Research Article
Retailer's EOQ Model with Limited Storage Space Under Partially Permissible Delay in Payments

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The main purpose of this paper wants to investigate the optimal retailer's lot-sizing policy with two warehouses under partially permissible delay in payments within the economic order quantity (EOQ) framework. In this paper, we want to extend that fully permissible delay in payments to the supplier would offer the retailer partially permissible delay in payments. That is, the retailer must make a partial payment to the supplier when the order is received. Then the retailer must pay off the remaining balance at the end of the permissible delay period. In addition, we want to add the assumption that the retailer's storage space is limited. That is, the retailer will rent the warehouse to store these exceeding items when the order quantity is larger than retailer's storage space. Under these conditions, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal cycle time and optimal order quantity. Three theorems are developed to efficiently determine the optimal replenishment policy for the retailer. Finally, numerical examples are given to illustrate these theorems and obtained a lot of managerial insights.

1. Introduction

The traditional economic order quantity (EOQ) model focuses on the buyer's view and makes several assumptions, for example, no stockouts, fixed demand rate, unlimited store space, zero lead time and must be paid for the items as soon as the items were received. But we know these assumptions are rarely met in real-life situation. For instance, in most business transactions, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of his/her products. This

credit term in financial management is denoted as "net 30." Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. So the assumption that the retailer must be paid for the items as soon as the items were received is debatable. The effect of supplier's trade credit policy on inventory problem has received the attention of many researchers.

Goyal [1] established a single-item inventory model for determining the economic ordering quantity in the case that the supplier offers the retailer the opportunity to delay his payment within a fixed time period. Chung [2] simplified the search of the optimal solution for the problem explored by Goyal [1]. Aggarwal and Jaggi [3] considered the inventory model with an exponential deterioration rate under the condition of trade credit. Chang et al. [4] extended this line of research to the varying rate of deterioration. Liao et al. [5] and Sarker et al. [6] investigated this topic with inflation. Jamal et al. [7] and Chang and Dye [8] extended this line of research with allowable shortage. Chang et al. [9] extended this line of research with linear demand. Chen and Chuang [10] investigated buyer's inventory policy under trade credit by the concept of discounted cash flow. Hwang and Shinn [11] developed the model for determining the retailer's optimal price and lot-size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is a function of constant price elasticity. Jamal et al. [12] and Sarker et al. [13] formulated a model where the retailer can pay the wholesaler either at the end of the credit period or later, incurring interest charges on the unpaid balances for the overdue period. They developed a retailer's policy for the optimal cycle and payment times for a retailer in a deteriorating-item inventory scenario, in which a wholesaler allows a specified credit period for payment without penalty. Teng [14] extended Goyal's [1] implicit assumption that the difference between unit selling price and unit purchasing price is equal to operations cost. That is, Goyal [1] implicitly assumed that unit selling price is equal to unit purchasing price. The important finding from Teng's study [14] is that it makes economic sense for a well-established retailer to order small lot sizes and so take more frequently the benefits of the permissible delay in payments. Chung et al. [15] discussed this issue under the assumptions that the selling price is not equal to the purchasing price and different payment rules are allowed. Shinn and Hwang [16] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price. Chung and Huang [17] extended Goyal [1] to consider the case that the units are replenished at a finite rate under permissible delay in payments and developed an efficient solution-finding procedure to determine the retailer's optimal ordering policy. Huang and Chung [18] extended Goyal's model [1] to discuss the replenishment and payment policies to minimize the annual total average cost under cash discount and payment delay from the retailer's point of view. They assumed that the supplier could adopt a cash discount policy to attract retailer to pay the full payment of the amount ofpurchasing at an earlier time as a means to shorten the collection

period. Salameh et al. [19] extended this issue to inventory decision under continuous review. Huang [20] extended one-level trade credit into two-level trade credit to develop the retailer's replenishment model from the viewpoint of the supply chain. He assumed that not only the supplier offers the retailer trade credit but also the retailer offers the trade credit to his/her customer. This viewpoint reflected more real-life situations in the supply chain model. Chang et al. [21] and Chung and Liao [24] dealt with the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity and developed an efficient solution-finding procedure to determine the retailer's optimal ordering policy. In this regard, Chang [23] extended Chung and Liao [24] by taking into account inflation and finite time horizon. Huang [25] considered the case in which the unit selling price and the unit purchasing price are not necessarily equal within the EPQ framework under supplier's trade credit policy.

In this paper, we want to extend that fully permissible delay in payments to the supplier would offer the retailer partially permissible delay in payments. That is, the retailer must make a partial payment to the supplier when the order is received. Then, the retailer must pay off the remaining balance at the end of the permissible delay period. From the viewpoint of supplier's marketing policy, the supplier can use the fraction of permissible delay in payments to agilely control the effects of stimulating the retailer's demand. So this topic is a realistic and new issue in this research field.

Such trade credit policy is one kind of encouragement of the retailer to order large quantities because a delay of payments indirectly reduces inventory cost. Hence, the retailer may purchase more goods than that can be stored in his/her own warehouse (OW). These excess quantities are stored in a rented warehouse (RW). The proposed model is applicable for the business of small- and medium-sized retailers since their storage capacities are small and limited. Especially, Taiwan has traditionally relied on its small- and medium-sized firms to compete in international markets since the 1970s. Therefore, this proposed model is more applicable for the special industrial environment in Taiwan. In general, the inventory holding charges in RW are higher than those in OW. When the demand occurs, it is first replenished from the RW which stores those exceeding items. This is done to reduce the inventory costs. It is further assumed that the transportation costs between warehouses are negligible. Several researchers have studied in this area such as Sarma [26], Pakkala and Achary [27], Benkherouf [28], Goswami and Chaudhuri [22], and Bhunia and Maiti [29].

Under these conditions, this paper tries to deal with the optimal retailer's lot-sizing policy with two warehouses under partially permissible delay in payments. We model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal cycle time and optimal order quantity. Three theorems are developed to efficiently determine the optimal replenishment policy for the retailer. This means that the operation/production department, market department, and finance department are in an enterprise jointly to determine the policy. Therefore, the policy involves inventory, marketing, and financing issues. So, we investigate that this integrated model is very important and valuable to the enterprise.

2. Model formulation

In this section, we want to develop the retailer's inventory model with two warehouses under partially permissible delay in payments. The following notation and assumptions will be used throughout.

Notation:

D = demand rate per year

A = ordering cost per order

W = retailer's storage capacity

c = unit purchasing price

h = unit stock holding cost per year excluding interest charges

k = unit stock holding cost of rented warehouse per year, $(k \geq h)$

I_e = interest earned per \$ per year by retailer

I_k = interest charged per \$ in stocks per year due to partially permissible delay in payment

M = the trade credit period in years

α = the fraction of the delay payments permitted by the supplier per order, $0 \leq \alpha \leq 1$

T = the cycle time in years; time interval between the two consecutive replenishment orders by retailer

TRC (T) = the annual total relevant cost, which is a function of T

T^* = the optimal cycle time of TRC(T)

Q^* = the optimal order quantity = DT^*.

Assumptions:

(1) Demand rate, D, is known and constant.

(2) Shortages are not allowed.

(3) Time horizon is infinite.

(4) $I_k \geq I_e$.

(5) If the order quantity is larger than retailer's storage capacity, the retailer will rent the warehouse to store these exceeding items. When the demand occurs, it is first replenished from the warehouse which stores those exceeding items. In addition, the transportation cost between retailer's warehouse and rented warehouse is negligible. Therefore, we define t_w = the rented warehouse time,

$$
t_w = \begin{cases} \dfrac{DT - W}{D} & \text{if } DT > W \left(T > \dfrac{W}{D} \right), \\ 0 & \text{if } DT \leq W \left(T \leq \dfrac{W}{D} \right). \end{cases}
\tag{2.1}
$$

(6) As the order is received, the retailer must make a partial payment $(1 - \alpha)cDT$ to the supplier. Then the retailer must pay off the remaining balance αcDT at the end of the trade credit period. Therefore, the retailer must pay for the interest charges with rate I_k under partially permissible delay in payments.

(7) During the time, the account is not settled, generated sales revenue is deposited in an interest-bearing account with rate I_e.

The model.   If the order quantity is larger than retailer's storage capacity, the retailer will rent the warehouse to store these exceeding items, $DT > W$ ($T > W/D$). Otherwise, the retailer has not necessarily to rent a warehouse to store items, $DT \leq W$ ($T \leq W/D$). Hence, the stock holding cost is different between these both situations. In addition, the interest charge is different between $M < (1 - \alpha)T(M/(1 - \alpha) < T)$ and $M \geq (1 - \alpha)T(M/(1 - \alpha) \geq T)$. Therefore, we will divide three cases to construct the annual total relevant cost: (1) $M \geq W/D$, (2) $M < W/D \leq M/(1 - \alpha)$, and (3) $M/(1 - \alpha) < W/D$.

Case 1. Suppose that $M \geq W/D$.
    (1) Annual ordering cost = A/T.
    (2) There are two cases to occur in annual stock holding cost (excluding interest charges).
       (i) $0 < T \leq W/D$.
       In this case, the order quantity is not larger than retailer's storage capacity. So the retailer has not necessarily to rent warehouse to store items. Hence, annual stock holding cost = $DTh/2$.
       (ii) $W/D < T$.
       In this case, the order quantity is larger than retailer's storage capacity. So the retailer needs to rent the warehouse to store those exceeding items. Hence, annual stock holding cost = annual stock holding cost of rented warehouse + annual stock holding cost of the own storage capacity = $kt_w(DT - W)/2T + h[Wt_w + W(T - t_w)/2]/T = k(DT - W)^2/2DT + hW(2DT - W)/2DT$.
    (3) According to assumption (6), there are three cases to occur in interest charged per year.
       (i) $M/(1 - \alpha) \leq T$.
       In this case, the retailer must make a partial payment $(1 - \alpha)cDT$ to the supplier as the order received. Hence, the retailer must pay the interest charge from amount $(1 - \alpha)cDT$ on $(0, M]$. In addition, the retailer pays off the remaining balance αcDT at the end of the trade credit period. So, the retailer pays the interest charge from amount αcDT on $[M, T]$. The total amount of interest payable is shown in Figure 2.1. Therefore, the annual interest payable = $cI_k(DT^2/2 - \alpha DTM)/T$.
       (ii) $M \leq T \leq M/(1 - \alpha)$.
       In this case, the retailer must make a partial payment $(1 - \alpha)cDT$ to the supplier as the order received. Hence, the retailer must pay the interest charge from amount $(1 - \alpha)cDT$ on $(0, (1 - \alpha)T]$. In addition, the retailer pays off the remaining balance αcDT at the end of the trade credit period. So, the retailer pays the interest charge from amount αcDT on $[M, T]$. The total amount of interest payable is shown in Figure 2.2. Therefore, the annual interest payable = $cI_k[(1 - \alpha)^2DT^2/2 + D(T - M)^2/2]/T = (cDI_k/2)[(1 - \alpha)^2T^2 + (T - M)^2]/T$.
       (iii) $W/D < T \leq M$, as shown in Figure 2.3.
       In this case, the retailer only makes a partial payment $(1 - \alpha)cDT$ to the supplier as the order received. Hence, the retailer must pay the interest
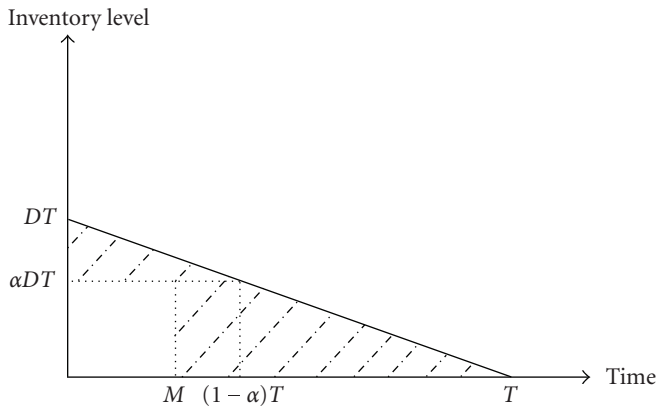
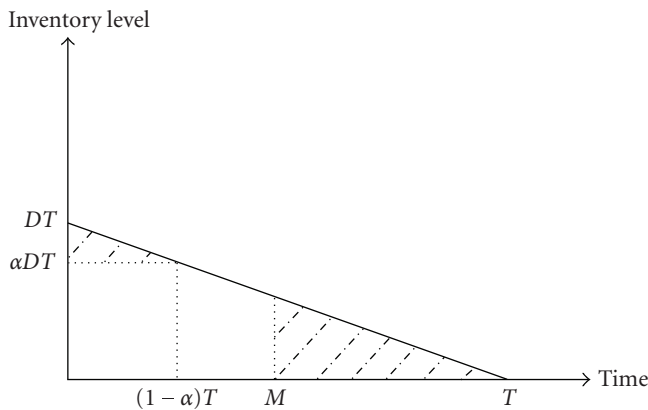FIGURE 2.1 The inventory level and the total accumulation of interest payable when $M/(1-\alpha) \leq T$.

FIGURE 2.2 The inventory level and the total accumulation of interest payable when $M \leq T \leq M/(1-\alpha)$.

charge from amount $(1-\alpha)cDT$ on $(0,(1-\alpha)T]$. The total amount of interest payable is shown in Figure 2.3. Therefore, the annual interest payable $= cI_k[(1-\alpha)^2DT^2/2]/T$.

(4) According to assumption (7), there are two cases to occur in interest earned per year.

(i) $M \leq T$.

In this case, the retailer can earn the interest from sales revenue on $(0,M]$. Therefore, the annual interest earned $= cI_e(DM^2/2)/T$.

(ii) $T \leq M$.

In this case, the retailer can earn the interest from sales revenue on $(0,M]$. The total amount of interest earned is shown in Figure 2.4. Therefore, the annual interest earned $= cI_e[DT^2/2 + DT(M-T)]/T = cI_eDT(M-T/2)/T$.
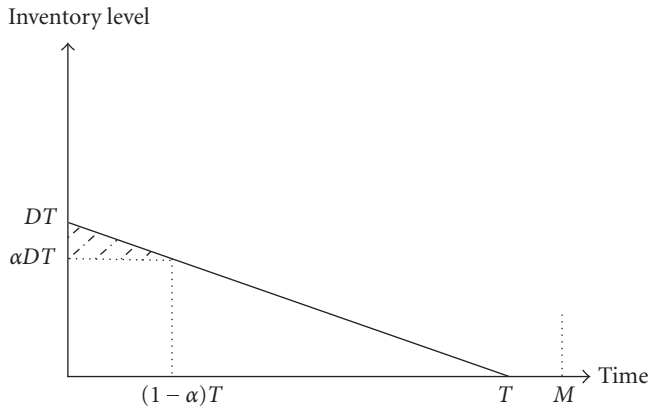
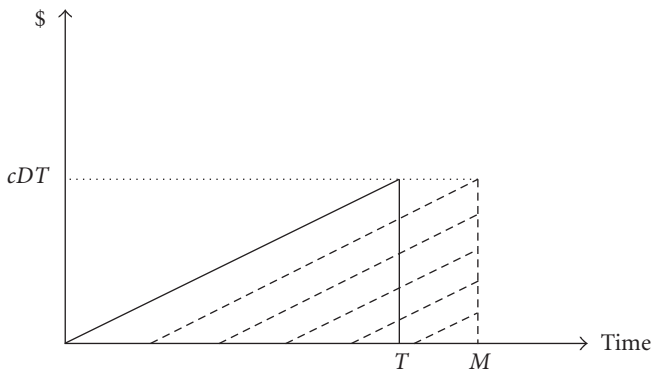FIGURE 2.3 The inventory level and the total accumulation of interest payable when $0 < T \leq M$.

FIGURE 2.4 The total accumulation of interest earned when $T \leq M$.

From the above arguments, the annual total relevant cost for the retailer can be expressed as TRC(T) = ordering cost + stock-holding cost + interest payable − interest earned:

$$
\text{TRC}(T) =
\begin{cases}
\text{TRC}_1(T) & \text{if } T \geq \dfrac{M}{1-\alpha}, \\[2ex]
\text{TRC}_2(T) & \text{if } M \leq T \leq \dfrac{M}{1-\alpha}, \\[2ex]
\text{TRC}_3(T) & \text{if } \dfrac{W}{D} < T \leq M, \\[2ex]
\text{TRC}_4(T) & \text{if } 0 < T \leq \dfrac{W}{D},
\end{cases}
\tag{2.2}
$$

where

$$\mathrm{TRC}_1(T) = \frac{A}{T} + \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT}$$
$$+ cI_kDT\left(\frac{T}{2} - \alpha M\right)\bigg/T - cI_eDM^2/2T, \tag{2.3}$$

$$\mathrm{TRC}_2(T) = \frac{A}{T} + \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT}$$
$$+ cI_kD[(1-\alpha)^2T^2 + (T-M)^2]/2T - cI_eDM^2/2T, \tag{2.4}$$

$$\mathrm{TRC}_3(T) = \frac{A}{T} + \frac{k(DT-W)^2}{2DT} + \frac{hW(2DT-W)}{2DT}$$
$$+ (1-\alpha)^2cI_kDT^2/2T - cI_eDT\left(M - \frac{T}{2}\right)\bigg/T, \tag{2.5}$$

$$\mathrm{TRC}_4(T) = \frac{A}{T} + \frac{DTh}{2} + (1-\alpha)^2cI_kDT^2/2T - cI_eDT\left(M - \frac{T}{2}\right)\bigg/T. \tag{2.6}$$

Case 2. Suppose that $M < W/D \le M/(1-\alpha)$.
   If $M < W/D \le M/(1-\alpha)$, (2.2) will be modified as

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T \ge \dfrac{M}{1-\alpha}, \\[2mm] \mathrm{TRC}_2(T) & \text{if } \dfrac{W}{D} < T \le \dfrac{M}{1-\alpha}, \\[2mm] \mathrm{TRC}_5(T) & \text{if } M \le T \le \dfrac{W}{D}, \\[2mm] \mathrm{TRC}_4(T) & \text{if } 0 < T \le M. \end{cases} \tag{2.7}$$

When $M < W/D \le M/(1-\alpha)$, the annual total relevant cost, $\mathrm{TRC}_5(T)$, consists of the following elements.
   (1) Annual ordering cost $= A/T$.
   (2) In this case, the order quantity is not larger than retailer's storage capacity. So the retailer will not necessarily rent warehouse to store items. Hence, annual stock holding cost $= DTh/2$.
   (3) Annual interest payable $= cI_k[(1-\alpha)^2DT^2/2 + D(T-M)^2/2]/T = (cDI_k/2)[(1-\alpha)^2T^2 + (T-M)^2]/T$.
   (4) Annual interest earned $= cI_e(DM^2/2)/T$.
Combining the above elements, we get

$$\mathrm{TRC}_5(T) = \frac{A}{T} + \frac{DTh}{2} + cI_kD[(1-\alpha)^2T^2 + (T-M)^2]/2T - cI_eDM^2/2T. \tag{2.8}$$

Case 3. Suppose that $M/(1-\alpha) < W/D$.

If $M/(1-\alpha) < W/D$, (2.2) will be modified as

$$\text{TRC}(T) = \begin{cases} \text{TRC}_1(T) & \text{if } T > \dfrac{W}{D}, \\[2mm] \text{TRC}_6(T) & \text{if } \dfrac{M}{1-\alpha} \le T \le \dfrac{W}{D}, \\[2mm] \text{TRC}_5(T) & \text{if } M \le T \le \dfrac{M}{1-\alpha}, \\[2mm] \text{TRC}_4(T) & \text{if } 0 < T \le M. \end{cases} \tag{2.9}$$

When $M/(1-\alpha) < W/D$, the annual total relevant cost, $\text{TRC}_6(T)$, consists of the following elements.

(1) Annual ordering cost $= A/T$.

(2) In this case, the order quantity is not larger than retailer's storage capacity. So the retailer will not necessarily rent a warehouse to store items. Hence, annual stock holding cost $= DTh/2$.

(3) Annual interest payable $= cI_k(DT^2/2 - \alpha DTM)/T$.

(4) Annual interest earned $= cI_e(DM^2/2)/T$.

Combining the above elements, we get

$$\text{TRC}_6(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k DT\left(\frac{T}{2} - \alpha M\right)\Big/T - cI_e DM^2/2T. \tag{2.10}$$

3. Decision rule of the optimal cycle time T^*

In this section, we will determine optimal cycle time for the above three cases under minimizing annual total relevant cost.

Case 1. Suppose that $M \ge W/D$.

From (2.3)–(2.6), derive $\text{TRC}_i'(T_i^*) = 0$ for all $i = 1, 2, 3, 4$. Then, we can obtain

$$T_1^* = \sqrt{\frac{2A + (W^2/D)(k-h) - cDM^2I_e}{D(k+cI_k)}} \quad \text{if } 2A + \frac{W^2}{D}(k-h) - cDM^2I_e > 0, \tag{3.1}$$

$$T_2^* = \sqrt{\frac{2A + (W^2/D)(k-h) + cDM^2(I_k - I_e)}{D\{k + cI_k[1 + (1-\alpha)^2]\}}}, \tag{3.2}$$

$$T_3^* = \sqrt{\frac{2A + (W^2/D)(k-h)}{D\{k + c[(1-\alpha)^2 I_k + I_e]\}}}, \tag{3.3}$$

$$T_4^* = \sqrt{\frac{2A}{D\{h + c[(1-\alpha)^2 I_k + I_e]\}}}. \tag{3.4}$$

Then, we derive $\text{TRC}_i''(T)$ for all $i = 1, 2, 3, 4$. We can find $\text{TRC}_i''(T) > 0$ for all $i = 2, 3, 4$. In addition, we can obtain $\text{TRC}_1''(T) > 0$ when $2A + (W^2/D)(k-h) - cDM^2I_e > 0$.

Therefore, all $\text{TRC}_i(T)$ are convex functions for all $i = 1, 2, 3, 4$ when $2A + (W^2/D)(k - h) - cDM^2I_e > 0$.

Equation (3.1) implies that the optimal value of T for the case of $T \geq M/(1 - \alpha)$ is $T_1^* \geq M/(1 - \alpha)$. We substitute (3.1) into $T_1^* \geq M/(1 - \alpha)$, then we can obtain that

$$\text{iff} - \left[2A + \frac{W^2}{D}(k - h) - cDM^2I_e\right] + D\left(\frac{M}{1 - \alpha}\right)^2(k + cI_k) \leq 0. \tag{3.5}$$

Similarly, (3.2) implies that the optimal value of T for the case of $M \leq T \leq M/(1 - \alpha)$ is $M \leq T_2^* \leq M/(1 - \alpha)$. We substitute (3.2) into $M \leq T_2^* \leq M/(1 - \alpha)$, then we can obtain that

$$\text{iff} - \left[2A + \frac{W^2}{D}(k - h) - cDM^2I_e\right] + D\left(\frac{M}{1 - \alpha}\right)^2(k + cI_k) \geq 0,$$
$$\text{iff} - \left[2A + \frac{W^2}{D}(k - h)\right] + DM^2\{k + c[(1 - \alpha)^2I_k + I_e]\} \leq 0. \tag{3.6}$$

Likewise, (3.3) implies that the optimal value of T for the case of $W/D < T \leq M$ is $W/D < T_3^* \leq M$. We substitute (3.3) into $W/D < T_3^* \leq M$, then we can obtain that

$$\text{iff} - \left[2A + \frac{W^2}{D}(k - h)\right] + DM^2\{k + c[(1 - \alpha)^2I_k + I_e]\} \geq 0,$$
$$\text{iff} - 2A + \frac{W^2}{D}\{h + c[(1 - \alpha)^2I_k + I_e]\} < 0. \tag{3.7}$$

Finally, (3.4) implies that the optimal value of T for the case of $T \leq W/D$, that is $T_4^* \leq W/D$. We substitute (3.4) into $T_4^* \leq W/D$, then we can obtain that

$$\text{iff} - 2A + \frac{W^2}{D}\{h + c[(1 - \alpha)^2I_k + I_e]\} \geq 0. \tag{3.8}$$

Furthermore, we let

$$\Delta_1 = -\left[2A + \frac{W^2}{D}(k - h) - cDM^2I_e\right] + D\left(\frac{M}{1 - \alpha}\right)^2(k + cI_k), \tag{3.9}$$

$$\Delta_2 = -\left[2A + \frac{W^2}{D}(k - h)\right] + DM^2\{k + c[(1 - \alpha)^2I_k + I_e]\}, \tag{3.10}$$

$$\Delta_3 = -2A + \frac{W^2}{D}\{h + c[(1 - \alpha)^2I_k + I_e]\}. \tag{3.11}$$

Equations (3.9)–(3.11) imply that $\Delta_1 \geq \Delta_2 \geq \Delta_3$. From above arguments, we can summarize the following results.

THEOREM 3.1. *Suppose that $M \geq W/D$. Then the following hold.*
    (A) *If $\Delta_3 \geq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_4^*)$ and $T^* = T_4^*$.*
    (B) *If $\Delta_2 > 0$ and $\Delta_3 < 0$, then $\text{TRC}(T^*) = \text{TRC}(T_3^*)$ and $T^* = T_3^*$.*
    (C) *If $\Delta_1 > 0$ and $\Delta_2 \leq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_2^*)$ and $T^* = T_2^*$.*
    (D) *If $\Delta_1 \leq 0$, then $\text{TRC}(T^*) = \text{TRC}(T_1^*)$ and $T^* = T_1^*$.*

Case 2. Suppose that $M < W/D \leq M/(1 - \alpha)$.

If $M < W/D \leq M/(1 - \alpha)$, we know TRC($T$) as follows from (2.7):

$$
\text{TRC}(T) = \begin{cases}
\text{TRC}_1(T) & \text{if } T \geq \dfrac{M}{1 - \alpha}, \\[2ex]
\text{TRC}_2(T) & \text{if } \dfrac{W}{D} < T \leq \dfrac{M}{1 - \alpha}, \\[2ex]
\text{TRC}_5(T) & \text{if } M \leq T \leq \dfrac{W}{D}, \\[2ex]
\text{TRC}_4(T) & \text{if } 0 < T \leq M.
\end{cases}
\tag{3.12}
$$

From (2.8), derive $\text{TRC}_5'(T_5^*) = 0$. Then, we can obtain

$$
T_5^* = \sqrt{\frac{2A + cDM^2(I_k - I_e)}{D\{h + cI_k[1 + (1 - \alpha)^2]\}}}.
\tag{3.13}
$$

Then, we derive $\text{TRC}_5''(T)$ and find $\text{TRC}_5''(T) > 0$. Therefore, $\text{TRC}_5(T)$ is a convex function.

Similar as above procedure in Case 1, we substitute (3.1) into $T_1^* \geq M/(1 - \alpha)$, then we can obtain that

$$
\text{iff} - \left[2A + \frac{W^2}{D}(k - h) - cDM^2 I_e\right] + D\left(\frac{M}{1 - \alpha}\right)^2 (k + cI_k) \leq 0.
\tag{3.14}
$$

Substitute (3.2) into $W/D < T_2^* \leq M/(1 - \alpha)$, then we can obtain that

$$
\begin{aligned}
&\text{iff} - \left[2A + \frac{W^2}{D}(k - h) - cDM^2 I_e\right] + D\left(\frac{M}{1 - \alpha}\right)^2 (k + cI_k) \geq 0, \\[1ex]
&\text{iff} - [2A + cDM^2(I_k - I_e)] + \frac{W^2}{D}\{h + cI_k[1 + (1 - \alpha)^2]\} < 0.
\end{aligned}
\tag{3.15}
$$

Substitute (3.13) into $M \leq T_5^* \leq W/D$, then we can obtain that

$$
\begin{aligned}
&\text{iff} - [2A + cDM^2(I_k - I_e)] + \frac{W^2}{D}\{h + cI_k[1 + (1 - \alpha)^2]\} \geq 0, \\[1ex]
&\text{iff} - 2A + DM^2\{h + c[(1 - \alpha)^2 I_k + I_e]\} \leq 0.
\end{aligned}
\tag{3.16}
$$

Substitute (3.4) into $T_4^* \leq M$, then we can obtain that

$$
\text{iff} - 2A + DM^2\{h + c[(1 - \alpha)^2 I_k + I_e]\} \geq 0.
\tag{3.17}
$$

Furthermore, we let

$$\Delta_4 = -[2A + cDM^2(I_k - I_e)] + \frac{W^2}{D}\{h + cI_k[1 + (1 - \alpha)^2]\}, \tag{3.18}$$

$$\Delta_5 = -2A + DM^2\{h + c[(1 - \alpha)^2 I_k + I_e]\}. \tag{3.19}$$

Equations (3.9), (3.18), and (3.19) imply that $\Delta_1 \geq \Delta_4 > \Delta_5$. From the above arguments, we can summarize the following results.

THEOREM 3.2. *Suppose that $M < W/D \leq M/(1 - \alpha)$. Then the following hold.*
    (A) *If $\Delta_5 \geq 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_4^*)$ and $T^* = T_4^*$.*
    (B) *If $\Delta_4 \geq 0$ and $\Delta_5 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_5^*)$ and $T^* = T_5^*$.*
    (C) *If $\Delta_1 > 0$ and $\Delta_4 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_2^*)$ and $T^* = T_2^*$.*
    (D) *If $\Delta_1 \leq 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_1^*)$ and $T^* = T_1^*$.*

Case 3. Suppose that $M/(1 - \alpha) < W/D$.
    If $M/(1 - \alpha) < W/D$, we know $\mathrm{TRC}(T)$ as follows from (2.9):

$$\mathrm{TRC}(T) = \begin{cases} \mathrm{TRC}_1(T) & \text{if } T > \dfrac{W}{D}, \\[2mm] \mathrm{TRC}_6(T) & \text{if } \dfrac{M}{1 - \alpha} \leq T \leq \dfrac{W}{D}, \\[2mm] \mathrm{TRC}_5(T) & \text{if } M \leq T \leq \dfrac{M}{1 - \alpha}, \\[2mm] \mathrm{TRC}_4(T) & \text{if } 0 < T \leq M. \end{cases} \tag{3.20}$$

From (2.10), derive $\mathrm{TRC}_6'(T_6^*) = 0$. Then, we can obtain

$$T_6^* = \sqrt{\frac{2A - cDM^2 I_e}{D(h + cI_k)}} \quad \text{if } 2A - cDM^2 I_e > 0. \tag{3.21}$$

Then, we derive $\mathrm{TRC}_6''(T)$ and find $\mathrm{TRC}_6''(T) > 0$ when $2A - cDM^2 I_e > 0$. Therefore, $\mathrm{TRC}_6(T)$ is a convex function when $2A - cDM^2 I_e > 0$.
    Similar as the above procedure in Cases 1 and 2, we substitute (3.1) into $T_1^* > W/D$, then we can obtain that

$$\text{iff } -[2A - cDM^2 I_e] + \frac{W^2}{D}(h + cI_k) < 0. \tag{3.22}$$

Substitute (3.21) into $M/(1 - \alpha) \leq T_6^* \leq W/D$, then we can obtain that

$$\text{iff } -[2A - cDM^2 I_e] + \frac{W^2}{D}(h + cI_k) \geq 0,$$

$$\text{iff } -[2A - cDM^2 I_e] + D\left(\frac{M}{1 - \alpha}\right)^2 (h + cI_k) \leq 0. \tag{3.23}$$

Substitute (3.13) into $M \leq T_5^* \leq M/(1-\alpha)$, then we can obtain that

$$
\begin{aligned}
&\text{iff } - [2A - cDM^2 I_e] + D\left(\frac{M}{1-\alpha}\right)^2 (h + cI_k) \geq 0, \\
&\text{iff } - 2A + DM^2 \{h + c[(1-\alpha)^2 I_k + I_e]\} \leq 0.
\end{aligned} \tag{3.24}
$$

Substitute (3.4) into $T_4^* \leq M$, then we can obtain that

$$
\text{iff } - 2A + DM^2 \{h + c[(1-\alpha)^2 I_k + I_e]\} \geq 0. \tag{3.25}
$$

Furthermore, we let

$$
\begin{aligned}
\Delta_6 &= -[2A - cDM^2 I_e] + \frac{W^2}{D}(h + cI_k), \\
\Delta_7 &= -[2A - cDM^2 I_e] + D\left(\frac{M}{1-\alpha}\right)^2 (h + cI_k).
\end{aligned} \tag{3.26}
$$

Equations (3.26) and (3.19) imply that $\Delta_6 > \Delta_7 \geq \Delta_5$. From above arguments, we can summarize the following results.

THEOREM 3.3. *Suppose that $M/(1-\alpha) < W/D$. Then the following hold.*
    (A) *If $\Delta_5 \geq 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_4^*)$ and $T^* = T_4^*$.*
    (B) *If $\Delta_7 \geq 0$ and $\Delta_5 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_5^*)$ and $T^* = T_5^*$.*
    (C) *If $\Delta_6 \geq 0$ and $\Delta_7 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_6^*)$ and $T^* = T_6^*$.*
    (D) *If $\Delta_6 < 0$, then $\mathrm{TRC}(T^*) = \mathrm{TRC}(T_1^*)$ and $T^* = T_1^*$.*

4. Special cases

4.1. Goyal's model. If $\alpha = 1$, it means that the supplier would offer the retailer fully permissible delay in payments. If $k = h$, it means that the unit stock holding cost of the rented warehouse and the unit stock holding cost of the retailer himself are equal. It implies that the retailer's storage capacity is unlimited. Therefore, when $\alpha = 1$ and $k = h$, let

$$
\begin{aligned}
\mathrm{TRC}_7(T) &= \frac{A}{T} + \frac{DTh}{2} + cI_k D(T - M)^2/2T - cI_e DM^2/2T, \\
\mathrm{TRC}_8(T) &= \frac{A}{T} + \frac{DTh}{2} - cI_e DT\left(M - \frac{T}{2}\right)\Big/ T,
\end{aligned} \tag{4.1}
$$

$$
\begin{aligned}
T_7^* &= \sqrt{\frac{2A + cDM^2(I_k - I_e)}{D(h + cI_k)}}, \\
T_8^* &= \sqrt{\frac{2A}{D(h + cI_e)}}.
\end{aligned} \tag{4.2}
$$

Step 1. If $\alpha = 1$ and $k = h$, then use Theorem 4.1. Otherwise, go to Step 2.

Step 2. If $M \geq W/D$, then use Theorem 3.1. Otherwise, go to Step 3.

Step 3. If $M < W/D \leq M/(1 - \alpha)$, then use Theorem 3.2. Otherwise, go to Step 4.

Step 4. If $M/(1 - \alpha) < W/D$, then use Theorem 3.3 and exit the procedure.

<div align="center">ALGORITHM 5.1</div>

Then $\text{TRC}_i'(T_i^*) = 0$ for $i = 7, 8$. Equations (2.2), (2.7), and (2.9) will be reduced as follows:

$$\text{TRC}(T) = \begin{cases} \text{TRC}_7(T) & \text{if } M \leq T, \\ \text{TRC}_8(T) & \text{if } 0 < T \leq M. \end{cases} \tag{4.3}$$

Equations (4.3) will be consistent with [1, equations (1) and (4)], respectively. Hence, Goyal [1] will be a special case of this paper. In addition, (3.2) and (3.3) will be reduced as (4.2), respectively. That is, T_2^* and T_3^* obtained in this paper will be reduced as T_7^* and T_8^* in (4.2), respectively. Then, (3.10) can be modified as $\Delta_2 = -2A + DM^2(h + cI_e)$. If we let $\Delta = -2A + DM^2(h + cI_e)$, Theorem 3.1 can be modified as follows.

THEOREM 4.1. (A) *If* $\Delta > 0$, *then* $T^* = T_8^*$.
    (B) *If* $\Delta < 0$, *then* $T^* = T_7^*$.
    (C) *If* $\Delta = 0$, *then* $T^* = T_7^* = T_8^* = M$.
   Theorem 4.1 has been discussed in Chung [2]. Hence, Theorem 1 in [2] is a special case of Theorem 3.1 of this paper.

4.2. EOQ model. When $\alpha = 1$, $k = h$, and $M = I_k = I_e = 0$, let

$$\text{TRC}_9(T) = \frac{A}{T} + \frac{DTh}{2}, \tag{4.4}$$

$$T_9^* = \sqrt{\frac{2A}{Dh}}. \tag{4.5}$$

Then $\text{TRC}_9'(T_9^*) = 0$. Equations (2.2), (2.7), and (2.9) will be reduced to $\text{TRC}_9(T)$. Equation (4.5) is the optimal time interval of EOQ model. Therefore, EOQ model is a special case of this paper.

5. The algorithm to determine T^*

Now, we will provide an algorithm to determine T^* based on all theorems developed in this paper.

6. Numerical examples

To illustrate the results, let us apply the proposed method to efficiently solve the following numerical examples. For convenience, the values of the parameters are selected randomly. The optimal cycle time and optimal order quantity for different parameters of W, α, and k are shown in Table 6.1. The following inferences can be made based on Table 6.1.

(1) For fixed α and k, increasing the value of W will result in a significant increase in the value of the optimal cycle time for the retailer. It means that the retailer will order more quantity since the retailer owns larger storage space to store more items.

(2) For fixed W and k, increasing the value of α will result in a significant increase in the value of the optimal cycle time for the retailer. It implies that the retailer will order a larger quantity since the retailer can enjoy greater benefits when the fraction of the delay payments permitted is increasing. So the supplier can use the policy of increasing α to stimulate the demands from the retailer. Consequently, the supplier's marketing policy under partially permissible delay in payments will be more agile than fully permissible delay in payments.

(3) Last, for fixed α and W, increasing the value of k will result in a significant decrease in the value of the optimal cycle time for the retailer. It means that the retailer will order less quantity to avoid renting expensive warehouse to store these exceeding items for a too long period when the retailer must rent warehouse.

7. Summary and conclusions

This paper extends the assumption of the fully permissible delay in payments in previously published results and considers two warehouses to reflect realistic business situations. We assume that the supplier will offer the retailer partially permissible delay in payments under two warehouses to model the retailer's inventory system. The retailer's policy involves inventory, marketing, and financing issues. We investigate this integrated model as it is very important and valuable to the retailer. Then we develop three effective and easy-to-use theorems to help the decision maker to find the optimal replenishment policy. Theorem 3.1 gives the decision rule of the optimal cycle time when $M \geq W/D$ after computing the numbers Δ_1, Δ_2, and Δ_3. Theorem 3.2 does the decision rule of the optimal cycle time when $M < W/D \leq M/(1 - \alpha)$ after computing the numbers Δ_1, Δ_4, and Δ_5. At last Theorem 3.3 gives the decision rule of the optimal cycle time when $M/(1 - \alpha) < W/D$ after computing the numbers Δ_5, Δ_6, and Δ_7. Then we deduce Goyal's model [1] and EOQ model as special cases of this paper. Finally, numerical examples are given to illustrate all effective theorems and obtained a lot of managerial insights.

A future study will further incorporate the proposed model into more realistic assumptions, such as finite replenishment rate, probabilistic demand, and allowable shortages. In addition, in this paper, we focus on retailer's inventory decisions in order quantity and cycle time under these conditions we assumed. In the future, we will develop an integrated-supplier-and-retailer inventory model to investigate optimal supplier's strategy and optimal retailer's strategy.

TABLE 6.1 Optimal cycle time and optimal order quantity.

Let A = \$100/order, D = 1000 units/year, c = \$15/unit, h = \$3/unit/year, I_k = \$0.1/\$/year,
I_e = \$0.07/\$/year, and M = 0.12 year

W	α	k	W/D	$M/(1-\alpha)$	Judgements of $\Delta_i (i=1\sim7)$			T^*	Q^*	TRC(T^*)	Theorem
		4	0.1	0.15	$\Delta_1 < 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_1^* = 0.18824$	188.2	899.3	1-(D)
	0.2	6	0.1	0.15	$\Delta_1 < 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_1^* = 0.16927$	169.3	933.49	1-(D)
		8	0.1	0.15	$\Delta_1 < 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_1^* = 0.15724$	157.2	957.77	1-(D)
		4	0.1	0.24	$\Delta_1 > 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_2^* = 0.19196$	192	847.75	1-(C)
100	0.5	6	0.1	0.24	$\Delta_1 > 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_2^* = 0.17329$	173.3	884.65	1-(C)
		8	0.1	0.24	$\Delta_1 > 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_2^* = 0.16116$	161.2	911.46	1-(C)
		4	0.1	0.6	$\Delta_1 > 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_2^* = 0.19732$	197.3	817.1	1-(C)
	0.8	6	0.1	0.6	$\Delta_1 > 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_2^* = 0.17686$	176.9	857.08	1-(C)
		8	0.1	0.6	$\Delta_1 > 0$	$\Delta_2 < 0$	$\Delta_3 < 0$	$T_2^* = 0.16379$	163.8	885.87	1-(C)
		4	0.2	0.15	$\Delta_5 < 0$	$\Delta_6 < 0$	$\Delta_7 < 0$	$T_1^* = 0.20221$	202.2	876.13	3-(D)
	0.2	6	0.2	0.15	$\Delta_5 < 0$	$\Delta_6 < 0$	$\Delta_7 < 0$	$T_1^* = 0.20162$	201.6	876.15	3-(D)
		8	0.2	0.15	$\Delta_5 < 0$	$\Delta_6 < 0$	$\Delta_7 < 0$	$T_1^* = 0.20128$	201.3	876.16	3-(D)
		4	0.2	0.24	$\Delta_1 > 0$	$\Delta_4 < 0$	$\Delta_5 < 0$	$T_2^* = 0.20483$	204.8	823.36	2-(C)
200	0.5	6	0.2	0.24	$\Delta_1 > 0$	$\Delta_4 < 0$	$\Delta_5 < 0$	$T_2^* = 0.20361$	203.6	823.44	2-(C)
		8	0.2	0.24	$\Delta_1 > 0$	$\Delta_4 < 0$	$\Delta_5 < 0$	$T_2^* = 0.20289$	202.9	823.49	2-(C)
		4	0.2	0.6	$\Delta_1 > 0$	$\Delta_4 < 0$	$\Delta_5 < 0$	$T_2^* = 0.21055$	210.6	790.65	2-(C)
	0.8	6	0.2	0.6	$\Delta_1 > 0$	$\Delta_4 < 0$	$\Delta_5 < 0$	$T_2^* = 0.20781$	207.8	791.05	2-(C)
		8	0.2	0.6	$\Delta_1 > 0$	$\Delta_4 < 0$	$\Delta_5 < 0$	$T_2^* = 0.2062$	206.2	791.28	2-(C)
		4	0.3	0.15	$\Delta_5 < 0$	$\Delta_6 > 0$	$\Delta_7 < 0$	$T_6^* = 0.20269$	202.7	876.12	3-(C)
	0.2	6	0.3	0.15	$\Delta_5 < 0$	$\Delta_6 > 0$	$\Delta_7 < 0$	$T_6^* = 0.20269$	202.7	876.12	3-(C)
		8	0.3	0.15	$\Delta_5 < 0$	$\Delta_6 > 0$	$\Delta_7 < 0$	$T_6^* = 0.20269$	202.7	876.12	3-(C)
		4	0.3	0.24	$\Delta_5 < 0$	$\Delta_6 > 0$	$\Delta_7 > 0$	$T_5^* = 0.2058$	205.8	823.29	3-(B)
300	0.5	6	0.3	0.24	$\Delta_5 < 0$	$\Delta_6 > 0$	$\Delta_7 > 0$	$T_5^* = 0.2058$	205.8	823.29	3-(B)
		8	0.3	0.24	$\Delta_5 < 0$	$\Delta_6 > 0$	$\Delta_7 > 0$	$T_5^* = 0.2058$	205.8	823.29	3-(B)
		4	0.3	0.6	$\Delta_1 > 0$	$\Delta_4 > 0$	$\Delta_5 < 0$	$T_5^* = 0.21279$	212.8	790.33	2-(B)
	0.8	6	0.3	0.6	$\Delta_1 > 0$	$\Delta_4 > 0$	$\Delta_5 < 0$	$T_5^* = 0.21279$	212.8	790.33	2-(B)
		8	0.3	0.6	$\Delta_1 > 0$	$\Delta_4 > 0$	$\Delta_5 < 0$	$T_5^* = 0.21279$	212.8	790.33	2-(B)

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