

Research Article

Closed-Form Solutions for a Mode-III Moving Interface Crack at the Interface of Two Bonded Dissimilar Orthotropic Elastic Layers

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An integral transform technique is used to solve the elastodynamic problem of a crack of fixed length propagating at a constant speed at the interface of two bonded dissimilar orthotropic layers of equal thickness. Two cases of practical importance are investigated. Firstly, the lateral boundaries of the layers are clamped and displaced in equal and opposite directions to produce antiplane shear resulting in a tearing motion along the leading edge of the crack, and secondly, the lateral boundaries of the layers are subjected to shear stresses. The analytic solution for a semi-infinite crack at the interface of two bonded dissimilar orthotropic layers has been derived. Closed-form expressions are obtained for stressing the intensity factor and other physical quantities in all cases.

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1. Introduction

First of all, Sih and Chen [1] solved the problem of a Griffith crack in an orthotropic layer under antiplane shear. They reduced the solution to a Fredholm integral equation of the second kind, and values for stress intensity factors were obtained by solving the Fredholm integral equation numerically. Singh et al. [2], and Tait and Moodie [3] obtained closed-form solutions for a finite length crack moving with constant velocity in a strip. In [2], an integral transform method was used, while in [3] complex variable techniques were used. Closed-form solutions for a finite length crack moving at constant velocity in an orthotropic layer of finite thickness were obtained by Danyluk and Singh [4], and that work was an extension of the work discussed in [1–3]. Making use of complex variable methods, Georgiadis [5] solved the problem of a cracked orthotropic strip, and the problem of steadily moving crack in an orthotropic material under antiplane shear stress has been studied by Piva [6, 7]. Recently,

Li [8] obtained a closed-form solution for a mode-III interface crack between two bonded dissimilar elastic layers. This paper is concerned with a mode-III moving crack interface between two bonded orthotropic dissimilar elastic layers whose closed-form solution has been obtained. Furthermore, the exact results for a semi-infinite interface crack in two bonded elastic orthotropic elastic layers have been obtained directly from those of a finite length crack results through a limiting process.

The results of this paper are more general than those of the paper of Li [8]. If we assume that the velocity of the moving crack is zero and assuming the constants $c_{44(j)} = c_{55(j)} = \mu_{(j)}$ ($j = 1, 2$), we get the results of the paper of Li [8], where $\mu_{(j)}$ are the shear moduli of the upper and lower layers and $c_{44(j)}$, $c_{55(j)}$ are defined in the paper.

The standard method for solving mixed boundary value problems is to reduce the solution into Fredholm integral equation of the second kind, where approximate solutions can be found. The aim of this paper is to obtain closed forms or exact solutions of the problems.

2. Basic equations and formulation of the crack problem

Consider two elastic layers of equal thickness h occupying the region $-\infty < X < \infty$, $-h < Y < h$, $-\infty < Z < \infty$, where $OXYZ$ is a fixed rectangular coordinate system. We assume that a crack of length $2a$ is moving at a constant velocity v in the X -direction at the interface of the two layers as shown in Figure 1. The purpose of this investigation is to determine the effect of orthotropy of the materials on the initial direction of propagation of the crack which is moving with constant speed. Assuming that there is a single nonvanishing displacement component in the Z -direction, we have

$$U_j = V_j = 0, \quad W_j = W_j(X, Y, t), \quad (2.1)$$

where U_j , V_j , W_j are displacement components in the X , Y , and Z directions, respectively, and $j = 1, 2$. Then

$$\sigma_{X(j)} = \sigma_{Y(j)} = \sigma_{Z(j)} = \sigma_{XY(j)} = 0, \quad \sigma_{XZ(j)} = c_{55(j)} \frac{\partial W_j}{\partial X}, \quad \sigma_{YZ(j)} = c_{44(j)} \frac{\partial W_j}{\partial Y}, \quad (2.2)$$

where $c_{44(j)}$ and $c_{55(j)}$ are the shear moduli in YZ and XZ planes, respectively, for both materials. The equation of motion for both layers is

$$\frac{\partial^2 W_j}{\partial X^2} + \frac{\partial^2 W_j}{\partial Y_j^2} = \frac{1}{C_j^2} \frac{\partial^2 W_j}{\partial t^2}, \quad (2.3)$$

where

$$Y_j = \frac{Y}{\sqrt{\beta_j}}, \quad \beta_j = \frac{c_{44(j)}}{c_{55(j)}}, \quad C_j = \sqrt{\frac{c_{44(j)}}{\rho_j}}, \quad (2.4)$$

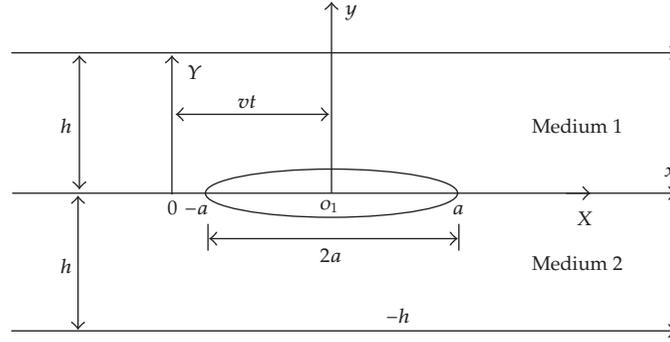


Figure 1: Two bonded layers of piezoelectric ceramic materials with crack at interface.

and C_j is the shear wave speed and ρ_j is the constant density of the material. For a crack moving with constant speed in the X -direction, it is convenient to introduce the Galilean transformation

$$x = X - vt, \quad y_j = Y_j, \quad z = Z. \quad (2.5)$$

Equation (2.3) now becomes

$$s_j^2 \frac{\partial^2 W_j}{\partial x^2} + \frac{\partial^2 W_j}{\partial y_j^2} = 0, \quad (2.6)$$

where

$$s_j^2 = 1 - \frac{v^2}{C_j^2}. \quad (2.7)$$

3. Solution of equilibrium equation

The solution of the equilibrium (2.6) may be written as

$$W_j(x, y) = F_c [A_j \exp(-\xi y_j s_j) + B_j \exp(\xi y_j s_j); \xi \rightarrow x], \quad (3.1)$$

where s_j is taken as (2.7) and

$$F_c [A_j(\xi); \xi \rightarrow x] = \sqrt{\frac{2}{\pi}} \int_0^\infty A_j(\xi) \cos(\xi x) d\xi, \quad (3.2)$$

$$F_s [B_j(\xi); \xi \rightarrow x] = \sqrt{\frac{2}{\pi}} \int_0^\infty B_j(\xi) \sin(\xi x) d\xi.$$

The nonzero stresses are given by

$$\sigma_{yz(j)}(x, y) = \frac{s_j c_{44(j)}}{\sqrt{\beta_j}} F_c [\xi B_j \exp(\xi y_j s_j) - \xi A_j \exp(-\xi y_j s_j); \xi \rightarrow x], \quad (3.3)$$

$$\sigma_{xz(j)}(x, y) = -c_{55(j)} F_s [\xi B_j \exp(\xi y_j s_j) + \xi A_j \exp(-\xi y_j s_j); \xi \rightarrow x]. \quad (3.4)$$

Now, we consider the two basic problems involving shear stress loading and displacement conditions on the surface of the layer.

Problem A. Let the antiplane shear stress be applied to the surfaces $Y = \pm h$, then the equivalent problem in this instance involves the application of a shear stress $-p(x)$ to the crack surfaces at $Y = 0$. The boundary conditions can then be written as

$$\sigma_{yz(1)}(x, 0^+) = -p(x), \quad |x| < a, \quad (3.5)$$

$$\sigma_{yz(1)}(x, 0^+) = \sigma_{yz(2)}(x, 0^-), \quad -\infty < x < \infty, \quad (3.6)$$

$$W_1(x, 0^+) = W_2(x, 0^-), \quad |x| > a, \quad (3.7)$$

$$\sigma_{yz(1)}(x, +h) = \sigma_{yz(2)}(x, -h) = 0, \quad (3.8)$$

where $p(x)$ is an even function. Problem A consists of solving (2.6) together with (3.5)–(3.8).

Problem B. Let the lateral boundaries of the layer $Y = \pm h$ be rigidly clamped and displaced by equal amount $p(x)$ in opposite directions which produce an antiplane shear motion in the Z -direction and while the crack moves in the positive X -direction at a constant speed. In order to use the integral transform technique, it is necessary to solve an alternative but equivalent problem.

The equivalent conditions are

$$\sigma_{yz(1)}(x, 0) = -\frac{c_{44(1)}}{h} p(x), \quad |x| < a, \quad (3.9)$$

$$\sigma_{yz(1)}(x, 0) = \sigma_{yz(2)}(x, 0), \quad -\infty < x < \infty, \quad (3.10)$$

$$W_1(x, 0) = W_2(x, 0), \quad |x| > a, \quad (3.11)$$

$$W_1(x, h) = W_2(x, -h) = 0. \quad (3.12)$$

4. Solution of Problem A

From (3.3) and (3.6), we find that

$$B_1(\xi) = A_1(\xi) \exp(-2\xi h_1 s_1), \quad B_2(\xi) = A_2(\xi) \exp(2\xi h_2 s_2), \quad (4.1)$$

where

$$h_1 = \frac{h}{\sqrt{\beta_1}}, \quad h_2 = \frac{h}{\sqrt{\beta_2}}. \quad (4.2)$$

From (3.1), (3.3), and (4.1), we find that

$$W_1(x, y) = 2F_c [A_1(\xi) e^{-s_1 h_1 \xi} \cos h \xi s_1 (h_1 - y_1); \xi \rightarrow x], \quad -\infty < x < \infty, \quad 0 < y \leq h, \quad (4.3)$$

$$W_2(x, y) = 2F_c [A_2(\xi) e^{s_2 h_2 \xi} \cos h \xi s_2 (h_2 - y_2); \xi \rightarrow x], \quad -\infty < x < \infty, \quad -h \leq y < 0, \quad (4.4)$$

$$\sigma_{yz_{(1)}}(x, y) = -\frac{2s_1 c_{44(1)}}{\sqrt{\beta_1}} F_c [\xi A_1(\xi) e^{-s_1 h_1 \xi} \sinh \xi s_1 (h_1 - y_1); \xi \rightarrow x], \quad -\infty < x < \infty, \quad 0 < y \leq h, \quad (4.5)$$

$$\sigma_{yz_{(2)}}(x, y) = \frac{2s_2 c_{44(2)}}{\sqrt{\beta_2}} F_c [\xi A_2(\xi) e^{s_2 h_2 \xi} \sinh \xi s_2 (h_2 + y_2); \xi \rightarrow x], \quad -\infty < x < \infty, \quad -h \leq y < 0. \quad (4.6)$$

From boundary condition (3.7), we find that

$$A_2 = -\left[\frac{s_1 \sqrt{\beta_2} \sinh(\xi s_1 h_1) c_{44(1)}}{s_2 \sqrt{\beta_1} \sinh(\xi s_2 h_2) c_{44(2)}} e^{-\xi h_1 s_1 - \xi h_2 s_2} \right] A_1(\xi). \quad (4.7)$$

From (4.3), (4.4), and (4.7), we find that

$$\begin{aligned} & W_1(x, 0^+) - W_2(x, 0^-) \\ &= 2F_c \left[\frac{e^{-\xi h_1 s_1} A_1(\xi)}{\sinh(\xi s_2 h_2)} \{ \cos h(\xi s_1 h_1) \sinh(\xi s_2 h_2) + P \sinh(\xi s_1 h_1) \cos h(\xi s_2 h_2) \}; \xi \rightarrow x \right], \end{aligned} \quad (4.8)$$

where

$$P = \frac{s_1 \sqrt{\beta_2} c_{44(1)}}{s_2 \sqrt{\beta_1} c_{44(2)}}. \quad (4.9)$$

Now, the boundary conditions (3.5) and (3.7) lead to the following dual integral equations:

$$\begin{aligned} F_c [\xi A_1(\xi) e^{-\xi h_1 s_1} \sinh(\xi s_1 h_1); \xi \rightarrow x] &= \frac{\sqrt{\beta_1}}{2s_1 c_{44(1)}} p(x), \quad 0 < x < a, \\ F_c \left[\frac{A_1(\xi) e^{-\xi h_1 s_1}}{\sinh(\xi s_2 h_2)} \{ \cos h(\xi s_1 h_1) \sinh(\xi s_2 h_2) + P \sinh(\xi s_1 h_1) \cos h(\xi s_2 h_2) \}; \xi \rightarrow x \right] &= 0, \\ & a < x < \infty. \end{aligned} \quad (4.10)$$

Closed-form solution of the dual integral equations (4.10) is difficult to obtain and only approximate solution of these dual integral equations can be obtained by changing them into a Fredholm integral equation of the second kind. For obtaining closed form, we assume that

$$\frac{s_1}{\sqrt{\beta_1}} = \frac{s_2}{\sqrt{\beta_2}}, \quad (4.11)$$

so that

$$P = \frac{c_{44(1)}}{c_{44(2)}}. \quad (4.12)$$

If we take

$$(1 + P)e^{-\xi h_1 s_1} \cos h(\xi s_1 h_1) A_1(\xi) = C(\xi), \quad (4.13)$$

then the dual integral equations (4.10) can be written in the form

$$F_c[\xi C(\xi) \tanh(\xi s_1 h_1); \xi \rightarrow x] = \frac{\sqrt{\beta_1}}{2s_1} (c_{44(1)}^{-1} + c_{44(2)}^{-1}) p(x) = p_1(x), \quad 0 < x < a, \quad (4.14)$$

$$F_s[C(\xi); \xi \rightarrow x] = 0, \quad a < x < \infty.$$

Following [4], the solution of the the dual integral equations (4.14) can be written in the form

$$C(\xi) = \frac{1}{\xi} \sqrt{\frac{\pi}{2}} \int_0^a \Phi(\tau) \sin(\xi \tau) d\tau, \quad (4.15)$$

where

$$\Phi(\tau) = \frac{-2c \sinh(2c\tau)}{\pi^2 (\sinh^2(ca) - \sinh^2(c\tau))^{1/2}} \int_0^a \frac{p_1(x) (\sinh^2(ca) - \sinh^2(cx))^{1/2}}{\sinh^2(cx) - \sinh^2(c\tau)} dx, \quad 0 < \tau < a, \quad (4.16)$$

$$c = \frac{\pi}{2h_1 s_1}.$$

For the particular case $p(x) = p_0$ when p_0 is a constant, we find that

$$\Phi(\tau) = \frac{\sqrt{\beta_1} ((c_{44(1)}^{-1} + c_{44(2)}^{-1}) p_0 \sinh(2c\tau))}{s_1 \pi^2 \cos h(ac) (\sinh^2(ca) - \sinh^2(c\tau))^{1/2}} \left[F\left(\frac{\pi}{2}, \tanh(ac)\right) - L(\tau) \right], \quad (4.17)$$

$$L(\tau) = \Pi\left(\frac{\pi}{2}, \frac{\sinh^2(ca)}{\sinh^2(ca) - \sinh^2(c\tau)} \tanh(ac)\right),$$

where F and Π are elliptic integrals of the first and third kinds, respectively, as defined in the table of Gradshteyn and Ryzhik [9].

The stress distribution along the crack is given by

$$\sigma_{yz(1)}(x, y) = \frac{p_0 \sinh(2cx)}{\pi \cos h(ac) (\sinh^2(ca) - \sinh^2(cx))^{1/2}} \left[F\left(\frac{\pi}{2}, \tanh(ac)\right) - L(\tau) \right], \quad a < x < \infty, \quad (4.18)$$

and the crack sliding displacement is

$$\begin{aligned} \Delta W(x) &= W_{(1)}(x, 0^+) - W_{(2)}(x, 0^-) \\ &= \frac{\sqrt{\beta_1} ((c_{44(1)}^{-1} + c_{44(2)}^{-1}) p_0 \sinh(2c\tau))}{\pi s_1 \cos h(ac) (\sinh^2(ca) - \sinh^2(cx))^{1/2}} \int_x^a \left[F\left(\frac{\pi}{2}, \tanh(ac)\right) - L(\tau) \right] d\tau, \quad 0 < x < a. \end{aligned} \quad (4.19)$$

The stress intensity factor can be written in the form

$$K_3 = \lim_{x \rightarrow a^+} \left[\sqrt{2(x-a)} \sigma_{yz(1)}(x, 0) \right] = \frac{2p_0}{\pi} \left[\frac{\tanh(ca)}{c} \right]^{1/2} F\left[\frac{\pi}{2}, \tanh(ca)\right]. \quad (4.20)$$

Assuming that under applied loading the crack tip advances along the crack plane from $x = a$ to $a + \delta a$ ($\delta a \ll a$), then the energy release rate per unit length during this process is given by

$$G_{III} = \lim_{\delta a \rightarrow 0} \frac{1}{2\delta a} \int_0^{\delta a} [\sigma_{yz(1)}(r, 0) \Delta W(\delta a - r, 0)] dr, \quad (4.21)$$

where r denotes the distance from the crack tip.

From (4.21), we find that

$$G_{III} = \frac{\sqrt{\beta_1}}{s} (c_{44(1)}^{-1} + c_{44(2)}^{-1}) \frac{p_0^2 \tanh(ca)}{\pi c} F\left[\frac{\pi}{2}, \tanh(ca)\right]^2. \quad (4.22)$$

We find that the shear stress displacement and intensity factors obtained above are in agreement with the corresponding results in Danyluk and Singh [4].

The energy release rate (4.22) is new, which has not been obtained in [4].

5. Solution of Problem B

For this problem, we find that the boundary condition (3.12) will be satisfied if

$$B_1(\xi) = -A_1(\xi) \exp(-2\xi h_1 s_1), \quad B_2(\xi) = -A_2(\xi) \exp(-2\xi h_2 s_2). \quad (5.1)$$

Using (5.1), (3.1), and (3.3), we find that

$$W_1(x, y) = 2F_c [e^{-\xi h_1 s_1} A_1(\xi) \sinh(\xi s_1 (h_1 - y_1)); \xi \rightarrow x] \quad 0 \leq y \leq h, \quad x > 0, \quad (5.2)$$

$$W_2(x, y) = 2F_c [e^{\xi h_2 s_2} A_2(\xi) \sinh(\xi s_2 (h_2 + y_2)); \xi \rightarrow x] \quad -h \leq y \leq 0, \quad x > 0, \quad (5.3)$$

$$\sigma_{yz(1)}(x, y) = -\frac{2s_1 c_{44(1)}}{\sqrt{\beta_1}} F_c [\xi A_1(\xi) e^{-s_1 h_1 \xi} \cos h[\xi s_1 (h_1 - y_1)]; \xi \rightarrow x], \quad 0 \leq y \leq h, \quad x > 0, \quad (5.4)$$

$$\sigma_{yz(2)}(x, y) = \frac{2s_2 c_{44(2)}}{\sqrt{\beta_2}} F_c [\xi A_2(\xi) e^{s_2 h_2 \xi} \cos h[\xi s_2 (h_2 + y_2)]; \xi \rightarrow x], \quad -h \leq y \leq 0, \quad x > 0. \quad (5.5)$$

From (5.4) and (5.5) and the boundary condition (3.10), we find that

$$A_1(\xi) \frac{s_1 c_{44(1)} e^{-\xi h_1 s_1} \cos h(\xi s_1 h_1)}{\sqrt{\beta_1}} = -\frac{s_2 c_{44(2)} e^{\xi h_2 s_2} \cos h(\xi s_2 h_2)}{\sqrt{\beta_2}} A_2(\xi). \quad (5.6)$$

Using (4.11) and (5.6), the boundary conditions (3.9) and (3.11) lead to the dual integral equations

$$F_c [\xi C_1(\xi) \coth(s_1 h_1 \xi); \xi \rightarrow x] = \frac{\sqrt{\beta_1}}{2hs_1} (c_{44(1)}^{-1} + c_{44(2)}^{-1}) c_{44(1)} p(x), \quad 0 < x < a, \quad (5.7)$$

$$F_c [C_1(\xi); \xi \rightarrow x] = 0, \quad a < x,$$

where

$$(1 + P) e^{-s_1 h_1 \xi} A_1(\xi) \sinh(\xi s_1 h_1) A_1(\xi) = C_1(\xi). \quad (5.8)$$

Following [4], the solution of the dual integral equations (5.7) for $p(x) = p_0$ can be written in the form

$$C_1(\xi) = \sqrt{\frac{\pi}{2}} \xi^{-1} \int_0^a \Psi(u) \sin(u\xi) du, \quad (5.9)$$

where

$$\Psi(u) = \frac{\sqrt{\beta_1} \sin(cu) c_{44(1)} (c_{44(1)}^{-1} + c_{44(2)}^{-1}) p_0}{\pi (hs_1) [\sinh^2(ca) - \sinh^2(cu)]^{1/2}}. \quad (5.10)$$

We can easily find that

$$\sigma_{yz(1)}(x, 0^+) = \frac{-c_{44(1)} p_0}{h} \left[1 - \frac{\sinh(cx)}{[\sinh^2(cx) - \sinh^2(ca)]^{1/2}} \right], \quad x > a. \quad (5.11)$$

The stress intensity factor at $x = a$ is given by

$$K_{III} = \lim_{x \rightarrow a^+} [\sqrt{2(x-a)} \sigma_{yz(1)}(x, 0)] = \frac{c_{44(1)}}{h} p_0 \left[\frac{\tanh(ca)}{c} \right]^{1/2}, \quad (5.12)$$

and the crack sliding displacement is

$$\Delta W(x) = W_{(1)}(x, 0^+) - W_{(2)}(x, 0^-) = \frac{p_0 \sqrt{\beta_1}}{s_1} \left(1 + \frac{c_{44(1)}}{c_{44(2)}} \right) \left[1 - \frac{2}{\pi} \sin^{-1} \left(\frac{\cos h(cx)}{\cos h(ca)} \right) \right]. \quad (5.13)$$

The energy release rate per unit length during the process is given by

$$G_{III} = \lim_{\delta \rightarrow 0} \frac{1}{2\delta} \int_0^\delta \sigma_{yz(1)}(r) \Delta w(\delta - r) dr = \frac{1}{2s_1 h} (c_{44(1)}^{-1} + c_{44(2)}^{-1}) p_0^2 c_{44(1)}^2 \tanh(ca). \quad (5.14)$$

6. Solution for a semi-infinite interface crack

For the case of clamped boundaries of the layers, the closed-form solution for a semi-infinite interface crack is obtained by taking $x = a + x_1$ in (5.11), (5.12), and (5.13) and then letting $a \rightarrow \infty$, so that we have

$$\begin{aligned} \sigma_{yz(1)}(x, 0^+) &= \sigma_{yz(1)}(x, 0^-) = -\frac{c_{44(1)} p_0}{h} \left[1 - \frac{1}{\sqrt{1 - e^{-(\pi x_1/h)}}} \right], \quad x_1 > 0, \\ \Delta W(x_1) &= \frac{p_0 \sqrt{\beta_1}}{s_1} \left(1 + \frac{c_{44(1)}}{c_{44(2)}} \right) \left[1 - \frac{2}{\pi} \sin^{-1} \left(e^{\pi x_1/2h} \right) \right], \quad x_1 < 0, \\ K_{III} &= \lim_{x_1 \rightarrow 0} [\sqrt{2x_1} \sigma_{yz(1)}(x_1, 0^+)] = c_{44(1)} p_0 \sqrt{\frac{2}{h\pi}}. \end{aligned} \quad (6.1)$$

The result for stress intensity factor for the special case of a stationary crack in an infinite strip coincides with the corresponding result obtained by Georgiadis [5] and Rice [10].

7. Conclusions

The closed-form solution provided in this paper is of less importance due to condition (4.11). Condition (4.11) can be written in the form

$$v^2 = \frac{c_{44(1)} c_{44(2)} [c_{55(1)} c_{44(2)} - c_{55(2)} c_{44(1)}]}{[\rho_1 c_{55(1)} c_{44(2)}^2 - \rho_2 c_{55(2)} c_{44(1)}^2]}. \quad (7.1)$$

If the velocity of the crack is known for particular values of the constants $c_{44(1)}$, $c_{55(1)}$, ρ_1 , we can substitute the values of constants $c_{44(2)}$, $c_{55(2)}$, ρ_2 so that (7.1) is satisfied. Due to this, the solutions of (7.1) exist for some layered specified orthotropic materials.

For the stationary crack, we assume $\nu = 0$. Then, from (7.1), we find that

$$\frac{c_{44(1)}}{c_{55(1)}} = \frac{c_{44(2)}}{c_{55(2)}}. \quad (7.2)$$

Equation (7.2) is very simple and hence we can easily find the solutions for stationary crack problems for upper and lower layers of orthotropic materials from (7.2), which has practical value. The solutions are already obtained by Li [8] for stationary crack in isotropic elastic layers, while due to condition (7.2), we can find solutions for stationary crack in orthotropic layers.

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