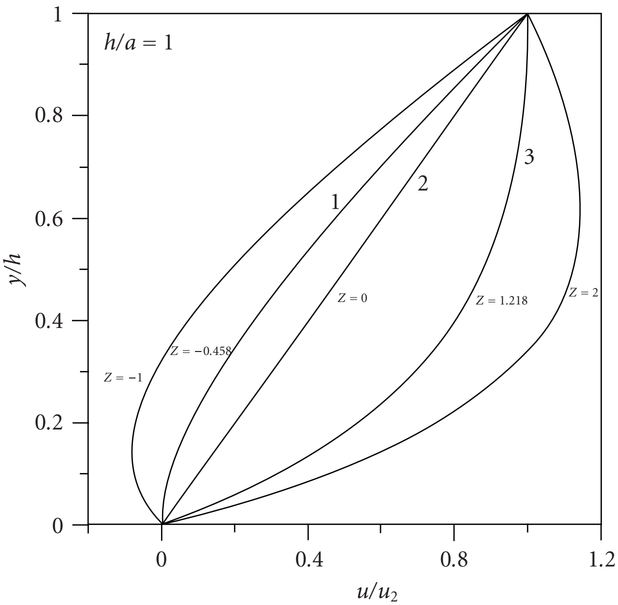


Figure 3.1. Velocity profiles for Couette flow with Lorentz force for $h/a = 1$ and different values of the Chandrasekhar number. The case $Z = 0$ corresponds to Couette flow with upper plate moving.

The wall shear stress τ_1 becomes zero when the Chandrasekhar number takes the value

$$Z = \left[-\frac{\pi h}{a} + 1 - \exp\left(-\frac{\pi}{a} h \right) \right]^{-1} \tag{3.7}$$

while τ_2 becomes zero when

$$Z = \left[1 - \left(\frac{\pi h}{a} + 1 \right) \exp\left(-\frac{\pi}{a} h \right) \right]^{-1}. \tag{3.8}$$

In Figure 3.1, some velocity profiles are presented for different values of the Chandrasekhar number and $h/a = 1$. The profile 1 corresponds to zero shear stress at the lower plate and this happens when $Z = -0.4577064$. Profile 3 corresponds to zero shear stress at the upper plate and this happens when $Z = 1.2179889$. Profile 2 corresponds to Couette flow.

When the lower plate, where the electromagnetic field is produced, is moving and the upper plate is motionless, the velocity is given by the following equation:

$$\frac{u}{u_1} = 1 - \frac{y}{h} + Z\left[1 - \exp\left(-\frac{\pi}{a} y \right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a} h \right) \right) \right], \tag{3.9}$$

where u_1 is the velocity of the lower plate and the Chandrasekhar number is based on velocity of the lower plate. Now the flow rate is defined as

$$M = \frac{1}{u_1 h} \int_0^h u\, dy \tag{3.10}$$

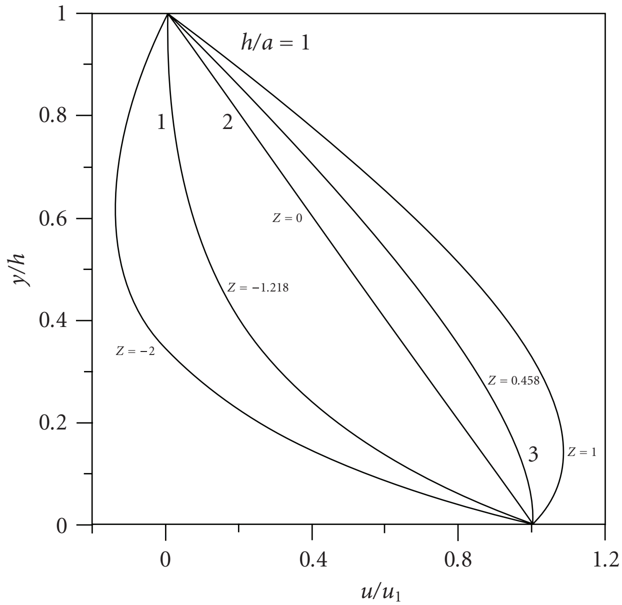


Figure 3.2. Velocity profiles for Couette flow with Lorentz force for $h/a = 1$ and different values of the Chandrasekhar number. The case $Z = 0$ corresponds to Couette flow with lower plate moving.

and (3.5) is valid also for this case. The wall shear stresses at the two plates are

$$\begin{aligned}\tau_1 &= -\frac{\mu u_1}{h} + \mu u_1 Z\left[\frac{\pi}{a} - \frac{1}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right],\\ \tau_2 &= -\frac{\mu u_1}{h} + \mu u_1 Z\left[-\frac{1}{h} + \left(\frac{\pi}{a} + \frac{1}{h}\right)\exp\left(-\frac{\pi}{a}h\right)\right].\end{aligned} \tag{3.11}$$

The wall shear stress τ_1 becomes zero when the Chandrasekhar number takes the value

$$Z = \left[\frac{\pi h}{a} - 1 + \exp\left(-\frac{\pi}{a}h\right)\right]^{-1}, \tag{3.12}$$

while τ_2 becomes zero when

$$Z = \left[-1 + \left(\frac{\pi h}{a} + 1\right)\exp\left(-\frac{\pi}{a}h\right)\right]^{-1}. \tag{3.13}$$

In Figure 3.2, some velocity profiles are presented for different values of the number and $h/a = 1$. Profile 1 corresponds to zero shear stress at the upper plate and this happens when $Z = -1.2179889$. Profile 3 corresponds to zero shear stress at the lower plate and this happens when $Z = 0.4577064$. Profile 2 corresponds to Couette flow.

3.2. The classical Poiseuille flow with Lorentz forces. Another kind of flow between parallel plates is the Poiseuille flow (Poiseuille [31]) which is caused by a constant pressure gradient along the plates while the plates are motionless. This flow is also included in

Fluid Mechanics books ([25, page 157], [27, page 125], [24, page 106]). The boundary conditions are

$$\begin{aligned} &\text{at } y = 0: && u = 0, \\ &\text{as } y = h: && u = 0. \end{aligned} \tag{3.14}$$

The analytical solution of (2.2) with boundary conditions (3.14) is

$$u = -\frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}\left(1 - \frac{y}{h}\right) + \frac{j_0 M_0 a^2}{8\pi\mu}\left[1 - \exp\left(-\frac{\pi}{a}y\right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right]. \tag{3.15}$$

The first term on the right is due to the pressure gradient (classical Poiseuille flow) and the second term due to the action of the Lorentz force. In the above equation, there are two characteristic velocities, the first one due to pressure gradient and the second due to Lorentz forces as follows:

$$\begin{aligned} u_P &= -\frac{h^2}{2\mu}\frac{dp}{dx}, \\ u_Z &= \frac{j_0 M_0 a^2}{8\pi\mu}. \end{aligned} \tag{3.16}$$

Taking into account these characteristic velocities, (3.15) becomes

$$u = u_P \frac{y}{h}\left(1 - \frac{y}{h}\right) + u_Z\left[1 - \exp\left(-\frac{\pi}{a}y\right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right] \tag{3.17}$$

and in dimensionless form,

$$\frac{u}{u_Z} = \frac{u_P}{u_Z}\frac{y}{h}\left(1 - \frac{y}{h}\right) + \left[1 - \exp\left(-\frac{\pi}{a}y\right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right]. \tag{3.18}$$

The ratio u_P/u_Z is a new dimensionless number which expresses the balance between the pressure forces to electromagnetic forces:

$$\mathrm{Pa}_p = \frac{u_P}{u_Z}. \tag{3.19}$$

The dimensionless velocity given by (3.18) is a function of Pa_p and h/a. The above combination of the classical Poiseuille flow with Lorentz forces is a new kind of parallel flow. We define the dimensionless flow rate as

$$M = \frac{1}{u_Z h}\int_0^h u\,dy \tag{3.20}$$

and M is

$$M = \frac{\mathrm{Pa}_p}{6} + \frac{1}{2\pi}\left[\left(\pi + 2\frac{a}{h}\right)\exp\left(-\frac{\pi}{a}h\right) + \pi - 2\frac{a}{h}\right]. \tag{3.21}$$

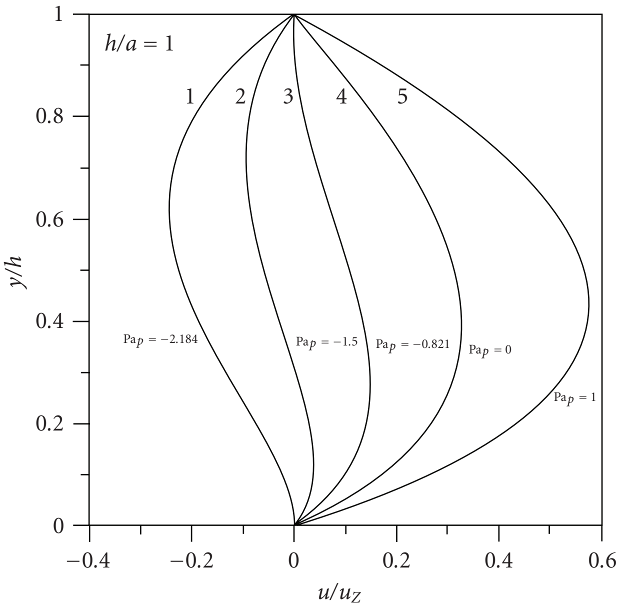


FIGURE 3.3. Velocity profiles for Poiseuille flow with Lorentz force for $h/a = 1$ and different values of the Pa_p parameter. The case $\mathrm{Pa}_p = 0$ corresponds to zero pressure gradient.

The wall shear stresses at the two plates are

$$\begin{aligned}\tau_1 &= \frac{\mu u_P}{h} + \mu u_Z\left[\frac{\pi}{a} - \frac{1}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right],\\ \tau_2 &= -\frac{\mu u_P}{h} + \mu u_Z\left[-\frac{1}{h} + \left(\frac{\pi}{a} + \frac{1}{h}\right)\exp\left(-\frac{\pi}{a}h\right)\right].\end{aligned} \tag{3.22}$$

The wall shear stress τ_1 becomes zero when the quantity Pa_p takes the value

$$\mathrm{Pa}_p = \left[-\frac{\pi h}{a} + 1 - \exp\left(-\frac{\pi}{a}h\right)\right] \tag{3.23}$$

while τ_2 becomes zero when

$$\mathrm{Pa}_p = \left[-1 + \left(\frac{\pi h}{a} + 1\right)\exp\left(-\frac{\pi}{a}h\right)\right]. \tag{3.24}$$

In Figure 3.3, we present some velocity profiles for $h/a = 1$ and different Pa_p values. Profile 1 corresponds to zero shear stress at the lower plate and this happens when $\mathrm{Pa}_p = -2.1848066$. Profile 3 corresponds to zero shear stress at the upper plate and this happens when $\mathrm{Pa}_p = -0.8210255$. Profile 4 corresponds to zero pressure gradient. Profiles 1, 2, and 3 are *S* shaped and each of them has an inflection point.

3.3. Flow between parallel plates due to Lorentz force only. If both plates are motionless and the pressure gradient is zero, we have a flow caused by the Lorentz force only.

From (3.15) we get the velocity of this flow by putting the pressure gradient zero. Thus, we have

$$u = \frac{j_0 M_0 a^2}{8\pi\mu}\left[1 - \exp\left(-\frac{\pi}{a}y\right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right] \tag{3.25}$$

and in a dimensionless form,

$$\frac{u}{u_Z} = \left[1 - \exp\left(-\frac{\pi}{a}y\right) - \frac{y}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right]. \tag{3.26}$$

This flow is completely new, it is presented here for the first time in the literature. The dimensionless flow rate is defined as

$$M = \frac{1}{u_Z h}\int_0^h u\,dy \tag{3.27}$$

and the flow rate is

$$M = \frac{1}{2\pi}\left[\left(\pi + 2\frac{a}{h}\right)\exp\left(-\frac{\pi}{a}h\right) + \pi - 2\frac{a}{h}\right]. \tag{3.28}$$

The wall shear stresses at the two plates are

$$\begin{aligned} \tau_1 &= \mu u_Z\left[\frac{\pi}{a} - \frac{1}{h}\left(1 - \exp\left(-\frac{\pi}{a}h\right)\right)\right], \\ \tau_2 &= \mu u_Z\left[-\frac{1}{h} + \left(\frac{\pi}{a} + \frac{1}{h}\right)\exp\left(-\frac{\pi}{a}h\right)\right]. \end{aligned} \tag{3.29}$$

In Figure 3.4, velocity profiles are shown for different values of h/a. This flow has some special characteristics. When $h/a \to \infty$, the maximum dimensionless velocity tends to 1, and the velocity profile tends to compose from two straight lines: one of them horizontal and the other with inclination equal to 45 degrees. When $h/a \to 0$, the dimensionless velocity tends to 0, and the velocity profile tends to become symmetric with its maximum at the centerline between the plates. We see also that the velocity maximum moves to the centerline as h/a decreases.

3.4. Film flow due to Lorentz forces. Another kind of simple parallel flow is that of a film falling down an inclined wall due to gravity and due to the action of a constant shear stress on the free surface (Bird et al. [32, page 45], [27, page 135]). Here we will treat the motion of a film due to Lorentz force ignoring gravity and retaining the action of the surface shear stress. The boundary conditions for this case are

$$\begin{aligned} &\text{at } y = 0: \quad u = 0, \\ &\text{as } y = h: \quad \mu\frac{\partial u}{\partial y} = \tau_2, \end{aligned} \tag{3.30}$$

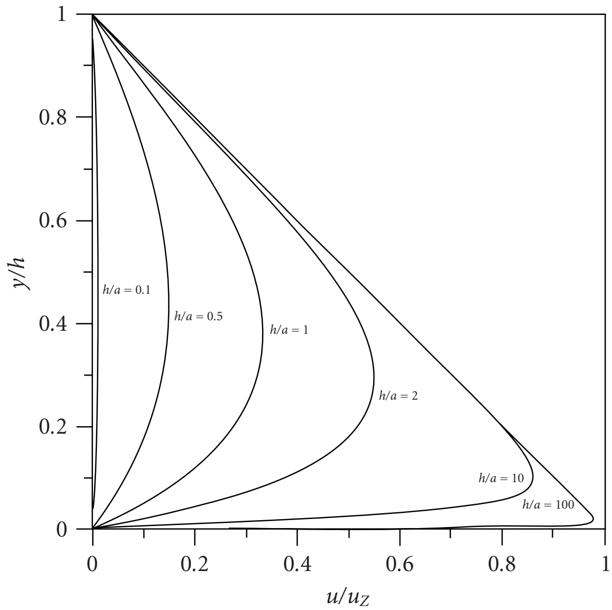


Figure 3.4. Velocity profiles for flow due to Lorentz force only for different values of h/a.

where τ_2 is known and constant. The analytical solution of (2.2), without pressure gradient, with boundary conditions (3.30) is

$$u = \frac{j_0 M_0 a^2}{8\pi\mu}\left[1-\exp\left(-\frac{\pi}{a}y\right)\right]+\left[\frac{\tau_2}{\mu}-\frac{j_0 M_0 a}{8\mu}\exp\left(-\frac{\pi}{a}h\right)\right]y \tag{3.31}$$

and in dimensionless form,

$$\frac{u}{u_Z} = \left[1-\exp\left(-\frac{\pi}{a}y\right)\right]+\left[\frac{\tau_2 h}{\mu u_Z}-\frac{\pi h}{a}\exp\left(-\frac{\pi}{a}h\right)\right]\frac{y}{h}. \tag{3.32}$$

The new dimensionless number is

$$\mathrm{Pa}_f = \frac{\tau_2 h}{\mu u_Z} \tag{3.33}$$

and (3.32) becomes

$$\frac{u}{u_Z} = \left[1-\exp\left(-\frac{\pi}{a}y\right)\right]+\left[\mathrm{Pa}_f-\frac{\pi h}{a}\exp\left(-\frac{\pi}{a}h\right)\right]\frac{y}{h}. \tag{3.34}$$

The dimensionless flow rate is defined as

$$M = \frac{1}{u_Z h}\int_0^h u\,dy \tag{3.35}$$

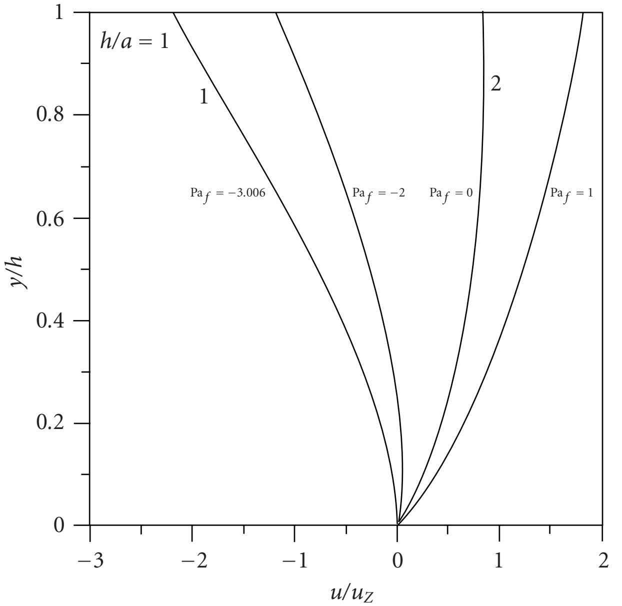


FIGURE 3.5. Velocity profiles for film flow for $h/a = 1$ and different values of the Pa_f number. The case $\mathrm{Pa}_f = 0$ corresponds to zero surface shear stress.

and the flow rate is

$$M = \frac{a}{\pi h}\left[\exp\left(-\frac{\pi}{a}h\right) - 1\right] + \frac{1}{2}\left[\mathrm{Pa}_f - \frac{\pi h}{a}\exp\left(-\frac{\pi}{a}h\right)\right] + 1. \tag{3.36}$$

The shear stress at the plate is

$$\tau_1 = \frac{j_0 M_0 a}{8}\left[1 - \exp\left(-\frac{\pi}{a}h\right)\right] + \tau_2. \tag{3.37}$$

The wall shear stress τ_1 becomes zero when the quantity Pa_f takes the value

$$\mathrm{Pa}_f = \frac{\pi h}{a}\left[\exp\left(-\frac{\pi}{a}h\right) - 1\right]. \tag{3.38}$$

In Figure 3.5, velocity profiles are shown for different values of Pa_f number and $h/a = 1$. Curve 1 corresponds to zero wall shear stress and this happens when $\mathrm{Pa}_f = -3.0058321$ while curve 2 corresponds to zero surface shear stress ($\mathrm{Pa}_f = 0$), that is, the flow is produced by the Lorentz force only.

3.5. Film flow due to Lorentz force only. If the shear stress at the surface is zero we have a film flow caused by the Lorentz force only. From (3.34) we get the velocity of this flow by putting $\mathrm{Pa}_f = 0$. Thus, we have

$$\frac{u}{u_Z} = \left[1 - \exp\left(-\frac{\pi}{a}y\right)\right] - \left[\frac{\pi h}{a}\exp\left(-\frac{\pi}{a}h\right)\right]\frac{y}{h}. \tag{3.39}$$

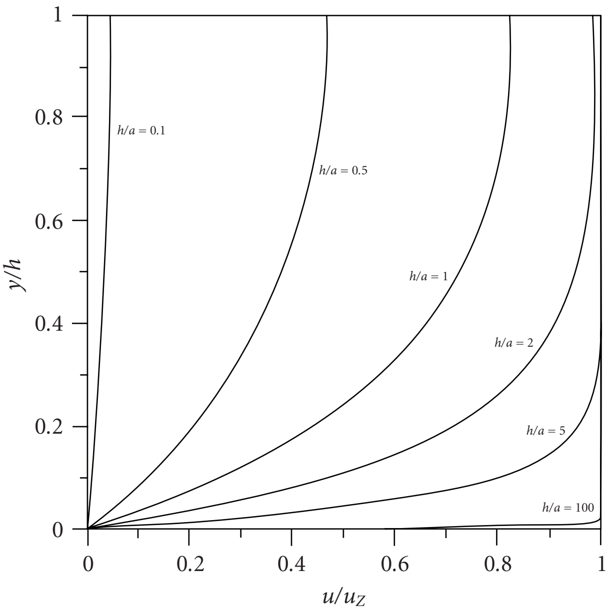


FIGURE 3.6. Velocity profiles for film flow due to Lorentz force only for different values of h/a.

The dimensionless flow rate is defined as

$$M = \frac{1}{u_Z h}\int_0^h u\,dy \tag{3.40}$$

and the flow rate is

$$M = \frac{a}{\pi h}\left[\exp\left(-\frac{\pi}{a}h\right) - 1\right] - \frac{1}{2}\left[\frac{\pi h}{a}\exp\left(-\frac{\pi}{a}h\right)\right] + 1. \tag{3.41}$$

The shear stress at the plate is

$$\tau_1 = \frac{j_0 M_0 a}{8}\left[1 - \exp\left(-\frac{\pi}{a}h\right)\right]. \tag{3.42}$$

In Figure 3.6, velocity profiles are shown for different values of h/a. All velocity profiles have zero gradient at the surface and meet the surface vertically. When $h/a \to \infty$, the maximum dimensionless velocity, which lies on the free surface, tends to 1 and the velocity profile tends to compose from two straight lines: one of them horizontal and the other vertical. When $h/a \to 0$, the dimensionless velocity tends to 0.

4. Conclusions

In this paper some new kinds of parallel flows have been presented and analyzed for weakly electrically conducting fluids. Exact analytical solutions have been given for velocity, flow rate, and wall shear stresses. The author believes that the results of the present work will enrich the list with the existing exact solutions of the Navier-Stokes equations and may help the investigation of flow of electrically conducting fluids in MHD.

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