Research Article

Convection-Radiation Heat Transfer in a Nonlinear Fluid with Temperature-Dependent Viscosity

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We will study the fully developed flow of a generalized second-grade fluid down an inclined plane. The viscosity is not only a function of the shear rate (shear-thinning or shear-thickening) but also depends on the temperature in an exponential way. The effects of radiation boundary condition at the free surface are considered. The boundary value problem is solved numerically, and the velocity and temperature profiles are obtained for various dimensionless numbers.

1. Introduction

Combined heat transfer processes, such as convection-radiation, play a significant role in many chemical processes involving combustion, drying, fluidization, MHD flows, and so forth (see [1, 2]). In general, the radiative process either occurs at the boundaries or as a term in the energy equation. The latter case is usually accomplished by a suggestion due to Rosseland (see [3]) where the radiative term is approximated as a flux in such a way that the term corresponding to radiation in the heat transfer (energy) equation now appears as a gradient term similar to Fourier's conduction term. This method has found much favor among many researchers especially those working with viscoelastic fluids and MHD flows (see [4–10]). Alternatively, radiation effects can be incorporated at the boundaries through appropriate (constitutive) assumption, such as the Stefan-Boltzmann condition. Free surface flows present a challenging problem to engineers as the combined convection-radiation at the boundaries has major applications in many industries.

Flow down an inclined plane occurs naturally as in the cases of avalanches and mudslides; it is also used for transporting and drying of bulk solids (such as agricultural

and pharmaceutical products). It is a viscometric flow (see [11]) and one which amends itself to fundamental theoretical and experimental studies. Studies have shown that if the effects of viscous dissipation are included, strong nonlinearities in the temperate profiles are observed (see [12]). There are many cases in polymer rheology and lubrication, for example, where viscous dissipation cannot be ignored (see [13–15]). An important combustion-related issue is the effectiveness of handling and injecting of many non-Newtonian fluids such as coal-water slurries or coal-oil slurries. It has been demonstrated (see [16]) that substantial performance benefits can be obtained if the coal-water mixture is preheated. The effects of temperature change on coal slurry properties are controlled to a great extent by changes in the properties of the fluid. The constitutive equation used, most often, in these studies is the power law model. Although the power law model adequately fits the shear stress and shear rate measurements for many non-Newtonian fluids, it cannot always be used to accurately predict the pressure loss data measured during transport of a coal-liquid mixture in a fuel delivery system (see [17]).

Fluids of differential type (especially second and third grade fluids) have been studied extensively. For example, Rajagopal and Na [18] analyzed the natural convection of a third-grade fluid between two infinite parallel vertical plates; Szeri and Rajagopal [14] studied the flow of a third-grade fluid between heated parallel plates, and Massoudi and Christie [19, 20] considered the natural and forced convection of a third-grade fluid in an annulus. One of the recent advances in the theoretical studies in rheology is the development of generalized differential grade models. The simplicity of the form and the fact that these modified constitutive relations can be used to study shear-thinning/thickening, the decrease/increase in viscosity with increasing/decreasing shear rate, as well as predicting normal stress differences have opened the way for the solution to a series of engineering problems (see [21–25]).

In this work, we study the behavior of coal slurries, assuming that they can be represented by a constitutive equation similar to that of a generalized second-grade fluid, where the viscosity is also a function of temperature. Specifically, we revisit the problem considered by Massoudi and Phuoc [24] who studied the flow of a generalized second-grade fluid down an inclined plane subject to convective heat transfer condition at the free surface, and consider the radiation boundary instead. The free surface is exposed to a high temperature and as a result a radiation boundary condition is used. The effects of viscous dissipation are also included in the heat transfer equation. The fully developed flow condition simplifies the problem and we obtain a set of three nonlinear ordinary differential equations which would have to be solved numerically. The equations are made dimensionless and the effects of different dimensionless numbers are discussed.

2. Governing Equations

The governing equations of motion are the conservation of mass, linear momentum, and energy equation. These are (see [26]) as follows.

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \text{div}\,(\rho \mathbf{u}) = 0, \tag{2.1}$$

where ρ is the density of the fluid, $\partial/\partial t$ is the partial derivative with respect to time, and $\mathbf{u}$ is the velocity vector. For an isochoric motion we have div $\mathbf{u} = 0$.

Conservation of linear momentum:

$$\rho \frac{d\mathbf{u}}{dt} = \operatorname{div} \mathbf{T} + \rho \mathbf{b}, \tag{2.2}$$

where $\mathbf{b}$ is the body force vector, $\mathbf{T}$ is the stress tensor, and d/dt is the total time derivative, given by $d(\cdot)/dt = \partial(\cdot)/\partial t + [\operatorname{grad}(\cdot)]\mathbf{u}$.

Conservation of energy:

$$\rho \frac{de}{dt} = \mathbf{T} \cdot \mathbf{L} - \operatorname{div} \mathbf{q} + \rho r, \tag{2.3}$$

where e is the specific internal energy, $\mathbf{L}$ is the gradient of velocity, $\mathbf{q}$ is the heat flux vector, and r is the radiant heating. Thermodynamical considerations require the application of the second law of thermodynamics or the entropy inequality. The various forms of this law are known as the Clausius-Duhem inequality, or the principle of dissipation [27, page 295] which states that

$$\frac{1}{\rho}\operatorname{tr}(\mathbf{TD}) + \theta\dot{\eta} - \dot{e} \geq 0, \tag{2.4}$$

where θ is the temperature field, $\mathbf{D}$ is the symmetric part of the velocity gradient, and η is the specific entropy. We will not consider the constraints imposed due to Clausius-Duhem inequality (entropy law). In order to "close" these equations, we need to provide constitutive relations for $\mathbf{T}, \mathbf{q}$, and r. The application of the second law to radiative heat processes is by no means an easy and straightforward matter (see [28]) as assumed or implied in books and research articles. In this paper, we use the thermodynamical results of Dunn and Fosdick [29] without applying (2.4) directly to our constitutive relation.

3. Constitutive Relations

Perhaps the simplest model which can describe the normal stress effects (which could lead to phenomena such as "die-swell", and "rod-climbing" which are manifestations of the stresses that develop orthogonal to planes of shear) is the second-grade fluid, or the Rivlin-Ericksen fluid of grade two (see [27, 30]). This model has also been used and studied extensively and is a special case of fluids of differential type. For a second-grade fluid the stress is given by

$$\mathbf{T} = -p\mathbf{1} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{3.1}$$

where p is the indeterminate part of the stress due to the constraint of incompressibility, μ is the coefficient of viscosity, α_1 and α_2 are material moduli which are commonly referred to as the normal stress coefficients. The kinematical tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined through $\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T$, $\mathbf{A}_2 = (d\mathbf{A}_1/dt) + \mathbf{A}_1\mathbf{L} + (\mathbf{L})^T\mathbf{A}_1$, and $\mathbf{L} = \operatorname{grad} \mathbf{u}$. The thermodynamics and stability of fluids of second grade have been studied in detail by Dunn and Fosdick [29]. They show that if the fluid is to be thermodynamically consistent in the sense that all motions of the fluid

meet the Clausius-Duhem inequality and that the specific Helmholtz free energy of the fluid be a minimum in equilibrium, then

$$
\begin{aligned}
\mu &\geq 0, \\
\alpha_1 &\geqq 0, \\
\alpha_1 + \alpha_2 &= 0.
\end{aligned}
\tag{3.2}
$$

It is known that for many non-Newtonian fluids which are assumed to obey (3.1), the experimental values reported for α_1 and α_2 do not satisfy the restriction $(3.2)_{2,3}$. In an important paper, Fosdick and Rajagopal [31] show that irrespective of whether $\alpha_1 + \alpha_2$ is positive, the fluid is unsuitable if α_1 is negative. In particular, they showed that if it is assumed that

$$
\mu > 0, \qquad \alpha_1 < 0, \qquad \alpha_1 + \alpha_2 \neq 0,
\tag{3.3}
$$

which as many experiments have reported to be the case "for those fluids which the experimentalists *assume* to be constitutively determined by (3.1), at least sufficiently well as a second-order approximation" [31, page 147], then certain anomalous results follow. Fosdick and Rajagopal [31] proved a theorem which indicates that if $(3.3)_{2,3}$ hold, then an unusual behavioral property not to be expected for any rheological fluid occurs, namely, the *larger* the viscosity, keeping everything else fixed, the *faster* that initial data is amplified in motions which take place under strict isolation. For further details on this and other relevant issues in fluids of differential type, we refer the reader to the review article by Dunn and Rajagopal [32]. In recent years, Rajagopal and colleagues (see, e.g., [33, 34]) have devised a thermodynamic framework, the Multiple Natural Configuration Theory, by appealing to the maximization of the rate of entropy production to obtain a class of constitutive relations for many different types of materials. Unlike the traditional thermodynamic approach (e.g., [29]) whereby a form for the stress is assumed (or derived) and restrictions on the material parameters are obtained by invoking the Clausius-Duhem inequality, in this new thermodynamic framework, they assume specific forms for the Helmholtz potential and the rate of dissipation—reflecting on how the energy is stored in the body and the way in which the body dissipates it. In a recent study, Rajagopal and Srinivasa [35] have modified their theory to obtain various forms for the fluid models of differential type, specially the second grade fluid; interestingly, they also arrive at the same conclusion about the restrictions imposed on α_1 and α_2, namely that $\alpha_1 + \alpha_2 = 0$.

In an effort to obtain a model that does exhibit both normal stress effects and shear-thinning/thickening, Man and Sun [36] and Man [21] modified the constitutive equation for a second-grade fluid by allowing the viscosity coefficient to depend upon the rate of deformation. Their model was later generalized by Gupta and Massoudi [22, 37] by allowing the shear viscosity to also be a function of temperature. That is,

$$
\mathbf{T} = -p\mathbf{1} + \mu(\theta)\Pi^{m/2}\,\mathbf{A_1} + \alpha_1\mathbf{A_2} + \alpha_2\mathbf{A_1^2},
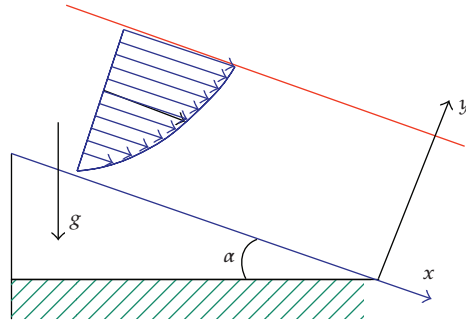\tag{3.4}
$$

Figure 1

where $\Pi = (1/2)\mathrm{tr}\,\mathbf{A}_1^2$ is the second invariant of the symmetric part of the velocity gradient, m is a material parameter, and $\mu(\theta)$ was assumed to obey the Reynolds viscosity model,

$$\mu(\theta) = \mu_0 e^{-M\theta}, \qquad (3.5)$$

where

$$M = n(\theta_2 - \theta_1), \qquad (3.6)$$

when $m < 0$, the fluid is shear-thinning, and if $m > 0$, the fluid is shear-thickening. This model can find applications in many processes where preheating of the fuel is used as a means to enhance heat transfer effects. In addition, for many fluids such as lubricants, polymers, and coal slurries where viscous dissipation is substantial, an appropriate constitutive relation where viscosity is a function of temperature should be used. In this paper, we will use (3.4) (see [38] for a recent review). It also needs to be mentioned that second-grade fluids (or higher order models) raise the order of differential equations by introducing higher-order derivates into the equations. As a result, in general, one needs additional boundary conditions; for a discussion of this issue, see Rajagopal [39] and Rajagopal and Kaloni [40].

The constitutive relation for the heat flux vector is given by Fourier's assumption where

$$\mathbf{q} = -K\nabla\theta, \qquad (3.7)$$

where K is the thermal conductivity, and is assumed to be constant in this paper. The effects of radiation are assumed to be at the free surface and therefore will be discussed in the next section along with other boundary conditions.

4. Flow Down an Inclined Plane

Whenever nonlinear constitutive relations are studied, the solution procedures, for solving the governing equations, whether analytical or computational, are more complicated. Exact
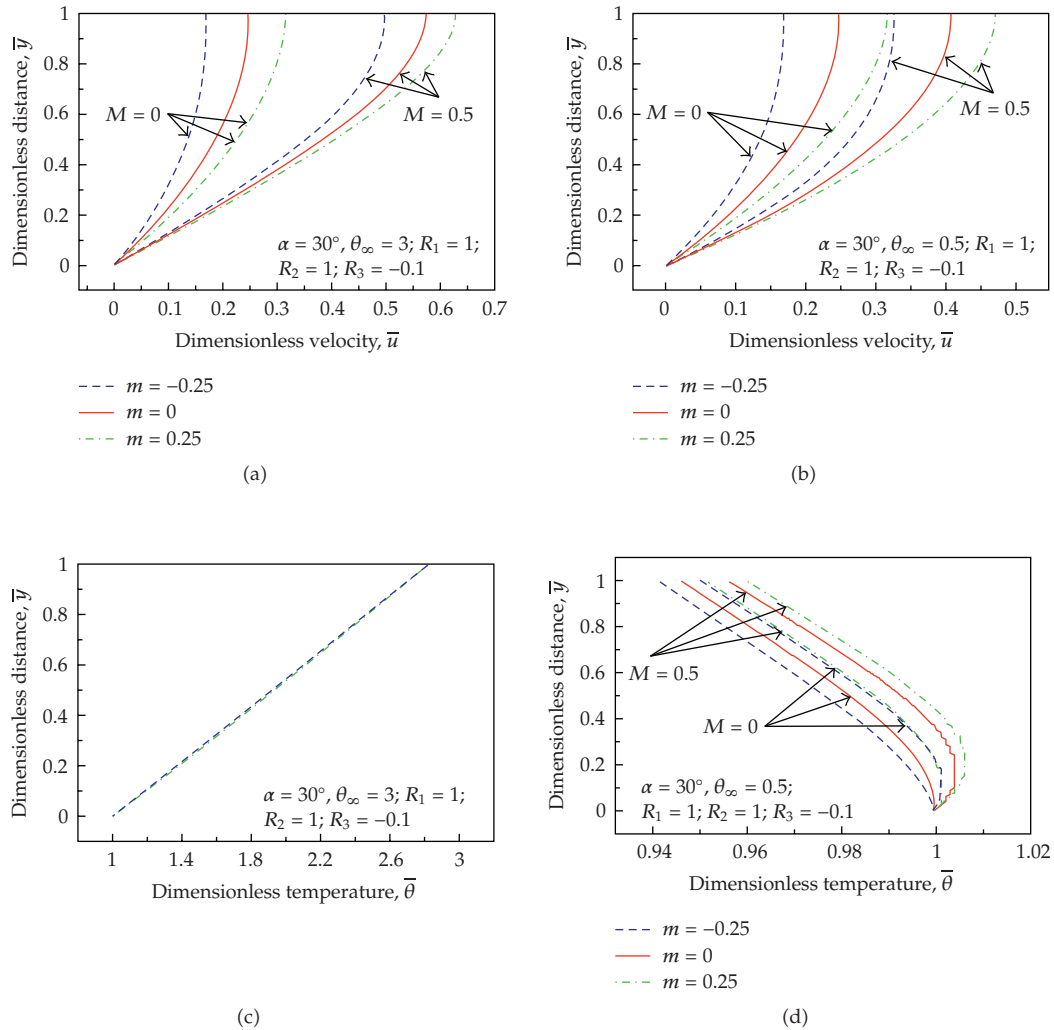
(a)

(b)

(c)

(d)

Figure 2: Velocity and temperature profiles.

solutions are very rare in heat transfer studies of nonlinear materials or multiphase flows. Next to the exact solutions, finding solutions to simple boundary value problems is extremely desirable. Most of the constitutive relations used in mechanics, whether non-Newtonian models, turbulence models, and so forth, when substituted in the general governing equations, that is, the balance laws, would produce a system of partial differential equations which at times are impossible to solve completely with the numerical techniques currently available. Therefore, from a modeling point of view, it is worthwhile to study problems where due to simplification of the kinematics of the flow or the boundary conditions, one obtains a system of (nonlinear) ordinary differential equations. The solution to these simpler problems would be useful for at least two different reasons: (i) They provide insight into the nature of these nonlinear constitutive relations, and (ii) they provide cases where the accuracy or convergence of solutions to the general multidimensional equations can be tested. Other interesting phenomena such as stability and uniqueness of solutions also sometimes arise.
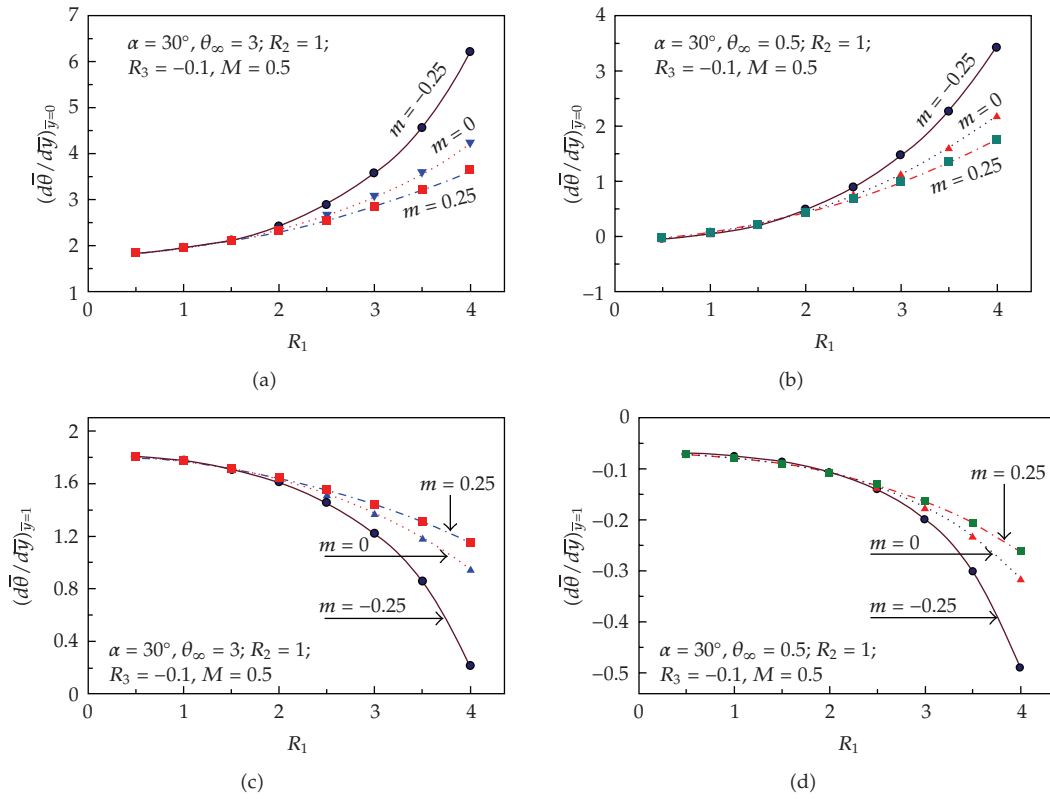
Figure 3: Heat transfer from the free surface: effect of R_1.

For the problem under consideration, we make the following assumptions:

(i) the motion is steady and fully developed,

(ii) the effect of radiant heating r is included at the free surface,

(iii) the constitutive equation for the stress tensor is given by (3.4) and the constitutive equation for the heat flux vector is that of Fourier's assumption,

(iv) the velocity and temperature fields are of the form:

$$\mathbf{u} = u(y)\mathbf{i},$$
$$\theta = \theta(y),$$

(4.1)

where y is the direction normal to the inclined plane, and $\mathbf{i}$ designates a unit vector in the direction of the flow (see Figure 1). The free surface is exposed to high ambient temperature and as a result a modified Stefan-Boltzmann correlation for radiation is used at that surface (see [41, page 331]).
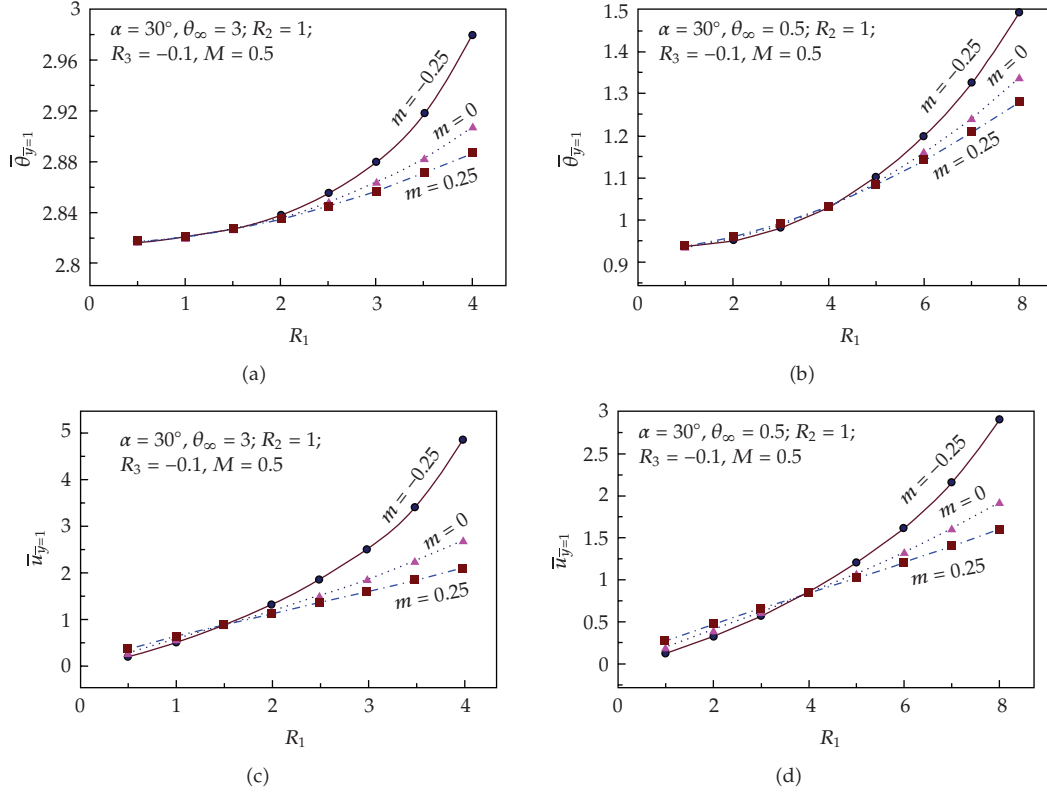
(a)

(b)

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Figure 4: Free surface temperature and velocity as a function of R_1.

With the above assumptions, the conservation of mass is automatically satisfied and the momentum equation (2.2) in its dimensionless form becomes (see [24, 42] for details), having used the Reynolds viscosity model and integrating once

$$\frac{d\overline{u}}{d\overline{y}} = R_1^{1/(m+1)}\left(e^{M\overline{\theta}}(1-\overline{y})\right)^{1/(m+1)}(\sin\alpha)^{1/(m+1)}. \tag{4.2}$$

The energy equation (2.3) reduces to

$$\frac{d^2\overline{\theta}}{d\overline{y}^2} + R_2\frac{1}{e^{M\overline{\theta}}}\left(\frac{d\overline{u}}{d\overline{y}}\right)^{m+2} = 0, \tag{4.3}$$

where the dimensionless distance $\overline{y}$, the velocity $\overline{u}$, and the temperature $\overline{\theta}$ are assumed to be given by the following equations (see [43]):

$$\overline{y} = \frac{y}{h}; \qquad \overline{u} = \frac{u}{V}; \qquad \overline{\theta} = \frac{\theta}{\theta_w} \qquad \mu^* = \frac{\mu}{\mu_o} \tag{4.4}$$

where V is a reference velocity, θ_w is the wall temperature, and h is the constant height to the free surface. The dimensionless numbers R_1 and R_2 are

$$R_1 = \frac{\rho g h^{m+2}}{V^{m+1}\mu_0},$$

$$R_2 = \mu_o \left(\frac{V}{h}\right)^{m+2} \frac{h^2}{\theta_w K},$$

(4.5)

Now, R_1 is the ratio of the viscous effects to the gravitational effects

$$R_1 = \frac{\rho g h^{m+2}}{V^{m+1}\mu_0} = \frac{gh}{V^2}\left(\frac{\rho h^{m+1} V^{1-m}}{\mu_0}\right) = \frac{\text{Re}}{(\text{Fr})^2},$$

(4.6)

related to the Reynolds and Froude numbers, and R_2 is a measure of viscous dissipation

$$R_2 = \mu_o \left(\frac{V}{h}\right)^{m+2} \frac{h^2}{\theta_w K} = \left(\left(\frac{c_p \mu}{K}\right)\left(\frac{V}{h}\right)^m\right)\left(\frac{V^2}{c_p \theta_w}\right) = \text{Pr Ec},$$

(4.7)

related to the Prandtl and Eckert numbers. At the wall of the inclined we assume the no-slip condition for the velocity and a constant temperature θ_w. At the free surface, the no-traction boundary condition is imposed on the stress tensor ($t_x|_{y=h} = T_{xx}n_x + T_{xy}n_y + T_{xz}n_z = \mu[|u'|^2]^{m/2}u' = 0$ and $t_y|_{y=h} = T_{yx}n_x + T_{yy}n_y + T_{yz}n_z = -p + (2\alpha_1 + \alpha_2)u'^2 = 0$, where by defining a modified pressure $p^* = p(2\alpha_1 + \alpha_2)u'^2$, the second relationship implies that $p^* = 0$ at $y = h$), (see [24]) , and for the temperature we apply the Stefan-Boltzmann condition (see also [44])

$$q = \varepsilon\sigma\left(\theta_h^4 - \theta_\infty^4\right),$$

(4.8)

where the surrounding temperature is designated as θ_∞ and the temperature at the free surface is θ_h. (The importance of radiation boundary condition for composite plates is discussed by Miller and Weaver [45], for fluid-particle flow by Chamkha [46], and for packed beds by Wu and Chu [47].)The boundary conditions become

$$\text{at } \overline{y} = 0 : \begin{cases} \overline{u} = 0, \\ \overline{\theta} = 1, \end{cases}$$

$$\text{at } \overline{y} = 1 : \begin{cases} \dfrac{d\overline{u}}{d\overline{y}} = 0, \\ \dfrac{d\overline{\theta}}{d\overline{y}} = R_3\left(\overline{\theta}_{\overline{y}=1}^4 - \overline{\theta}_\infty^4\right), \end{cases}$$

(4.9)

where

$$R_3 = -\frac{h\varepsilon\sigma\theta_w^3}{K},$$
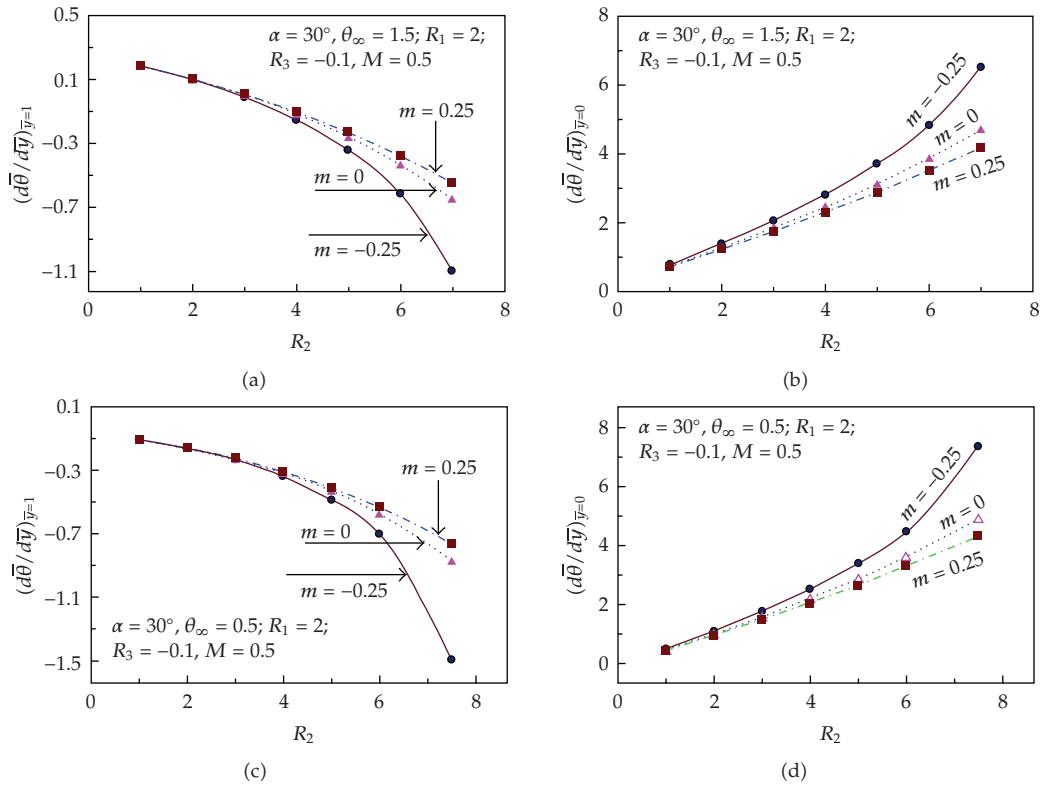
(4.10)

(a)

(b)

(c)

(d)

Figure 5: Heat transfer from the free surface: effect of R_2.

is a measure of the emissivity to the thermal conductivity and

$$\overline{\theta}_\infty = \frac{\theta_\infty}{\theta_w}. \tag{4.11}$$

It can be seen that the following material parameters m, M, the angle of inclination α, a dimensionless temperature at infinity $\overline{\theta}_\infty$, and three dimensionless numbers R_1, R_2, and R_3 determine the flow and temperature patterns.

5. Numerical Solutions

Based on our previous experience we use the following values:

$$
\begin{aligned}
m &= -0.25, 0, 0.25, \\
M &= 0, 0.5, 3.5, \\
\alpha &= 30 \, \text{degrees}, \\
R_1 &= 0.5, 2, 4, \\
R_2 &= 1, 5, 10, \\
R_3 &= -0.1, \\
\overline{\theta}_\infty &= 0.5, 1.5, 3.
\end{aligned}
\tag{5.1}
$$

When the viscosity does not depend on the temperature, that is, $M = 0$, the solutions are given analytically as

$$\overline{u} = (R_1 \sin \alpha)^{1/(m+1)} \frac{m+1}{m+2} \left[1 - (1 - \overline{y})^{(m+2)/(m+1)}\right], \tag{5.2}$$

$$\frac{d^2\overline{\theta}}{d\overline{y}^2} = -R_2 (R_1 \sin \alpha)^{(m+2)/(m+1)} (1 - \overline{y})^{(m+2)/(m+1)}, \tag{5.3}$$

$$\frac{d\overline{\theta}}{d\overline{y}} = R_2 (R_1 \sin \alpha)^{(m+2)/(m+1)} \frac{m+1}{2m+3} (1 - \overline{y})^{(2m+3)/(m+1)} + R_3 \left(\overline{\theta}_{\overline{y}=1}^4 - \overline{\theta}_\infty^4\right), \tag{5.4}$$

$$\theta = R_2 (R_1 \sin \alpha)^{(m+2)/(m+1)} \frac{m+1}{2m+3} \frac{m+1}{3m+4} \left[1 - (1 - \overline{y})^{(3m+4)/(m+1)}\right] + R_3 \left(\overline{\theta}_{\overline{y}=1}^4 - \overline{\theta}_\infty^4\right) y + 1. \tag{5.5}$$

When $M \neq 0$, (4.2) and (4.3) are coupled and they must be solved simultaneously. Following the numerical procedures reported by Massoudi and Phuoc [24], the dimensionless temperature and velocity are obtained for $m = -0.25, 0.0,$ and 0.25 by keeping $\alpha = 30°, R_3 = -0.1,$ and $M = 0.5$ and using $\overline{\theta}_\infty, R_1,$ and R_2 as parameters. Figure 2 shows the velocity distribution for both heating and cooling conditions for the case of temperature-dependent ($M = 3.5$) and temperature-independent ($M = 0.0$) viscosity. For the heating case we use $\overline{\theta}_\infty = 3.5$ and for the cooling case $\overline{\theta}_\infty = 0.5$. All other parameters are kept constant ($\alpha = 30$, $R_1 = 0.5$; $R_2 = 1$; $R_3 = -0.1$). The results are calculated for a shear-thinning fluid ($m = -0.25$), the Newtonian fluid, ($m = 0$) and a shear-thickening fluid ($m = 0.25$). When $M = 0$, the velocity does not depend on the temperature and therefore the distribution is the same for $\overline{\theta}_\infty = 0.5$ and $\overline{\theta}_\infty = 3$. In this case the temperature profile depends significantly on $\overline{\theta}_\infty$. In the cooling case where $\overline{\theta}_\infty \prec \overline{\theta}_w$ the temperature profiles are affected whether the fluid is shearthinning, Newtonian, or shear-thickening. When $\overline{\theta}_\infty > \overline{\theta}_w$, the effects of m are minimal due to the viscosity decrease with higher temperature.

The effects of the dimensionless parameter R_1 on the heat transfer, the dimensionless temperature and velocity at the free surface are presented in Figures 3 and 4. The calculations are made using $\overline{\theta}_\infty = 3.0$ and 0.5 while all other parameters are kept constant. For the conditions used here, the heat transfer at the wall (the surface of the incline), as well as from the free surface is seen to increase with R_1. In the heating case, $\overline{\theta}_\infty = 3.0$, the wall temperature is always lower than that of the surrounding, and it increases as R_1 increases. In the cooling case, $\overline{\theta}_\infty = 0.5$, the free surface temperature is lower than the wall temperature when R_1 is less than about 1.75 indicating that the fluid is heated. For R_1 higher than this value, the free surface temperature is higher than both the wall temperature and the temperature of the surrounding. This is due to the fact that R_1 represents the effect of the gravity force such that an increase in R_1 leads to an increase the dimensionless velocity of the free surface as shown in Figure 4. As a result, the fluid is heated due to the kinetic energy that is converted into thermal energy.

The effects of R_2 on the heat transfer, temperature, and velocity at the free surface are shown in Figures 5 and 6 for the case where $\alpha = 30°$, $R_1 = 2$, $R_3 = -0.1$, $M = 0.5$, and $\overline{\theta}_\infty = 1.5$ and 0.5. When $\overline{\theta}_\infty = 0.5$ as R_2 increases, $(d\theta/d\overline{y})_{\overline{y}=1}$ is negative and it continues to decrease while $(d\theta/d\overline{y})_{\overline{y}=0}$ is positive and increases indicating that the maximum temperature is
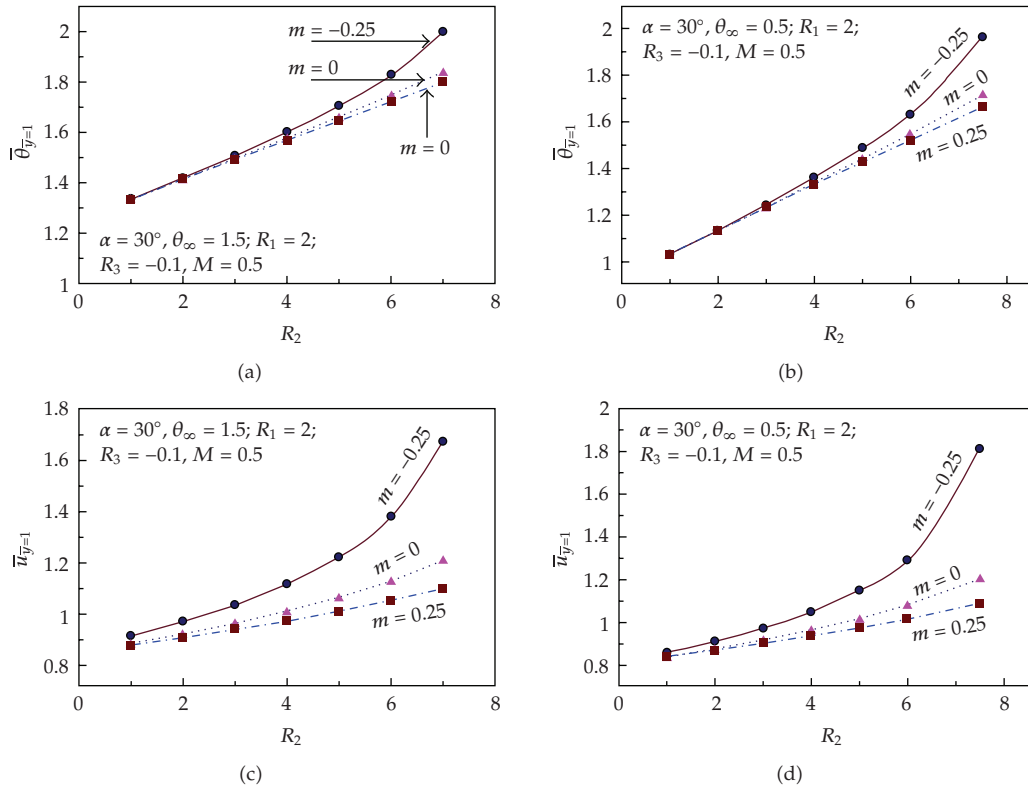
Figure 6: Free surface temperature and velocity as a function of R_2.

higher than both the wall (incline surface) temperature and the surrounding temperature; this maximum value is located somewhere in the flow field and heat is transferred from the fluid to both the wall and the free surface. When $\overline{\theta}_\infty = 1.5$, as R_2 increases, $(d\theta/d\overline{y})_{\overline{y}=0}$ is positive and increases while $(d\theta/d\overline{y})_{\overline{y}=1}$ is positive and decreases when R_2 is less than 3.0. The free surface temperature is lower than that of the surrounding. Thus, in this range of R_2 the highest temperature is that of the surrounding and the fluid is heated. When R_2 is increased to a higher value than 3.0, $(d\theta/d\overline{y})_{\overline{y}=1}$ becomes negative and the free surface temperature is higher than that of the surrounding indicating that the fluid is heated (due to the effect of R_2) and the excess heat is transferred to both the wall and the surrounding. As shown by (4.3), the temperature is not only affected by the surrounding temperature but also by parameter R_2 which represents the effect of viscous dissipation. Increasing R_2 can lead to a situation that the viscous dissipation becomes a significant source in the energy equation.

6. Conclusions

In this paper we have studied the fully developed flow of a generalized second-grade fluid down an inclined plane. The viscosity is a function of the shear rate (shear-thinning or shear-thickening) and it also depends on the temperature in an exponential way. The effects of radiation boundary condition at the free surface are considered. The boundary value problem is solved numerically, and the velocity and temperature profiles are obtained for

various dimensionless numbers. The following material parameters m (power-law exponent accounting for shear-rate dependence of the viscosity), M (an exponent accounting for temperature dependence of viscosity), the angle of inclination α, a dimensionless temperature at infinity $\bar{\theta}_\infty = \theta_\infty/\theta_w$, and three dimensionless numbers R_1, R_2, and R_3, where R_1 is the ratio of the viscous effects to the gravitational effects (related to the Reynolds and Froude numbers), R_2 is a measure of viscous dissipation (related to the Prandtl and Eckert numbers), and R_3 is a measure of the emissivity to the thermal conductivity.

When $M=0$, the velocity does not depend on the temperature and, therefore, the distribution is the same for $\bar{\theta}_\infty = 0.5$ and $\bar{\theta}_\infty = 3$. In this case the temperature profile depends significantly on $\bar{\theta}_\infty$. In the cooling case where $\bar{\theta}_\infty < \bar{\theta}_w$, the temperature profiles are affected whether the fluid is shear-thinning, Newtonian, or shear-thickening. When $\bar{\theta}_\infty > \bar{\theta}_w$ the effects of m are minimal. When $\bar{\theta}_\infty = 0.5$ and 3.0, the heat transfer at the wall (surface of the incline) as well as from the free surface is seen to increase with R_1. In the heating case, $\bar{\theta}_\infty = 3.0$, the wall temperature is always lower than that of the surrounding and it increases as R_1 increases. In the cooling case, $\bar{\theta}_\infty = 0.5$, the free surface temperature is lower than the wall temperature when R_1 is less than about 1.75 indicating that the fluid is heated.

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