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Research Article

Nonlinear Dynamic Response of Functionally Graded Rectangular Plates under Different Internal Resonances

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The nonlinear dynamic response of functionally graded rectangular plates under combined transverse and in-plane excitations is investigated under the conditions of 1 : 1, 1 : 2 and 1 : 3 internal resonance. The material properties are assumed to be temperature-dependent and vary along the thickness direction. The thermal effect due to one-dimensional temperature gradient is included in the analysis. The governing equations of motion for FGM rectangular plates are derived by using Reddy's third-order plate theory and Hamilton's principle. Galerkin's approach is utilized to reduce the governing differential equations to a two-degree-of-freedom nonlinear system including quadratic and cubic nonlinear terms, which are then solved numerically by using 4th-order Runge-Kutta algorithm. The effects of in-plane excitations on the internal resonance relationship and nonlinear dynamic response of FGM plates are studied.

1. Introduction

Functionally graded materials (FGMs) are new engineering materials. Due to their advantages of being able to withstand severe high-temperature gradient while maintaining structural integrity, FGMs are considered to be advanced composite materials in high temperature and vibration environments [1, 2].

With the increasing use of FGMs, it is important to understand the nonlinear vibration behavior of FGM structures. Quite a few studies in this area have been conducted. Praveen and Reddy [3] analyzed the nonlinear static and dynamic response of functionally graded ceramic-metal plates in a steady temperature field based on the first-order shear deformation plate theory.Sundararajan et al. [4] carried out finite element analysis of nonlinear-free vibration of both rectangular and skew FGM plates. Yang et al. [5] investigated the large amplitude vibration of pre-stressed FGM plates composed of a functionally graded layer and two surface-mounted piezoelectric actuator layers.

A semi analytical method and Galerkin technique were employed to predict the nonlinear vibration behavior of FGM-laminated plates. The parametric resonance of functionally graded rectangular plates under harmonic in-plane loading was investigated by Ng et al. [6]. Using a higher-order shear and normal deformable plate theory (HOSNDPT) and a meshless local Petrov-Galerkin (MLPG) method, Qian et al. [7] analyzed the static deformation, and free and forced vibrations of a thick rectangular functionally graded plate.Vel and Batra [8] gave a three-dimensional exact solution for the linear free and forced vibration of simply supported FGM rectangular plates. Woo and Meguid [9] studied the nonlinear deflection of FGM plates and shells under transverse mechanical loads and a temperature field. Hao et al. [10] reported a nonlinear dynamic analysis of a simply supported FGM rectangular plate subjected to transversal and in-plane excitations. The resonant case considered in their work is 1 : 1 internal resonance and principal parametric resonance. The asymptotic perturbation method is used to obtain four-dimensional nonlinear averaged equation. It was found that periodic, and quasiperiodic solutions and chaotic motions occur under some conditions. It is known that for a two-degree-of-freedom nonlinear vibration system, different internal resonance between two modes, such as 1 : 1, 1 : 2, and 1 : 3 internal resonances, can exist in some cases. To the best of the authors' knowledge, there is still no literature concerning nonlinear dynamic behavior of FGM plates with different cases of internal resonances.

The present work aims to investigate the nonlinear dynamic response of a simply supported FGM rectangular plate subjected to transversal and in-plane excitations in a thermal environment. The cases considered in this paper include 1 : 1, 1 : 2, and 1 : 3 internal resonances and principal parametric resonance-1/2 subharmonic resonance. It is assumed that the material properties of the plate are graded in the thickness direction according to a power-law distribution. The analysis is based on the nonlinear dynamic governing equations derived in our previous work [10]. The influences of the in-plane excitations on the internal resonance relationship and nonlinear dynamic response of the FGM plate are studied in numerical examples.

2. Theoretical Formulation

2.1. Material Properties

It is assumed that the bottom surface of the plate is metal rich, whereas the top surface is ceramic rich. The material properties P, such as Young's modulus E, the coefficient of thermal expansion α, thermal conductivity κ, and mass density ρ, can be expressed as a function of temperature as [11]

$$P_i = P_0\left(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right), \tag{2.1}$$

where P_0, P_{-1}, P_1, P_2, and P_3 are temperature coefficients.

The effective material properties P of the FGM plate can be expressed as

$$P = P_t V_c + P_b V_m, \tag{2.2}$$

where subscripts "t" and "b" represent the top and bottom surfaces of the FGMs plate, respectively, and V_c and V_m are the volume fraction of ceramic and metal which add to unity

$$V_c + V_m = 1. \tag{2.3}$$

The metal volume fraction V_m is defined as

$$V_m(z) = \left(\frac{2z + h}{2h}\right)^N, \tag{2.4}$$

where exponent N is a real number that characterizes the material profile along plate thickness.

From (2.2)–(2.4), the effective values of E, α, ρ, and κ at an arbitrary point of the plate can be expressed as

$$\begin{aligned} E &= (E_b - E_t)\ V_m + E_t, \\ \alpha &= (\alpha_b - \alpha_t)\ V_m + \alpha_t, \\ \rho &= (\rho_b - \rho_t)\ V_m + \rho_t, \\ \kappa &= (\kappa_b - \kappa_t)\ V_m + \kappa_t. \end{aligned} \tag{2.5}$$

It is also assumed that the plate is initially stress free at T_0 and is subjected to a uniform temperature variation $\Delta T = T - T_0$ that is constant in the xy plane of the plate while varies in the thickness direction only. In this case, the temperature distribution along plate thickness can be obtained from a steady-state heat transfer equation:

$$-\frac{d}{dz}\left[\kappa(z)\frac{dT}{dz}\right] = 0. \tag{2.6}$$

This equation is solved by imposing boundary condition of $T = T_b$ at $z = h/2$ and $T = T_t$ at $z = -h/2$. As a special case, the solution of (2.6) for isotropic homogeneous material, may be expressed as

$$T(z) = \frac{T_t + T_b}{2} + \frac{T_b - T_t}{h} z. \tag{2.7}$$

2.2. Equations of Motion

A simply supported four-edges FGMs rectangular plate of length a, width b and thickness h, which is subjected to the in-plane and transversal excitations is considered, as shown in

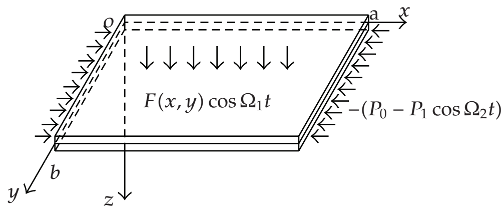


Figure 1: The model of a FGMs rectangular plate and the coordinate system.

Figure 1. The in-plane excitation of the FGMs plate is distributed along the y direction at $x = 0$ and $x = a$ and is of the form $p_0 - p_1 \cos\Omega_2 t$. The transversal excitation subject to the FGMs plate is represented by $F(x, y)\cos\Omega_1 t$. Here the Ω_1 and Ω_2 are the frequencies of the transversal excitation and the in-plane excitation, respectively.

As usual, the coordinate $Oxyz$ has its origin at the corner of the plate on the middle plane. Assume that (u, v, w) and (u_0, v_0, w_0) represent the displacements of an arbitrary point and a point in the middle surface of the FGMs rectangular plate in the x, y and z directions, respectively. It is also assumed that ϕ_x and ϕ_y, respectively, represent the mid-plane rotations of two transverse normals about the x and y axes. With Reddy's third-order shear deformation plate theory [12], the displacements of the FGM plate can be expressed as follows:

$$\begin{aligned}
u(x,y,t) &= u_0(x,y,t) + z\phi_x(x,y,t) - z^3\frac{4}{3h^2}\left(\phi_x + \frac{\partial w_0}{\partial x}\right),\\
v(x,y,t) &= v_0(x,y,t) + z\phi_y(x,y,t) - z^3\frac{4}{3h^2}\left(\phi_y + \frac{\partial w_0}{\partial y}\right),\\
w(x,y,t) &= w_0(x,y,t)
\end{aligned} \tag{2.8}$$

Based on the nonlinear strains-displacement relation and the above displacement field, we obtain

$$\begin{gathered}
\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\\
\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right),\\
\gamma_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \quad \gamma_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),
\end{gathered} \tag{2.9}$$

$$\begin{gathered}
\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z\begin{Bmatrix}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{xy}^{(1)}\end{Bmatrix} + z^3\begin{Bmatrix}\varepsilon_{xx}^{(3)}\\ \varepsilon_{yy}^{(3)}\\ \gamma_{xy}^{(3)}\end{Bmatrix},\\
\begin{Bmatrix}\gamma_{yz}\\ \gamma_{zx}\end{Bmatrix} = \begin{Bmatrix}\gamma_{yz}^{(0)}\\ \gamma_{zx}^{(0)}\end{Bmatrix} + z^2\begin{Bmatrix}\gamma_{yz}^{(2)}\\ \gamma_{zx}^{(2)}\end{Bmatrix},
\end{gathered} \tag{2.10}$$

where

$$
\begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{zx}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \phi_y + \dfrac{\partial w_0}{\partial y} \\ \phi_x + \dfrac{\partial w_0}{\partial x} \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{zx}^{(2)} \end{Bmatrix} = -c_2 \begin{Bmatrix} \phi_y + \dfrac{\partial w_0}{\partial y} \\ \phi_x + \dfrac{\partial w_0}{\partial x} \end{Bmatrix},
$$

$$
\begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{Bmatrix}, \qquad \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = -c_1 \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix}, \tag{2.11}
$$

$$
\begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} = -c_1 \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} + \dfrac{\partial^2 w_0}{\partial x^2} \\ \dfrac{\partial \phi_y}{\partial y} + \dfrac{\partial^2 w_0}{\partial y^2} \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} + 2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},
$$

$$
c_2 = 3c_1, \qquad c_1 = \frac{4}{3}h^2.
$$

Taking into account the thermal effects, the linear stress-strain constitutive relationship is

$$
\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix} = \begin{Bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{Bmatrix} \left\{ \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 0 \\ 0 \\ 2\alpha_{xy} \end{Bmatrix} \Delta T \right\}, \tag{2.12}
$$

where Q are elastic stiffness elements [12].

According to Reddy's third-order shear deformation theory and Hamilton's principle, the nonlinear governing equations of motion for the FGM rectangular plate are given as [10]

$$
N_{xx,\,x} + N_{xy,\,y} = I_0 \ddot{u}_0 + (I_1 - c_1 I_3)\ddot{\phi}_x - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial x},
$$

$$
N_{yy,\,y} + N_{xy,x} = I_0 \ddot{v}_0 + (I_1 - c_1 I_3)\ddot{\phi}_y - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial y},
$$

$$
\begin{aligned}
&N_{yy,\,y}\frac{\partial w_0}{\partial y}+N_{yy}\frac{\partial^2 w_0}{\partial y^2}+N_{xy,\,x}\frac{\partial w_0}{\partial y}+N_{xy,\,y}\frac{\partial w_0}{\partial x}+2N_{xy}\frac{\partial^2 w_0}{\partial y\partial x}\\
&\qquad+N_{xx,\,x}\frac{\partial w_0}{\partial x}+N_{xx}\frac{\partial^2 w_0}{\partial x^2}+c_1\left(P_{xx,\,xx}+2P_{xy,xy}+P_{yy,\,yy}\right)\\
&\qquad+\left(Q_{x,\,x}-c_2R_{x,\,x}\right)+\left(Q_{y,\,y}-c_2R_{y,\,y}\right)+F-\gamma\dot{w}_0\\
&\quad=I_0\ddot{w}_0+c_1I_3\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial x}\right)+c_1\left(I_4-c_1I_6\right)\left(\frac{\partial\ddot{\phi}_x}{\partial x}+\frac{\partial\ddot{\phi}_y}{\partial y}\right),\\
&M_{xx,\,x}+M_{xy,\,y}-c_1P_{xx,\,x}-c_1P_{xy,\,y}-\left(Q_x-c_2R_x\right)\\
&\quad=\left(I_1-c_1I_3\right)\ddot{u}_0+\left(I_2-2c_1I_4+c_1^2I_6\right)\ddot{\phi}_x-c_1\left(I_4-c_1I_6\right)\frac{\partial\ddot{w}_0}{\partial x},\\
&M_{yy,\,y}+M_{xy,\,x}-c_1P_{yy,\,y}-c_1P_{xy,\,x}-\left(Q_y-c_2R_y\right)\\
&\quad=\left(I_1-c_1I_3\right)\ddot{v}_0+\left(I_2-2c_1I_4+c_1^2I_6\right)\ddot{\phi}_y-c_1\left(I_4-c_1I_6\right)\frac{\partial\ddot{w}_0}{\partial y},
\end{aligned}
\tag{2.13}
$$

where γ is the damping coefficient, a comma denotes the partial differentiation with respect to a specified coordinate, and a super dot implies the partial differentiation with respect to time.

All kinds of inertias in (2.13) are calculated by

$$
I_i=\int_{-h/2}^{h/2}z^i p(z)dz,\quad (i=0,\ 1,\ 2,\ 3,\ 4,\ 6).
\tag{2.14}
$$

the stress resultants are represented as follows

$$
\begin{gathered}
\begin{Bmatrix}\mathbf{N}\\ \mathbf{M}\\ \mathbf{P}\end{Bmatrix}=\begin{Bmatrix}\mathbf{A}&\mathbf{B}&\mathbf{E}\\ \mathbf{B}&\mathbf{D}&\mathbf{F}\\ \mathbf{E}&\mathbf{F}&\mathbf{H}\end{Bmatrix}\begin{Bmatrix}\boldsymbol{\varepsilon}^{(0)}\\ \boldsymbol{\varepsilon}^{(1)}\\ \boldsymbol{\varepsilon}^{(3)}\end{Bmatrix}+\begin{Bmatrix}\mathbf{N}^T\\ \mathbf{M}^T\\ \mathbf{P}^T\end{Bmatrix},\\
\begin{Bmatrix}\mathbf{Q}\\ \mathbf{R}\end{Bmatrix}=\begin{Bmatrix}\mathbf{A}&\mathbf{D}\\ \mathbf{D}&\mathbf{F}\end{Bmatrix}\begin{Bmatrix}\gamma^{(0)}\\ \gamma^{(2)}\end{Bmatrix},
\end{gathered}
\tag{2.15}
$$

where the membrane stress resultants, moments, higher-order moments, transverse shear stress resultants, and their higher-order counterparts are represented as follows:

$$
\begin{gathered}
\mathbf{N}=\left[N_{xx},N_{yy},N_{xy}\right]^{\mathbf{T}},\qquad \mathbf{M}=\left[M_{xx},M_{yy},M_{xy}\right]^{\mathbf{T}},\\
\mathbf{P}=\left[P_{xx},P_{yy},P_{xy}\right]^{\mathbf{T}},\quad \mathbf{Q}=\left[Q_{yy},Q_{xx}\right]^{\mathbf{T}},\quad \mathbf{R}=\left[R_{yy},R_{xx}\right]^{\mathbf{T}}.
\end{gathered}
\tag{2.16}
$$

The stiffness elements of the FGMs plate are denoted by

$$\begin{gathered}
\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1,\ z,\ z^2,\ z^3,\ z^4,\ z^6\right) dz, \quad (i, j = 1,\ 2,\ 6), \\
\left(A_{ij}, D_{ij}, F_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}\left(1,\ z^2,\ z^4\right) dz, \quad (i, j = 4,\ 5).
\end{gathered} \tag{2.17}$$

And the thermal stress resultants in (2.16) can be represented as

$$\left\{\mathbf{N}^T, \mathbf{M}^T, \mathbf{P}^T\right\} = \begin{Bmatrix} N_{xx}^T & M_{xx}^T & P_{xx}^T \\ N_{yy}^T & M_{yy}^T & P_{yy}^T \\ N_{xy}^T & M_{xy}^T & P_{xy}^T \end{Bmatrix} = \int_{-h/2}^{h/2} \left[A_{xx}, A_{yy}, A_{xy}\right]^{\mathbf{T}} \left(1,\ z^2,\ z^3\right) \Delta T\, dz, \tag{2.18}$$

where

$$\begin{Bmatrix} A_{xx} \\ A_{yy} \\ A_{xy} \end{Bmatrix} = -\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha \end{bmatrix}. \tag{2.19}$$

The nonlinear governing equations of motion for the FGM rectangular plate can be expressed in ters of displacements $(u_0,\ v_0,\ w_0, \phi_x, \phi_y)$ by substituting for the force and moments resultants. The equations of motion are very complicate nonlinear partial differential equations that can be seen in the conference [10].

The boundary conditions for the simply supported FGM plate requires that

at $x = 0$ and $x = a$,

$$w = \phi_y = M_{xx} = P_{xx} = N_{xy} = 0, \tag{2.20}$$

at $y = 0$ and $y = b$,

$$\begin{gathered}
w = \phi_x = M_{yy} = P_{yy} = N_{xy} = 0, \qquad N_{yy}\big|_{y=0,b} = 0, \\
\int_0^b N_{xx}|_{x=0,a}\, dy = -\int_0^b \left(p_0 - p_1 \cos \Omega_2 t\right) dy.
\end{gathered} \tag{2.21}$$

The present study focuses on the nonlinear transverse oscillations of FGM plates in the first two modes. It is then reasonable to construct deflection functions as a combination of the first two vibration mode shapes as follows:

$$w(x, y, t) = w_1(t) \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b} + w_2(t) \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b}, \tag{2.22}$$

where w_1 and w_2 are the amplitudes of two modes, respectively.

The transverse excitation can be represented as

$$F(x,y,t) = F_1(t)\sin\frac{\pi x}{a}\sin\frac{3\pi y}{b} + F_2(t)\sin\frac{3\pi x}{a}\sin\frac{\pi y}{b}, \tag{2.23}$$

where F_1 and F_2 represent the amplitudes of the transverse forcing excitation corresponding to the two nonlinear modes.

Based on research given in [13, 14], neglecting all inertia terms on u, v, ϕ_x, and ϕ_y in (2.13), we can obtain the displacements u, v, ϕ_x, and ϕ_y with respect to w. Then by the Galerkin procedure, the governing differential equations of transverse motion of the FGMs rectangular plate are obtained

$$\begin{aligned}
&\ddot{w}_1 + \omega_1^2 w_1 + a_1\dot{w}_1 + a_2 w_1\cos\Omega_2 t + a_3 w_1^2 + a_4 w_2^2 + a_5 w_1 w_2^2 \\
&\quad + a_6 w_1^3 + a_7 w_1 w_2 = f_1\cos\Omega_1 t, \\
&\ddot{w}_2 + \omega_2^2 w_2 + b_1\dot{w}_2 + b_2 w_2\cos\Omega_2 t + b_3 w_1 w_2 + b_4 w_1^2 + b_5 w_2^2 \\
&\quad + b_6 w_2 w_1^2 + b_7 w_2^3 = f_2\cos\Omega_1 t,
\end{aligned} \tag{2.24}$$

where w_1 and w_2 are the vibration amplitudes of the first two modes, respectively. f_1 and f_2 are the amplitudes of the transverse excitation force corresponding to the two nonlinear modes. The lengthy expressions of constants a_1-a_7, b_1-b_7 and the transverse excitation force f_1 and f_2 are not given here for brevity.

The present study focuses on the transverse nonlinear oscillations of a simply supported FGM rectangular plate in the first two modes.

The first two linear frequencies of this nonlinear dynamic system can be rewritten as

$$\begin{aligned}
\omega_1^2 &= -\frac{m_{007} + p_0 m_{008}}{m_{001}}, \\
\omega_2^2 &= -\frac{n_{007} + P_0 n_{008}}{n_{002}},
\end{aligned} \tag{2.25}$$

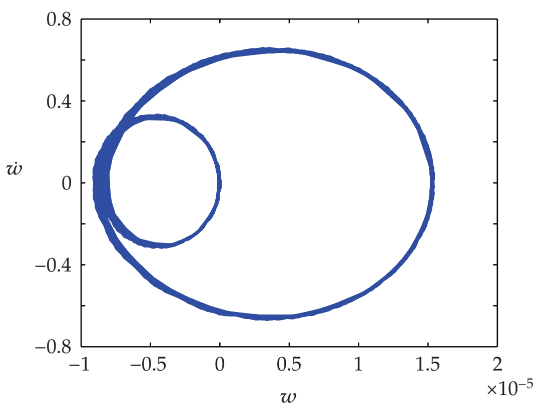
where p_0 is the static component in the in-plane excitation. The other coefficients in (2.13) are functions of geometric and physical parameters, in-plane excitations, and temperature field. That means that under different conditions, the system can have different internal resonance and exhibit different dynamic response.

It is seen that the in-plane stationary excitation p_0 can change the type of internal resonance.

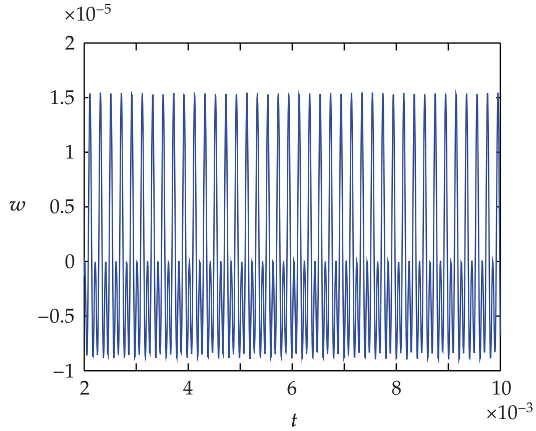
When ω_1 is close to ω_2, the one-to-one internal resonance occurs and p_0 is as follows:

$$p_{01} = \frac{m_{007}n_{002} - m_{001}n_{007}}{m_{001}n_{008} - m_{008}n_{002}}. \tag{2.26}$$

When $\omega_2 \approx 2\omega_1$ or $\omega_2 \approx 3\omega_1$, the one-to-two or one-to-three internal resonance occurs.



(a)



(b)

Figure 2: Effect of in-plane excitation on the dynamic response of the FGM plate with 1 : 1 internal resonance.

The in-plane forces in these cases are given by (2.27)

$$p_{02} = \frac{m_{007}n_{002} - 4m_{001}n_{007}}{4m_{001}n_{008} - m_{008}n_{002}},$$
$$p_{03} = \frac{m_{007}n_{002} - 9m_{001}n_{007}}{9m_{001}n_{008} - m_{008}n_{002}}. \tag{2.27}$$

3. Numerical Results

The influence of in-plane stationary excitation on internal resonance is studied. The fourth-order Runge-Kutta algorithm is employed to numerically solve (2.11) and (2.12) to obtain the nonlinear dynamic response of the FGM rectangular plate subjected to thermal and mechanical loads with various internal resonance and primary parametric resonance.

Aluminum oxide and Ti-6Al-4V are chosen to be the constituent materials of the plate ($a = b = 1\,\mathrm{m}$, $h = a/20$). The volume fraction exponent is $n = 0.2$. The transverse load amplitude is $-10^6\,\mathrm{N/m^2}$. In addition, the plate is subjected to a temperature field where the aluminum oxide rich top surface is held at 900 K and the Ti-6Al-4V rich bottom surface is held at 300 K. Their temperature-dependent material properties evaluated at $T_0 = 300\,\mathrm{K}$ are as follows.

Ti-6Al-4V:

$$E = 105.7\,\mathrm{GPa}, \qquad \nu = 0.2981, \qquad \rho = 4429\,\frac{\mathrm{kg}}{\mathrm{m}^3}. \tag{3.1}$$

Aluminum oxide:

$$E = 320.24\,\mathrm{GPa}, \qquad \nu = 0.2600, \qquad \rho = 3750\,\frac{\mathrm{kg}}{\mathrm{m}^3}. \tag{3.2}$$

Figures 2–4 depict, respectively, nonlinear dynamic response of FGM plates. The plots of phase portrait for the cases of 1 : 1, 1 : 2 and 1 : 3 internal resonance with different in-plane

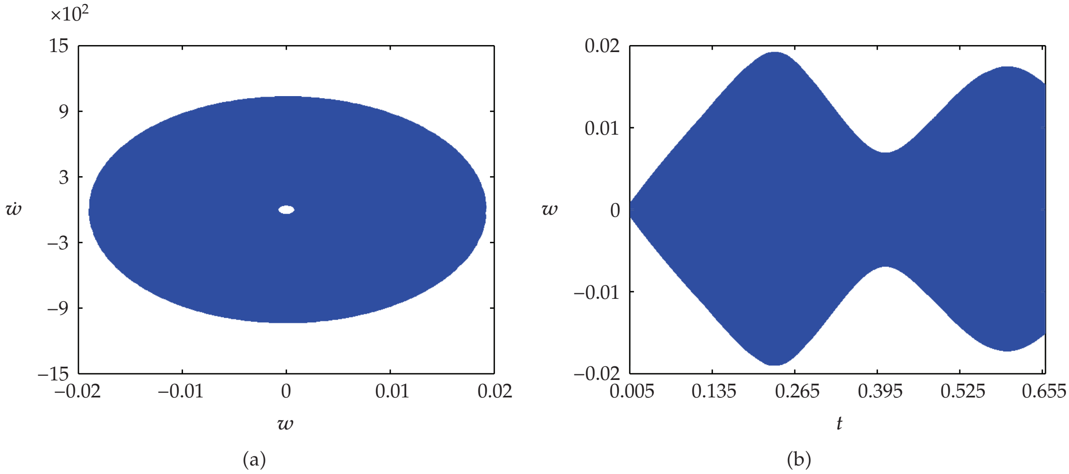


Figure 3: Effect of in-plane excitation on the dynamic response of the FGM plate with 1 : 2 internal resonance.

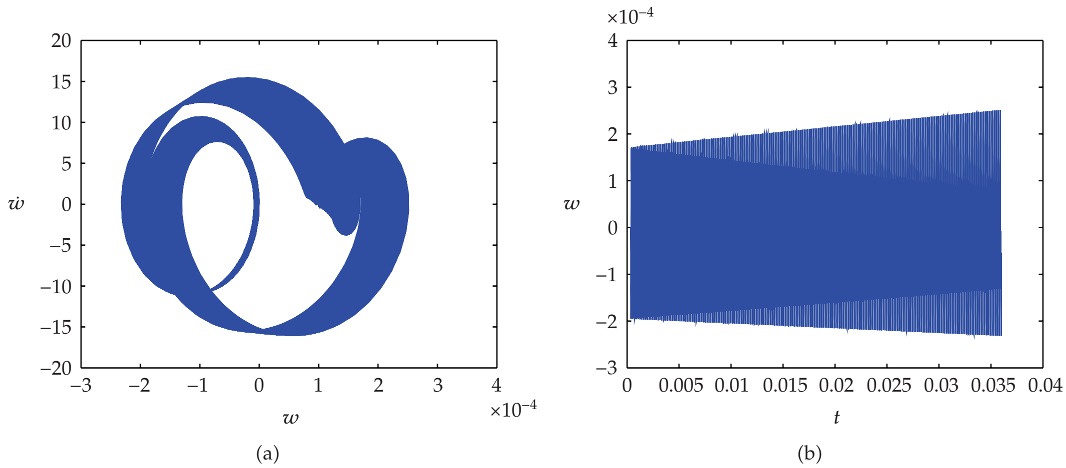


Figure 4: Effect of in-plane excitation on the dynamic response of the FGM plate with 1 : 3 internal resonance.

stationary loading are shown in Figures 2(a), 3(a), and 4(a) and the central deflection versus time curve is displayed in Figures 2(b), 3(b), and 4(b). The combinational resonance of the additive type is

$$\omega_1 = \frac{\Omega_1}{2}, \qquad \Omega_2 = \Omega_1. \tag{3.3}$$

It is observed that the central deflections are reduced by increasing the ratio of the two frequencies. In the case of 1 : 2 internal resonance the amplitude of the central deflection is larger than the one at other two frequency ratios. The case of internal resonance can be controlled by changing the in-plane excitation force, indicating that in the different case of internal resonance there is a different fundamental frequency.

Obviously, Figure 2 illustrates that the periodic response of the FGM rectangular plate occurs at 1 : 1 internal resonance when the p_0 is as 7.33×10^9 N/m. Figures 3 and 4 show that the beat vibration and quasiperiod dynamic response take place at 1 : 2 internal resonance when p_0 is as 6.24×10^{10} N/m and 1 : 3 internal resonance when p_0 is as 1.11×10^{11} N/m, respectively.

4. Conclusions

The nonlinear dynamics response of FGM rectangular plates under combined transverse and in-plane excitations is investigated in the cases of 1 : 1, 1 : 2 and 1 : 3 internal resonance. The material properties are assumed to be temperature-dependent. Based on Reddy's third-order shear deformation plate theory, the governing equations of motion for the FGMs rectangular plate are derived using Hamilton's principle. Galerkin's approach is used to reduce the governing equations of motion to a two-degree-of-freedom nonlinear system including the quadratic and cubic nonlinear terms. 1 : 1, 1 : 2 and 1 : 3 internal resonance and principal parametric resonance-1/2 subharmonic resonance are considered and solutions are obtained by using fourth-order Runge-Kutta method.

Numerical results show that plate geometry parameter, in-plane excitation and temperature field play important role in the internal resonance relationship and the nonlinear dynamic behavior of the FGM plate. In the case of 1 : 2 internal resonance and principal parametric resonance-1/2 subharmonic resonance, the vibration amplitude at the plate center is much greater than the one at other two cases of internal resonance. So in the actual condition, it is necessary to analyze what kinds of internal resonance may occur and how to control them.

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