Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2009, Article ID 486470, 22 pages
doi:10.1155/2009/486470

Research Article

Robust Stabilization Approach and H_∞ Performance via Static Output Feedback for a Class of Nonlinear Systems

Neila Bedioui, Salah Salhi, and Mekki Ksouri

Laboratoire d'Analyse et de Commande des Systèmes (LACS), Ecole Nationale des Ingénieurs de Tunis (ENIT), P. B. 37, Le Belvédère, CP 1002 Tunis, Tunisia

Correspondence should be addressed to Salah Salhi, salhis@lycos.com

Received 17 November 2008; Revised 18 February 2009; Accepted 12 March 2009

Recommended by Shijun Liao

This paper deals with the stability and stabilization problems for a class of discrete-time nonlinear systems. The systems are composed of a linear constant part perturbated by an additive nonlinear function which satisfies a quadratic constraint. A new approach to design a static output feedback controller is proposed. A sufficient condition, formulated as an LMI optimization convex problem, is developed. In fact, the approach is based on a family of LMI parameterized by a scalar, offering an additional degree of freedom. The problem of performance taking into account an H_∞ criterion is also investigated. Numerical examples are provided to illustrate the effectiveness of the proposed conditions.

1. Introduction

Modeling a real process is generally a complex and difficult task. Even if in numerous cases, a linear model can capture the main dynamical characteristics of a process. In some situations, it is necessary to take into account model uncertainties in order to design an efficient control law.

There exists an extensive literature dealing with this problem which is in fact the main problem in robust control design [1–8].

Among the numerous solutions allowing taking into account model uncertainties, a way which has been frequently investigated in literature consists in adding to the linear part of the model a nonlinear one which captures model uncertainties and frequently referred in literature as nonlinear systems with separated nonlinearity.

Nonlinear systems with separated nonlinearity are a class of nonlinear systems composed of a linear constant part to which another nonlinear function part is added. This function depends on both time and state and satisfies a quadratic constraint [9–16]. This class of nonlinear system can be considered as a generalized model for linear systems with parametric uncertainties where uncertainties can be norm bounded [2, 17] or polytopic [9, 10, 18, 19].

Many papers have investigated robust stability, analysis, and synthesis using essentially Lyapunov theory which has proved to be efficient in this context. Recent proposed approaches [9–13] are based on convex optimization problems involving linear matrix inequality (LMI) where the objective is to maximize the bounds on the nonlinearity that systems can tolerate without unstabilities. In particular, sufficient conditions where developed in the context of static state feedback and static or dynamic output feedback controllers [9–13, 15].

Even if the static output feedback stabilization (SOF) problem is considered as NP-hard [20] and still one of the most important open questions in the control theory, it concentrates the efforts of many researchers. SOF gains, which stabilize the system, are not easy to find due to the nonconvexity of the SOF formulation. In some papers the design of SOF controllers for a class of discrete-time nonlinear systems is proposed [11, 15, 17]. In this paper, we propose a new approach for robust static output feedback stabilization of a class of discrete-time nonlinear systems using LMI techniques. In fact, our approach is based on the introduction of a relaxation scheme to the SOF problem similar to [15, 21–23]. Our major objective is to maximize the admissible bounds on the nonlinearity guaranteeing the stability of system, with a prescribed degree μ. The main contribution is the possibility of decoupling the Lyapunov matrix to the SOF gain leading to less restrictive conditions.

The problem of performance is also treated in the context of H_∞ settings. A new H_∞ norm characterization is proposed for this class of nonlinear discrete time systems in terms of LMI formulation.

The paper is organized as follows. Section 2 presents robust stability condition for the class of nonlinear discrete time systems. Then, we develop our main results for robust stabilization by SOF. In Section 3, the problem of robust H_∞ synthesis via SOF is presented. Section 4 presents numerical examples for robust stabilization and H_∞ synthesis to illustrate the potential of the proposed conditions.

Notation 1. For conciseness the following notations are used: $\mathrm{sym}(A) = A + A^T$, $\mathrm{diag}(A, B) = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, $\begin{bmatrix} A & B \\ \bullet & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$, and $P = P^T > 0$ is a symmetric and positive definite.

2. Preliminaries

In this section, we consider nonlinear discrete-time system with the following state-space representation:

$$x(k+1) = Ax(k) + f(k, x), \tag{2.1}$$

where $x(k) \in \Re^n$ is the state vector of the system. $A \in \Re^{n\times n}$ is a constant matrix and $f(k, x)$ a nonlinear function in both arguments k and x satisfying $f(k, 0) = 0$. This means that the origin is an equilibrium point of the system.

The nonlinear function f is bounded by the following quadratic constraints:

$$f^T(k, x) f(k, x) \le \alpha^2 x^T M^T M x, \tag{2.2}$$

where $\alpha > 0$ is the bounding parameter of the nonlinear function f and M is a constant matrix of appropriate dimensions.

The parameter α can be defined as a degree of robustness, because its maximization leads to an increase of robustness against uncertain perturbations. Note that constraint (2.2) is equivalent to

$$\begin{bmatrix} x \\ f \end{bmatrix}^T \begin{bmatrix} -\alpha^2 M^T M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} \leq 0. \tag{2.3}$$

Remark 2.1. The nonlinear function $f(k, x)$, which satisfies the quadratic constraint (2.2), can be considered as parameter uncertainty [17].

In the sequel, we will use the following definition to present the concept of robust stability of the system (2.1), (2.2).

Definition 2.2. System (2.1) is robustly stable with degree $\alpha > 0$ if the equilibrium $x = 0$ is globally asymptotically stable for all $f(k, x)$ satisfying constraint (2.2).

In this section, we develop a method for studying robust stability of system (2.1). Before, we introduce some instrumentals tools which will be used in the proof of characterization of stability of system (2.1).

Lemma 2.3 (S-procedure lemma [1]). *Let $\Omega_0(x)$ and $\Omega_1(x)$ be two arbitrary quadratic forms over $\Re^n$, then $\Omega_0(x) < 0$ for all $x \in \Re^n - \{0\}$ satisfying $\Omega_1(x) \leq 0$ if and only if there exist $\tau \geq 0$:*

$$\Omega_0(x) - \tau\Omega_1(x) < 0, \quad \forall x \in \Re^n - \{0\}. \tag{2.4}$$

Proof. See [1]. □

Lemma 2.4 (Projection lemma [1]). *Given a symmetric matrix $\psi \in \Re^{n\times n}$, and two matrices P, Q of column dimensions n, there exists X such that the following LMI holds:*

$$\psi + \operatorname{sym}\left(P^T X^T Q\right) < 0, \tag{2.5}$$

if and only if the projection inequalities with respect to X are satisfied:

$$\mathcal{N}_P \psi \mathcal{N}_p^T < 0, \qquad \mathcal{N}_Q^T \psi \mathcal{N}_Q < 0, \tag{2.6}$$

where $\mathcal{N}_P$ and $\mathcal{N}_Q$ denote arbitrary bases of the nullspaces of P and Q, respectively.

Proof. See [1]. □

Lemma 2.5. *Let Φ a symmetric matrix and N, J be matrices of appropriate dimensions. The following statements are equivalent:*

(i) $\Phi < 0$ *and* $\Phi + NJ^T + JN^T < 0$,

(ii) *there exists a matrix G such that*

$$\begin{bmatrix} \Phi & J + NG \\ J^T + G^T N^T & -G - G^T \end{bmatrix} < 0. \tag{2.7}$$

Proof. The proof is obtained remarking that (2.7) can be developed as follows:

$$\begin{bmatrix} \Phi & J + NG \\ J^T + G^T N^T & -G - G^T \end{bmatrix} = \begin{bmatrix} \Phi & J \\ J^T & 0 \end{bmatrix} + \text{sym}\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} G^T \begin{bmatrix} N^T & -I \end{bmatrix} \right\} < 0, \tag{2.8}$$

and by applying lemma 2. □

2.1. Stability Characterization

We first introduce the following theorem which gives a robust stability conditions for system (2.1). In fact, it is described by a convex optimization problem where we try to maximize the nonlinear bounding parameter without loss of the stability of system.

Theorem 2.6. *System* (2.1) *is robustly stable with degree $\alpha > 0$ if there exist a positive definite symmetric matrix Q and a positive scalar $\beta = 1/\sqrt{\alpha}$ such that:*

$$\begin{cases} \textit{minimize } \beta \\ \begin{bmatrix} -Q & 0 & QA^T & QM^T \\ \bullet & -I & I & 0 \\ \bullet & \bullet & -Q & 0 \\ \bullet & \bullet & \bullet & -\beta I \end{bmatrix} < 0. \end{cases} \tag{2.9}$$

Proof. See [13]. □

Remark 2.7. The stability condition given by Theorem 2.6 is equivalent to the one introduced in [11] and [13].

A new condition for robust stability is proposed in the following theorem.

Theorem 2.8. *System* (2.1) *is stable with degree $\alpha > 0$ if there exist a positive definite symmetric matrix Q, a matrix G of appropriate dimensions, and a positive scalar $\beta = 1/\sqrt{\alpha}$, such that the following optimization problem:*

$$\begin{cases} \text{minimize } \beta \\ \begin{bmatrix} -Q & 0 & \mu Q & 0 & Q \\ \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & -Q & 0 & (A-\mu I)G \\ \bullet & \bullet & \bullet & -\beta I & MG \\ \bullet & \bullet & \bullet & \bullet & -G-G^T \end{bmatrix} < 0, \end{cases} \tag{2.10}$$

is feasible for any prescribed scalar $\mu \in]-1\ 1[$.

Proof. Inequality (2.9) can be expressed as follows:

$$\begin{bmatrix} -Q & 0 & QA^T & QM^T \\ \bullet & -I & I & 0 \\ \bullet & \bullet & -Q & 0 \\ \bullet & \bullet & \bullet & -\beta I \end{bmatrix} = \underbrace{\begin{bmatrix} -Q & 0 & \mu Q & 0 \\ \bullet & -I & I & 0 \\ \bullet & \bullet & -Q & 0 \\ \bullet & \bullet & \bullet & -\beta I \end{bmatrix}}_{\phi} + \text{sym}\left(\underbrace{\begin{bmatrix} 0 \\ 0 \\ A-\mu I \\ M \end{bmatrix}}_{N} \underbrace{[Q\ 0\ 0\ 0]}_{J} \right) < 0. \tag{2.11}$$

It is not difficult to proof that $\phi < 0$ for any $\mu \in]-1\ 1[$. Now writing:

(i) $N = \begin{bmatrix} 0 \\ 0 \\ A-\mu I \\ M \end{bmatrix}$,

(ii) $J = [Q\ 0\ 0\ 0]^T$,

and by lemma 2, there exists a matrix G of appropriate dimensions such that inequality is satisfied. □

Remark 2.9. The two optimization problems (2.9) and (2.10) are equivalent. In the case of stability analysis or state feedback control synthesis, no improvement is obtained by problem (2.10). The main advantage of problem (2.10) will appear when dealing with static output feedback. In that case, we will see that it theoretically improves the obtained results.

2.2. Static Output Feedback Control

In this section, we investigate the static output feedback stabilization problem for nonlinear discrete systems.

We consider the nonlinear discrete-time system described as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + f(k,x,u) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \tag{2.12}$$

where $u(k) \in \Re^m$ is the control input, $y(k) \in \Re^p$ is the measured output, and $B \in \Re^{n\times m}$ and $C \in \Re^{p\times n}$ are constant matrices. We assume that the pair (A, B) is stabilizable and C is full rank matrices. Also $f(k,x,u)$ is a nonlinear function which satisfies the following quadratic constraints:

$$f^T(k,x,u)f(k,x,u) \le \alpha^2\left(x^T F^T F x + u^T H^T H u\right), \tag{2.13}$$

where $\alpha > 0$ is the bounding parameter of the function f and F and H are constant matrices of appropriate dimensions.

The objective is to find a static output feedback control law such as

$$u(k) = Ky(k), \tag{2.14}$$

where $K \in \Re^{m\times p}$.

The closed loop system is given by the following state space representation:

$$x(k+1) = (A + BKC)x(k) + f(k,x,u), \tag{2.15}$$

and function f satisfies:

$$f^T(k,x,u)f(k,x,u) \le \alpha^2 x^T\left(F^T F + (HKC)^T HKC\right)x. \tag{2.16}$$

Note that in this case, the constraint (2.16) is equivalent to

$$\begin{bmatrix} x \\ f \end{bmatrix}^T \begin{bmatrix} -\alpha^2\left(F^T F + (HKC)^T HKC\right) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} \le 0. \tag{2.17}$$

To establish a robust stabilization theory for system (2.12) with (2.13) by SOF, we give in the following theorem, an optimization problem which allows to stabilize the linear part of system (2.15) with (2.17) and at the same time to maximize the value of parameter α.

Theorem 2.10. *System* (2.12) *is asymptotically stable by static output feedback with degree* $\alpha > 0$ *if there exist a positive definite symmetric matrix* Q*, a matrix* $R \in^{m\times p}$*, and a positive scalar* $\beta = 1/\sqrt{\alpha}$ *such that the following optimization problem is solvable:*

$$\begin{cases} \text{minimize } \beta \\ \begin{bmatrix} -Q & 0 & (AQ+BRC)^T & QF^T & (HRC)^T \\ \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I \end{bmatrix} < 0, \end{cases} \tag{2.18}$$

where

$$Q = V\begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} V^T. \tag{2.19}$$

The static output feedback gain is given by

$$K = RUC_0Q_1^{-1}C_0^{-1}U^T, \tag{2.20}$$

with $U \in \Re^{p\times p}$ *and* $V \in \Re^{n\times n}$ *are unitary matrices, and* $C_0 \in \Re^{p\times p}$ *matrix which are obtained by using the singular value decomposition of the matrix* C*:*

$$C = U\begin{bmatrix} C_0 & 0 \end{bmatrix} V^T. \tag{2.21}$$

Proof. According to the Theorem 2.6, system (2.15) is robustly stable if there exist a positive definite symmetric matrix Q and a positive scalar $\beta = 1/\sqrt{\alpha}$ such that the following optimization problem is solvable:

$$\begin{cases} \text{minimize } \beta \\ \begin{bmatrix} -Q & 0 & Q\tilde{A}^T & QM^T \\ \bullet & -I & I & 0 \\ \bullet & \bullet & -Q & 0 \\ \bullet & \bullet & \bullet & -\beta I \end{bmatrix} < 0. \end{cases} \tag{2.22}$$

Now we defin

(i) $\widetilde{A} = A + BKC$,

(ii) $M = \binom{F}{HKC}$,

(iii) Q is replaced by (2.19) with: $Q_1 \in \Re^{p\times p}$, $Q_2 \in \Re^{(n-p)\times(n-p)}$.

Equation (2.22) becomes:

$$\begin{cases} \textit{minimize } \beta \\ \begin{bmatrix} -Q & 0 & Q(A+BKC)^T & QF^T & QC^TK^TH^T \\ \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I \end{bmatrix} < 0. \end{cases} \tag{2.23}$$

Unfortunately, (2.23) is not convex in K and Q and cannot be solved by the LMI tools. We can introduce some transformations to simplify the KCQ term of the inequality (2.23) by using (2.19) and (2.21) as follows:

$$\begin{aligned} KCQ &= KU\left[C_0 \;\; 0\right]V^TV\begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}V^T \\ &= \underbrace{KUC_0Q_1C_0^{-1}U^{-1}}_{R}\underbrace{U\left[C_0 \;\; 0\right]V^T}_{C}, \end{aligned} \tag{2.24}$$

where $R \in \Re^{m\times p}$.

With this transformation we obtain the optimization problem given in Theorem 2.10. □

Remark 2.11. The optimization problem given by Theorem 2.10 presents a sufficient condition for the robust stabilization by SOF of discrete-time nonlinear system (2.12). In fact to solve the BMI problem, we impose a diagonal structure to Lyapunov matrix Q as in [24] and we obtain a LMI convex problem.

2.3. Main Results

In this section, we introduce a new approach for robust stabilization by SOF of nonlinear discrete time system (2.12). The following results present solutions to static output feedback problem in which an improved sufficient condition is presented. In fact, this approach is derived from Theorem 2.8.

Theorem 2.12. *The system* (2.12) *is robustly stable by static output feedback with degree $\alpha > 0$, for an arbitrary prescribed number μ in* $]-1\ 1[$ *if there exist a positive definite symmetric matrix Q,* *matrices* $R \in \Re^{m\times p}$, $G \in \Re^{n\times n}$, *and a positive scalar* $\beta = 1/\sqrt{\alpha}$ *such that the following optimization problem is solvable:*

$$\begin{cases} \text{minimize } \beta \\ \begin{bmatrix} -Q & 0 & \mu Q & 0 & 0 & Q \\ \bullet & -I & I & 0 & 0 & 0 \\ \bullet & \bullet & -Q & 0 & 0 & AG + BRC - \mu G \\ \bullet & \bullet & \bullet & -\beta I & 0 & FG \\ \bullet & \bullet & \bullet & \bullet & -\beta I & HRC \\ \bullet & \bullet & \bullet & \bullet & \bullet & -G - G^T \end{bmatrix} < 0, \end{cases} \tag{2.25}$$

where

$$G = V\begin{bmatrix} G_1 & 0 \\ G_2 & G_3 \end{bmatrix} V^T. \tag{2.26}$$

The static output feedback gain is given by

$$K = RUC_0G_1^{-1}C_0^{-1}U^T, \tag{2.27}$$

with U, V, *and* C_0 *are given in* (2.21).

Proof. According to Theorem 2.8, the closed loop system (2.15) is robustly stable if there exist a positive definite symmetric matrix Q, a matrix G of appropriate dimensions, and a positive scalar $\beta = 1/\sqrt{\alpha}$ such that

$$\begin{cases} \text{minimize } \beta \\ \begin{bmatrix} -Q & 0 & \mu Q & 0 & Q \\ \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & -Q & 0 & \left(\tilde{A} - \mu I\right)G \\ \bullet & \bullet & \bullet & -\beta I & MG \\ \bullet & \bullet & \bullet & \bullet & -G - G^T \end{bmatrix} < 0, \end{cases} \tag{2.28}$$

where the following hold:

(i) $\tilde{A} = A + BKC$,

(ii) $M = \binom{F}{HKC}$,

(iii) G replaced by (2.26) where $G_1 \in \Re^{p\times p}$, $G_2 \in \Re^{(n-p)\times p}$, $G_3 \in \Re^{(n-p)\times(n-p)}$,

with U, V, and C_0 are given by (2.21).

Then, inequality (2.28) becomes

$$\begin{bmatrix} -Q & 0 & \mu Q & 0 & 0 & Q \\ \bullet & -I & I & 0 & 0 & 0 \\ \bullet & \bullet & -Q & 0 & 0 & AG+BKCG-\mu G \\ \bullet & \bullet & \bullet & -\beta I & 0 & FG \\ \bullet & \bullet & \bullet & \bullet & -\beta I & HKCG \\ \bullet & \bullet & \bullet & \bullet & \bullet & -G-G^T \end{bmatrix} < 0. \quad (2.29)$$

Unfortunately, (2.29) is not convex and cannot be solved by the LMI tools.

For this reason, we introduce some transformations to simplify the KCG term of the inequality (2.29) by using (2.26) and (2.21) as follows:

$$\begin{aligned} KCG &= KU\left[C_0 \ \ 0\right]V^T V \begin{bmatrix} G_1 & 0 \\ G_2 & G_3 \end{bmatrix} V^T \\ &= \underbrace{KUC_0G_1C_0^{-1}U^{-1}}_{R}\underbrace{U\left[C_0 \ \ 0\right]V^T}_{C}, \end{aligned} \quad (2.30)$$

where $R \in \Re^{m\times p}$.

With this transformation, we obtain the optimization problem given by Theorem 2.12. □

The following lemma gives a connection of the results of Theorem 2.10 with the one of Theorem 2.12.

Lemma 2.13. *If the SOF stabilization problem is solvable by Theorem 2.10, then it is solvable by Theorem 2.12.*

Proof. If we consider the optimization problem (2.25) with $G = Q$ and $\mu = 0$, we obtain the optimization problem (2.18) with Q satisfying (2.19). Therefore, if (2.25) is feasible, then (2.19) is feasible too. □

3. Nonlinear Discrete-Time H_∞ Norm Characterization

Stability is the minimum requirement and in practice a performance level has to be guaranteed. Performance objectives can be achieved via H_∞ norm optimization. In this section, we study the H_∞ control problem for the following nonlinear discrete-time system:

$$\begin{aligned} \mathbf{x}(\mathbf{k}+1) &= \mathbf{A}\mathbf{x}(\mathbf{k}) + \mathbf{B}\mathbf{w}(\mathbf{k}) + \mathbf{f}(\mathbf{k},\mathbf{x}), \\ \mathbf{z}(\mathbf{k}) &= \mathbf{C}\mathbf{x}(\mathbf{k}) + \mathbf{D}\mathbf{w}(\mathbf{k}), \end{aligned} \quad (3.1)$$

where $w(k) \in \Re^q$ is the exogenous disturbance and $z(k) \in \Re^r$ is the controlled output. A, B, C, and D are known constant matrices of appropriate dimensions. $f(k,x)$ is a nonlinear function satisfying the following quadratic constraints:

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{w} \\ \mathbf{f} \end{bmatrix}^{\mathbf{T}} \begin{bmatrix} -\alpha^2\mathbf{M}^{\mathbf{T}}\mathbf{M} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{w} \\ \mathbf{f} \end{bmatrix} \leq 0. \tag{3.2}$$

We note T_{wz} the transfer matrix from input w to output z when $f(k,x) = 0$ is expressed by

$$\mathbf{T_{wz}} = \mathbf{C}(\mathbf{sI} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}. \tag{3.3}$$

Theorem 3.1. *System* (3.1) *is robustly stable with* $\|T_{wz}\|_\infty < \gamma$ *for a prescribed constant value* $\gamma > 0$, *if there exist a positive definite symmetric matrix* Q *and positive scalars* $\tau > 0$ *and* $\beta = 1/\sqrt{\alpha}$ *such that the following optimization problem is feasible:*

$$\begin{cases} \min \ \beta \\ \begin{bmatrix} -Q & 0 & 0 & QA^T & QM^T & QC^T \\ \bullet & -\tau\gamma^2 I & 0 & \tau B^T & 0 & \tau D^T \\ \bullet & \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} < 0. \end{cases} \tag{3.4}$$

Proof. Define the Lyapunov function:

$$V(x) = x^T(k)Px(k), \tag{3.5}$$

where $P = P^T > 0$.

By using the dissipative theory, we show that

$$V(k+1) - V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) < 0, \tag{3.6}$$

where $\gamma > 0$ is a prescribed scalar such that

$$\|T_{wz}\|_\infty < \gamma_\infty < \gamma, \tag{3.7}$$

where γ_∞ is the corresponding bounding norm bound when $f = 0$.

Evaluating (3.6) leads to

$$\begin{bmatrix} x^T \\ w^T \\ f^T \end{bmatrix} \begin{bmatrix} A^T PA - P + C^T C & A^T PB + C^T D & A^T P \\ B^T PA + D^T C & B^T PB + D^T D - \gamma^2 I & B^T P \\ PA & PB & P \end{bmatrix} \begin{bmatrix} x^T \\ w^T \\ f^T \end{bmatrix}^T < 0. \tag{3.8}$$

Now applying the S-procedure Lemma to (3.8) with (3.2), there exits $\tau > 0$ such that

$$\begin{bmatrix} A^T \widetilde{P} A - \widetilde{P} + \tau^{-1} C^T C + \alpha^2 M^T M & A^T \widetilde{P} B + \tau^{-1} C^T D & A^T \widetilde{P} \\ B^T \widetilde{P} A + \tau^{-1} D^T C & B^T \widetilde{P} B + \tau^{-1} D^T D - \tau^{-1} \gamma^2 I & B^T \widetilde{P} \\ \widetilde{P} A & \widetilde{P} B & \widetilde{P} - I \end{bmatrix} < 0, \tag{3.9}$$

where $\widetilde{P} = P/\tau$.

By Schur complement, (3.9) is equivalent to

$$\begin{bmatrix} -P & 0 & 0 & A^T & M^T & C^T \\ \bullet & -\tau^{-1}\gamma^2 I & 0 & B^T & 0 & D^T \\ \bullet & \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & \bullet & -P^{-1} & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} < 0, \tag{3.10}$$

where $\beta = \alpha^{-2}$.

Multiplying by $\mathrm{diag}(Q, \tau, I, I, I)$ with $Q = \widetilde{P}^{-1}$ the both sides of (3.10), we obtain the optimization problem (3.4). □

Now, we introduce the following theorem which can be seen as an alternate characterization of upper bounds of the H_∞ norm of.

Theorem 3.2. *System* (3.1) *is robustly stable with* $\|T_{wz}\|_\infty < \gamma$ *for a prescribed constant value* $\gamma > 0$, *if there exist a positive definite symmetric matrix* Q, *a matrix* G *of appropriate dimensions, and positive* *scalars $\tau > 0$ and $\beta = 1/\sqrt{\alpha}$ such that for any prescribed scalar μ in $]-1\ 1[$, the following optimization problem is feasible:*

$$\begin{cases} \min\ \beta \\ \begin{bmatrix} -Q & 0 & 0 & \mu Q & 0 & 0 & Q \\ \bullet & -\tau\gamma^2 I & 0 & \tau B^T & 0 & \tau D^T & 0 \\ \bullet & \bullet & -I & I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 & (A-\mu I)G \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & MG \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I & CG \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -G-G^T \end{bmatrix} < 0. \end{cases} \tag{3.11}$$

Proof. Inequality (3.4) can be written as follows:

$$\begin{aligned} &\begin{bmatrix} -Q & 0 & 0 & QA^T & QM^T & QC^T \\ \bullet & -\tau\gamma^2 I & 0 & \tau B^T & 0 & \tau D^T \\ \bullet & \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} = \begin{bmatrix} -Q & 0 & 0 & \mu Q & 0 & 0 \\ \bullet & -\tau\gamma^2 I & 0 & \tau B^T & 0 & \tau D^T \\ \bullet & \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} \\ &\qquad + \operatorname{sym}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ A-\mu I \\ M \\ C \end{bmatrix} \begin{bmatrix} Q & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right) < 0. \end{aligned} \tag{3.12}$$

By lemma 2, denoting

$$\phi = \begin{bmatrix} -Q & 0 & 0 & \mu Q & 0 & 0 \\ \bullet & -\tau\gamma^2 I & 0 & \tau B^T & 0 & \tau D^T \\ \bullet & \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} < 0, \quad \text{for any } \mu \in]-1\ \ 1[$$

$$N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ A - \mu I \\ M \\ C \end{bmatrix}, \qquad J = \begin{bmatrix} Q & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.13}$$

there exists a matrix G of appropriate dimensions such that (3.11) holds where γ, β, and τ are scalars previously defined. □

3.1. Static Output Feedback H_∞ Synthesis

In this section, we consider the static output feedback stabilization problem for the following nonlinear discrete system (3.14):

$$\begin{aligned} x(k+1) &= Ax(k) + B_u u(k) + B_w w(k) + f(k, x, u), \\ z(k) &= C_z x(k) + D_{zu} u(k) + D_{zw} w(k), \\ y(k) &= C_y x(k), \end{aligned} \tag{3.14}$$

where $u(k) \in \Re^m$ is the control input, $w(k) \in \Re^q$ is the exogenous disturbance, $z(k) \in \Re^r$ is the controlled output, and $y(k) \in \Re^p$ is the measured output. Also, $B_u, B_w, C_z, D_{zu}, D_{zw}$ and C_y are known constant matrices of appropriate dimensions.

The system closed by SOF is written as

$$\begin{aligned} x(k+1) &= A_{cl} x(k) + B_{cl} w(k) + f(k, x, u), \\ z(k) &= C_{cl} x(k) + D_{cl} w(k), \end{aligned} \tag{3.15}$$

where the following hold:

$$\begin{aligned} A_{cl} &= A + B_u K C_y, \\ B_{cl} &= B_w, \\ C_{cl} &= C_z + D_{zu} K C_y, \\ D_{cl} &= D_{zw}. \end{aligned} \tag{3.16}$$

The objective of this section is to design static output feedback H_∞ controllers for nonlinear discrete time system (3.14).

Theorem 3.3. *System* (3.14) *is robustly static output feedback stabilizable with* $\|T_{wz}\|_\infty < \gamma$ *for a prescribed constant value* $\gamma > 0$ *if there exist a positive definite symmetric matrix* Q*, a matrix* R *of* *appropriate dimensions, and positive scalars* $\tau > 0$ *and* $\beta = 1/\sqrt{\alpha}$ *such that the following optimization problem is feasible:*

$$\begin{cases} \min\ \beta \\ \begin{bmatrix} -Q & 0 & 0 & QA^T + (B_uRC_y)^T & QF^T & (HRC_y)^T & QC_z^T + (D_{zu}RC_y)^T \\ \bullet & -\tau\gamma^2 I & 0 & \tau B_w^T & 0 & 0 & \tau D_{zw}^T \\ \bullet & \bullet & -I & I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} < 0, \end{cases} \quad (3.17)$$

where

$$Q = V \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} V^T. \quad (3.18)$$

The static output feedback gain is then given by

$$K = RUC_0Q_1^{-1}C_0^{-1}U^T, \quad (3.19)$$

where U, V*, and* C_0 *are given in* (2.21).

Proof. According to Theorem 3.1, system (3.14) is robustly stable with $\|T_{wz}\|_\infty < \gamma$ for a prescribed constant value $\gamma > 0$ if there exist a positive definite symmetric matrix Q and positive scalars $\tau > 0$ and $\beta = 1/\sqrt{\alpha}$ such that the following optimization problem is feasible:

$$\begin{cases} \min\ \beta \\ \begin{bmatrix} -Q & 0 & 0 & QA_{cl}^T & QM^T & QC_{cl}^T \\ \bullet & -\tau\gamma^2 I & 0 & \tau B_{cl}^T & 0 & \tau D_{cl}^T \\ \bullet & \bullet & -I & I & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I \end{bmatrix} < 0. \end{cases} \quad (3.20)$$

Now:

(i) A_{cl}, B_{cl}, C_{cl} and D_{cl} are replaced by (3.16),

(ii) $M = \binom{F}{HKC_y}$,

(iii) Q is replaced by (3.18) where : $Q_1 \in \Re^{p\times p}$, $Q_2 \in \Re^{(n-p)\times(n-p)}$.

After same direct developments the result follows. □

3.2. *An Improved Approach of Static Output Feedback Synthesis for* H_∞ *Robust Control*

In this paragraph, an H_∞ robust control for nonlinear systems (3.14) improving the previous approach is proposed.

Theorem 3.4. *System* (3.14) *is robustly SOF stabilisable with* $\|T_{wz}\|_\infty < \gamma$ *for a prescribed constant value* $\gamma > 0$ *if there exist a positive definite symmetric matrix* Q*, matrices* G *and* R *of appropriate dimensions, and positive scalars* $\tau > 0$*, and* $\beta = 1/\sqrt{\alpha}$ *such that the following optimization problem is feasible for any prescribed scalar* μ *in* $]-1\ 1[$:

$$\begin{bmatrix} -Q & 0 & 0 & \mu Q & 0 & 0 & 0 & Q \\ \bullet & -\tau\gamma^2 I & 0 & \tau B_w^T & 0 & 0 & \tau D_{zw}^T & 0 \\ \bullet & \bullet & -I & I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 & 0 & AG - \mu G + B_u RC_y \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & 0 & FG \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & HRC_y \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I & C_z G + D_{zu} RC_y \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -G - G^T \end{bmatrix} < 0, \tag{3.21}$$

where

$$G = V \begin{bmatrix} G_1 & 0 \\ G_2 & G_3 \end{bmatrix} V^T. \tag{3.22}$$

The static output feedback gain is given by

$$K = RUC_0 G_1^{-1} C_0^{-1} U^T, \tag{3.23}$$

with $R \in \Re^{m\times p}$*,* U, V*, and* C_0 *are given in* (2.21).

Proof. According to the Theorem 3.3, the system (3.14) is robustly stable with $\|T_{wz}\|_\infty < \gamma$ for a prescribed degree $\gamma > 0$ if there exist a positive definite symmetric matrix Q, a matrix G of appropriate dimensions, and positive scalars $\tau > 0$ and $\beta = 1/\sqrt{\alpha}$, for any prescribed scalar μ in $]-1\ 1[$, such that the following optimization problem is feasible:

$$\begin{cases} \min\ \beta \\ \begin{bmatrix} -Q & 0 & 0 & \mu Q & 0 & 0 & Q \\ \bullet & -\tau\gamma^2 I & 0 & \tau B_{cl}^T & 0 & \tau D_{cl}^T & 0 \\ \bullet & \bullet & -I & I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 & (A_{cl} - \mu I)G \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & MG \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I & C_{cl}G \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -G - G^T \end{bmatrix} < 0, \end{cases} \tag{3.24}$$

where

(i) A_{cl}, B_{cl}, C_{cl}, and D_{cl} are replaced by (3.16),

(ii) $M = \binom{F}{HKC_y}$,

(iii) G replaced by (3.22), where $G_1 \in \mathfrak{R}^{p\times p}$, $G_2 \in \mathfrak{R}^{(n-p)\times p}$.

The optimization problem (3.24) is expressed as follows:

$$\begin{cases} \min\ \beta < 0 \\ \begin{bmatrix} -Q & 0 & 0 & \mu Q & 0 & 0 & 0 & Q \\ \bullet & -\tau\gamma^2 I & 0 & \tau B_w^T & 0 & 0 & \tau D_{zw}^T & 0 \\ \bullet & \bullet & -I & I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -Q & 0 & 0 & 0 & AG + B_u KC_y G - \mu G \\ \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & 0 & FG \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\beta I & 0 & HKC_y G \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\tau I & C_z G + D_{zu} KC_y \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -G - G^T \end{bmatrix} < 0. \end{cases} \tag{3.25}$$

For this reason, we express differently the KC_yG term of the inequality (3.25), by using (3.22) and (2.21), in the same way as in (2.30). Consequently, we obtain

$$KC_y G = RC_y. \tag{3.26}$$

With this transformation, we obtain the optimization problem given in Theorem 3.4. □

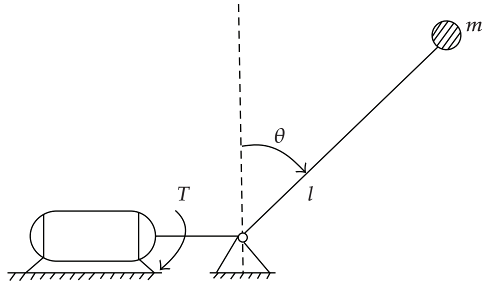

Figure 1: Inverted pendulum scheme.

4. Numerical Examples

We present in this section two numerical examples to illustrate the proposed theory for SOF synthesis.

Our approach Theorem 2.12 is compared to the methods presented in [11].

Example 4.1 (see [5]). We consider the nonlinear discrete-time system of inverted pendulum (see Figure 1).

The system can be described by

$$\begin{aligned} x(k+1) &= Ax(k) + B\big(u(k) + \alpha \sin y(t)\big), \\ y(k) &= Cx(k), \end{aligned} \tag{4.1}$$

where $x(k) = [x_1(k)\ x_2(k)]^T$, $x_1(k)$ and $x_2(k)$ being, respectively, the angular displacement and velocity; $u(k)$ represents the field current of the DC motor; the matrices $A \in \mathfrak{R}^{2\times 2}, B \in \mathfrak{R}^{2\times 1}$, and $C \in \mathfrak{R}^{1\times 2}$are given by:

$$A = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix}; \quad B = \begin{bmatrix} 0.048 \\ 0.952 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{4.2}$$

The nonlinear function f is given by the following expression:

$$f(k,x) = \alpha B \sin y\,(k) = \alpha\, B\ \sin\ x_1(k). \tag{4.3}$$

This function satisfies

$$\begin{aligned} f^T(k,x) f(k,x) &= \alpha^2 B^T B \sin^2 x_1(k) \\ &= 0.9\alpha^2 \sin^2 x_1(k) \\ &\le \alpha^2 x^T \begin{bmatrix} 0.95 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.95 & 0 \\ 0 & 0 \end{bmatrix} x. \end{aligned} \tag{4.4}$$

In this case, we take $F = [\begin{smallmatrix} 0.95 & 0 \\ 0 & 0 \end{smallmatrix}];\ H = 0$.

Table 1: Numerical Evaluation for Example 4.1.

Approach	$\alpha_{\max}$	K
Theorem 2.10 condition	$0.8254 \cdot 10^{-4}$	−1.9987
Theorem 2.12 condition for $\mu = 0.7$	0.06688	−0.3101

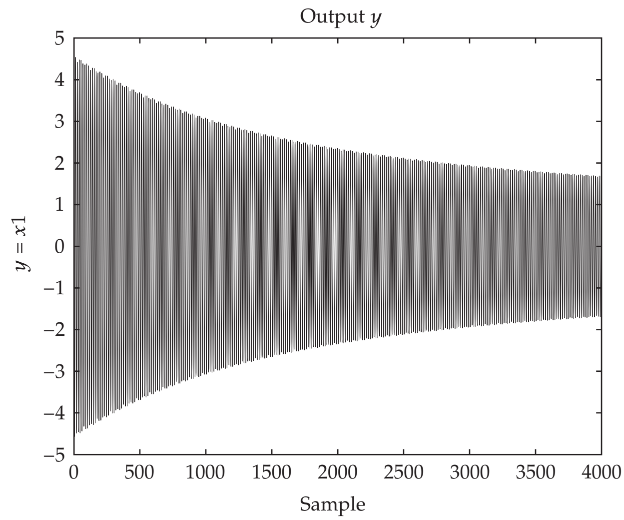

Figure 2: Output $y = x_1$.

The nonlinear system is unstable. The matrix A is unstable which means that the linear part of systems is unstable. Therefore, we apply the approach given by Theorems 2.10 and 2.12 to stabilize system by SOF. We summarize in Table 1 the obtained results.

Figures 2 and 3 show the output trajectory obtained respectively by applying Theorems 2.10 and 2.12 to system (3.26) with $\alpha = \alpha_{\max}$.

We can see a drastic improvement obtained by Theorem 2.12.

Example 4.2. We consider now the nonlinear discrete-time system:

$$x(k+1) = \begin{bmatrix} -0.0725 & 0.1957 & 1.5931 \\ 0.0301 & 0.0404 & 0.6084 \\ 0.3764 & -0.1635 & 0.9024 \end{bmatrix} x(k) + \begin{bmatrix} 1.2123 & 0.2895 \\ 0.6174 & 0.0651 \\ 0.9379 & 0.5110 \end{bmatrix} u(k) + f(k, x(k), u(k)),$$
$$y(k) = \begin{bmatrix} 0.9313 & -0.7534 & -0.0335 \\ 1 & 2 & 0.5 \end{bmatrix} x(k), \tag{4.5}$$

with the nonlinear function f satisfying the constraint (2.17), where $F = I_3$ and $H = I_2$. The results obtained byapproaches of Theorem 2.12 and in [11] are given in Table 2.

The new approach (Theorem 2.12) improves the results obtained by [11].

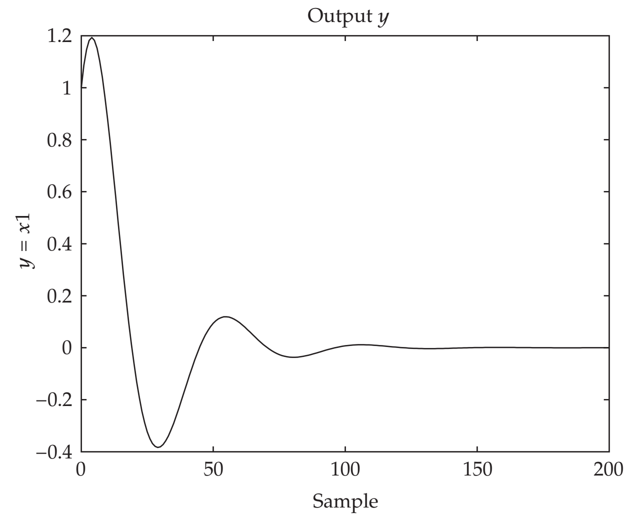

Figure 3: Output $y = x_1$.

Table 2: Numerical Evaluation for Example 4.2.

Approach	$\alpha_{\max}$	K
[11]	0.1954	$\begin{bmatrix} -0.1329 & -0.1268 \\ -0.4309 & -0.0405 \end{bmatrix}$
New approach, (2.25), Theorem 2.12 for $\mu = 0$	0.3084	$\begin{bmatrix} -0.0019 & -0.2766 \\ -1.1542 & 0.1067 \end{bmatrix}$
New approach, (2.25), Theorem 2.12 for $\mu = -0.35$	0.3329	$\begin{bmatrix} 0.0357 & -0.3141 \\ -0.9812 & 0.0924 \end{bmatrix}$

Example 4.3. We consider the nonlinear discrete-time system (3.14) with

$$A = \begin{bmatrix} 0.8189 & 0.0863 & 0.0900 & 0.0813 \\ 0.2524 & 1.0033 & 0.0313 & 0.2004 \\ -0.0545 & 0.0102 & 0.7901 & -0.2580 \\ -0.1918 & -0.1034 & 0.1602 & 0.8604 \end{bmatrix}, \quad B_u = \begin{bmatrix} 0.0045 & 0.0044 \\ 0.1001 & 0.0100 \\ 0.0003 & -0.0136 \\ -0.0051 & 0.0926 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.0953 & 0 & 0 \\ 0.0145 & 0 & 0 \\ 0.0862 & 0 & 0 \\ -0.0011 & 0 & 0 \end{bmatrix},$$

$$C_z = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{zu} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_{yw} = 0, \tag{4.6}$$

with the nonlinear function f satisfying the constraint (3.2), where $F = I_4$ and $H = I_4$.

In Table 3, we present numerical result for H_∞ performance via SOF by applying Theorem 3.3 and for different values of γ which satisfy (3.7).

In Table 4, we present numerical result for H_∞ performance via SOF by applying Theorem 3.4 and for different values of γ which satisfy (3.7).

Table 3: Numerical Evaluation for Example4.3 by Theorem 3.3.

Approach	$\alpha_{\max}$	K
Approach, (3.17), Theorem 3.3 for $\gamma_\infty = 1.9918$, $\gamma = 2.2361$	0.0032	$\begin{bmatrix} -2.0416 & -0.1796 \\ 0.6545 & -1.0362 \end{bmatrix}$
Approach, (3.17), Theorem 3.3 for $\gamma_\infty = 1.9918$, $\gamma = 5$	0.0288	$\begin{bmatrix} -2.1323 & -0.0808 \\ 1.1624 & -1.2381 \end{bmatrix}$

Table 4: Numerical Evaluation for Example 4.3 by Theorem 3.4.

Approach	$\alpha_{\max}$	K
Approach, (3.21), Theorem 3.4 for $\gamma_\infty = 1.9918$, $\gamma = 2.2361$, $\mu = 0.22$	0.3643	$\begin{bmatrix} -0.2993 & -0.1328 \\ -0.0685 & -0.0743 \end{bmatrix}$
Approach, (3.21), Theorem 3.4 for $\gamma_\infty = 1.9918$, $\gamma = 5, \mu = 0.2400$	0.4087	$\begin{bmatrix} -0.3376 & -0.1185 \\ -0.0660 & -0.0737 \end{bmatrix}$

5. Conclusion

In this paper, the stabilization problem by static output feedback (SOF) for a particular class of nonlinear discrete time systems is investigated. A new sufficient condition is elaborated by using Lyapunov theory and formulated by LMI constraints. We obtain a convex optimization problem for maximizing the bound of the nonlinearity preserving the stability of the systems.

Finally, the proposed controller design method was extended to incorporate H_∞ synthesis. An optimization problem, which is linear both in the admissible nonlinearity bound and the disturbance attenuation, is developed. Numerical comparisons with existing methods in literature illustrate the improvement obtained by our approaches. All of them can also be extended to the dynamic output feedback.

Other classes of nonlinear discrete time nonlinear systems exist and present some interesting characteristic from a practical point of view. For example, the ones where the linear part is affected by polytopic uncertainties. It would be interesting to extend the results of this paper to those classes. This will be exploited in a near future.

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