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Research Article

Solving Heat and Wave-Like Equations Using He's Polynomials

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We use He's polynomials which are calculated form homotopy perturbation method (HPM) for solving heat and wave-like equations. The proposed iterative scheme finds the solution without any discretization, linearization, or restrictive assumptions. Several examples are given to verify the reliability and efficiency of the method. The fact that suggested technique solves nonlinear problems without using Adomian's polynomials is a clear advantage of this algorithm over the decomposition method.

1. Introduction

The heat and wave-like models are the integral part of applied sciences and arise in various physical phenomena. Several techniques including spectral, characteristic, modified variational iteration and Adomian's decomposition have been used for solving these problems; see [1–3] and the references therein. Most of these techniques encounter a considerable size of difficulty. He [4–11] developed and formulated homotopy perturbation method (HPM) by merging the standard homotopy and perturbation. The homotopy perturbation method (HPM) proved to be compatible with the versatile nature of the physical problems and has been applied to a wide class of functional equations; see [1, 4–19] and the references therein. In this technique, the solution is given in an infinite series usually converging to an accurate solution; see [1, 4–19] and the references therein. It is worth mentioning that HPM is applied without any discretization, restrictive assumption or transformation and is free from round off errors. The HPM is applied for all the nonlinear terms in the problem without discretizing either by finite difference or by spline techniques at the nodes and involves laborious calculations coupled with a strong possibility of the ill-conditioned resultant equations which are a complicated problem to solve. Moreover, unlike the method of separation of variables that requires initial and boundary conditions, the homotopy perturbation method (HPM) provides an analytical solution by using the initial conditions only. The fact that HPM solves nonlinear problems without using Adomian's polynomials is a clear advantage of this technique over the decomposition method. It is worth mentioning that [12, 13] introduced He's polynomials by splitting the nonlinear term and proved their compatibility with Adomian's polynomials coupled with the conclusion that He's polynomials are easier to calculate, are more user friendly, and are independent of the complexities arising in calculating the so-called Adomian's polynomials. It is to be highlighted that He's polynomials are calculated from homotopy perturbation method (HPM). Inspired and motivated by the ongoing research in this area, we use He's polynomials for solving heat and wave-like equations. It is worth mentioning that Noor and Mohyud-Din [20] introduced a homotopy approach which involves an additional term and consequently leads towards laborious and redundant calculations, whereas the approach used in Section 2 is more precise and easier to implement. Moreover, it reduces the huge unnecessary calculation arising in [20]. Several examples are given to verify the reliability and efficiency of the algorithm.

2. Homotopy Perturbation Method and He's Polynomials

To explain the homotopy perturbation method, we consider a general equation of the type

$$L(u) = 0, \tag{2.1}$$

where L is any integral or differential operator. We define a convex homotopy $H(u, p)$ by

$$H(u, p) = (1 - p)F(u) + pL(u), \tag{2.2}$$

where $F(u)$ is a functional operator with known solutions v_0, which can be obtained easily. It is clear that, for

$$H(u, p) = 0, \tag{2.3}$$

we have

$$H(u, 0) = F(u), \qquad H(u, 1) = L(u). \tag{2.4}$$

This shows that $H(u, p)$ continuously traces an implicitly defined curve from a starting point $H(v_0, 0)$ to a solution function $H(f, 1)$. The embedding parameter monotonically increases from zero to unit as the trivial problem $F(u) = 0$ is continuously deforms the original problem $L(u) = 0$. The embedding parameter $p \in (0, 1]$ can be considered as an expanding parameter [1, 2, 4–19]. The homotopy perturbation method uses the homotopy parameter p as an expanding parameter [4–11] to obtain

$$u = \sum_{i=0}^{\infty} p^i u_i = u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \cdots , \tag{2.5}$$

if $p \to 1$, then (2.5) corresponds to (2.2) and becomes the approximate solution of the form

$$f = \lim_{p \to 1} u = \sum_{i=0}^{\infty} u_i. \tag{2.6}$$

It is well known that series (2.6) is convergent for most of the cases and also the rate of convergence is dependent on $L(u)$; see [4–11]. We assume that (2.6) has a unique solution. The comparisons of like powers of p give solutions of various orders. In sum, according to [12, 13], He's HPM considers the solution, $u(x)$, of the homotopy equation in a series of p as follows:

$$u(x) = \sum_{i=0}^{\infty} p^i u_i = u_0 + p u_1 + p^2 u_2 + \cdots, \tag{2.7}$$

and the method considers the nonlinear term $N(u)$ as

$$N(u) = \sum_{i=0}^{\infty} p^i H_i = H_0 + p H_1 + p^2 H_2 + \cdots, \tag{2.8}$$

where H_n's are the so-called He's polynomials [12, 13], which can be calculated by using the formula

$$H_n(u_0, \ldots, u_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left(N\left(\sum_{i=0}^{n} p^i u_i \right) \right)_{p=0}, \quad \text{n} = 0, 1, 2, \ldots. \tag{2.9}$$

3. Numerical Applications

In this section, we use He's polynomials which are calculated from homotopy perturbation method (HPM) for solving heat and wave-like equations.

Example 3.1 ([1, 2]). Consider the one-dimensional initial boundary value problem which describes the heat-like models

$$u_t = \frac{1}{2} x^2 u_{xx}, \quad 0 < x < 1, \; t > 0, \tag{3.1}$$

with boundary conditions

$$u(0, t) = 0, \qquad u(1, t) = e^t, \tag{3.2}$$

and initial conditions

$$u(x, 0) = x^2. \tag{3.3}$$

Apply the convex homotopy

$$u_0 + pu_1 + p^2u_2 + \cdots = x^2 + \int_0^t \frac{1}{2}x^2\left(\frac{\partial^2 u_0}{\partial x^2} + p\frac{\partial^2 u_1}{\partial x^2} + p^2\frac{\partial^2 u_2}{\partial x^2} + \cdots\right)dt. \tag{3.4}$$

Compare the coefficient of like powers of p

$$\begin{aligned}
p^{(0)} &: u_0(x,t) = x^2,\\
p^{(1)} &: u_1(x,t) = x^2 t,\\
p^{(2)} &: u_2(x,t) = x^2\frac{t^2}{2!},\\
p^{(3)} &: u_3(x,t) = x^2\frac{t^3}{3!},\\
p^{(4)} &: u_4(x,t) = x^2\frac{t^4}{4!},\\
&\vdots
\end{aligned} \tag{3.5}$$

where $p^{(i)}s$ are He's polynomials. The series solution is given by

$$u(x,t) = x^2\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right), \tag{3.6}$$

and in a closed form by

$$u(x,t) = x^2 e^t. \tag{3.7}$$

Example 3.2 ([1, 2]). Consider the two-dimensional initial boundary value problem which describes the heat-like models

$$u_t = \frac{1}{2}\left(y^2 u_{xx} + x^2 u_{yy}\right), \quad 0 < x,\ y < 1,\ t > 0, \tag{3.8}$$

with boundary conditions

$$\begin{aligned}
u_x(0,y,t) &= 0, \qquad u_x(1,y,t) = 2\sinh t,\\
u_y(x,0,t) &= 0, \qquad u_y(x,1,t) = 2\cosh t,
\end{aligned} \tag{3.9}$$

and initial conditions

$$u(x,y,0) = y^2. \tag{3.10}$$

Apply the convex homotopy method

$$u_0 + pu_1 + p^2u_2 + \cdots = y^2 + \frac{1}{2}\int_0^t \left(y^2\left(\frac{\partial^2 u_0}{\partial x^2} + p\frac{\partial^2 u_1}{\partial x^2} + p^2\frac{\partial^2 u_2}{\partial x^2} + \cdots \right) + x^2\left(\frac{\partial^2 u_0}{\partial y^2} + p\frac{\partial^2 u_1}{\partial y^2} + p^2\frac{\partial^2 u_2}{\partial y^2} + \cdots \right)\right) ds. \tag{3.11}$$

Compare the coefficient of like powers of p

$$\begin{aligned}
p^{(0)} &: u_0(x,y,t) = y^2, \\
p^{(1)} &: u_1(x,y,t) = x^2 t, \\
p^{(2)} &: u_2(x,y,t) = y^2\frac{t^2}{2!}, \\
p^{(3)} &: u_3(x,y,t) = x^2\frac{t^3}{3!}, \\
p^{(4)} &: u_4(x,y,t) = y^2\frac{t^4}{4!}, \\
&\vdots
\end{aligned} \tag{3.12}$$

where $p^{(i)}s$ are He's polynomials. The series solution is given by

$$u(x,y,t) = x^2\left(t + \frac{t^3}{3!} + \frac{t^5}{5!} + \cdots \right) + y^2\left(1 + \frac{t^2}{2!} + \frac{t^4}{4!} + \cdots \right), \tag{3.13}$$

and in a closed form by

$$u(x,y,t) = x^2 \sinh t + y^2 \cosh t. \tag{3.14}$$

Example 3.3 ([1, 2]). Consider the three-dimensional inhomogeneous initial boundary value problem which describes the heat-like models

$$u_t = x^4y^4z^4 + \frac{1}{36}\left(x^2u_{xx} + y^2u_{yy} + z^2u_{zz} \right), \quad 0 < x, y, z < 1, \ t > 0, \tag{3.15}$$

subject to the boundary conditions

$$\begin{aligned}
u(0,y,z,t) &= 0, & u(1,y,z,t) &= y^4z^4(e^t - 1), \\
u(x,0,z,t) &= 0, & u(x,1,z,t) &= x^4z^4(e^t - 1), \\
u(x,y,0,t) &= 0, & u(x,y,1,t) &= x^4y^4(e^t - 1),
\end{aligned} \tag{3.16}$$

and the initial conditions

$$u(x,y,z,0) = 0. \tag{3.17}$$

Apply the convex homotopy method

$$\begin{aligned} u_0 + pu_1 + p^2u_2 + \cdots = x^4y^4z^4t + \frac{1}{36}p\int_0^t x^2\left(\frac{\partial^2 u_0}{\partial x^2} + p\frac{\partial^2 u_1}{\partial x^2} + p^2\frac{\partial^2 u_2}{\partial x^2} + \cdots\right)dt \\ + \frac{1}{36}p\int_0^t \left(y^2\left(\frac{\partial^2 u_0}{\partial y^2} + p\frac{\partial^2 u_1}{\partial y^2} + p^2\frac{\partial^2 u_2}{\partial y^2} + \cdots\right)\right. \\ \left. + z^2\left(\frac{\partial^2 u_0}{\partial z^2} + p\frac{\partial^2 u_1}{\partial z^2} + \cdots\right)\right)dt. \end{aligned} \tag{3.18}$$

Compare the coefficient of like powers of p

$$\begin{aligned} p^{(0)} &: u_0(x,y,z,t) = x^4y^4z^4t, \\ p^{(1)} &: u_1(x,y,z,t) = x^4y^4z^4\frac{t^2}{2!}, \\ p^{(2)} &: u_2(x,y,z,t) = x^4y^4z^4\frac{t^3}{3!}, \\ p^{(3)} &: u_3(x,y,z,t) = x^4y^4z^4\frac{t^4}{4!}, \\ &\vdots \end{aligned} \tag{3.19}$$

where $p^{(i)}s$ are He's polynomials. The series solution is given by

$$u(x,y,t) = x^4y^4z^4\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right), \tag{3.20}$$

and in a closed form by

$$u(x,y,z,t) = x^4y^4z^4(e^t - 1). \tag{3.21}$$

Example 3.4. Consider the following nonlinear heat-like model:

$$u_t = u_{xx} + \frac{k}{x}u_x - (2+2k)u - 4u\,\ln(u), \tag{3.22}$$

subject to the initial conditions

$$u(0,t)=\exp\left(\exp(-4t)\right). \tag{3.23}$$

Apply the convex homotopy method

$$\begin{aligned}
&u_0+pu_1+p^2u_2+\cdots\\
&\quad=\exp\left(\exp\left(-4t\right)\right)+p\int_0^t\left(\frac{\partial^2u_0}{\partial x^2}+p\frac{\partial^2u_1}{\partial x^2}+p^2\frac{\partial^2u_2}{\partial x^2}+\cdots\right)dt\\
&\qquad+p\int_0^t\left(\frac{k}{x}\left(\frac{\partial u_0}{\partial x}+p\frac{\partial u_1}{\partial x}+p^2\frac{\partial u_2}{\partial x}+\cdots\right)-(2+2k)\left(u_0+pu_1+p^2u_2+\cdots\right)\right)dt\\
&\qquad-4p\int_0^t\left(u_0+pu_1+p^2u_2+\cdots\right)(\ln u_0+p\ln u_1+\cdots)dt.
\end{aligned} \tag{3.24}$$

Compare the coefficient of like powers of p

$$\begin{aligned}
p^{(0)}&:u_0(x,t)=e^{e^{-4t}},\\
p^{(1)}&:u_1(x,t)=\frac{x^2}{1!}e^{e^{-4t}},\\
p^{(2)}&:u_2(x,t)=\frac{x^4}{2!}e^{e^{-4t}},\\
p^{(3)}&:u_3(x,t)=\frac{x^6}{3!}e^{e^{-4t}},\\
p^{(4)}&:u_4(x,t)=\frac{x^8}{4!}e^{e^{-4t}},\\
p^{(5)}&:u_2(x,t)=\frac{x^{10}}{5!}e^{e^{-4t}},\\
&\vdots
\end{aligned} \tag{3.25}$$

where $p^{(i)}s$ are He's polynomials. The series solution is given by

$$u(x,t)=\left(1+\frac{x^2}{1!}+\frac{x^4}{2!}+\frac{x^6}{3!}+\frac{x^8}{4!}+\frac{x^{10}}{5!}+\cdots\right)e^{e^{-4t}}, \tag{3.26}$$

and the closed form solution is given as

$$u(x,t)=e^{x^2+e^{-4t}}. \tag{3.27}$$

Example 3.5 ([1, 2]). Consider the one-dimensional initial boundary value problem which describes the wave-like models

$$u_{tt} = \frac{1}{2}x^2 u_{xx}, \quad 0 < x < 1, \ t > 0, \tag{3.28}$$

subject to the boundary conditions

$$u(x,t) = x, \qquad u(1,t) = 1 + \sinh t, \tag{3.29}$$

and initial conditions

$$u(x,0) = x, \qquad u_t(x,0) = x^2. \tag{3.30}$$

Apply the convex homotopy method

$$u_0 + pu_1 + p^2u_2 + \cdots = x^2 + \frac{1}{2}\iint_0^t x^2\left(\frac{\partial^2 u_0}{\partial x^2} + p\frac{\partial^2 u_1}{\partial x^2} + p^2\frac{\partial^2 u_2}{\partial x^2} + \cdots\right)dt\ dt. \tag{3.31}$$

Compare the coefficient of like powers of p

$$\begin{aligned}
p^{(0)} &: u_0(x,t) = x + x^2 t, \\
p^{(1)} &: u_1(x,t) = x^2\frac{t^3}{3!}, \\
p^{(2)} &: u_2(x,t) = x^2\frac{t^5}{5!}, \\
p^{(3)} &: u_3(x,t) = x^2\frac{t^7}{7!}, \\
&\vdots
\end{aligned} \tag{3.32}$$

where $p^{(i)}s$ are He's polynomials. The series solution is given by

$$u(x,t) = x + x^2\left(t + \frac{t^3}{3!} + \frac{t^5}{5!} + \frac{t^7}{7!} + \ldots\right), \tag{3.33}$$

and in a closed form by

$$u(x,t) = x + x^2 \sinh t. \tag{3.34}$$

Example 3.6 ([1, 2]). Consider the two-dimensional initial boundary value problem which describes the wave-like models

$$u_{tt} = \frac{1}{12}\left(x^2 u_{xx} + y^2 y_{yy}\right), \quad 0 < x, y < 1, \ t > 0, \tag{3.35}$$

subject to the Neumann boundary conditions

$$\begin{aligned} u_x(0, y, t) &= 0, \qquad u_x(1, y, t) = 4\cosh t, \\ u_y(x, 0, t) &= 0, \qquad u_y(x, 1, t) = 4\sinh t, \end{aligned} \tag{3.36}$$

and initial conditions

$$u(x, y, 0) = x^4, \qquad u_t(x, y, 0) = y^4. \tag{3.37}$$

Apply the convex homotopy method

$$\begin{aligned} u_0 + pu_1 + p^2u_2 + \cdots = \left(x^4 + y^4 t\right) &+ \frac{1}{12}\iint_0^t x^2\left(\frac{\partial^2 u_0}{\partial x^2} + p\frac{\partial^2 u_1}{\partial x^2} + p^2\frac{\partial^2 u_2}{\partial x^2} + \cdots\right) dt\, dt \\ &+ \frac{1}{12}\iint_0^t y^2\left(\frac{\partial^2 u_0}{\partial y^2} + p\frac{\partial^2 u_1}{\partial y^2} + p^2\frac{\partial^2 u_2}{\partial y^2} + \cdots\right) dt\, dt. \end{aligned} \tag{3.38}$$

Compare the coefficient of like powers of p

$$\begin{aligned} p^{(0)} &: u_0(x, y, t) = x^4 + y^4 t, \\ p^{(1)} &: u_1(x, y, t) = x^4\frac{t^2}{2!} + y^4\frac{t^3}{3!}, \\ p^{(2)} &: u_2(x, y, t) = x^4\frac{t^4}{4!} + y^4\frac{t^5}{5!}, \\ p^{(3)} &: u_3(x, y, t) = x^4\frac{t^6}{6!} + y^4\frac{t^7}{7!}, \\ &\vdots \end{aligned} \tag{3.39}$$

where $p^{(i)}s$ are He's polynomials. The series solution is given by

$$u(x, y, t) = x^4\left(1 + \frac{t^2}{2!} + \frac{t^4}{4!} + \cdots\right) + y^4\left(1 + \frac{t^3}{3!} + \frac{t^5}{5!} + \cdots\right), \tag{3.40}$$

and in a closed form by

$$u(x, y, t) = x^4\cosh t + y^4\sinh t. \tag{3.41}$$

Example 3.7 ([1, 2]). Consider the three-dimensional inhomogeneous initial boundary value problem which describes the wave-like models

$$u_{tt} = \left(x^2 + y^2 + z^2\right) + \frac{1}{2}\left(x^2 u_{xx} + y^2 u_{yy} + z^2 u_{zz}\right), \quad 0 < x, y, z < 1, \ t > 0, \tag{3.42}$$

subject to the boundary conditions

$$\begin{aligned} u(0,y,z,t) &= y^2\left(e^t - 1\right) + z^2\left(e^{-t} - 1\right), \qquad u(1,y,z,t) = \left(1 + y^2\right)\left(e^t - 1\right) + z^2\left(e^{-t} - 1\right), \\ u(x,0,z,t) &= x^2\left(e^t - 1\right) + z^2\left(e^{-t} - 1\right), \qquad u(x,1,z,t) = \left(1 + x^2\right)\left(e^t - 1\right) + z^2\left(e^{-t} - 1\right), \\ u(x,y,0,t) &= \left(x^2 + y^2\right)\left(e^t - 1\right), \qquad u(x,y,1,t) = \left(x^2 + y^2\right)\left(e^t - 1\right) + \left(e^{-t} - 1\right), \end{aligned} \tag{3.43}$$

and the initial conditions

$$u(x,y,z,0) = 0, \qquad u_t(x,y,z,0) = x^2 + y^2 - z^2. \tag{3.44}$$

Apply the convex homotopy method

$$\begin{aligned} u_0 + pu_1 + p^2 u_2 + \cdots = \left(x^2 + y^2 - z^2\right) t &+ \frac{1}{2}\iint_0^t x^2\left(\frac{\partial^2 u_0}{\partial x^2} + p\frac{\partial^2 u_1}{\partial x^2} + p^2\frac{\partial^2 u_2}{\partial x^2} + \cdots\right) dt\, dt \\ &+ \frac{1}{2}\iint_0^t \left(y^2\left(\frac{\partial^2 u_0}{\partial y^2} + p\frac{\partial^2 u_1}{\partial y^2} + p^2\frac{\partial^2 u_2}{\partial y^2} + \cdots\right)\right. \\ &\qquad \left. + z^2\left(\frac{\partial^2 u_0}{\partial z^2} + p\frac{\partial^2 u_1}{\partial z^2} + p^2\frac{\partial^2 u_2}{\partial z^2} + \cdots\right)\right) dt\, dt. \end{aligned} \tag{3.45}$$

Compare the coefficient of like powers of p and proceede as before, the series solution is given by

$$u(x,y,t) = \left(x^2 + y^2\right)\left(t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right) + z^2\left(-t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right), \tag{3.46}$$

and in a closed form by

$$u(x,y,z,t) = \left(x^2 + y^2\right)e^t + z^2 e^{-t} - \left(x^2 + y^2 + z^2\right). \tag{3.47}$$

4. Conclusion

In this paper, we use He's polynomials which are calculated from homotopy perturbation method (HPM) for solving heat and wave-like equations. The method is applied in a direct way without using linearization, transformation, discretization, or restrictive assumptions. It may be concluded that the proposed scheme is very powerful and efficient in finding the analytical solutions for a wide class of boundary value problems. The method gives more realistic series solutions that converge very rapidly in physical problems. It is also observed that He's polynomials are compatible with Adomian's Polynomials but are easier to calculate and are more user friendly.

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