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Research Article

Spin-Stabilized Spacecrafts: Analytical Attitude Propagation Using Magnetic Torques

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An analytical approach for spin-stabilized satellites attitude propagation is presented, considering the influence of the residual magnetic torque and eddy currents torque. It is assumed two approaches to examine the influence of external torques acting during the motion of the satellite, with the Earth's magnetic field described by the quadripole model. In the first approach is included only the residual magnetic torque in the motion equations, with the satellites in circular or elliptical orbit. In the second approach only the eddy currents torque is analyzed, with the satellite in circular orbit. The inclusion of these torques on the dynamic equations of spin stabilized satellites yields the conditions to derive an analytical solution. The solutions show that residual torque does not affect the spin velocity magnitude, contributing only for the precession and the drift of the spacecraft's spin axis and the eddy currents torque causes an exponential decay of the angular velocity magnitude. Numerical simulations performed with data of the Brazilian Satellites (SCD1 and SCD2) show the period that analytical solution can be used to the attitude propagation, within the dispersion range of the attitude determination system performance of Satellite Control Center of Brazil National Research Institute.

1. Introduction

This paper aims at analyzing the rotational motion dynamics of spin-stabilized Earth's artificial satellites, through derivation of an analytical attitude prediction. Emphasis is placed on modeling the torques steaming from residual magnetic and eddy currents perturbations, as well as their influences on the satellite angular velocity and space orientation. A spherical coordinated system fixed in the satellite is used to locate the spin axis of the satellite in relation to the terrestrial equatorial system. The directions of the spin axis are specified by the right ascension (α) and the declination (δ) as represented in Figure 1. The magnetic residual torque occurs due to the interaction between the Earth magnetic field and the residual magnetic moment along the spin axis of the satellite. The eddy currents torque appears due to the interaction of such currents circulating along the satellite structure chassis and the Earth's magnetic field.

The torque analysis is performed through the quadripole model for the Earth's magnetic field and the satellite in circular and elliptical orbits. Essentially an analytical averaging method is applied to determine the mean torque over an orbital period.

To compute the average components of both the residual magnetic and eddy current torques in the satellite body frame reference system (satellite system), an average time in the fast varying orbit element, the mean anomaly, is utilized. This approach involves several rotation matrices, which are dependent on the orbit elements, right ascension and declination of the satellite spin axis, the magnetic colatitudes, and the longitude of ascending node of the magnetic plane.

Unlike the eddy currents torques, it is observed that the residual magnetic torque does not have component along the spin axis; however, it has nonzero components in satellite body x-axis and y-axis. Afterwards, the inclusion of such torques on the rotational motion differential equations of spin-stabilized satellites yields the conditions to derive an analytical solution [1]. The theory is developed accounting also for orbit elements time variation, not restricted to circular orbits, giving rise to some hundreds of curvature integrals solved analytically.

In order to validate the analytical approach, the theory developed has been applied for the spin-stabilized Brazilian Satellites (SCD1 and SCD2), which are quite appropriated for verification and comparison of the theory with the data generated and processed by the Satellite Control Center (SCC) of Brazil National Research Institute (INPE). The oblateness of the orbital elements is taken into account.

The behaviors of right ascension, declination, and spin velocity of the spin axis with the time are presented and the results show the agreement between the analytical solution and the actual satellite behavior.

2. Geomagnetic Field

It is well known that the Earth's magnetic field can be obtained by the gradient of a scalar potential V [2]; it means that

$$\vec{B} = -\nabla V, \tag{2.1}$$

with the magnetic potential V given by

$$V(r', \phi, \theta) = r_T \sum_{n=1}^{k} \left(\frac{r_T}{r}\right)^{n+1} \sum_{m=0}^{n} \left(g_n^m \cos m\theta + h_n^m \operatorname{sen} m\theta\right) P_n^m(\phi), \tag{2.2}$$

where r_T is the Earth's equatorial radius, g_n^m, h_n^m are the Gaussian coefficients, $P_n^m(\phi)$ are the Legendre associated polynomial and r, ϕ, θ mean the geocentric distance, the local colatitudes, and local longitude, respectively.

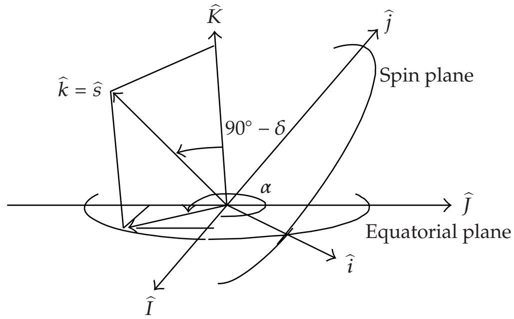

Figure 1: Orientation of the spin axis ($\hat{s}$): equatorial system ($\hat{I}, \hat{J}, \hat{K}$), satellite body frame reference system ($\hat{i}, \hat{j}, \hat{k}$), right ascension ($\alpha$), and declination ($\delta$) of the spin axis.

In terms of spherical coordinates, the geomagnetic field can be expressed by [2],

$$\overrightarrow{B} = B_r\hat{r} + B_\phi\hat{\phi} + B_\theta\hat{\theta}, \tag{2.3}$$

with

$$B_r = -\frac{\partial V}{\partial r}, \qquad B_\phi = -\frac{1}{r}\frac{\partial V}{\partial \phi}, \qquad B_\theta = -\frac{1}{r\,\text{sen}\,\phi}\frac{\partial V}{\partial \theta}. \tag{2.4}$$

For the quadripole model, it is assumed that n equals 1 and 2 and m equals 0, 1 and 2 in (2.2). After straightforward computations, the geomagnetic field can be expressed by [3, 4]

$$B_r = 2\left(\frac{r_T}{r}\right)^3 f_1(\theta,\phi) + 3\left(\frac{r_T}{r}\right)^4 f_2(\theta,\phi), \tag{2.5}$$

$$B_\phi = -\left(\frac{r_T}{r}\right)^3 f_3(\theta,\phi) - \left(\frac{r_T}{r}\right)^4 f_4(\theta,\phi), \tag{2.6}$$

$$B_\theta = -\frac{1}{\text{sen}\,\phi}\left\{\left(\frac{r_T}{r}\right)^3 f_5\,(\theta,\phi) + \left(\frac{r_T}{r}\right)^4 f_6(\theta,\phi) + 2\left(\frac{r_T}{r}\right)^4 f_7(\theta,\phi)\right\}, \tag{2.7}$$

where the functions f_i, $i = 1, 2, \ldots, 7$, are shown in [3] and depend on the Gaussian coefficients g_2^2, h_1^1, h_2^1, h_2^2 .

In the Equator reference system, the geomagnetic field is expressed by [2]

$$B_X = \left(B_r \cos\overline{\delta} + B_\phi \text{sen}\,\overline{\delta}\right)\cos\overline{\alpha} - B_\theta \text{sen}\,\overline{\alpha}, \tag{2.8}$$

$$B_Y = \left(B_r \cos\overline{\delta} + B_\phi \text{sen}\,\overline{\delta}\right)\text{sen}\,\overline{\alpha} - B_\theta \cos\overline{\alpha}, \tag{2.9}$$

$$B_Z = B_r \text{sen}\,\overline{\delta} + B_\phi \cos\overline{\delta}, \tag{2.10}$$

where $\overline{\alpha}$ and $\overline{\delta}$ are the right ascension and declination of the satellite position vector, respectively, which can be obtained in terms of the orbital elements; B_r, B_ϕ, and B_θ are given by (2.5), (2.6), and (2.7), respectively.

In a satellite reference system, in which the axis z is along the spin axis, the geomagnetic field is given by [4, 5]

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}, \tag{2.11}$$

where

$$\begin{aligned} B_x &= -\ B_X \operatorname{sen}\alpha + B_Y\cos\alpha, \\ B_y &= -B_X \operatorname{sen}\delta\ \cos\alpha - B_Y \operatorname{sen}\delta \operatorname{sen}\alpha + B_Z\cos\delta, \\ B_z &= -\ B_X\ \cos\delta\cos\alpha - B_Y\cos\delta \operatorname{sen}\alpha + B_Z \operatorname{sen}\delta, \end{aligned} \tag{2.12}$$

with B_X, B_Y, and B_Z given by (2.8)–(2.10).

3. Residual and Eddy Currents Torques

Magnetic residual torques result from the interaction between the spacecraft's residual magnetic field and the Earth's magnetic fields. If $\vec{m}$ is the magnetic moment of the spacecraft and $\vec{B}$ is the geomagnetic field, then the residual magnetic torques are given by [2]

$$\vec{N}_r = \vec{m} \times \vec{B}. \tag{3.1}$$

For the spin-stabilized satellite, with appropriate nutation dampers, the magnetic moment is mostly aligned along the spin axis and the residual torque can be expressed by [5]

$$\vec{N}_r = M_s\hat{k} \times \vec{B}, \tag{3.2}$$

where M_s is the satellite magnetic moment along its spin axis and $\hat{k}$ is the unit vector along the spin axis of the satellite.

By substituting the geomagnetic field (2.11) in (3.1), the instantaneous residual torque is expressed by

$$\vec{N}_r = M_s\left(-\ B_y\hat{i} + B_x\hat{j}\right). \tag{3.3}$$

On the other hand, the eddy currents torque is caused by the spacecraft spinning motion. If $\vec{W}$ is the spacecraft's angular velocity vector and p is the Foucault parameter representing the geometry and material of the satellite chassis [2], then this torque may be modeled by [2]

$$\vec{N}_i = p\vec{B} \times \left(\vec{\mathbf{B}}\ \times \vec{W}\right). \tag{3.4}$$

For a spin-stabilized satellite, the spacecraft's angular velocity vector and the satellite magnetic moment, along the z-axis and induced eddy currents torque, can be expressed by [5, 6]

$$\vec{N}_i = pW\left(-B_x\,B_z\hat{i} - B_yB_z\,\hat{j} + \left(B_y^2 + B_x^2\right)\hat{k}\right). \tag{3.5}$$

4. Mean Residual and Eddy Currents Torques

In order to obtain the mean residual and eddy currents torques, it is necessary to integrate the instantaneous torques $\vec{N}_r$ and $\vec{N}_i$, given in (3.3) and (3.5), over one orbital period T as

$$\vec{N}_{r_m} = \frac{1}{T}\int_{t_i}^{t_i+T}\vec{N}_r dt, \qquad \vec{N}_{i_m} = \frac{1}{T}\int_{t_i}^{t_i+T}\vec{N}_i dt, \tag{4.1}$$

where t is the time t_i the initial time, and T the orbital period. Changing the independent variable to the fast varying true anomaly, the mean residual and eddy currents torque can be obtained by [4]

$$\vec{N}_{r_m} = \frac{1}{T}\int_{\upsilon_i}^{\upsilon_i+2\pi}\vec{N}_r\frac{r^2}{h}d\upsilon, \qquad \vec{N}_{i_m} = \frac{1}{T}\int_{\upsilon_i}^{\upsilon_i+2\pi}\vec{N}_i\frac{r^2}{h}d\upsilon, \tag{4.2}$$

where υ_i is the true anomaly at instant t_i, r is the geocentric distance, and h is the specific angular moment of orbit.

To evaluate the integrals of (4.2), we can use spherical trigonometry properties, rotation matrix associated with the references systems, and the elliptic expansions of the true anomaly in terms of the mean anomaly [7], including terms up to first order in the eccentricity (e). Without losing generality, for the sake of simplification of the integrals, we consider the initial time for integration equal to the instant that the satellite passes through perigee. After extensive but simple algebraic developments, the mean residual and eddy currents torques can be expressed by [3, 6]

$$\vec{N}_{r_m} = N_{rxm}\hat{i} + N_{rym}\hat{j}, \qquad \vec{N}_{i_m} = \frac{pW}{2\pi}\left(N_{ixm}\hat{i} + N_{iym}\hat{j} + N_{izm}\hat{k}\right), \tag{4.3}$$

with

$$\begin{aligned} N_{rxm} &= \frac{M_s}{2\pi}(A\,\mathrm{sen}\,\delta\cos\alpha + B\,\mathrm{sen}\,\delta\,\mathrm{sen}\,\alpha - C\,\mathrm{sen}\,\delta), \\ N_{rym} &= \frac{M_s}{2\pi}(-\,D\,\mathrm{sen}\,\alpha + E\,\cos\,\delta) \end{aligned} \tag{4.4}$$

and N_{ixm}, N_{iym}, N_{izm} as well as the coefficients A, B, C, D, and E are presented in the appendix. It is important to observe that the mean components of these torques depend on the attitude angles (δ, α) and the orbital elements (orbital major semi-axis: a, orbital eccentricity: e, longitude of ascending node: Ω, argument of perigee: ω, and orbital inclination: i).

5. The Rotational Motion Equations

The variations of the angular velocity, the declination, and the ascension right of the spin axis for spin-stabilized artificial satellites are given by Euler equations in spherical coordinates [5] as

$$\dot{W} = \frac{1}{I_z} N_z,$$
$$\dot{\delta} = \frac{1}{I_z W} N_y, \tag{5.1}$$
$$\dot{\alpha} = \frac{1}{I_z W \operatorname{Cos} \delta} N_x,$$

where I_z is the moment of inertia along the spin axis and N_x, N_y, N_z are the components of the external torques in the satellite body frame reference system. By substituting N_{rm}, given in (4.3), in (5.1), the equations of motion are

$$\frac{dW}{dt} = 0, \tag{5.2}$$
$$\frac{d\delta}{dt} = \frac{N_{rym}}{I_z W}, \tag{5.3}$$
$$\frac{d\alpha}{dt} = \frac{N_{rxm}}{I_z W \cos \delta}, \tag{5.4}$$

where it is possible to observe that the residual torque does not affect the satellite angular velocity (because its z-axis component is zero).

By substituting N_{im}, given in (4.3), in (5.1), the equations of motion are

$$\frac{dW}{dt} = \frac{pW}{2\pi I_z} N_{izm}, \tag{5.5}$$
$$\frac{d\delta}{dt} = \frac{pW}{2\pi I_z} N_{iym}, \tag{5.6}$$
$$\frac{d\alpha}{dt} = \frac{p}{2\pi I_z \cos \delta} N_{ixm}. \tag{5.7}$$

The differential equations of (5.2)–(5.4) and (5.5)–(5.7) can be integrated assuming that the orbital elements (I, Ω, w) are held constant over one orbital period and that all other terms on right-hand side of equations are equal to initial values.

6. Analysis of the Angular Velocity Magnitude

The variation of the angular velocity magnitude, given by (5.5), can be expressed as:

$$\frac{dW}{W} = k\,dt, \quad \text{with } k = \frac{N_{izm}p}{2\,\pi I_z}. \tag{6.1}$$

If the parameter k is considered constant for one orbital period, then the analytical solution of (6.1) is

$$W = W_0 e^{kt}, \tag{6.2}$$

where W_0 is the initial angular velocity. If the coefficient $k < 0$ in (6.2), then the angular velocity magnitude decays with an exponential profile.

7. Analysis of the Declination and Right Ascension of Spin Axis

For one orbit period, the analytical solutions of (5.3)-(5.4) and (5.6)-(5.7) for declination and right ascension of spin axis, respectively, can simply be expressed as,

$$\delta = k_1 t + \delta_0, \tag{7.1}$$

$$\alpha = k_2 t + \alpha_0, \tag{7.2}$$

with:

(i) for the case where the residual magnetic torque is considered in the motion equations,

$$k_1 = \frac{N_{rym}}{I_z W_o}, \qquad k_2 = \frac{N_{rym}}{I_z W_o \cos\delta_o}, \tag{7.3}$$

(ii) for the case where the eddy currents torque is considered in the motion equations,

$$k_1 = \frac{pN_{iym}}{2\pi I_z}, \qquad k_2 = \frac{pN_{iym}}{2\pi I_z \cos\delta_o}, \tag{7.4}$$

where W_0, δ_0, and α_0 are the initial values for spin velocity, declination, and right ascension of spin axis.

Table 1: INPE's Satellite Control Center Data (index SCC) and computed results with the satellite in elliptical orbit and under the influence of the residual magnetic torque (index QER) for declination and right ascension and SCD1 (in degrees).

Day	α_{SCC}	α_{QER}	$\alpha_{SCC} - \alpha_{QER}$	δ_{SCC}	δ_{QER}	$\delta_{SCC} - \delta_{QER}$
22/08/93	282.70	282.7000	0.0000	79.64	79.6400	0.0000
23/08/93	282.67	282.7002	−0.0302	79.35	79.6399	−0.2899
24/08/93	283.50	282.6999	0.8001	79.22	79.6394	−0.4194
25/08/93	283.01	282.7004	0.3096	78.95	79.6395	−0.6895
26/08/93	282.43	282.7015	−0.2715	78.70	79.6399	−0.9399
27/08/93	281.76	282.7019	−0.9419	78.48	79.6398	−1.1598
28/08/93	281.01	282.7019	−1.6919	78.27	79.6393	−1.3693
29/08/93	280.18	282.7024	−2.5224	78.08	79.6392	−1.5592
30/08/93	279.29	282.7036	−3.4136	77.91	79.6396	−1.7296
31/08/93	278.34	282.7043	−4.3643	77.78	79.6397	−1.8597
01/09/93	277.36	282.7044	−5.3444	77.67	79.6391	−1.9691

The solutions presented in (7.1) and (7.2), for the spin velocity magnitude, declination and right ascension of the spin axis, respectively, are valid for one orbital period. Thus, for every orbital period, the orbital data must be updated, taking into account at least the main influences of the Earth's oblateness. With this approach, the analytical theory will be close to the real attitude behavior of the satellite.

8. Applications

The theory developed has been applied to the spin-stabilized Brazilian Satellites (SCD1 and SCD2) for verification and comparison of the theory against data generated by the Satellite Control Center (SCC) of INPE. Operationally, SCC attitude determination comprises [8, 9] sensors data preprocessing, preliminary attitude determination, and fine attitude determination. The preprocessing is applied to each set of data of the attitude sensors that collected every satellite that passes over the ground station. Afterwards, from the whole preprocessed data, the preliminary attitude determination produces estimates to the spin velocity vector from every satellite that passes over a given ground station. The fine attitude determination takes (one week) a set of angular velocity vector and estimates dynamical parameters (angular velocity vector, residual magnetic moment, and Foucault parameter). Those parameters are further used in the attitude propagation to predict the need of attitude corrections. Over the test period, there are not attitude corrections. The numerical comparison is shown considering the quadripole model for the geomagnetic field and the results of the circular and elliptical orbits. It is important to observe that, by analytical theory that included the residual torque, the spin velocity is considered constant during 24 hours. In all numerical simulations, the orbital elements are updated, taking into account the main influences of the Earth's oblateness.

9. Results for SCD1 Satellite

The initial conditions of attitude had been taken on 22 of August of 1993 to the 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC). Tables 1, 2, and 3 show the results

Table 2: INPE's Satellite Control Center Data (index SCC) and computed results with the satellite in circular orbit and under the influence of the residual magnetic torque (index QCR) for declination and right ascension and SCD1 (in degrees).

Day	α_{SCC}	α_{QCR}	$\alpha_{SCC} - \alpha_{QCR}$	δ_{SCC}	δ_{QCR}	$\delta_{SCC} - \delta_{QCR}$
22/08/93	282.70	282.7000	0	79.64	79.6400	0
23/08/93	282.67	282.7216	−0.0516	79.35	79.6251	−0.2751
24/08/93	283.50	282.7151	0.7849	79.22	79.6172	−0.3972
25/08/93	283.01	282.6737	0.3363	78.95	79.6182	−0.6682
26/08/93	282.43	282.5877	−0.1577	78.70	79.6303	−0.9303
27/08/93	281.76	282.4526	−0.6926	78.48	79.6552	−1.1752
28/08/93	281.01	282.2631	−1.2531	78.27	79.6940	−1.4240
29/08/93	280.18	282.0158	−1.83588	78.08	79.7473	−1.6673
30/08/93	279.29	281.7091	−2.4191	77.91	79.8140	−1.9040
31/08/93	278.34	281.3439	−3.0039	77.78	79.8962	−2.1162
01/09/93	277.36	280.9240	−3.5640	77.67	79.9892	−2.3191

with the data from SCC and computed values by the present analytical theory, considering the quadripole model for the geomagnetic field and the satellite in circular and elliptical orbit, under influence of the residual and eddy currents torques.

The mean deviation errors for the right ascension and declination are shown in Table 4 for different time simulations. The behavior of the SCD1 attitude over 11 days is shown in Figure 2. It is possible to note that mean error increases with the time simulation. For more than 3 days, the mean error is bigger than the required dispersion range of SCC.

Over the 3 days of test period, better results are obtained for the satellite in circular orbit with the residual torque. In this case, the difference between theory and SCC data has mean deviation error in right ascension of 0.2444° and −0.2241° for the declination. Both are within the dispersion range of the attitude determination system performance of INPE's Control Center.

In Table 5 is shown the computed results to spin velocity when the satellite is under influence of the eddy currents torque, and its behavior over 11 days is shown in Figure 3. The mean error deviation for the spin velocity is shown in Table 6 for different time simulation. For the test period of 3 days, the mean deviation error in spin velocity was of −0.0312 rpm and is within the dispersion range of the attitude determination system performance of INPE's Control Center.

10. Results for SCD2 Satellite

The initial conditions of attitude had been taken on 12 February 2002 at 00:00:00 GMT, supplied by the SCC. In the same way for SCD1, Tables 7, 8, and 9 presented the results with the data from SCC and computed values by circular and elliptical orbits with the satellite under the influence of the residual magnetic torque and eddy currents torque.

The mean deviation errors are shown in Table 10 for different time simulations. For this satellite, there is no significant difference between the circular and elliptical orbits when considering the residual magnetic torque. The behavior of the SCD2 attitude over 12 days is shown in Figure 4.

Table 3: INPE's Satellite Control Center Data (index SCC) and computed results with the satellite in circular orbit and under the influence of the eddy current torque (index QCI) for declination and right ascension and SCD1 (in degrees).

Day	α_{SCC}	α_{QCI}	$\alpha_{SCC} - \alpha_{QCI}$	δ_{SCC}	δ_{QCI}	$\delta_{SCC} - \delta_{QCI}$
22/08/93	282.70	282.7000	0.0000	79.64	79.6400	0.0000
23/08/93	282.67	282.6848	−0.0148	79.35	79.6490	−0.2990
24/08/93	283.50	282.6723	0.8277	79.22	79.6457	−0.4257
25/08/93	283.01	282.6621	0.3479	78.95	79.6352	−0.6852
26/08/93	282.43	282.6538	−0.2238	78.70	79.6220	−0.9220
27/08/93	281.76	282.6468	−0.8868	78.48	79.6092	−1.1292
28/08/93	281.01	282.6412	−1.6312	78.27	79.6001	−1.3301
29/08/93	280.18	282.6366	−2.4566	78.08	79.5942	−1.5142
30/08/93	279.29	282.6331	−3.3431	77.91	79.5913	−1.6813
31/08/93	278.34	282.6305	−4.2905	77.78	79.5904	−1.8104
01/09/93	277.36	282.6287	−5.2687	77.67	79.5909	−1.9209

Table 4: Mean deviations for different time simulations for declination and right ascension and SCD1 (in degrees).

Time Simulation (days)	11	8	3	2
$\alpha_{SCC} - \alpha_{QER}$	−1.5882	−0.5435	0.2566	−0.0151
$\alpha_{SCC} - \alpha_{QCR}$	−1.0779	−0.3587	0.2444	−0.0258
$\alpha_{SCC} - \alpha_{QCI}$	−1.5400	−0.5047	0.2710	0.0074
$\delta\alpha_{SCC} - \delta\alpha_{QER}$	−1.0896	−0.8034	−0.2364	−0.1449
$\delta_{SCC} - \delta_{QCR}$	−1.1707	−0.8172	−0.2241	−0.1376
$\delta_{SCC} - \delta_{QCI}$	−1.0653	−0.7882	−0.2416	−0.1495

Table 5: INPE's Satellite Control Center Data (index SCC) and computed results for spin velocity, with the satellite in circular orbit and under the influence of the eddy currents torque (index QCI) (in rpm).

Day	W_{SCC}	W_{QCI}	$W_{SCC} - W_{QCI}$
22/08/93	86.2100	86.2100	0.0000
23/08/93	86.0400	86.3156	−0.2756
24/08/93	85.8800	86.4985	−0.6185
25/08/93	85.8000	86.7144	−0.9144
26/08/93	85.7300	86.9439	−1.2139
27/08/93	85.6600	87.1719	−1.5119
28/08/93	85.5800	87.3631	−1.7831
29/08/93	85.5100	87.5296	−2.0196
30/08/93	85.4400	87.6657	−2.2257
31/08/93	85.3700	87.7658	−2.3958
01/09/ 93	85.3100	87.8426	−2.5326

Table 6: Mean deviations for different time simulations for spin velocity and SCD1 (in degrees).

Time Simulation (days)	11	8	3	2
$W_{SCC} - W_{QCI}$ (rpm)	−0.1475	−0.1091	−0.0312	−0.0144

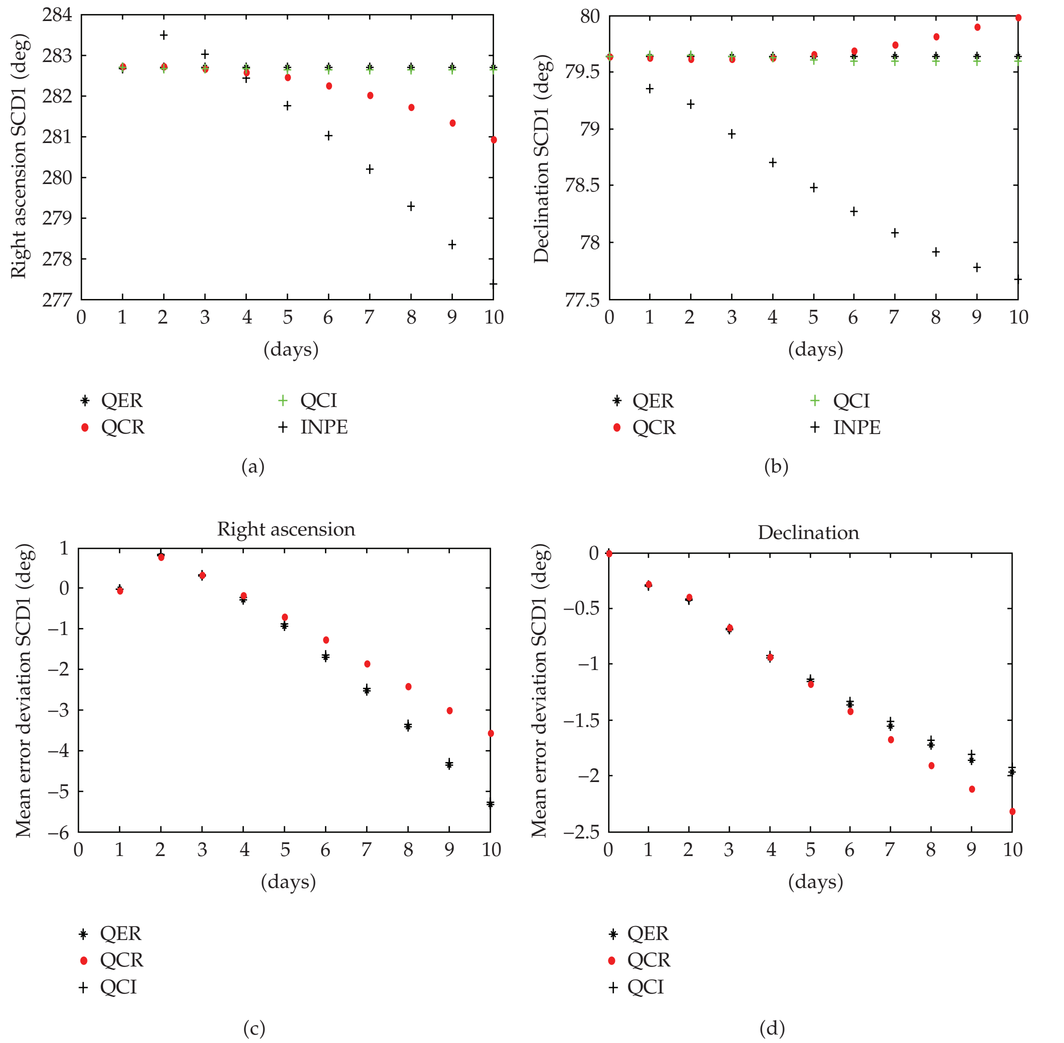

Figure 2: Evolution of the declination (δ) and right ascension (α) of satellite spin axis for SCD1 and its mean deviation error.

Over the test period of the 12 days with the satellite in elliptical orbit and considering the residual magnetic torque, the difference between theory and SCC data has mean deviation error in right ascension of −0.1266 and −0.1358 in the declination. Both torques are within the dispersion range of the attitude determination system performance of INPE's Control Center, and the solution can be used for more than 12 days.

In Table 11 the computed results to spin velocity are shown when the satellite is under the influence of the eddy currents torque. The mean deviation error for the spin velocity is shown in Table 12 for different time simulation. For the test period, the mean deviation error in spin velocity was of 0.0253 rpm and it is within the dispersion range of the attitude determination system performance of INPE's Control Center. The behavior of the spin velocity is shown in Figure 5.

Table 7: INPE's Satellite Control Center Data (index SCC) and computed results with the satellite in elliptical orbit and under the influence of the residual magnetic torque (index QER) for declination and right ascension and SCD2 (in degrees).

Day	α_{SCC}	α_{QER}	$\alpha_{SCC} - \alpha_{QER}$	δ_{SCC}	δ_{QER}	$\delta_{SCC} - \delta_{QER}$
12/02/02	278.71	278.710000	0.0000	63.47	63.470000	0.0000
13/02/02	278.73	278.709999	0.0200	63.45	63.469998	−0.0200
14/02/02	278.74	278.710000	0.0300	63.42	63.470002	−0.0500
15/02/02	278.74	278.710000	0.0300	63.39	63.470005	−0.0800
16/02/02	278.72	278.709999	0.0100	63.36	63.470002	−0.1100
17/02/02	278.68	278.709999	−0.0300	63.33	63.470000	−0.1400
18/02/02	278.63	278.710000	−0.0800	63.31	63.470003	−0.1600
19/02/02	278.57	278.710001	−0.1400	63.29	63.470006	−0.1800
20/02/02	278.50	278.710000	−0.2100	63.27	63.470004	−0.2000
21/02/02	278.42	278.709999	−0.2900	63.25	63.470000	−0.2200
22/02/02	278.33	278.710000	−0.3800	63.24	63.470002	−0.2300
23/02/02	278.23	278.710002	−0.4800	63.23	63.470006	−0.2400

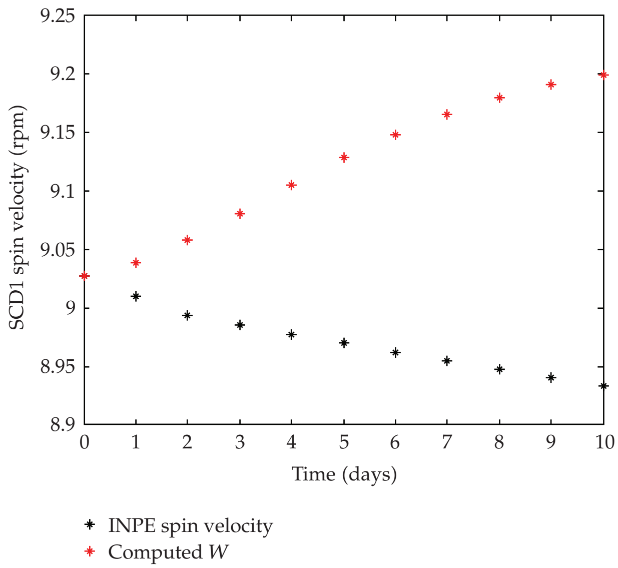

Figure 3: Evolution of the spin velocity (W) for SCD1.

11. Mean Pointing Deviation

For the tests, it is important to observe the deviation between the actual SCC supplied and the analytically computed attitude, for each satellite. It can be computed by

$$\theta = \cos^{-1}\left(\hat{i}\,\hat{i}_c + \hat{j}\,\hat{j}_c + \hat{k}\,\hat{k}_c\right), \tag{11.1}$$

where $(\hat{i}, \hat{j}, \hat{k})$ indicates the unity vectors computed by SCC and $(\hat{i}_c, \hat{j}_c, \hat{k}_c)$ indicates the unity vector computed by the presented theory.

Table 8: INPE's Satellite Control Center Data (index SCC) and computed results with the satellite in circular orbit and under the influence of the residual magnetic torque (index QCR) for declination and right ascension and SCD2 (in degrees).

Day	α_{SCC}	α_{QCR}	$\alpha_{SCC}-\alpha_{QCR}$	δ_{SCC}	δ_{QCR}	$\delta_{SCC}-\delta_{QCR}$
12/02/02	278.71	278.710000	0	63.47	63.470000	0
13/02/02	278.73	278.7113	0.01870	63.45	63.4692	−0.0192
14/02/02	278.74	278.7127	0.02733	63.42	63.4683	−0.0482
15/02/02	278.74	278.7141	0.0259	63.39	63.4673	−0.0773
16/02/02	278.72	278.7155	0.0045	63.36	63.4664	−0.1064
17/02/02	278.68	278.7168	−0.0368	63.33	63.4654	−0.1354
18/02/02	278.63	278.7180	−0.0880	63.31	63.4646	−0.1546
19/02/02	278.57	278.7191	−0.1491	63.29	63.4638	−0.1738
20/02/02	278.50	278.7200	−0.2200	63.27	63.4631	−0.1931
21/02/02	278.42	278.7207	−0.3007	63.25	63.4625	−0.2125
22/02/02	278.33	278.7212	−0.3913	63.24	63.4621	−0.2221
23/02/02	278.23	278.7215	−0.4916	63.23	63.4618	−0.2318

Table 9: INPE's Satellite Control Center Data (index SCC) and computed results with the satellite in circular orbit and under the influence of the eddy currents torque (index QCI) for declination and right ascension and SCD2 (in degrees).

Day	α_{SCC}	α_{QCI}	$\alpha_{SCC}-\alpha_{QCI}$	δ_{SCC}	δ_{QCI}	$\delta_{SCC}-\delta_{QCI}$
12/02/02	278.71	278.7100	0.0000	63.47	63.4700	0.0000
13/02/02	278.73	278.7170	0.0130	63.45	63.4921	−0.0421
14/02/02	278.74	278.7261	0.0139	63.42	63.5119	−0.0919
15/02/02	278.74	278.7371	0.0029	63.39	63.5268	−0.1368
16/02/02	278.72	278.7497	−0.0296	63.36	63.5352	−0.1752
17/02/02	278.68	278.7635	−0.0835	63.33	63.5370	−0.2070
18/02/02	278.63	278.7772	−0.1472	63.31	63.5345	−0.2245
19/02/02	278.57	278.7912	−0.2212	63.29	63.5302	−0.2402
20/02/02	278.50	278.8044	−0.3043	63.27	63.5285	−0.2585
21/02/02	278.42	278.8159	−0.3959	63.25	63.5334	−0.2834
22/02/02	278.33	278.8253	−0.4953	63.24	63.5477	−0.3077
23/02/02	278.23	278.8321	−0.6021	63.23	63.5724	−0.3423

Table 10: Mean deviations for different time simulation for declination and right ascension and SCD2 (in degrees).

Time Simulation (days)	12	8	5	2
$\alpha_{SCC}-\alpha_{QER}$	−0.1266	−0.0200	0.0180	0.0100
$\alpha_{SCC}-\alpha_{QCR}$	−0.1334	−0.0247	−0.0153	−0.0093
$\alpha_{SCC}-\alpha_{QCI}$	−0.1875	−0.0565	−0.0139	0.0065
$\delta\alpha_{SCC}-\delta\alpha_{QER}$	−0.1358	−0.0925	−0.0520	−0.0099
$\delta_{SCC}-\delta_{QCR}$	−0.1312	−0.0894	−0.0502	−0.0096
$\delta_{SCC}-\delta_{QCI}$	−0.1925	−0.1397	−0.1088	−0.0210

Table 11: INPE's Satellite Control Center Data (index SCC) and computed results of spin velocity, with the satellite in circular orbit and under the influence of the eddy currents torque (index QCI) (in rpm).

Day	W_{SCC}	W_{QCI}	$W_{SCC} - W_{QCI}$
12/02/02	34.4800	34.4800	0.0000
13/02/02	34.4200	34.4942	−0.0742
14/02/02	34.3700	34.4572	−0.0872
15/02/02	34.3100	34.3561	−0.04617
16/02/02	34.2600	34.1831	0.0769
17/02/02	34.2000	33.9323	0.2678
18/02/02	34.1400	34.6059	−0.4659
19/02/02	34.0800	34.2108	−0.1308
20/02/02	34.0200	33.7703	0.2497
21/02/02	33.9600	33.3067	0.6533
22/02/02	33.9000	32.8493	1.0508
23/02/02	33.8300	32.4199	1.4101

Table 12: Mean deviations for different time simulations for spin velocity and SCD1 (in degrees).

Time simulation (days)	12	8	5	2
$W_{SCC} - W_{QCI}$	0.0253	−0.0060	−0.0027	−0.0039

Figures 6 and 7 present the pointing deviations for the test period. The mean pointing deviation for the SCD1 for different time simulations are presented in Table 13. Over the test period of 11 days, the mean pointing deviation with the residual magnetic torque and elliptical orbit was 1.1553°, circular orbit was 1.2003°, and eddy currents torque with circular orbit was 1.1306°. The test period of SCD1 shows that the pointing deviation is higher than the precision required for SCC. Therefore for SCD1, this analytical approach should be evaluated by a time less than 11 days.

For SCD2, the mean pointing deviation considering the residual magnetic torque and elliptical orbit was 0.1538, residual magnetic torque and circular orbit was 0.1507, and eddy current torque was 0.2160. All the results for SCD2 are within the dispersion range of the attitude determination system performance of INPE's Control Center of 0.5°.

12. Summary

In this paper an analytical approach was presented to the spin-stabilized satellite attitude propagation taking into account the residual and eddy currents torque. The mean components of these torques in the satellite body reference system have been obtained and the theory shows that, unlike the eddy currents torque, there is no residual torque component along the spin axis (z-axis). Therefore this torque does not affect the spin velocity magnitude, but it can cause a drift in the satellite spin axis.

The theory was applied to the spin-stabilized Brazilian satellites SCD1 and SCD2 in order to validate the analytical approach, using quadripole model for geomagnetic field and the satellite in circular and elliptical orbits.

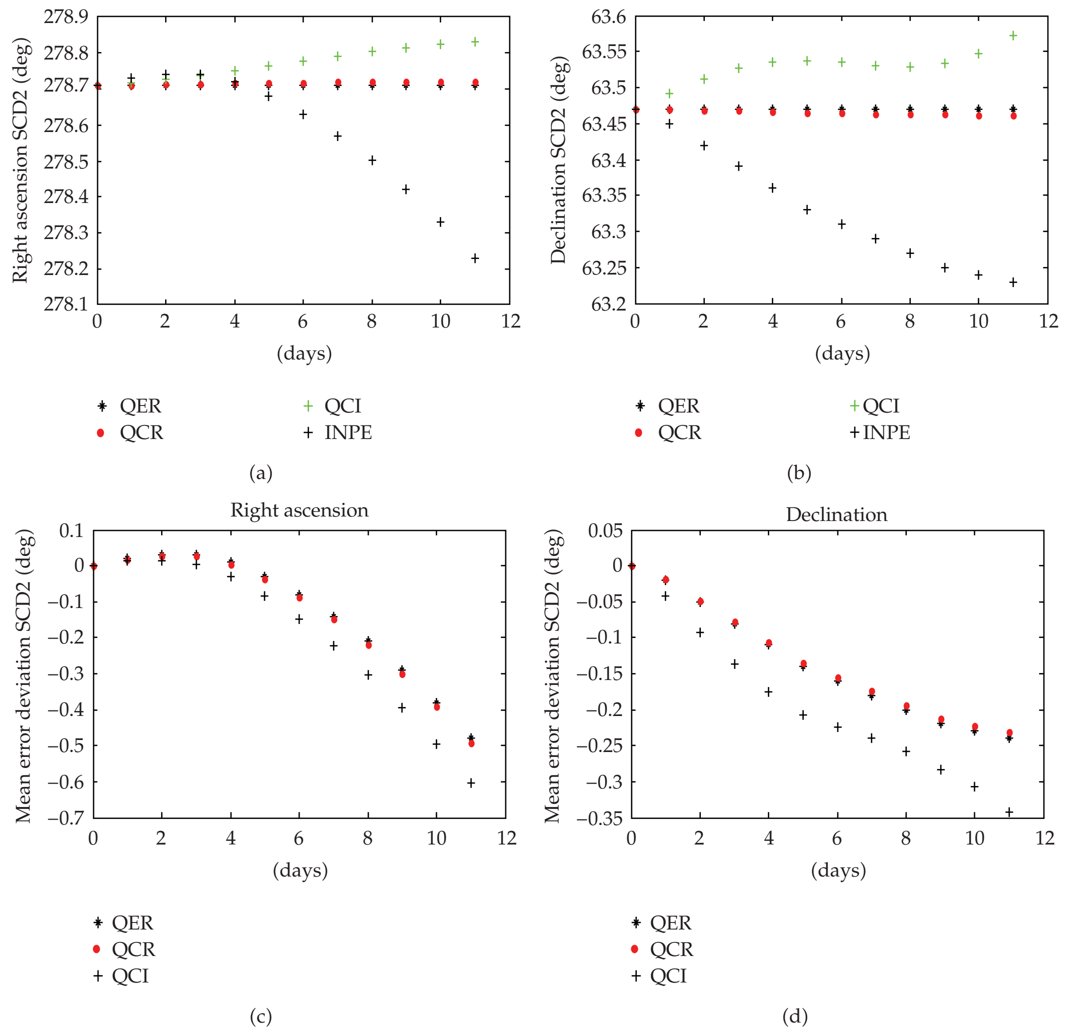

Figure 4: Evolution of the declination (δ) and right ascension (α) of satellite spin axis for SCD2 and its mean deviation error.

Table 13: Mean pointing deviation for SCD1.

Time simulation (days)	11	8	3	2
θ_{QER}	1.1553	0.8226	0.2448	0.1450
θ_{QCR}	1.2003	0.8288	0.2326	0.1376
θ_{QCI}	1.1306	0.8071	0.2503	0.1495

The result of the 3 days of simulations of SCD1, considering the residual magnetic torque, shows a good agreement between the analytical solution and the actual satellite behavior. For more than 3 days, the pointing deviation is higher than the precision required for SCC (0.5°).

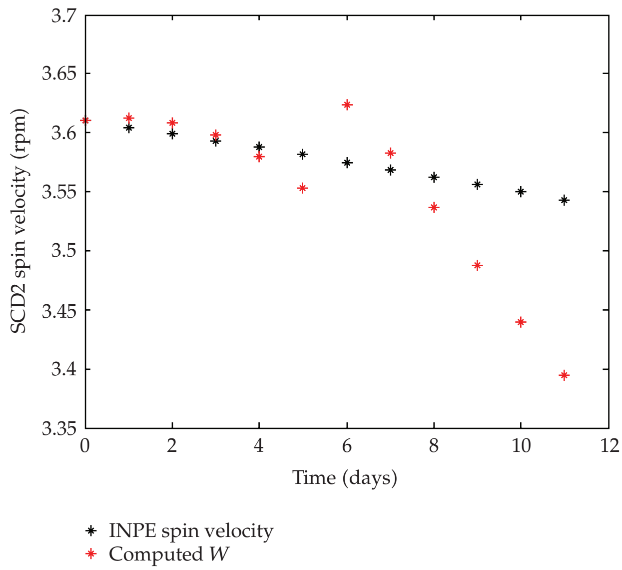

Figure 5: Evolution of the spin velocity magnitude (W) for SCD2.

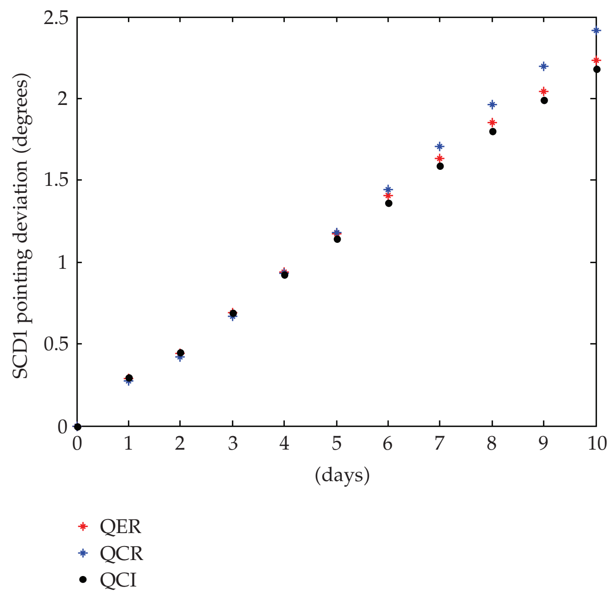

Figure 6: Pointing deviation evolution (in degrees) for SCD1.

For the satellite SCD2, over the test period of the 12 days, the difference between theory (when considering the residual or eddy currents torque) and SCC data is within the dispersion range of the attitude determination system performance of INPE's Control Center.

Thus the procedure is useful for modeling the dynamics of spin-stabilized satellite attitude perturbed by residual or eddy currents torques but the time simulation depends on the precision required for satellite mission.

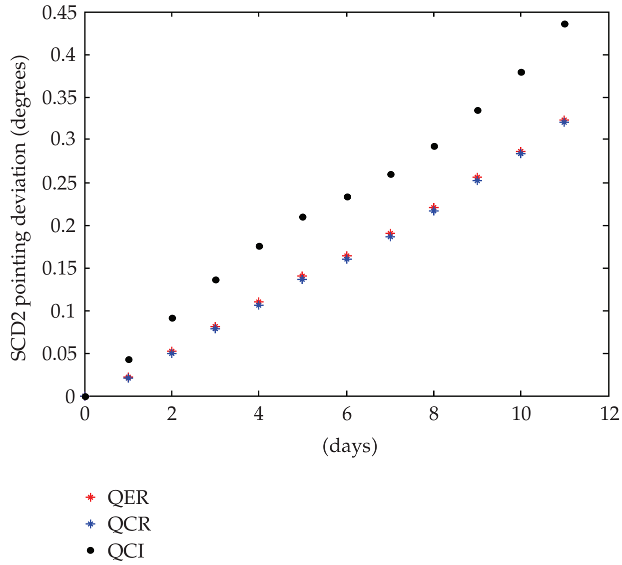

Figure 7: Pointing deviation evolution (in degrees) for SCD2.

Appendix

The coefficients of the mean components of the residual magnetic torques, given by (2.9), are expressed by

$$A = \sum_{i=1}^{7} a_{ia} + \sum_{i=1}^{7} a_{ib}, \qquad B = \sum_{i=1}^{7} b_{ia} + \sum_{i=1}^{7} b_{ib}, \qquad C = \sum_{i=1}^{7} c_{ia} + \sum_{i=1}^{7} c_{ib},$$
$$D = \sum_{i=1}^{7} a_{ia} + \sum_{i=1}^{7} a_{ib}, \qquad E = \sum_{i=1}^{7} b_{ia} + \sum_{i=1}^{7} b_{ib}, \tag{A.1}$$

where a_{ib}, b_{ib}, c_{jb}, $i = 1, 2, \ldots, 7$; $j = 1, \ldots, 4$, can be got by Garcia in [3]. It is important to note that the parcel b_{ib} is associated with the quadripole model and the satellite in an elliptical orbit. For circular, orbit, b_{ib} is zero.

The mean components N_{ixm}, N_{iym}, N_{izm} of the eddy currents torque are expressed by

$$N_{ixm} = \sum_{i=1}^{14\,712} \operatorname{tr} x(i) + \sum_{i=1}^{18\,426} Nx(i),$$
$$N_{iym} = \sum_{i=1}^{14\,712} \operatorname{tr} y(i) + \sum_{i=1}^{53\,765} Ny(i), \tag{A.3}$$
$$N_{izm} = \sum_{i=1}^{7\,350} \operatorname{tr} z(i) + \sum_{i=1}^{21\,435} Nz(i),$$

where $\operatorname{tr} x(i)$, $\operatorname{tr} y(i)$, $\operatorname{tr} z(i)$, $Nx(i)$, $Ny(i)$, and $Nz(i)$ are presented by Pereira [6].

The terms a_{ib}, b_{ib}, c_{jb}, $\mathrm{tr}\, x(i)$, $\mathrm{tr}\, y(i)$, $\mathrm{tr}\, z(i)$, $Nx(i)$, $Ny(i)$, and $Nz(i)$ depend on orbital elements (a, e, I, Ω, w) and attitude angles (δ, α).

Acknowledgment

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