Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2009, Article ID 471540, 18 pages
doi:10.1155/2009/471540

Research Article

An LPV Fractional Model for Canal Control

Rubén Martínez-González,[1] Yolanda Bolea,[1] Antoni Grau,[1] and Herminio Martínez-García[2]

[1] *Automatic Control Department, Technical University of Catalonia, Pau Gargallo 5, 08028 Barcelona, Spain*
[2] *Electronics Engineering Department, Technical University of Catalonia, c/Urgell 187, 08036 Barcelona, Spain*

Correspondence should be addressed to Antoni Grau, antoni.grau@upc.edu

Received 6 February 2009; Revised 27 April 2009; Accepted 24 May 2009

Recommended by José Roberto Castilho Piqueira

An LPV rational order control model of an irrigation canal is derived from system identification experiments. This model is experimentally obtained by using the described LPV fractional identification procedure. This procedure consists of the identification of a rational order model in each operation point in an experimental test canal. Global LPV model is obtained from polynomial interpolation of local model parameters. Validation results demonstrate that rational order models are more accurate than integer order models. Therefore rational order control models have an important role to play in management and efficient use of water resources.

1. Introduction

Water is becoming a precious and very scarce resource in many countries due to the increase of industrial and agricultural demands, as well as population growth. Irrigation is the main water consuming activity in the world, as it represents about 80% of the available fresh water consumption. There is growing interest for the application of advanced management methods that prevent wastage and facilitate the efficient use of this vital resource [1].

Unfortunately, for control design purposes control techniques and their implementation are directly proportional to the complexity of proposed control models. Then, it is essentially a noncomplex and simple control model that represents in a precise way water behavior of open-flow canals. However, this type of systems corresponds to long distributed systems with complex dynamics. Furthermore, these systems involve mass energy transport phenomena which behave as intrinsically distributed parameter systems, and their characteristics are very complex such as the variation of parameters with operation points, large delays that vary with operation point, and numerous interactions between different consecutive subsystems and strong nonlinearity. Their complete dynamics is represented by nonlinear partial differential hyperbolic equations (PDEs) that depend on the time as well as the spatial coordinates: Saint-Venant's equations. This equation system has unknown analytical solution in real geometry and it has to be solved numerically (characteristic method, Preissman implicit scheme, etc.) [2].

Resulting time consuming simulation models are therefore suitable for scientific purposes but they are too complex for on-line applications and control needs. Moreover, linearizations or simplifications of Saint-Venant's equations are currently studied by irrigation control research community [3]. Distributed parameters systems, considered as systems with a very large number of states could be approximated with low-order linear time invariant (LTI) models in order to use classical linear control design tools, as an usual practice in control engineering. There are two main approaches that are followed to obtain a linear model for irrigation main canals: the use of linearized Saint-Venant equations [4, 5] and the use of identification methods [6–8]. In case of open canal hydraulic system, identification is a classical method because their operational data are widely available and resulting models are suitable for design control.

Normally, classical identification methods [9] are used to obtain LTI discrete models which describe dynamics of irrigation water. However, in such systems LTI models lose information about these characteristics (non-linearity, coupling between pools, dynamics parameters changing over operation time in a wide range variationĚ). Then, a simplified control model structure that still preserve their information is needed. Such a structure can be provided by linear parameter varying (LPV) models consisting of a linear lumped parameter model in which parameters are not constant, but they depend on external parameters and/or system states and/or operating conditions of the system.

One of the main motivations for using LPV gain scheduling control versus classical gain scheduling control is that the former, as opposed to the latter, rigorously guarantees system stability [10]. Gain scheduling control is a heuristic method that consists in dividing the parameter space into small regions, in which the plant is observed as an LTI system, and LTI controllers are designed for every fixed set of parameters to achieve a synthetic controller with the use of interpolation or other techniques as switching techniques or fuzzy control. Heuristic gain scheduling controllers normally guarantee control system stability when parameters perform a slow variation [11] but sometimes may lead to unstability or chaotic behavior [12]. Furthermore, benefits of using gainscheduling techniques instead of robust control are obvious in this type of systems because of conservative results of robust control since model errors are partly due to non-linear effects and partly to the strong unknown perturbations considered as uncertainties [13]. Then, it is convenient to identify an LPV model for control canal purposes. Mainly, there are two approaches of identifying LPV models: since an LPV model is essentially a parameterized family of LTI models, a first identification approach is to collect data enough at each operating point to identify its corresponding LTI model [14]. Identified LTI coefficients are used to interpolate LPV coefficients as polynomial functions of scheduling variables.

Alternatively, a second approach that can be carried out in "one shot" , by assuming a linear dependence of parameters with operating points. Here, according to [15], identification problem can be reduced to a linear regression that may be solved using an extended regressor in the Least Mean Square (LMS) algorithm. In general, both methods lead to similar models. These identified LTI integer models do fit good enough with the dynamics of the canal system in each operating point in order to lineary control the system in such points. But, due to (i) that recently some control researchers have used fractional control methods for canal control purposes with satisfactory results [16] and (ii) noninteger models describe completely the behaviour of distributed systems [17], such as irrigation canals, in this article authors have carried out an LPV fractional identification using the former mentioned LPV approach. This fractional identification approach has been developed to model an irrigation prototype canal. Some properties of fractional calculus are applied in order to obtain a noninteger order model in each operation point.

2. LPV Noninteger Order Modeling for Irrigation Canal Pool

The last two decades have witnessed considerable development in the use of fractional differentiation in various fields. Fractional control is now mature enough and is widely used to design control for representing systems that present diffusive phenomena, electromechanical diffusion, and transport phenomena. This last phenomenon corresponds to the case of irrigation pools. In this section, LPV identification methodology used for the experimental modelling of a pilot canal plant is described.

2.1. Pilot Canal Plant Description

An experimental canal prototype (this experimental test canal is a part of a more complex laboratory research canal available at Automatic Control Dept, UPC, Barcelona) is used in the research presented in this paper (Figure 1). This plant consists on two tanks, P_1 and P_2 (Figure 2), with a top side view shown in Figure 3. On one side of pool P_1 there is a pump (B_2, 1.3 kl/h) to empty the pool. The output-flowing liquid of B_2 is collected in P_2, where there is a second pump (B_3, 1.3 kl/h) to empty the pool. The output-flowing liquid of B_3 is collected in a reservoir, R, located under P_2. The reservoir supplies flow to the pool P_1 by another pump (B_1, 3.8 kl/h). In fact, the plant is a closed system, where the liquid that arrives to the reservoir from the pool P_2 returns to the pool P_1 via the pump. Lengthening the water path, tank plant is easily converted into a canal plant. The water path can vary placing methacrylate plates along the structure, (Figure 3). Here, the plates are separated 2 cm away creating a zigzag path. Then, pools are enlarged from 2 m to 12 m long, 15 cm wide, and the maximum allowed level is 25 cm. To know pools' levels after the zigzag path, that is, the pool level at the end of their path, two ultrasonic level sensors, y_1 and y_2, with a precision of 1 mm are used. The sensors are attached to the canal metallic structure.

2.2. Preliminary Definitions in Fractional Modeling

The mathematical definition of fractional derivatives has been the subject of several different approaches [17]. In this paper the following definition of fractional discrete derivative,

$$\Delta^{\alpha} y_k = \sum_{j=0}^{k} w_j^{\alpha} y_{k-j} \quad 0 < \alpha < 1, \tag{2.1}$$

where

$$w_j^{\alpha} = (-1)^j \binom{\alpha}{j} \tag{2.2}$$

will be used; α is the order of the fractional difference.

Figure 1: Frontal view of the experimental prototype canal.

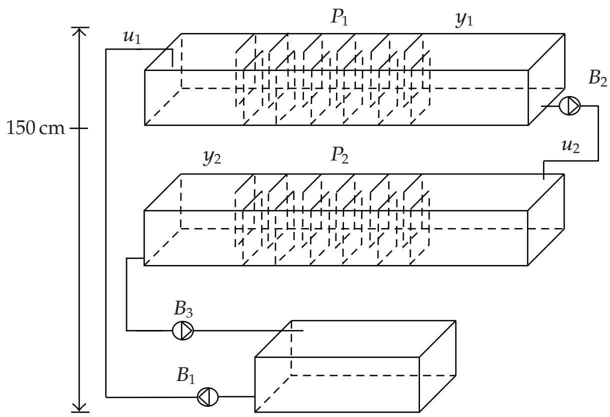

Figure 2: Full structure of the plant.

The fractional order models are clasificated in commensurable and non-commensurable order models. In this work, commensurable models are used.

Definition 2.1. A system is of commensurable order if it can be represented by a differential equation where all the orders of derivation are integers multiple of an order basis, α, that is, systems where the next condition is fulfilled:

$$a_n \Delta^{\gamma_n} y(t) + a_{n-1} \Delta^{\gamma_{n-1}} y(t) + \cdots + a_0 \Delta^{\gamma_0} y(t) = b_m \Delta^{\beta_m} u(t) + b_{m-1} \Delta^{\beta_{m-1}} u(t) + \cdots + b_0 \Delta^{\beta_0} u(t)$$
$$\gamma_k, \beta_k = k\alpha, \quad \alpha \in \mathbb{R}^+. \tag{2.3}$$

So, the differential equation (2.3) can be written as follow:

$$\sum_{k=0}^{n} a_k \Delta^{k\alpha} y(t) = \sum_{k=0}^{m} b_k \Delta^{k\alpha} u(t). \tag{2.4}$$

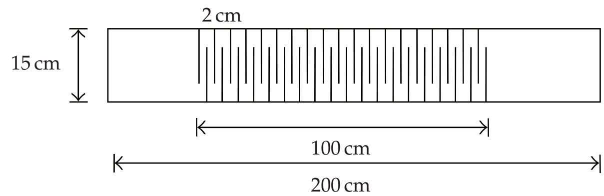

Figure 3: Top side view of the tank, converted into a pool.

Definition 2.2. A system is of rational order, if it is a commensurable order system and besides fulfills the condition of $\alpha = 1/q$ *for all* $q \in \mathbb{N} \mid q \neq 0$.

From the previous definition and based on the property of "q", *an integer order system is a particular case of rational order systems, where* $q = 1$.

Consider the fractional discrete linear system, described by the state-space equations

$$\Delta^{\alpha} x_{k+1} = Ax_k + Bu_k; \quad k \in \mathbb{Z}^{+}, \qquad y_k = Cx_k, \tag{2.5}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k \in \mathbb{R}^p$ are the state, input, and output vectors and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$. Using Definition 2.1, equations (2.5) can be written in the form

$$x_{k+1} + \sum_{j=1}^{k+1} w_j^{\alpha} x_{k-j+1} = Ax_k + Bu_k, y_k = Cx_k. \tag{2.6}$$

2.3. LPV Fractional Identification Methodology

LPV identification method used in this article is a two-step procedure where (1) nonfractional models are identified at several different equilibrium (operating condition) by classical methods [9]; (2) a global multimodel is obtained by interpolating among the local nonfractional models [14]. In this paper, a nonlinear leastsquares estimation method, based on Levenberg-Marquardt [18, 19], is used to obtain the parameters of the rational identified model in each operation point [20]. Local identification method forces rational local models to fit the system separately and locally. This local identification procedure (in each operation point) is standard and it can be itemized as follows: (1) design of the experiment and collection of input-output data in each operation mode from the process to be identified; (2) model structure selection in each operation point; (3) parameter estimation in each operation point; (4) model validation in each operation point.

As the LPV model is interpolated between local rational models, varying parameters of LPV model can be locally interpreted as parameters of the interpolated rational model. Varying parameters in each operation point are interpolated in a polynomical way. This polynomial depends on a scheduling parameter vector $\theta \in \mathbb{R}_+^2$, in this case $\theta = [u_1, u_2]$, that corresponds to the integral of pump activation in each canal that changes in their operating ranges. These values correspond to the upstream levels and are proportional to the upstream flow of each pool. Once the LPV model is obtained, it is validated globally.

Table 1: Operation points for pool P_1.

Pool P_1	Operation range [cm]; $u_2 = 0.5$
$OP_{1_{P_1}}$	$u_1 \in [0.0000, 1.5398]$
$OP_{2_{P_1}}$	$u_1 \in [1.5398, 3.1241]$
$OP_{3_{P_1}}$	$u_1 \in [3.1241, 4.6063]$
$OP_{4_{P_1}}$	$u_1 \in [4.6063, 6.3979]$
$OP_{5_{P_1}}$	$u_1 \in [6.3979, 8.3671]$

Table 2: Operation points for pool P_2.

Pool P_2	Operation range [cm]; $u_1 = 0.5$
$OP_{1_{P_2}}$	$u_2 \in [0.0000, 0.9396]$
$OP_{2_{P_2}}$	$u_2 \in [0.9396, 1.8679]$
$OP_{3_{P_2}}$	$u_2 \in [1.8679, 2.8067]$
$OP_{4_{P_2}}$	$u_2 \in [2.8067, 3.7535]$
$OP_{5_{P_2}}$	$u_2 \in [3.7535, 4.6989]$
$OP_{6_{P_2}}$	$u_2 \in [4.6989, 5.6261]$
$OP_{7_{P_2}}$	$u_2 \in [5.6261, 6.5445]$
$OP_{8_{P_2}}$	$u_2 \in [6.5445, 7.4333]$

In this paper, this system identification procedure is used to obtain a reliable dynamic model of a main irrigation canal when the design of a model-based control system is requested.

3. Experiment Design and Model Structure Selection

For identification of the pilot canal system different experiments have been carried out. These canal pools are operated by means of a downstream water level regulation method. Available measurements are downstream water levels (y_1 for pool P_1 and y_2 for pool P_2) and pump voltage (u_{P_1} for pump B_1 and u_{P_2} for pump B_2). Then, for the identification of the control model canal, as output variables, downstream levels are used, and as input variables integral pump voltage variables (u_1 and u_2) are used. According to literature [5, 21, 22], this model obtained after identification corresponds to a first-order model with delay with an integrator or to a second order model with delay with an integrator, depending on the geometry of the pool.

The appearance of integrator pole, or in other words, the fact that a reach has similarities with a swimming pool or a tank, is not a real surprise and is, in some case, expected. As mentioned before, this pole appears clearly in the uniform case regime and has been successfully included in several simplified models proposed in other works (Integrator Delay (ID) model [23], Integrator Delay Zero (IDZ) model [4], etc.). It is known that the identification of a system with integrators is very erratic about the exact localization of its poles. For this reason, the identified model relates the downstream levels (model outputs) and the integral of pump voltages (model inputs: u_1 for pool P_1 and u_2 for pool P_2).

3.1. Experiment Design

To obtain data containing the maximum information about the canal pools dynamic behaviour, pools must be excited with a persistent input signal that contains the largest number of frequencies representative of the system dynamics [9]. Then a pseudorandom binary sequence (PRBS) is a kind of signal that fulfills these conditions. Since these signals are suitable to identify linear systems and our system is nonlinear and timevarying, a PBRS is used in each operating point within the working range of the system. These signals are integrated (because the system has implicitly an integrator [5]) generating the input for the identification process, u_1 and u_2.

The sampling time T was selected to be 0.5 second because it is enough dueto the system dynamics. Pools act in different operating points. As the pool dynamics are different (due to their input pumps) five points have been selected for pool P_1 ($OP_{k_{P_1}}$, $k = 1, \ldots, 5$) and eight points for pool P_2 ($OP_{k_{P_2}}$, $k = 1, \ldots, 8$); see Tables 1 and 2.

3.2. Model Structure Selection

The model structure selection constitutes one of the most important and difficult decisions in system identification procedure because model complexity influences the accuracy of the description of the real process and the control schemes. Saint-Venant equations [24] represent the dynamics of an open flow canal in a precise and complete manner. This pair of partialdifferential equations constitutes a nonlinear hyperbolic system, which has no analytic solution for arbitrary geometry. However, such equations are not useful for designing a controller using linear theory as already noticed by [4, 25]. In these references, a simplified control-oriented model methodology is proposed that describes an n-pool canal system. In this methodology each pool is modeled around a given operating point using the transfer function matrices:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix}, \tag{3.1}$$

where $Y_1(s)$ and $Y_2(s)$ are the upstream and downstream water levels of pools, respectively, and $Q_1(s)$ and $Q_2(s)$ are the upstream and downstream flow levels of pools considered here. $P_{12} = 0$ and $P_{21} = 0$ because, normally, control models do not take into account the strong coupling between canals because SISO controllers and decouplers are used [26], and the model structure for each pool:

$$P_{ij}(s) = \frac{1}{s} \frac{k}{S_{ij}s^2 + M_{ij}s + 1} e^{-\tau_{ij}s}, \tag{3.2}$$

where $i = j$ and $i = 1, 2$ are transfer functions relating downstream flows with upstream levels. Additionally, there is a relationship between discharge flow and pump voltage. The upstream flow of each pool can be related with its upstream level equivalent to the integral

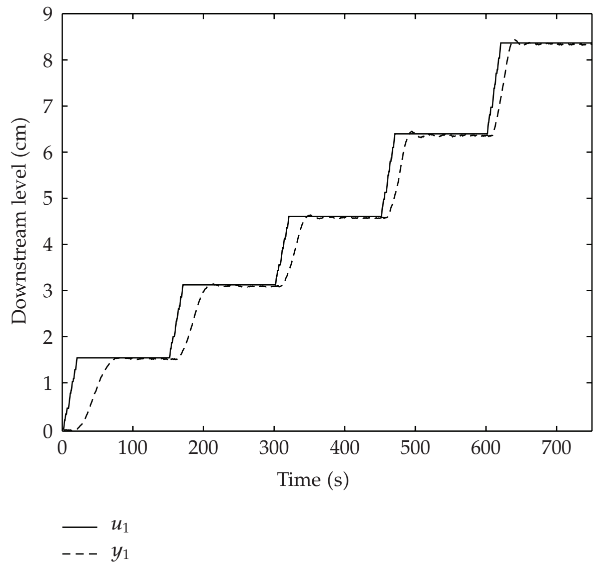

Figure 4: Downstream level for pool P_1, y_1 [cm], and pump input voltage integral, u_1 [cm].

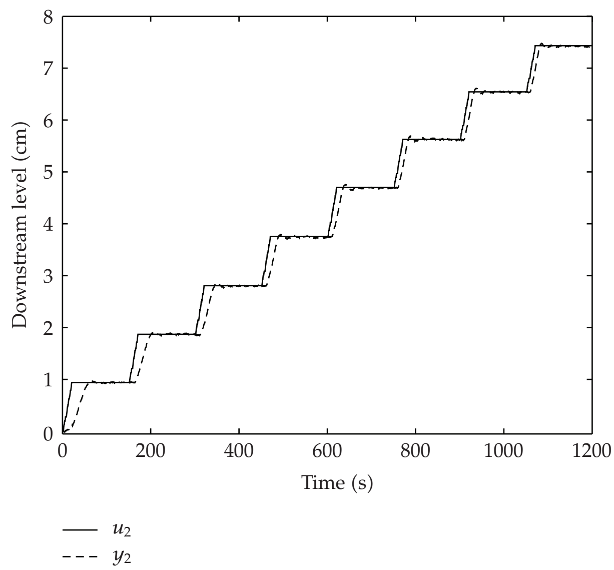

Figure 5: Downstream level for pool P_2, y_2 [cm], and pump input voltage integral, u_2 [cm].

of pump voltages, respectively in a linear way. The following additional relationship should be considered [27]:

$$Q_i(s) = \alpha_i U_i(s). \tag{3.3}$$

The second-order system behaviour can be clearly observed in Figures 4 and 5 when the integral of pump voltage is used as input of the identification model. As it is studied in literature, in backwater part of each pool the dynamics are complicated: waves move up and down and reflect against the boundaries. However, at low frequencies, the water level "integrates" flow variations in the backwater part. In other words, the backwater can be considered to behave as an integrator or reservoir for low frequencies, and for this reason the integrator is included in the control model.

In order to identify the canal system, the continuous model is discretized by using zero-order hold method. Furthermore we assume that control model is LPV (as it is explained in Section 2.3). For each operating point in each pool ($OP_{k_{P_1}}$ for pool P_1 and $OP_{k_{P_2}}$ for pool P_2, see Tables 1 and 2), the discretized model can be expressed as

$$P_d(z,\theta) = \frac{a_3(\theta)z + a_4(\theta)}{z^{-2} + a_1(\theta)z^{-1} + a_2(\theta)} z^{-\tau(\theta)/T}. \tag{3.4}$$

Observing and analyzing the PRBS responses obtained at each operation point (see Figures 4 and 5) in our prototype canal, the canal dynamics can be represented by a seconds order equation with delay, as it is often used in the literature by Hayami model in linear and integer control [1]. As canals are systems that vary according to the operation point, an LPV Hayami model is more suitable [28]. Besides, as canals are nonlinear systems and with distributed parameters, fractional control models are suitable because they yield a more accurate behavior representation. It is desirable to hold the maximum degree of the dynamical equation (second order). So, our models in each operation point are of n-rational order with $n\alpha = 2$. Then, as defined by (2.5), the proposed model structure for $\alpha = 0.5$ and $n = 4$ is

$$\begin{aligned} \Delta^{0.5} x_{k+1} &= A_{0.5}(\theta)x_k + B_{0.5}(\theta)u_k, \\ y_k &= C_{0.5}(\theta)x_k, \end{aligned} \tag{3.5}$$

where $x_k \in \mathbb{R}^4$, $u_k \in \mathbb{R}$, $y_k \in \mathbb{R}$ and

$$A_{0.5}(\theta) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -b_4(\theta) & -b_3(\theta) & -b_2(\theta) & -b_1(\theta) \end{bmatrix}, \tag{3.6}$$

$B_{0.5}(\theta) = [0\ \ 0\ \ 0\ \ b_4(\theta)]^T$ and $C_{0.5}(\theta) = [1\ \ b_7(\theta)\ \ b_6(\theta)\ \ b_5(\theta)]$.

For $\alpha = 0.25$ and $n = 8$, the proposed model structure is

$$\begin{aligned} \Delta^{0.25} x_{k+1} &= A_{0.25}(\theta)x_k + B_{0.25}(\theta)u_k, \\ y_k &= C_{0.25}(\theta)x_k, \end{aligned} \tag{3.7}$$

Table 3: Model parameters obtained by identification in each operating point $OP_{k_{P_1}}$: pool P_1

Parameters	$OP_{1_{P_1}}$	$OP_{2_{P_1}}$	$OP_{3_{P_1}}$	$OP_{4_{P_1}}$	$OP_{5_{P_1}}$
τ	17	10	8	6	5
a_1	−1.9527	−1.9287	−1.9008	−1.8960	−1.8659
a_2	0.9537	0.9309	0.9051	0.9011	0.8747
a_3	0.0010	0.0022	0.0043	0.0051	0.0088
a_4	0.0203	−0.0261	−0.0409	0.0063	−0.0028
b_1	−0.1313	−0.1144	−0.0757	−0.0667	−0.0080
b_2	0.0370	0.0333	0.0261	0.0226	0.0212
b_3	0.0019	0.0052	0.0104	0.0139	0.0198
$b_4(\times 10^{-3})$	0.0995	0.1224	0.1793	0.1969	0.3952
b_5	633.48	606.00	553.53	544.37	229.55
b_6	−345.72	−352.92	−335.57	−363.27	−179.48
b_7	55.0820	74.408	82.686	95.255	62.955
c_1	−2.1318	−1.7484	−2.1485	−2.2626	−2.0778
c_2	2.0729	1.8516	2.1183	2.3379	2.0016
c_3	−1.1719	−1.3840	−1.2185	−1.4044	−1.1296
c_4	0.4176	0.7331	0.4440	0.5309	0.4104
c_5	−0.0948	−0.2552	−0.1036	−0.1270	−0.0985
c_6	0.0133	0.0560	0.0151	0.0185	0.0162
c_7	−0.0010	−0.0071	−0.0013	−0.0015	−0.0021
$c_8(\times 10^{-3})$	0.0464	0.6397	0.0990	0.1061	0.2949
c_9	−1.9148	−1.5150	0.9473	0.9293	1.8894
c_{10}	−2.0112	−1.5179	0.8899	0.8545	1.1611
c_{11}	−1.3216	−0.8523	0.9887	0.9411	0.6570
c_{12}	0.3424	0.5942	1.4846	1.4310	0.9577
c_{13}	2.2736	2.1238	2.0835	2.0224	1.8793
c_{14}	2.1646	1.7134	1.2674	1.1950	1.5309
c_{15}	−3.1753	−2.8745	−2.6283	−2.6231	−2.6879

where $x_k \in \mathbb{R}^8$, $u_k \in \mathbb{R}$, $y_k \in \mathbb{R}$ and

$$A_{0.25}(\theta) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \\ -c_8(\theta) & -c_7(\theta) & \cdots & -c_1(\theta) \end{bmatrix}_{8\times 8},$$
$$B_{0.25}(\theta) = \begin{bmatrix} 0 & \cdots & 0 & c_8(\theta) \end{bmatrix}^T_{8\times 1},$$
$$C_{0.25}(\theta) = \begin{bmatrix} 1 & c_{15}(\theta) & \cdots & c_9(\theta) \end{bmatrix}_{1\times 8} \tag{3.8}$$

$a(\theta) \in \mathbb{R}^4$, $b(\theta) \in \mathbb{R}^7$, $c(\theta) \in \mathbb{R}^{15}$, and $\tau(\theta)$ are the coefficients to be determined in operation points proposed in Tables 1 and 2. As it can be appreciated in (3.4), (3.5), and (3.7), both canals have been considered uncoupled, a widely common practice in literature [26].

Table 4: Model parameters obtained by identification in each operating point $OP_{k_{P_2}}$: pool P_2

Parameters	$OP_{1_{P_2}}$	$OP_{2_{P_2}}$	$OP_{3_{P_2}}$	$OP_{4_{P_2}}$	$OP_{5_{P_2}}$
τ	11	10	9	8	7
a_1	−1.9419	−1.9244	−1.9115	−1.8951	−1.8830
a_2	0.9434	0.9270	0.9153	0.9005	0.8904
a_3	0.0015	0.0026	0.0038	0.0055	0.0074
a_4	−0.0188	−0.0190	0.0075	0.0147	0.0205
b_1	−0.1515	−0.1931	−0.1361	−0.0535	−0.0244
b_2	0.0377	0.0484	0.0356	0.0178	0.0100
b_3	0.0032	0.0017	0.0069	0.0154	0.0199
$b_4(\times 10^{-3})$	0.1347	0.3226	0.2270	0.1694	0.1214
b_5	603.9000	117.29	307.20	726.79	989.96
b_6	−299.6300	−72.221	−204.48	−496.96	−734.07
b_7	51.8040	15.713	48.386	122.65	207.87
c_1	−2.0539	−1.8107	−2.1661	−2.3208	−2.3517
c_2	1.9327	1.6503	2.1372	2.4464	2.5364
c_3	−1.0584	−0.9557	−1.2253	−1.4919	−1.5938
c_4	0.3661	0.4019	0.4440	0.5688	0.6308
c_5	−0.0808	−0.1250	−0.1032	−0.1363	−0.1579
c_6	0.0111	0.0283	0.0151	0.0197	0.0241
c_7	−0.0008	−0.0042	−0.0014	−0.0016	−0.0021
$c_8(\times 10^{-3})$	0.0434	0.4807	0.1050	0.1112	0.1667
c_9	2.1585	−1.5600	−0.6518	−0.1523	1.1259
c_{10}	1.4147	−1.7388	−0.9192	−0.5187	0.0930
c_{11}	0.8165	−1.0839	−0.4348	−0.2286	−0.4165
c_{12}	1.0115	0.6744	1.1870	1.1744	0.5429
c_{13}	1.7673	2.4956	2.8789	2.7318	2.4285
c_{14}	0.9012	1.3293	1.2377	1.1205	1.6084
c_{15}	−2.2559	−2.7775	−2.8406	−2.7515	−2.8369

Parameters of models (3.4)–(3.7) in each operation points and pools are independently identified. To test the improvement of these rational order models (3.5) and (3.7) with respect to the LTI model with delay (3.4) in each pool, a parametric estimation of each model has been carried out. This estimation consists in the computation of parameter vectors $a(\theta)$ and $\tau(\theta)$ for integer model, $b(\theta)$ and $c(\theta)$ for non-integer models ($\alpha = 0.5$ and $\alpha = 0.25$, resp.).

The estimation method used in this work is the previously mentioned in Section 2.3 (see [18, 19]). This methodology guarantees robust convergence, even when the parameters are initialized with values far from the optimal value.

In the case of integer model, there exists a delay which is estimated using correlation analysis [9], providing an estimation of the canal impulse response with regard to the integral of pump activation. This method computes intervals for the delay with a given confidence, and only the nominal values are chosen.

Parameters of models obtained in both pools, P_1 and P_2, are gathered in Tables 3, 4, and 5, respectively.

Parameters are estimated experimentally by applying the set of input PRBSs, explained in Section 3.1, sweeping all the operating points in each pool (Figures 4 and 5).

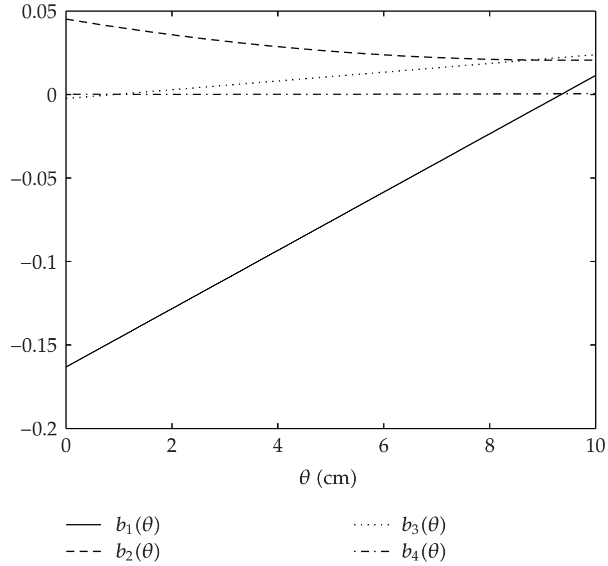

Figure 6: Polynomial aproximations of $b_1(\theta) - b_4(\theta)$ in pool P_1.

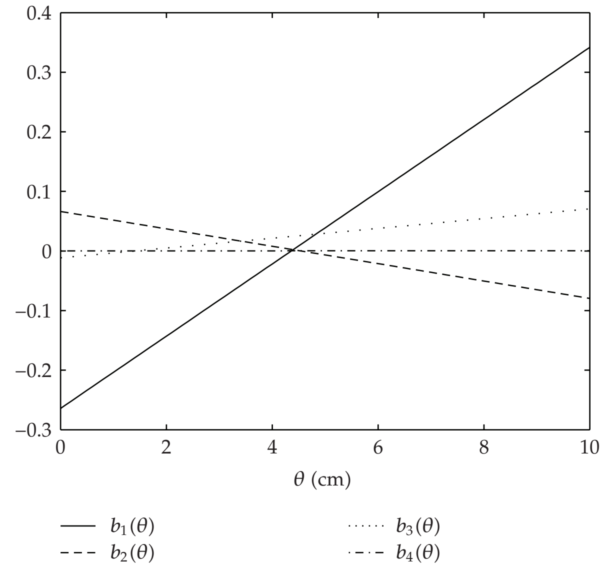

Figure 7: Polynomial aproximations of $b_1(\theta) - b_4(\theta)$ in pool P_2.

Each linear varying parameter depends on the gain scheduling variable $\theta = [u_1, u_2]$. Hence, it is assumed that the variation of parameters $a(\theta)$ and $\tau(\theta)$ for integer model and $b(\theta)$ and $c(\theta)$ for non-integer models according to the scheduling variable θ can be approximated by a polynomial function of θ, where $\theta = u_1$ with $u_2 = 0.5$ cm for pool P_1, and $\theta = u_2$ with $u_1 = 0.5$ cm for pool P_2.

Table 5: Model parameters obtained by identification in each operating Point $OP_{k_{P_2}}$: pool P_2 (cont.)

Parameters	$OP_{6_{P_2}}$	$OP_{7_{P_2}}$	$OP_{8_{P_2}}$
τ	5	4	3
a_1	−1.8826	−1.8804	−1.8697
a_2	0.8907	0.8891	0.8791
a_3	0.0081	0.0088	0.0094
a_4	0.0309	0.0447	0.0720
b_1	0.1722	0.2216	0.0919
b_2	−0.0398	−0.0470	−0.0235
b_3	0.0434	0.0509	0.0437
$b_4(\times 10^{-3})$	0.3951	0.4421	0.2546
b_5	722.21	566.36	757.35
b_6	−515.24	−476.59	−674.80
b_7	133.11	136.39	203.67
c_1	−2.2296	−2.3769	−2.4777
c_2	2.2776	2.6077	2.8311
c_3	−1.3542	−1.6760	−1.8925
c_4	0.5203	0.6903	0.8029
c_5	−0.1375	−0.1888	−0.2194
c_6	0.0282	0.0369	0.0390
c_7	−0.0050	−0.0059	−0.0048
$c_8(\times 10^{-3})$	0.7180	0.7576	0.4943
c_9	5.6247	3.8395	13.4920
c_{10}	2.3606	2.4912	−11.8110
c_{11}	−2.0125	−1.2561	−1.2178
c_{12}	−3.8188	−3.8690	7.8987
c_{13}	0.1248	−0.6176	−8.5361
c_{14}	5.8407	6.5926	8.5167
c_{15}	−4.1413	−4.4149	−4.4914

Table 6: Values of p for each $b_i(\theta)$: pool P_1.

Coefficients	p_1	p_2	p_3
b_1	0	0.01746	−0.1632
b_2	0.0002788	−0.005239	0.04517
b_3	0	0.002626	−0.002383
b_4	6.889×10^{-6}	-2.84×10^{-5}	0.0001364

For instance, for non-integer model $\alpha = 0.5$. Figures 6 and 7 graphically depict polynomial approximations of $b_1(\theta) - b_4(\theta)$ in both pools that correspond to the following functions:

$$b_i(\theta) = p_1\theta^2 + p_2\theta + p_3, \tag{3.9}$$

where the values of p_j $(j = 1, \dots, 3)$ are shown in Tables 6 and 7.

Table 7: Values of p for each $b_i(\theta)$: pool P_2.

Coefficients	p_1	p_2	p_3
b_1	0	0.0606	−0.1632
b_2	0	−0.01459	0.06633
b_3	0	0.008233	−0.01152
b_4	0	2.42×10^{-5}	0.0001565

Table 8: Mean absolute error (MAE) in every operation point: pool P_1.

Operation points	$\hat{y}_i(\alpha = 1)$	$\hat{y}_i(\alpha = 0.5)$	$\hat{y}_i(\alpha = 0.25)$
$OP_{1_{P_1}}$	0.0294	0.0152	0.0148
$OP_{2_{P_1}}$	0.0228	0.0145	0.0067
$OP_{3_{P_1}}$	0.0164	0.0120	0.0163
$OP_{4_{P_1}}$	0.0191	0.0168	0.0180
$OP_{5_{P_1}}$	0.0187	0.0172	0.0179

Table 9: Mean absolute error (MAE) in every operation point: pool P_2.

Operation points	$\hat{y}_i(\alpha = 1)$	$\hat{y}_i(\alpha = 0.5)$	$\hat{y}_i(\alpha = 0.25)$
$OP_{1_{P_2}}$	0.0160	0.0093	0.0074
$OP_{2_{P_2}}$	0.0133	0.0095	0.0073
$OP_{3_{P_2}}$	0.0121	0.0098	0.0188
$OP_{4_{P_2}}$	0.0133	0.0123	0.0106
$OP_{5_{P_2}}$	0.0117	0.0114	0.0108
$OP_{6_{P_2}}$	0.0129	0.0124	0.0093
$OP_{7_{P_2}}$	0.0128	0.0125	0.0099
$OP_{8_{P_2}}$	0.0101	0.0097	0.0077

4. Model Validation

Model validation is the core of the identification problem because it makes possible to evaluate the model quality, that is, if the method fits the measured experimental data with accuracy enough, if it is valid for its purpose, and if the model describes correctly the real process [9]. Figures 8 and 10 show the performance in all the operation points for rational models as well as for integer model in pools P_1 and P_2, respectively. Globally, in Figures 8 –11 it can be appreciated that rational models track better measured downstream level in transitory case and also in permanent regime case than integer models.

In order to assess how suitable models respect validation data set, mean absolute error (MAE) is quantified as

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{y}_i(\alpha) - y_i\right| = \frac{1}{n}\sum_{i=1}^{n}|e_i|. \tag{4.1}$$

As its name suggests, the mean absolute error is an average of absolute errors $e_i = \hat{y}_i(\alpha) - y_i$, where $\hat{y}_i(\alpha)$ is the prediction value and y_i the real value. The values of MAE for operating points in each pool are shown in Tables 8 and 9, being $\hat{y}_i(\alpha = 1)$ the integer case (3.4) and

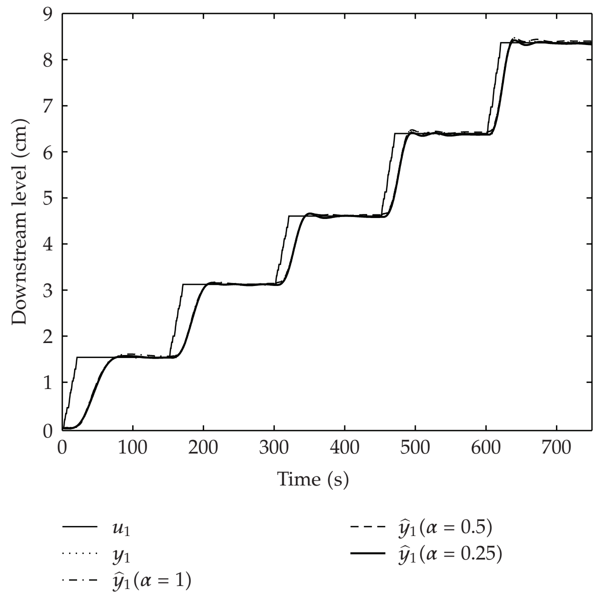

Figure 8: Model output in pool P_1.

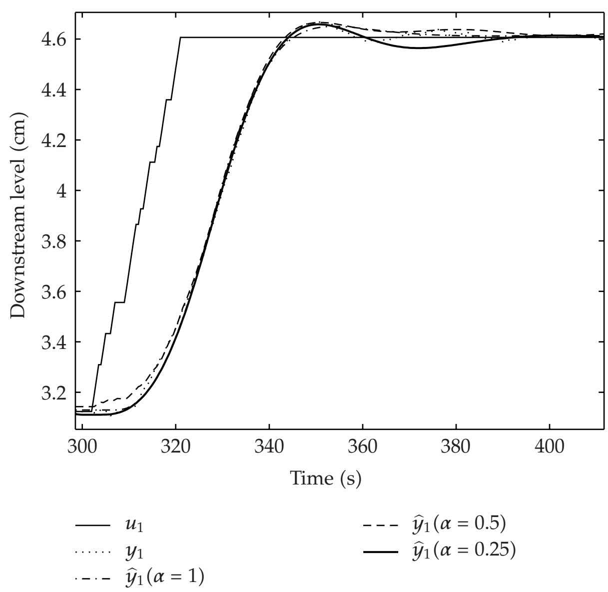

Figure 9: Model output in operation point $OP_{3_{P_1}}$ in pool P_1.

$\hat{y}_i(\alpha = 0.5)$ and $\hat{y}_i(\alpha = 0.25)$ the rational models (3.5) and (3.7), respectively. As it can be observed, most of errors in the integer case are higher than errors in the rational case, indicating that rational models give an improvement in the accuracy in each control model.

However, the lower the value of α is, the higher is the number of coefficients to be determined (see Tables 3, 4, and 5).

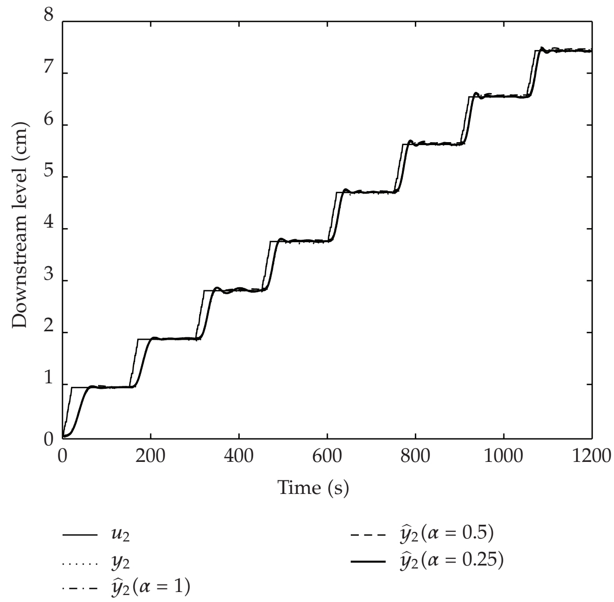

Figure 10: Model output in pool P_2.

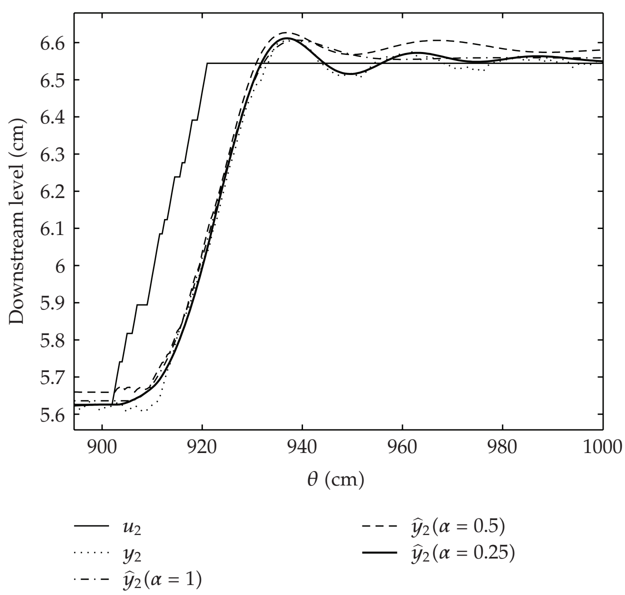

Figure 11: Model output in operation point $OP_{7_{P_2}}$ in pool P_2.

5. Conclusions

In this article, an LPV rational order model-based control-oriented system identification procedure for irrigation canals has been developed. This identification procedure has been applied in an experimental prototype canal. In this case, rational local models for an irrigation pool in different operation points have been obtained and interpolated to reach the complete model: the LPV rational model. Resulting LPV rational order control model normally describes the plant with a lower error than the corresponding LPV integer order control model. The lower the α value (degree of the rational order models) is, the lower the error is. Nevertheless, there exists a relevant trade-off between α values and model complexity for control purposes, because the lower the α values are, the higher is the number of coefficients to be computed. This amount of data increases controller computational complexity but on the other hand controller design techniques become simpler.

Acknowledgments

Authors would like to thank Inter-Ministerial Commission of Spanish Government (DPI2004-5414, MCYT), Catalan Autonomous Government (VIS, Consolidated Research Group), European Commission (FP6-2005-IST-6, URUS-045062), and Cooperation Spanish Agency for International Development (AECID) for funding this research. Authors also want explicitly to thank reviewers' comments that have largely improved this article.

References

[1] X. Litrico, "Modélisation et commande robuste de systèmes barrage-rivière," *La Houille Blanche*, no. 3-4, pp. 86–90, 2001.

[2] J. A. Cunge, F. M. Holly, and A. Verwey, *Practical Aspects of Computational River Hydraulics*, Pitman Advanced Publishing, London, UK, 1980.

[3] X. Litrico and V. Fromion, "Variations autour de la Performance Optimale de Systèmes Hydrauliques à Surface Libre," in *Colloque Automatique et Agronomie*, pp. 197–207, INRA, Montpellier, France, 2003.

[4] X. Litrico and V. Fromion, "Analytical approximation of open-channel flow for controller design," *Applied Mathematical Modelling*, vol. 28, no. 7, pp. 677–695, 2004.

[5] X. Litrico and V. Fromion, "Frequency modeling of open-channel flow," *Journal of Hydraulic Engineering*, vol. 130, no. 8, pp. 806–815, 2004.

[6] Y. Bolea, J. Blesa, and V. Puig, "LPV modelling and identification of an open canal for control," in *Proceedings of the European Control Conference (ECC '07)*, Kos, Greece, 2007.

[7] S. K. Ooi, M. P. M. Krutzen, and E. Weyer, "On physical and data driven modelling of irrigation channels," *Control Engineering Practice*, vol. 13, no. 4, pp. 461–471, 2005.

[8] E. Weyer, "System identification of an open water channel," *Control Engineering Practice*, vol. 9, no. 12, pp. 1289–1299, 2001.

[9] L. Ljung, *System Identification: Theory for the User*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1987.

[10] D. J. Leith and W. E. Leithead, "Survey of gain-scheduling analysis and design," *International Journal of Control*, vol. 73, no. 11, pp. 1001–1025, 2000.

[11] W. J. Rugh and J. S. Shamma, "Research on gain scheduling," *Automatica*, vol. 36, no. 10, pp. 1401–1425, 2000.

[12] D. J. Leith, R. N. Shorten, W. E. Leithead, O. Mason, and P. Curran, "Issues in the design of switched linear control systems: a benchmark study," *International Journal of Adaptive Control and Signal Processing*, vol. 17, no. 2, pp. 103–118, 2003.

[13] X. Litrico and D. Georges, "Robust continuous-time and discrete-time flow control of a dam-river system. (II) Controller design," *Applied Mathematical Modelling*, vol. 23, no. 11, pp. 829–846, 1999.

[14] J. Abonyi and R. Babuska, "Local and global identification and interpretation of parameters in Takagi-Sugeno fuzzy models," in *Proceedings of the IEEE International Conference on Fuzzy Systems*, vol. 2, pp. 835–840, San Antonio, Tex, USA, 2000.

[15] B. Bamieh and L. Giarré, "Identification of linear parameter varying models," *International Journal of Robust and Nonlinear Control*, vol. 12, no. 9, pp. 841–853, 2002.

[16] V. Feliu, R. Rivas, and F. J. Castillo, "Fractional robust control to delay changes in main irrigation canals," in *Proceedings of the 16th IFAC World Congress*, Prague, Czech Republic, 2005.

[17] I. Podlubny, *Fractional Differential Equations*, vol. 198 of *Mathematics in Science and Engineering*, Academic Press, San Diego, Calif, USA, 1999.

[18] D. W. Marquardt, “An algorithm for least-squares estimation of nonlinear parameters,” *SIAM Journal on Applied Mathematics*, vol. 11, pp. 431–441, 1963.
[19] J. J. Moré, “The Levenberg-Marquardt algorithm: implementation and theory,” in *Numerical Analysis*, G. A. Watson, Ed., vol. 630 of *Lecture Notes in Mathematics*, pp. 105–116, Springer, Berlin, Germany, 1978.
[20] R. Malti, S. Victor, O. Nicolas, and A. Oustaloup, “System identification using fractional models: state of the art,” in *Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (DETC ’07)*, pp. 295–304, Las Vegas, Nev, USA, 2007.
[21] Y. Bolea, V. Puig, J. Blesa, M. Gómez, and J. Rodellar, “An LPV model for canal control,” in *Proceedings of the 10th IEEE International Conference on Methods and Models in Automation and Robotics (MMAR ’04)*, Miedzyzdroje, Poland, 2004.
[22] E. F. Camacho, *Model Predictive Control*, Springer, New York, NY, USA, 1999.
[23] J. Schuurmans, *Control of water levels in open-channels*, Ph.D. thesis, Delft University of Technology, Delft, The Nederlands, 1997.
[24] V. T. Chou, *Open Channel Hydraulics*, MacGraw-Hill, New York, NY, USA, 1959.
[25] J. Schuurmans, O. H. Bosgra, and R. Brouwer, “Open-channel flow model approximation for controller design,” *Applied Mathematical Modelling*, vol. 19, no. 9, pp. 525–530, 1995.
[26] X. Litrico, V. Fromion, and J.-P. Baume, “Tuning of robuts distant downstream PI controllers for an irrigation canal pool: (II) implementation issues,” *Journal of Irrigation and Drainage Engineering*, vol. 132, no. 4, pp. 369–379, 2006.
[27] R. S. Sánchez-Peña, Y. Bolea, and V. Puig, “MIMO Smith predictor: global and structured robust performance analysis,” *Journal of Process Control*, vol. 19, no. 1, pp. 163–177, 2009.
[28] Y. Bolea, V. Puig, and J. Blesa, “Gain-scheduled smith PID controllers for LPV systems with time varying delay: application to an open-flow canal,” in *Proceedings of the 17th IFAC World Congress*, Seoul, South Korea, 2008.