Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2010, Article ID 898720, 17 pages
doi:10.1155/2010/898720

Research Article

Unsteady Flow and Heat Transfer of a Dusty Fluid through a Rectangular Channel

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Received 3 August 2010; Accepted 30 December 2010

Academic Editor: J. Jiang

The present discussion deals with the study of an unsteady flow and heat transfer of a dusty fluid through a rectangular channel under the influence of pulsatile pressure gradient along with the effect of a uniform magnetic field. The analytical solutions of the problem are obtained using variable separable and Fourier transform techniques. The graphs are drawn for the velocity fields of both fluid and dust phases under the effect of Reynolds number. Further, changes in the Nusselt number are shown graphically, and, on the basis of these, the conclusions and discussions are given.

1. Introduction

The concept of an unsteady flow and heat transfer of a dusty fluid *has a wide* range of applications in refrigeration, air conditioning, space heating, power generation, chemical processing, pumps, accelerators, nuclear reactors, filtration and geothermal systems, and so forth. One common *example of heat* transfer is the radiator in a car, in which the hot radiator fluid is cooled by the flow of air over the radiator surface. On this basis many mathematicians were attracted by this field.

Saffman [1] has formulated the governing equations for the flow of dusty fluid and has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Datta et al. [2] have obtained the solution of unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel. Heat transfer in unsteady laminar flow through a channel was analyzed by Ariel [3]. Ghosh et al. [4] have made the solution for hall effects on MHD flow in a rotating system with heat transfer characteristics. Ezzat et al. [5] analyzed a space approach to the hydromagnetic flow of a dusty fluid through a porous medium.

Some researchers like Anjali Devi and Jothimani [6] have discussed the heat transfer in unsteady MHD oscillatory flow. Further, Malashetty et al. [7] have investigated the convective magnetohydrodynamic two phase flow and heat transfer of a fluid in an inclined channel. Palani and Ganesan [8] have discussed the heat transfer effects on dusty gas flow past a semi-infinite inclined plate. Attia [9] has investigated an unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties.

Unsteady hydromagnetic flow and heat transfer from a nonisothermal stretching sheet immersed in a porous medium was discussed by Chamkha [10]. Mishra et al. [11] have studied the two-dimensional transient conduction and radiation heat transfer with temperature-dependent thermal conductivity. MHD flow and heat transfer of a dusty visco-elastic stratified fluid down an inclined channel in porous medium under variable viscosity was analyzed by Chakraborty [12]. Shawky [13] has investigated the solution for pulsatile flow with heat transfer of dusty magnetohydrodynamic Ree-Eyring fluid through a channel.

Gireesha et al. [14] have obtained the analytical solutions for velocity fields using variable separable method for an unsteady flow of dusty fluid through a rectangular channel under the influence of pulsatile pressure gradients and in the absence of a magnetic field. In continuation of this paper and with the help of the above cited papers we have studied an unsteady flow and heat transport in a dusty fluid through a rectangular channel under the influence of a pulsatile pressure gradient in the presence of uniform magnetic field and viscous dissipation term. Further, heat transfer analysis and the effect of Reynolds number, Prandtl number, and Nusselt number have been considered. This paper presents three methods of solution, namely, perturbation technique, Fourier decomposition, and finite Fourier transform, to obtain useful results on the problem. Finally, the graphical representation of velocity fields of both fluid and dust phases and changes in the Nusselt number are drawn for different values of Reynolds number and Prandtl number.

2. Equations of Motion

The governing equations of motion and energy for two phases are given by [1] the following.

For fluid phase,

$$\nabla \cdot \vec{u} = 0,$$
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{u} + \frac{f}{\tau_v}(\vec{v} - \vec{u}) - \frac{\sigma B_0^2}{\rho}\vec{u}, \tag{2.1}$$
$$\rho\left\{\frac{\partial E}{\partial t} + (\vec{u} \cdot \nabla E)\right\} = Q + (\vec{v} - \vec{u}) \cdot F + k\nabla \cdot (\nabla T) + \Phi_f.$$

For dust phase,

$$\nabla \cdot \vec{v} = 0,$$
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{1}{\tau_v}(\vec{u} - \vec{v}), \tag{2.2}$$
$$N\left\{\frac{\partial E_p}{\partial t} + (\vec{v} \cdot \nabla E_p)\right\} = -Q - \Phi_d.$$

We have following nomenclature. $E = c_p T$, $E_p = c_m T_p$, $Q = Nc_p(T_p - T)/\tau_T$ is the thermal interaction between fluid and dust particle phases, $F = N(\vec{v} - \vec{u})/\tau_v$ is the velocity interaction force between the fluid and dust particle phase, $\tau_v = m/6\pi a\mu = m/K$ is the velocity relaxation time of the dust particles, $\tau_T = mc_p/4\pi ak$ is the thermal relaxation time of the dust particles, $k\nabla.\ (\nabla T)$ is the rate of heat added to the fluid by conduction in unit volume, Φ_f and Φ_d are the viscous dissipation of fluid and dust particles. $\vec{u}$, ρ, p, ν, T, c_p, and k are, respectively, the velocity vector, density, pressure, kinematic viscosity, temperature, specific heat, and thermal conductivity of the fluid, $\vec{v}$, N, T_p, c_m, and m are, respectively, the velocity vector, number density, temperature, specific heat, mass concentration of dust particles, $K = 6\pi a\mu$ is the Stoke's resistance coefficient, and t is the time.

3. Formulation of the Problem

Consider an unsteady flow of an incompressible, viscous, electrically conducting fluid with uniform distribution of dust particles through a rectangular channel. It is assumed that the flow is due to the time-dependent pressure gradient and applied uniform magnetic field. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. The flow is taken along z-axis, and it is as shown in Figure 1. For the above described flow the velocities of both fluid and dust particles are given by

$$\vec{u} = u(x, y, t)\hat{k}, \qquad \vec{v} = v(x, y, t)\hat{k}. \tag{3.1}$$

4. Solution of the Problem

The governing equations from (2.1) and (2.2) can be decomposed as follows.

For fluid phase,

$$\begin{aligned}
\frac{\partial u}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{f}{\tau_v}(v - u) - \frac{\sigma B_0^2}{\rho}u, \\
\rho c_p \frac{\partial T}{\partial t} &= \frac{Nc_p}{\tau_T}(T_p - T) + \frac{N}{\tau_v}(v - u)^2 + k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \\
&\quad + \mu\left[\frac{4}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right].
\end{aligned} \tag{4.1}$$

For dust phase,

$$\begin{aligned}
\frac{\partial v}{\partial t} &= \frac{1}{\tau_v}(u - v), \\
c_m \frac{\partial T_p}{\partial t} &= \frac{c_p}{\tau_T}(T - T_p) - \frac{\mu_p}{N}\left[\frac{4}{3}\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right],
\end{aligned} \tag{4.2}$$

where u and v denote the velocity of the fluid and the dust phases, respectively.

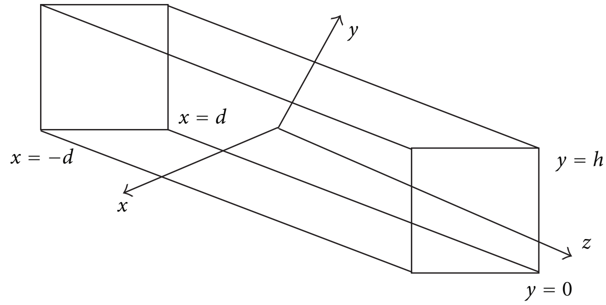


Figure 1: Schematic diagram of dusty fluid flow in a rectangular channel.

The boundary conditions of the given problem are taken as

$$\begin{aligned} &u = 0, \quad v = 0, \quad T = T_0 \quad \text{at } y = 0, \\ &u = 0, \quad v = 0, \quad T = T_1 \quad \text{at } y = h, \\ &u = 0, \quad v = 0, \quad T = T_2 \quad \text{at } x = -d, \\ &u = 0, \quad v = 0, \quad T = T_3 \quad \text{at } x = d. \end{aligned} \tag{4.3}$$

Since we have assumed that the pulsatile pressure gradient has influence on the flow, we have that

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = A\left[1 + \epsilon e^{i\omega t}\right], \tag{4.4}$$

where ϵ is a small quantity and A and ω are constants.

To make the above system dimensionless, introduce the following nondimensional variables:

$$\begin{gathered} \overline{u} = \frac{u\omega}{A}, \quad \overline{v} = \frac{v\omega}{A}, \quad \bar{t} = t\omega, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \\ \xi = \frac{x}{h}, \quad \eta = \frac{y}{h}, \quad \varphi = \frac{z}{h}, \quad \overline{p} = \frac{p}{A\rho h}, \quad \theta_p = \frac{T_p - T_0}{T_1 - T_0}, \\ P_r = \frac{\mu c_p}{k}, \quad \mathrm{Re} = \frac{\omega h^2}{\nu}, \quad \mathrm{Re}_p = \frac{\omega h^2}{\nu_p}, \quad \mathrm{Ec} = \frac{A^2}{\omega^2 c_p (T_1 - T_0)}, \end{gathered} \tag{4.5}$$

where h is the distance between plates, Ec the Eckert number, P_r the Prandtl number, Re and Re_p the Reynolds numbers of fluid and dust phases, x and y the space coordinates along and perpendicular to the plates, θ and θ_p the dimensionless fluid and dust phase temperatures, μ the viscosity of fluid, and ν_p and ρ_p the kinematic viscosity and density of the dust particles.

Using the above nondimensional variables in (4.1) and (4.2) and dropping the bars, one can get that

$$\begin{aligned}
\frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial \psi} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2}\right) + f\alpha(v-u) - \beta u, \\
\frac{\partial \theta}{\partial t} &= \frac{1}{\mathrm{Re}\,P_r}\left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2}\right) + \alpha_1\beta_1(\theta_p - \theta) + \alpha\beta_1 \mathrm{Ec}(v-u)^2 \\
&\quad + \frac{\mathrm{Ec}}{\mathrm{Re}}\left[\frac{4}{3}\left(\frac{\partial u}{\partial \xi}\right)^2 + \left(\frac{\partial u}{\partial \eta}\right)^2\right], \\
\frac{\partial v}{\partial t} &= \alpha(u-v), \\
\frac{\partial \theta_p}{\partial t} &= \gamma\alpha_1(\theta - \theta_p) - \frac{\gamma \mathrm{Ec}}{\mathrm{Re}_p \beta_2}\left[\frac{4}{3}\left(\frac{\partial v}{\partial \xi}\right)^2 + \left(\frac{\partial v}{\partial \eta}\right)^2\right],
\end{aligned} \tag{4.6}$$

where $\alpha = 1/\omega\tau_v$, $\beta = \sigma B_0^2/\rho\omega$, $\alpha_1 = 1/\omega\tau_T$, $\beta_1 = N/\rho$, $\beta_2 = N/\rho_p$, and $\gamma = c_p/c_m$.

The dimensionless boundary conditions are

$$\begin{aligned}
u = 0, \quad v = 0, \quad \theta = 0 \quad &\text{at } \eta = 0, \\
u = 0, \quad v = 0, \quad \theta = 1 \quad &\text{at } \eta = 1, \\
u = 0, \quad v = 0, \quad \theta = T_a \quad &\text{at } \xi = -r, \\
u = 0, \quad v = 0, \quad \theta = T_b \quad &\text{at } \xi = r,
\end{aligned} \tag{4.7}$$

where $T_a = (T_2 - T_0)/(T_1 - T_0)$, $T_b = (T_3 - T_0)/(T_1 - T_0)$, and $r = d/h$.

The nondimensional form of pressure gradient is given by

$$-\frac{\partial p}{\partial \psi} = \left[1 + \epsilon e^{it}\right]. \tag{4.8}$$

Now assume the solutions of the velocities and temperature of both fluid and dust phases as

$$\begin{aligned}
u(\xi,\eta,t) &= u_0(\xi,\eta) + \epsilon u_1(\xi,\eta)e^{it}, \\
v(\xi,\eta,t) &= v_0(\xi,\eta) + \epsilon v_1(\xi,\eta)e^{it}, \\
\theta(\xi,\eta,t) &= \theta_0(\xi,\eta) + \epsilon\theta_1(\xi,\eta)e^{it} + \epsilon^2\theta_2(\xi,\eta)e^{2it}, \\
\theta_p(\xi,\eta,t) &= \theta_{p0}(\xi,\eta) + \epsilon\theta_{p1}(\xi,\eta)e^{it} + \epsilon^2\theta_{p2}(\xi,\eta)e^{2it}.
\end{aligned} \tag{4.9}$$

Substituting (4.8) and (4.9) in (4.6) and equating the coefficient of the similar powers of ϵ on both sides, then we obtain the following set of equations.

Steady part (coefficient of ϵ^0):

$$\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u_0}{\partial \xi^2}+\frac{\partial^2 u_0}{\partial \eta^2}\right]+f\alpha(v_0-u_0)-\beta u_0=-1, \tag{4.10}$$

$$\alpha(u_0-v_0)=0, \tag{4.11}$$

$$\begin{gathered}\frac{1}{\mathrm{Re}\,P_r}\left[\frac{\partial^2 \theta_0}{\partial \xi^2}+\frac{\partial^2 \theta_0}{\partial \eta^2}\right]+\alpha_1\beta_1(\theta_{p0}-\theta_0)+\alpha\beta_1\mathrm{Ec}(v_0-u_0)^2+\frac{\mathrm{Ec}}{\mathrm{Re}}\left[\frac{4}{3}\left(\frac{\partial u_0}{\partial \xi}\right)^2+\left(\frac{\partial u_0}{\partial \eta}\right)^2\right]=0,\\ \gamma\alpha_1(\theta_0-\theta_{p0})-\frac{\gamma\mathrm{Ec}}{\mathrm{Re}_p\beta_2}\left[\frac{4}{3}\left(\frac{\partial v_0}{\partial \xi}\right)^2+\left(\frac{\partial v_0}{\partial \eta}\right)^2\right]=0.\end{gathered} \tag{4.12}$$

Unsteady part (coefficient of ϵ):

$$\begin{gathered}\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u_1}{\partial \xi^2}+\frac{\partial^2 u_1}{\partial \eta^2}\right]+f\alpha(v_1-u_1)-(\beta+i)u_1=-1,\\ \alpha(u_1-v_1)-iv_1=0,\end{gathered} \tag{4.13}$$

$$\begin{gathered}\frac{1}{\mathrm{Re}\,P_r}\left[\frac{\partial^2 \theta_1}{\partial \xi^2}+\frac{\partial^2 \theta_1}{\partial \eta^2}\right]+\alpha_1\beta_1(\theta_{p1}-\theta_1)+2\alpha\beta_1\mathrm{Ec}(v_0-u_0)(v_1-u_1)\\ +\frac{2\mathrm{Ec}}{\mathrm{Re}}\left[\frac{4}{3}\frac{\partial u_0}{\partial \xi}\frac{\partial u_1}{\partial \xi}+\frac{\partial u_0}{\partial \eta}\frac{\partial u_1}{\partial \eta}\right]-i\theta_1=0,\end{gathered} \tag{4.14}$$

$$\gamma\alpha_1[\theta_1-\theta_{p1}]-\frac{2\gamma\mathrm{Ec}}{\mathrm{Re}_p\beta_2}\left[\frac{4}{3}\frac{\partial v_0}{\partial \xi}\frac{\partial v_1}{\partial \xi}+\frac{\partial v_0}{\partial \eta}\frac{\partial v_1}{\partial \eta}\right]-i\theta_{p1}=0. \tag{4.15}$$

Unsteady part (coefficient of ϵ^2):

$$\begin{gathered}\frac{1}{\mathrm{Re}\,P_r}\left[\frac{\partial^2 \theta_2}{\partial \xi^2}+\frac{\partial^2 \theta_2}{\partial \eta^2}\right]+\alpha_1\beta_1(\theta_{p2}-\theta_2)+\alpha\beta_1\mathrm{Ec}(v_1-u_1)^2\\ +\frac{\mathrm{Ec}}{\mathrm{Re}}\left[\frac{4}{3}\left(\frac{\partial u_1}{\partial \xi}\right)^2+\left(\frac{\partial u_1}{\partial \eta}\right)^2\right]-2i\theta_2=0,\end{gathered} \tag{4.16}$$

$$\gamma\alpha_1[\theta_2-\theta_{p2}]-\frac{\gamma\mathrm{Ec}}{\mathrm{Re}_p\beta_2}\left[\frac{4}{3}\left(\frac{\partial v_1}{\partial \xi}\right)^2+\left(\frac{\partial v_1}{\partial \eta}\right)^2\right]-2i\theta_{p2}=0. \tag{4.17}$$

Now, the corresponding dimensionless boundary conditions are as follows:

$$\begin{aligned} &u_0 = 0, \quad v_0 = 0, \quad \theta_0 = T_a \quad \text{at } \xi = -r, \\ &u_0 = 0, \quad v_0 = 0, \quad \theta_0 = T_b \quad \text{at } \xi = r, \\ &u_0 = 0, \quad v_0 = 0, \quad \theta_0 = 0 \quad \text{at } \eta = 0, \\ &u_0 = 0, \quad v_0 = 0, \quad \theta_0 = 1 \quad \text{at } \eta = 1; \end{aligned} \tag{4.18}$$

$$\begin{aligned} &u_1 = 0, \quad v_1 = 0, \quad \theta_1 = 0 \quad \text{at } \xi = -r, \\ &u_1 = 0, \quad v_1 = 0, \quad \theta_1 = 0 \quad \text{at } \xi = r, \\ &u_1 = 0, \quad v_1 = 0, \quad \theta_1 = 0 \quad \text{at } \eta = 0, \\ &u_1 = 0, \quad v_1 = 0, \quad \theta_1 = 0 \quad \text{at } \eta = 1; \end{aligned} \tag{4.19}$$

$$\begin{aligned} &\theta_2 = 0 \quad \text{at } \xi = -r, \\ &\theta_2 = 0 \quad \text{at } \xi = r, \\ &\theta_2 = 0 \quad \text{at } \eta = 0, \\ &\theta_2 = 0 \quad \text{at } \eta = 1. \end{aligned} \tag{4.20}$$

By substituting (4.11) in (4.10), one can get

$$\frac{\partial^2 u_0}{\partial \xi^2} + \frac{\partial^2 u_0}{\partial \eta^2} - \beta \operatorname{Re} u_0 = -\operatorname{Re}. \tag{4.21}$$

To solve (4.21), we assume that the solution is in the form

$$u_0(\xi, \eta) = X(\xi, \eta) + Y(\xi). \tag{4.22}$$

Substituting $u_0(\xi, \eta)$ in (4.21), then we obtain that

$$\frac{\partial^2 X}{\partial \xi^2} + \frac{\partial^2 Y}{\partial \eta^2} + \frac{\partial^2 X}{\partial \eta^2} - \beta \operatorname{Re}(X + Y) = -\operatorname{Re}. \tag{4.23}$$

so that

$$\frac{\partial^2 Y}{\partial \eta^2} - \beta \operatorname{Re} Y + \operatorname{Re} = 0, \tag{4.24}$$

$$\frac{\partial^2 X}{\partial \xi^2} + \frac{\partial^2 X}{\partial \eta^2} - \beta \operatorname{Re} X = 0. \tag{4.25}$$

The corresponding boundary conditions will become

$$u_0(-r,\eta) = X(-r,\eta) + Y(-r) = 0, \qquad u_0(r,\eta) = X(r,\eta) + Y(r) = 0,$$
$$u_0(\xi,0) = X(\xi,0) + Y(\xi) = 0, \qquad u_0(\xi,1) = X(\xi,0) + Y(\xi) = 0. \tag{4.26}$$

By solving (4.24) and using the method of separation of variables to (4.25), we obtain the solution in the form

$$\begin{aligned} u_0(\xi,\eta) &= \frac{1}{\beta}\left[1 - \frac{\cosh\left(\sqrt{\beta \operatorname{Re}}\,\xi\right)}{\cosh\left(\sqrt{\beta \operatorname{Re}}\,r\right)}\right] + \frac{2}{\beta}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{r}\xi\right)\left[\frac{\sinh A_1(\eta-1) - \sinh(A_1\eta)}{\sinh A_1}\right] \\ &\quad \times \left\{\frac{1}{n\pi} - \frac{n\pi}{r^2 A_1^2 \cosh\left(\sqrt{\beta \operatorname{Re}}\,r\right)} - \frac{\beta \operatorname{Re}}{n\pi A_1^2}(-1)^n\right\}, \\ v_0(\xi,\eta) &= u_0(\xi,\eta), \end{aligned} \tag{4.27}$$

where $A_1 = \sqrt{(\beta \operatorname{Re} r^2 + n^2\pi^2)/r^2}$.

Here one can observe that the velocity of steady part of the fluid and the dust phases is the same.

In a similar manner, by the method of separation of variables, the solution of (4.13), and using the boundary conditions (4.19), one can obtain that

$$\begin{aligned} u_1(\xi,\eta) &= \frac{\operatorname{Re}}{Q^2}\left[1 - \frac{\cosh(Q\xi)}{\cosh(Qr)}\right] + \frac{2\operatorname{Re}}{Q^2}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{r}\xi\right)\left[\frac{\sinh B(\eta-1) - \sinh(B\eta)}{\sinh B}\right] \\ &\quad \times \left\{\frac{1}{n\pi} - \frac{n\pi}{r^2 B^2 \cosh(Qr)} - \frac{Q^2}{n\pi B^2}(-1)^n\right\}, \\ v_1 &= \left(\frac{\alpha}{\alpha + i}\right)u_1, \end{aligned} \tag{4.28}$$

where $Q^2 = \operatorname{Re}[f\alpha i/(\alpha+i) + (\beta+i)]$ and $B = \sqrt{(Q^2 r^2 + n^2\pi^2)/r^2}$.

We define the finite Fourier sine transform of θ and θ_p as

$$F_s(\theta) = \int_{-r}^{r}\theta(\xi,\eta)\sin\left(\frac{n\pi}{r}\xi\right), \qquad F_s(\theta_p) = \int_{-r}^{r}\theta_p(\xi,\eta)\sin\left(\frac{n\pi}{r}\xi\right). \tag{4.29}$$

Eliminating θ_{p0} from (4.12), we get that

$$\frac{\partial^2\theta_0}{\partial\xi^2} + \frac{\partial^2\theta_0}{\partial\eta^2} = H_1\left[\frac{4}{3}\left(\frac{\partial u_0}{\partial\xi}\right)^2 + \left(\frac{\partial u_0}{\partial\eta}\right)^2\right], \tag{4.30}$$

$$\theta_{p0}(\xi,\eta) = \theta_0 - \frac{\operatorname{Ec}}{\operatorname{Re}_p\alpha_1\beta_2}\left[\frac{4}{3}\left(\frac{\partial v_0}{\partial\xi}\right)^2 + \left(\frac{\partial v_0}{\partial\eta}\right)^2\right], \tag{4.31}$$

where $H_1 = \operatorname{Ec}\operatorname{Re} P_r(\beta_1/\operatorname{Re}_p\beta_2 - 1/\operatorname{Re})$.

Applying the finite fourier sine transform to (4.30) with respect to the variable ξ and to boundary conditions, one obtains that

$$\frac{d^2F_s(\theta_0)}{d\eta^2} - \frac{n^2\pi^2}{r^2}F_s(\theta_0) = \sum_{n=1}^{\infty}\frac{n\pi}{r}\left[(T_b - T_a)(-1)^n\right] + \frac{4H_1}{3}\frac{8\pi^2\sqrt{\beta\,\mathrm{Re}}}{\beta^2}$$
$$\times\sum_{n=1}^{\infty}\frac{n^2}{\beta\,\mathrm{Re}\,r^2 + 4n^2\pi^2}\frac{\sinh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)}{\cosh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)}$$
$$\times\left(\frac{1}{n\pi} - \frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)} - \frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)$$
$$\times\left[\frac{\sinh A_1(\eta-1) - \sinh(A_1\eta)}{\sinh A_1}\right], \tag{4.32}$$

$$F_s(\theta_0(\xi,0)) = 0, \qquad F_s(\theta_0(\xi,1)) = 0. \tag{4.33}$$

The temperature of fluid is obtained by solving (4.32) with the help of boundary conditions (4.33) as

$$F_s(\theta_0) = \left[\sum_{n=1}^{\infty}\frac{r}{n\pi}\left[(T_b - T_a)(-1)^n\right] + \frac{32\pi^2\sqrt{\beta\,\mathrm{Re}}H_1}{3\beta^2}\sum_{n=1}^{\infty}\frac{n^2}{\beta\,\mathrm{Re}\,r^2 + 4n^2\pi^2}\right.$$
$$\left.\times\frac{r^2}{A_1^2r^2 - n^2\pi^2}\frac{\sinh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)}{\cosh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)}\left(\frac{1}{n\pi} - \frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)} - \frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)\right]$$
$$\times\left\{\cosh\left(\frac{n\pi}{r}\eta\right) + \frac{1-\cosh((n\pi/r)\eta)}{\sinh(n\pi/r)}\sinh\left(\frac{n\pi}{\mathrm{r}}\eta\right)\right\}$$
$$-\sum_{n=1}^{\infty}\frac{r}{n\pi}\left[(T_b - T_a)(-1)^n\right] + \frac{32\pi^2\sqrt{\beta\,\mathrm{Re}}H_1}{3\beta^2}\sum_{n=1}^{\infty}\frac{n^2}{\beta\,\mathrm{Re}\,r^2 + 4n^2\pi^2}$$
$$\times\frac{r^2}{A_1^2r^2 - n^2\pi^2}\frac{\sinh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}{\cosh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}\left(\frac{1}{n\pi} - \frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)} - \frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)$$
$$\times\left[\frac{\sinh A_1(\eta-1) - \sinh(A_1\eta)}{\sinh A_1}\right]. \tag{4.34}$$

Now taking the inverse finite Fourier sine transform to (4.34), one can obtain that

$$\begin{aligned}\theta_0(\xi,\eta) = \sum_{r=1}^{\infty}\frac{2}{r}\Bigg\{\Bigg[&\sum_{n=1}^{\infty}\frac{r}{n\pi}\left[(T_b-T_a)(-1)^n\right]+\frac{32\pi^2\sqrt{\beta\,\mathrm{Re}}\,H_1}{3\beta^2}\\
&\times\sum_{n=1}^{\infty}\frac{n^2}{\beta\,\mathrm{Re}\,r^2+4n^2\pi^2}\frac{r^2}{A_1^2r^2-n^2\pi^2}\frac{\sinh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}{\cosh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}\\
&\times\left(\frac{1}{n\pi}-\frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}-\frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)\Bigg]\\
&\times\left\{\cosh\left(\frac{n\pi}{r}\eta\right)+\frac{1-\cosh((n\pi/r)\eta)}{\sinh(n\pi/r)}\sinh\left(\frac{n\pi}{r}\eta\right)\right\}\\
&-\sum_{n=1}^{\infty}\frac{r}{n\pi}\left[(T_b-T_a)(-1)^n\right]+\frac{32\pi^2\sqrt{\beta\,\mathrm{Re}}\,H_1}{3\beta^2}\sum_{n=1}^{\infty}\frac{n^2}{\beta\,\mathrm{Re}\,r^2+4n^2\pi^2}\\
&\times\frac{r^2}{A_1^2r^2-n^2\pi^2}\frac{\sinh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}{\cosh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}\left[\frac{\sinh A_1(\eta-1)-\sinh(A_1\eta)}{\sinh A_1}\right]\\
&\times\left(\frac{1}{n\pi}-\frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\,r\right)}-\frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)\Bigg\}\sin\left(\frac{n\pi}{r}\xi\right).\end{aligned}\tag{4.35}$$

The temperature of dust θ_{p0} is obtained by substituting θ_0 in (4.31).

Using (4.15) in (4.14) and boundary conditions (4.19), with the help of finite fourier sine transform technique, one can get the solution for θ_1 as

$$\begin{aligned}&\theta_1(\xi,\eta)\\
&=\sum_{r=1}^{\infty}\frac{2}{r}\Bigg\{\frac{16\,\mathrm{Re}\,\pi^2H_3}{3\beta}\Bigg[\frac{1}{Q}\sum_{n=1}^{\infty}\frac{n^2}{A_1^2-q_1^2}\frac{1}{Q^2r^2+4n^2\pi^2}\frac{\sinh(Qr)}{\cosh(Qr)}\\
&\qquad\times\left(\frac{1}{n\pi}-\frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)}-\frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)\\
&\qquad+\frac{\sqrt{\beta\,\mathrm{Re}}}{Q^2}\sum_{n=1}^{\infty}\frac{n^2}{(B^2-q_1^2)}\frac{1}{\beta\,\mathrm{Re}\,r^2+4n^2\pi^2}\\
&\qquad\times\left(\frac{1}{n\pi}-\frac{n\pi}{(Q^2r^2+n^2\pi^2)\cosh(Qr)}-\frac{Q^2(-1)^n}{n\pi B^2}\right)\frac{\sinh\sqrt{\beta\,\mathrm{Re}}\;r}{\cosh\sqrt{\beta\,\mathrm{Re}}\;r}\Bigg]\\
&\qquad\times\left\{\cosh(q_1\eta)+\frac{(1-\cos hq_1)}{\sinh q_1}\sinh(q_1\eta)\right\}\end{aligned}$$

$$+\frac{16\,\mathrm{Re}\,\pi^2 H_3}{3\beta}\left[\frac{1}{Q}\sum_{n=1}^{\infty}\frac{n^2}{A_1^2-q_1^2}\,\frac{1}{Q^2r^2+4n^2\pi^2}\right.$$
$$\times\left(\frac{1}{n\pi}-\frac{n\pi}{r^2A_1^2\cosh\left(\sqrt{\beta\,\mathrm{Re}}\;r\right)}-\frac{\beta\,\mathrm{Re}\,(-1)^n}{n\pi A_1^2}\right)\frac{\sinh(Qr)}{\cosh(Qr)}$$
$$\times\left[\frac{\sinh A_1(\eta-1)-\sinh(A_1\eta)}{\sinh A_1}\right]+\frac{\sqrt{(\beta\,\mathrm{Re})}}{Q^2}\sum_{n=1}^{\infty}\frac{n^2}{(B^2-q_1^2)}\,\frac{1}{\beta\,\mathrm{Re}\,r^2+4n^2\pi^2}$$
$$\times\left(\frac{1}{n\pi}-\frac{n\pi}{(Q^2r^2+n^2\pi^2)\cosh(Qr)}-\frac{Q^2(-1)^n}{n\pi B^2}\right)\frac{\sinh\sqrt{\beta\,\mathrm{Re}}\,r}{\cosh\sqrt{\beta\,\mathrm{Re}}\,r}$$
$$\left.\left.\times\left(\frac{\sinh B(\eta-1)-\sinh(B\eta)}{\sinh B}\right)\right]\right\}\sin\left(\frac{n\pi}{r}\xi\right). \tag{4.36}$$

Using θ_1, we get the expression for θ_{p1} as

$$\theta_{p1}(\xi,\eta)=\left(\frac{\gamma\alpha_1}{\gamma\alpha_1+i}\right)\theta_1-\frac{2\gamma\mathrm{Ec}}{\mathrm{Re}_p\beta_2(\gamma\alpha_1+i)}\left[\frac{4}{3}\frac{\partial v_0}{\partial\xi}\frac{\partial v_1}{\partial\xi}+\frac{\partial v_0}{\partial\eta}\frac{\partial v_1}{\partial\eta}\right], \tag{4.37}$$

where

$$q_1=\sqrt{H_2+\frac{n^2\pi^2}{r^2}},\qquad H_2=\mathrm{Re}\,P_r\left[\frac{\alpha_1\beta_1 i}{\gamma\alpha_1+i}+i\right],$$
$$H_3=2\,\mathrm{Ec}\ \mathrm{Re}\ P_r\left[\frac{\alpha_1\beta_1\gamma}{\mathrm{Re}_p\beta_2(\gamma\alpha_1+i)}\,\frac{\alpha}{(\alpha+i)}-\frac{1}{\mathrm{Re}}\right]. \tag{4.38}$$

Similarly, the solutions of (4.16) and (4.17) using boundary conditions (4.20) are obtained as

$$\theta_2(\xi,\eta)=\sum_{r=1}^{\infty}\frac{2}{r}\left\{\frac{8\mathrm{Re}^2}{Q^3}\left[\frac{H_5}{Q}\sum_{n=1}^{\infty}\left(r-\frac{1}{Q}\right)+\frac{4\pi^2H_6}{3}\sum_{n=1}^{\infty}\frac{n^2}{Q^2r^2+4n^2\pi^2}\right]\frac{1}{B^2-q_2^2}\right.$$
$$\times\frac{\sinh(Qr)}{\cosh(Qr)}\left(\frac{1}{n\pi}-\frac{n\pi}{Q^2r^2+n^2\pi^2}\frac{1}{\cosh(Qr)}-\frac{Q^2(-1)^n}{n\pi B^2}\right)$$
$$\left.+\left[\cosh(q_2\eta)\frac{(1-\cosh q_2)}{\sinh q_2}\sinh(q_2\eta)+\frac{\sinh B(\eta-1)-\sinh(B\eta)}{\sinh B}\right]\right\}$$
$$\times\sin\left(\frac{n\pi}{r}\xi\right). \tag{4.39}$$

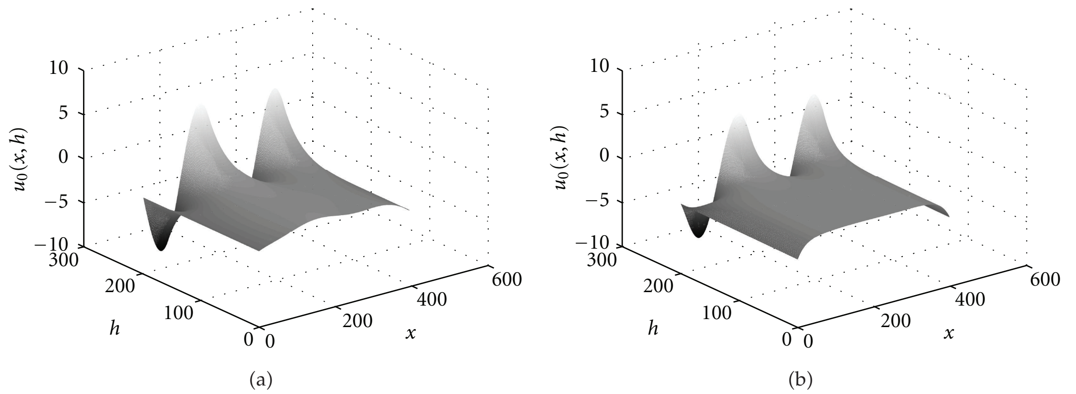


Figure 2: Variation of fluid velocity $u_0(\xi,\eta)$ with ξ and η (for Re = 1 and Re = 10).

From (4.17), one can get that

$$\theta_{p2}(\xi,\eta) = \left(\frac{\gamma\alpha_1}{\gamma\alpha_1+2i}\right)\theta_2 - \frac{\gamma \mathrm{Ec}}{\mathrm{Re}_p\beta_2(\gamma\alpha_1+2i)}\left[\frac{4}{3}\left(\frac{\partial v_1}{\partial\xi}\right)^2 + \left(\frac{\partial v_1}{\partial\eta}\right)^2\right], \tag{4.40}$$

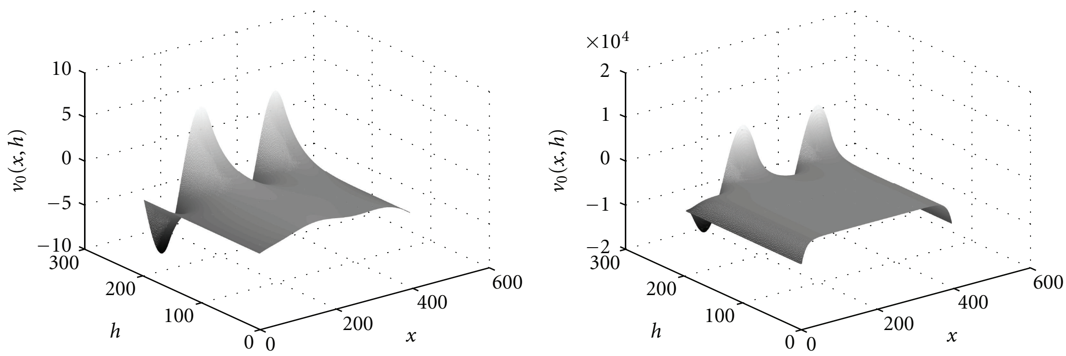
where

$$\begin{aligned} q_2 &= \sqrt{H_4 + \frac{n^2\pi^2}{r^2}}, \qquad H_4 = \mathrm{Re}\,P_r\left[\frac{2i\alpha_1\beta_1}{\gamma\alpha_1+2i}+2i\right], \\ H_5 &= -\frac{\mathrm{Ec}\,\mathrm{Re}\,P_r\alpha\beta_1}{(\alpha+i)^2}, \qquad H_6 = \mathrm{Ec}\,\mathrm{Re}\,P_r\left[\frac{\alpha_1\beta_1\gamma}{\mathrm{Re}_p\beta_2(\gamma\alpha_1+2i)}\left(\frac{\alpha}{\alpha+i}\right)^2 - \frac{1}{\mathrm{Re}}\right]. \end{aligned} \tag{4.41}$$

5. Results and Discussion

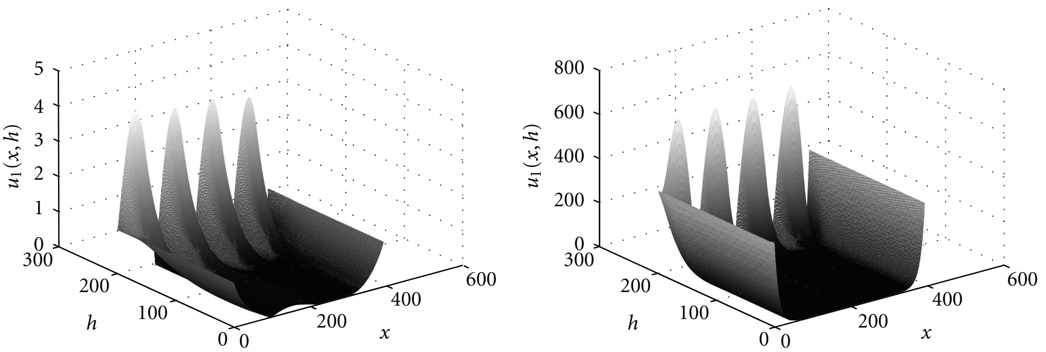
Figures 2, 3, 4, 5, 6, 7, and 8 represent the velocity and temperature fields, respectively, for the fluid and dust particles, which are parabolic in nature. Here we can see that the path of fluid particles is much steeper than that of dust particles. Further, one can see that if the dust is very fine, that is, the mass of the dust particles is negligibly small, then the relaxation time of dust particle decreases and ultimately as $\tau_v \rightarrow 0$ the velocities of fluid and dust particles will be the same. Also we see that the fluid particles will reach the steady state earlier than the dust particles. Further, one can observe the impressive effect of Reynolds number on the velocity fields. It means that the Reynolds number is favorable to the velocity fields, that is, the velocity profiles for both fluid and dust particles increases as the Reynolds number increases.

The graphs are drawn for the following values: $\omega = 0.5$, $N = 0.4$, $\beta = 2$, $\sigma = 1$, $\mathrm{Ec} = 0.02$, $P_r = 0.72$, $T_0 = 0.5$, $T_1 = 1$, $T_2 = 1.5$, $T_3 = 2$, $\tau_v = \tau_T = 0.15$, and $\gamma = 1.4$.



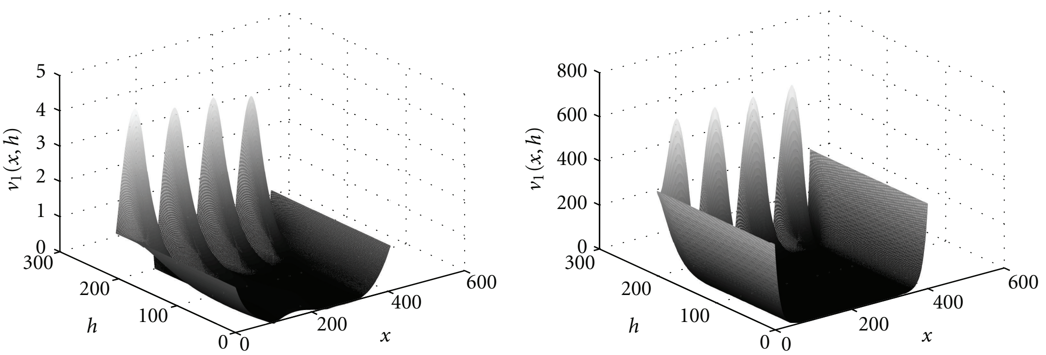
(a) (b)

Figure 3: Variation of dust velocity $v_0(\xi,\eta)$ with ξ and η (for Re = 1 and Re = 10).



(a) (b)

Figure 4: Variation of fluid velocity $u_1(\xi,\eta)$ with ξ and η (for Re = 1 and Re = 10).



(a) (b)

Figure 5: Variation of dust velocity $v_1(\xi,\eta)$ with ξ and η (for Re = 1 and Re = 10).

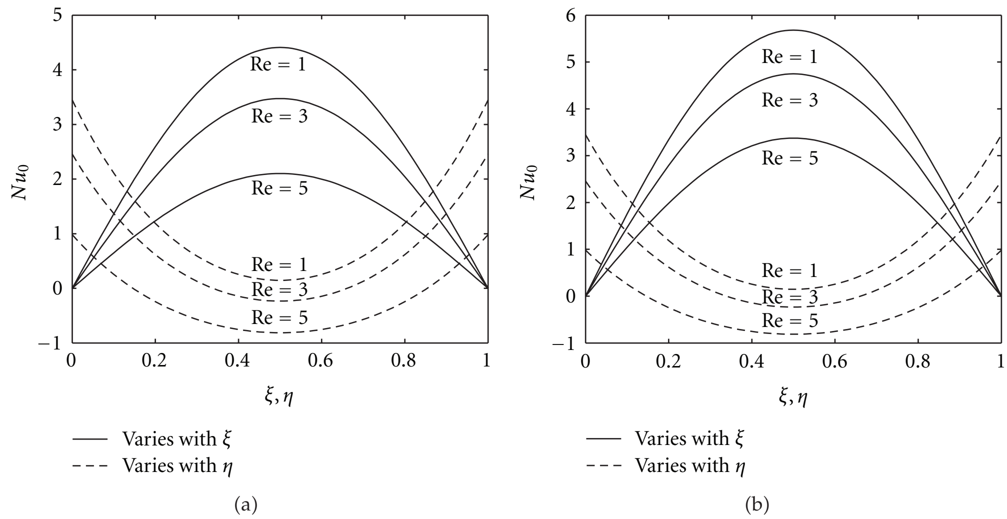


Figure 6: Steady part of the Nusselt number (Nu_0) versus ξ and η (for $\xi = r$ and $\eta = 0$ and $\xi = -r$ and $\eta = 1$).

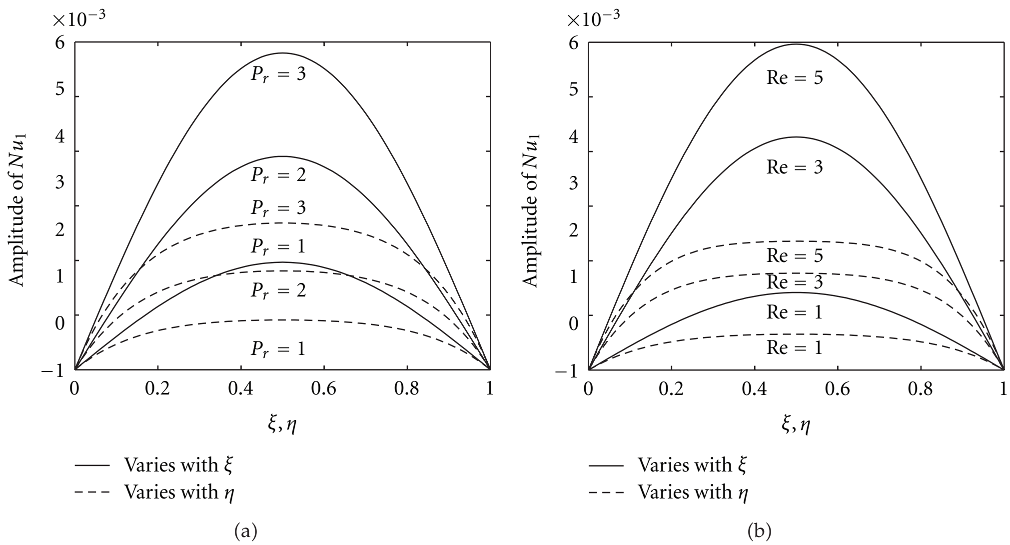


Figure 7: Unsteady part of Nusselt number (Nu_1) versus ξ and η (for $\xi = r$ and $\eta = 0$) and $\xi = -r$ and $\eta = 1$).

Now we discuss the heat transfer at the vertical walls, so we consider the Nusselt number (Nu) of the fluid as

$$\begin{aligned}
\mathrm{Nu} &= -\left.\frac{\partial \theta}{\partial \xi}\right|_{\text{at } \xi=r \text{ or } \xi=-r} \\
&= -\left[\frac{d\theta_0}{d\xi} + \epsilon e^{it}\frac{d\theta_1}{d\xi} + \epsilon^2 e^{2it}\frac{d\theta_2}{d\xi}\right]_{\text{at } \xi=r \text{ or } \xi=-r} \\
&= -\left[\mathrm{Nu}_0 + \epsilon e^{it}\mathrm{Nu}_1 + \epsilon^2 e^{2it}\mathrm{Nu}_2\right]_{\text{at } \xi=r \text{ or } \xi=-r}.
\end{aligned} \tag{5.1}$$

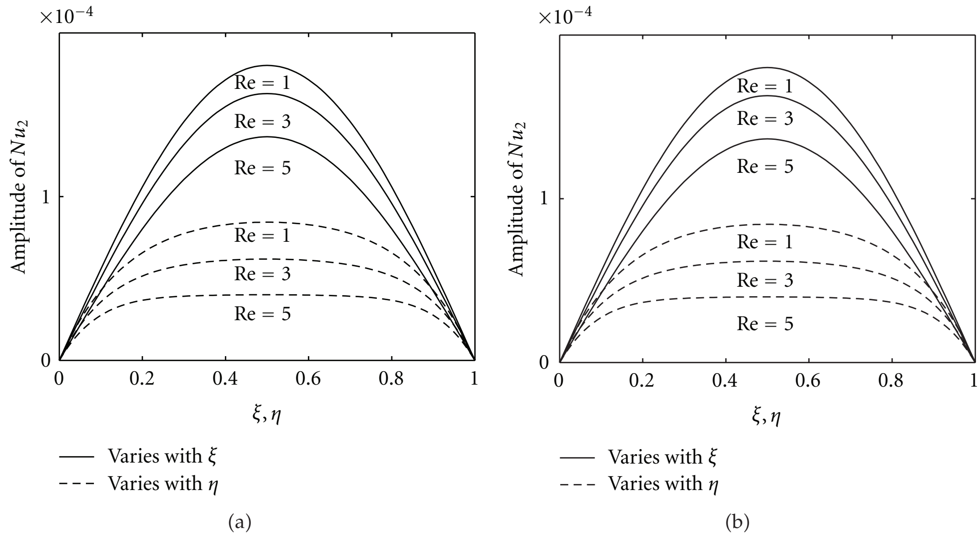


Figure 8: Unsteady part of Nusselt number (Nu_2) versus ξ and η (for $\xi = r$ and $\eta = 0$ and $\xi = -r$ and $\eta = 1$).

Next to discuss is the heat transfer at the horizontal walls, so we consider the Nusselt number (Nu) of the fluid as

$$\begin{aligned}\mathrm{Nu} &= -\frac{\partial\theta}{\partial\eta}\bigg|_{\text{at } \eta=0 \text{ or } \eta=1} \\ &= -\left[\frac{d\theta_0}{d\eta} + \epsilon e^{it}\frac{d\theta_1}{d\eta} + \epsilon^2 e^{2it}\frac{d\theta_2}{d\eta}\right]_{\text{at } \eta=0 \text{ or } \eta=1} \\ &= -\left[\mathrm{Nu}_0 + \epsilon e^{it}\mathrm{Nu}_1 + \epsilon^2 e^{2it}\mathrm{Nu}_2\right]_{\text{at } \eta=0 \text{ or } \eta=1},\end{aligned} \tag{5.2}$$

where Nu_0, Nu_1, and Nu_2 denote the Nusselt number for steady part, unsteady part for coefficient of ϵ, and unsteady part for coefficient of ϵ^2, respectively.

The graphs of steady part of Nusselt number (Nu_0) against ξ and η (at $\eta = 0$ and $\xi = r$ or at $\eta = 1$ and $\xi = -r$) has been drawn in Figure 6. It shows that for different values of Prandtl number, Nusselt number increases with increase in ξ and η.

Figure 7 shows the unsteady part of amplitude of Nusselt number (Nu_1) against ξ and η (at $\eta = 0$ and $\xi = r$ or at $\eta = 1$ and $\xi = -r$). It reveals that for different values of Prandtl number, amplitude of Nusselt number increases with increase of ξ and η. The unsteady part of the amplitude of Nusselt number (Nu_2) against ξ and η has been drawn in Figure 8. Here, one can see that the amplitude of Nusselt number increases with increase of ξ and η for different values of Prandtl number.

6. Conclusions

A detailed analytical study has been carried out for the unsteady flow and heat transfer of a dusty fluid through a rectangular channel. Here, one can see that the flow of fluid particles is parallel to that of dust. Further, one can see that the fluid particles will reach the steady state earlier than the dust particles. From the graphs the impressive effect of Reynolds number on the velocity fields of both fluid and dust phases is evident. It is clear that the effect of Reynolds number on velocity fields is favorable, that is, the velocity profiles for both fluid and dust particles increase as Reynolds number increases.

Further, one can observe the changes in the steady and unsteady parts of amplitude of Nusselt number. It is clear that for different values of Prandtl number steady part of Nusselt number (Nu_0) increases with increase of ξ and η. In the same manner unsteady parts of amplitude of Nusselt number (Nu_1) and (Nu_2) increases with increase of ξ and η for different values of Prandtl number.

Acknowledgments

The authors wish to express their thanks to DST (Department of Science and Technology), india for granting a Major Research Project (no: SR/S4/MS: 470/07, 25-08-2008). They are also grateful to the referees for useful comments.

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