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Research Article

Robust Sensor Fault Reconstruction for Lipschitz Nonlinear Systems

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We extend existing theory on robust nonlinear observer design to the class of nonlinear Lipschitz systems where the systems are subject to sensor faults and disturbances. The designed observer is used for robust reconstruction of fault signals. Allowing bounded unknown disturbances to model system uncertainties, it is shown that by adjusting a design parameter we can trade off between fault reconstruction and disturbance attenuation. An LMI procedure solvable using commercially available softwares is presented. Two examples are presented to illustrate the application of the results.

1. Introduction

Modern control systems strongly rely on actuators, sensors, and data acquisition/interface components to ensure a proper interaction between the physical controlled system and control devices. Any faults in sensors and/or actuators may cause process performance degradation, process shutdown, or a fatal accident. For instance, in feedback control applications, faulty sensors give wrong information about the system status, which could cause disastrous results as the system may go unstable. On the other hand, even if the system is stable, inaccurate sensor values can introduce poor regulation or tracking performance, which may be highly undesirable for many high precision control applications. Similarly, faulty actuators may severely affect the overall system performance. Therefore, there is a growing demand for reliability, safety, and fault tolerance in modern control systems. To improve the reliability and safety, much effort has been made to develop model-based fault detection and isolation (FDI) techniques (see, e.g., [1–3], and the references therein for recent advances). One of the particular interesting techniques among all model-based techniques for FDI is an observer-based fault detection filter design. The goal has been to utilize the underlying system model to generate a *residual signal* [4–6]. This signal is then processed to detect the occurrence of a fault and possibly identification of its type (bias, drift, noise, or complete failure) and location. However, the magnitude of the fault cannot be provided by FDI. The process of estimating the magnitude of the fault is called *fault reconstruction*. This approach is however different from the residual generation techniques in the sense that it not only detects and isolates the faults, but provides an estimate of the faults that can be then used to design a fault tolerant controller (FTC) which stabilizes the closed-loop system and guarantees a prescribed performance level in the presence of faults; for example, see [7–10]. For example, if the magnitude of the sensor fault can be obtained, the correct measurement can be obtained by subtracting the fault from the faulty measurement. Thus, the controller and observer can continue to function normally without the need of reconfiguration. Clearly, sensor and/or actuator fault reconstruction plays a key role in the FTC design.

During the last decade, a number of results have been reported on sensor and/or actuator fault reconstruction for Linear Time-Invariant (LTI) systems: pseudo-inverse [11], discrete max-min approach [12], sliding mode-based techniques [13–20], frequency weighted approach [21], adaptive techniques [22, 23], and descriptor approach [24]. In contrast to the LTI case, however, the nonlinear problem lacks a universal approach and is currently an active area of research, for example, see [25–39] for some important nonlinear results. The main obstacle in the solution of the observer-based nonlinear fault reconstruction problem is the lack of a universal approach for nonlinear observer synthesis.

A class of nonlinear systems that has recieved much attention in the literature is the class of Lipschitz nonlinear systems [40]. In recent years, different approaches to the observer-based FDI problem for this class of nonlinear systems have been reported and is currently an active area of research, for example, see descriptor system approach [10], unknown input observers (UIO) [35], adaptive techniques [37], sensor fault diagnosis using dynamic observers [38], and high-gain observers [39]. It is worth mentioning that the technique proposed for fault reconstruction in this paper is different from [10, 39]. The main idea in these works is to include the fault model in the state variables and try to estimate the states of the resulting augmented system. These techniques can however increase the order of the augmented system. In addition, they are only applicable for a special class of faults; for example, constant-like faults, step-like faults and ramp-like faults. In particular, the technique presented in [10] is valid provided that the kth derivative of the fault signal f is bounded, even when k is high. The use of high-gain observer for fault reconstruction as discussed in Proposition 3.1 in [39] is limited due to strict conditions on the system dynamics. For example in [39], the system dynamic must be in a semitriangular form and the states must remain bounded. In [35], an unknown input observer is designed for estimation of disturbances and faults, but there is no systematic solution for computing observer gain, for example, in terms of linear matrix inequalities (LMIs).

The main contribution of this paper is the generalization of the obtained results for sensor fault reconstruction in [16, 19] for linear time-invariant (LTI) systems to continuous-time Lipschitz nonlinear systems in the presence of disturbances and measurement noises. This generalization is based on robust $\mathscr{H}_\infty$ observer design recently reported in [41]. In this direction, the sensor fault reconstruction problem is formulated as an LMI feasibility problem whose solution is easily generated by using commercial softwares [42, 43]. As it will be shown, by adjusting a single design parameter, it becomes possible to trade off between fault reconstruction performance and robustness to unknown disturbances and noises. The proposed approach is practical for real systems and FTC design.

This paper is organized as follows. In Section 2, the sensor fault reconstruction problem is formulated. In Section 3, an easily implementable design algorithm summarizes the proposed methodology for fault reconstruction. In Section 4, this algorithm is applied to two numerical examples and simulation results are presented. Concluding remarks are given in Section 5.

The notation used in this paper is fairly standard. For a given matrix A, A^T denotes its transpose. I denotes unity matrix with appropriate dimension. If A and B are symmetric matrices, $A \geq B$ (resp., $A > B$) denotes $A - B$ positive semidefinite (resp., positive definite) and $A \leq B$ (resp., $A < B$) denotes $A - B$ negative semidefinite (resp., negative definite). $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote minimum and maximum eigenvalue of M, respectively. The space $\mathcal{L}_2[0,\infty]$ represents the set of all signals $\omega(t)$ which are square integrable and satisfy $\int_0^\infty \omega(t)^T\omega(t)dt < \infty$; and the $\mathcal{L}_2$-norm of $\omega(t) \in \mathcal{L}_2$ is defined by $\|\omega\|_2 := (\int_0^\infty \omega(t)^T\omega(t)dt)^{1/2}$. The following result is used in the paper.

Lemma 1.1 (see [22]). *Let D, S and F be real matrices of appropriate dimensions and F satisfying $F^TF \leq I$. Then for any scalar $\epsilon > 0$ and vectors $x, y \in \mathbb{R}^n$, we have*

$$2x^TDFSy \leq \epsilon^{-1}x^TDD^Tx + \epsilon y^TS^TSy. \tag{1.1}$$

2. Problem Formulation

We consider the nonlinear systems given by S:

$$\mathbf{S} : \begin{cases} \dot{x} = Ax + \Gamma(y,u,t) + B_\phi\phi(x,u,t) + Bw, \\ y = Cx + Du + Ew + Ff, \end{cases} \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control, $y \in \mathbb{R}^p$ is the output, and $\Gamma(y,u,t)$ is a known nonlinear vector function. The input $w \in \mathbb{R}^l$ is assumed to be the unknown disturbance which can also be used to represent a general class of modeling errors. In any case, w is assumed to be an unknown exogenous disturbance/noise. Here, sensor faults are described by the vector $f \in \mathbb{R}^q$, assumed to be zero prior to the failure time nonzero after the fault occurrence. A, B, B_ϕ, C, D, E and F are assumed to be known constant matrices of appropriate dimensions. It is worth noting that the distribution matrix B_ϕ indicates how the system (2.1) is affected by the nonlinearity ϕ. We assume that $\operatorname{rank}(C) = p$, $\operatorname{rank}(F) = q$, and $p \geq q$. Without loss of generality, it can be assumed that the outputs of the system have been reordered (and scaled if necessary) so that the matrix F has a structure

$$F = \begin{pmatrix} 0 \\ F_2 \end{pmatrix}, \tag{2.2}$$

where $F_2 \in \mathbb{R}^{q\times q}$ is a nonsingular matrix. As mentioned in [16], the assumption that only certain sensors are fault prone is a limitation. However in practical situations, some sensors may be more vulnerable to damage or may be more sensitive or delicate in terms of construction than others, and so such a situation is not unrealistic. Also certain key sensors may have backups (hardware redundancy) and so essentially a fault-free signal can be assumed from a certain subset of the sensors.

Finally, we assume that the system (2.1) is locally Lipschitz in a region Ω containing the origin, uniformly in u, that is:

$$\left\|\phi(x_1,u,t)-\phi(x_2,u,t)\right\| \le \alpha\|x_1-x_2\|, \tag{2.3}$$

for all $u \in \mathbb{R}^m$, for all $t \in \mathbb{R}^+$, for all x_1 and $x_2 \in \Omega$. Here, the parameter $\alpha > 0$ is referred to as the Lipschitz constant and is independent of x, u, and t. Many nonlinearities are locally Lipschitz. Examples include trigonometric nonlinearities occurring in robotics, nonlinearities which are square or cubic in nature, and so forth. The function ϕ can also be considered as a perturbation affecting the system; see [40] for more details about nonlinear Lipschitz systems.

Scaling the output y and partitioning appropriately yields

$$\begin{aligned} y_1 &= C_1x + D_1u + E_1w, \\ y_2 &= C_2x + D_2u + E_2w + F_2f, \end{aligned} \tag{2.4}$$

where $y_1 \in \mathbb{R}^{p-q}$ and C_1, C_2, D_1, D_2, E_1, and E_2 are appropriate matrices depending on C, D and E. The output vector has now been partitioned into nonfaulty (y_1) and potentially faulty (y_2). Notice now that the subsystem (2.1) and y_1 in (2.4) makes up a fault-free system. Assume further that (A, C_1) is detectable.

Consider a nonlinear observer for the fault-free system defined by (2.1) and y_1 in(2.4)

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \Gamma(y,u,t) + B_\phi\phi(\hat{x},u,t) + L(y_1 - C_1\hat{x} - D_1u), \\ \hat{y} &= C\hat{x} + Du, \end{aligned} \tag{2.5}$$

where $\hat{x} \in \mathbb{R}^n$ is an estimate for the state x and $L \in \mathbb{R}^{n\times(p-q)}$ is the observer gain. Define $e := x - \hat{x}$ as the state estimation error. Equations (2.1), (2.4), and (2.5) are combined to yield

$$\dot{e} = (A - LC_1)e + B_\phi\tilde{\phi} + (B - LE_1)w, \tag{2.6}$$

where $\tilde{\phi} = \phi(x,u,t) - \phi(\hat{x},u,t)$. A well-known result in [44] states that the error system (2.6) is asymptotically stable for all ϕ in (2.3) with a Lipschitz constant α if the observer gain L can be chosen in such a way that

$$\alpha < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}, \tag{2.7}$$

where

$$(A - LC_1)^T P + P(A - LC_1) = -Q, \quad Q > 0. \tag{2.8}$$

The ratio in (2.7) is maximized when $Q = I$ [45]. The problem is then reduced to that of choosing L to satisfy

$$\alpha < \frac{1}{2\lambda_{\max}(P)}. \tag{2.9}$$

As shown in [41], using Schur's complement lemma, the inequality (2.9) is equivalent to

$$\begin{pmatrix} \frac{1}{2\alpha}I & P \\ P & \frac{1}{2\alpha}I \end{pmatrix} > 0. \tag{2.10}$$

In the following, motivated by the development in [16, 19] for the LTI systems, an approach for reconstructing the sensor fault f from the residual for Lipschitz nonlinear systems in (2.1) is proposed. Define a reconstruction for the sensor fault f

$$\hat{f} = K\nu, \tag{2.11}$$

where $\nu = y - \hat{y}$ is the residual and

$$K = \begin{pmatrix} K_1 & F_2^{-1} \end{pmatrix}, \tag{2.12}$$

with $K_1 \in \mathbb{R}^{q\times(p-q)}$ being a weighting matrix. It is easy to show that combining (2.1), (2.2), (2.11), and (2.12) yields

$$e_f = H_1 e + H_2 w, \tag{2.13}$$

where $e_f = f - \hat{f}$ is the fault reconstruction error, and

$$H_1 = -KC, \qquad H_2 = -KE. \tag{2.14}$$

Equations (2.6) and (2.13) show the effect of the disturbance w on the quality of the fault reconstruction error, that is,

$$\boldsymbol{\Sigma} : \begin{cases} \dot{e} = (A - LC_1)e + B_\phi\tilde{\phi} + (B - LE_1)w, \\ e_f = H_1 e + H_2 w. \end{cases} \tag{2.15}$$

The objective now would be to minimize the effect of w on e_f. To achieve unknown disturbance attenuation and fault reconstruction, the following problem can be formulated: *find the gain L such that the system* Σ *in* (2.15) *be asymptotically stable and the* $\mathcal{L}_2$*-gain from the* *disturbance w to the fault reconstruction error e_f is less than or equal to a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, that is,*

$$\|e_f\|_2 < \gamma\|w\|_2. \tag{2.16}$$

This motivates us to consider the following optimization problem.

Problem 1. Given $\gamma > 0$, find the gain L such that the error dynamic system in (2.15) be asymptotically stable and

$$J := \int_0^\infty \left(e_f{}^T e_f - \gamma^2 w^T w\right) dt < 0. \tag{2.17}$$

In the next section, a solution is proposed to Problem 1 in terms of LMIs.

3. Fault Reconstruction

Following the lines of [41] for robust nonlinear observer design, we propose an LMI-based solution to Problem 1 that leads to a constructive algorithm for sensor fault reconstruction. The following result summarizes the main result of this section.

Theorem 3.1. *Consider the nonlinear system* (2.1)*. Given Lipschitz constant $\alpha > 0$ and $\gamma > 0$, there exists an nth-order nonlinear observer in the form* (2.5) *which solves Problem 1, if there exist $\beta > 0$ and the solutions $P = P^T > 0$ and Z such that the following LMIs have a solution*

$$\begin{pmatrix} \Omega & PB_\phi & H_1{}^T H_2 + PB - ZE_1 \\ B_\phi^T P & -\beta I & 0 \\ H_2{}^T H_1 + B^T P - E_1{}^T Z^T & 0 & H_2{}^T H_2 - \gamma^2 I \end{pmatrix} < 0,$$
$$\begin{pmatrix} \frac{1}{2\alpha} I & P \\ P & \frac{1}{2\alpha} I \end{pmatrix} > 0, \tag{3.1}$$

where

$$\Omega = A^T P + PA - C_1{}^T Z^T - ZC_1 + \beta\alpha^2 I + H_1{}^T H_1. \tag{3.2}$$

Once the problem is solved

$$L = P^{-1} Z. \tag{3.3}$$

Proof. Define a Lyapunov function $V = e^T P e$, where $P = P^T > 0$ satisfies in (2.10). From the error system (2.6), we have

$$\begin{aligned} \dot{V} &= \dot{e}^T P e + e^T P \dot{e} \\ &= -e^T Q e + 2\tilde{\phi}^T {B_\phi}^T P e + w^T (B - LE_1)^T P e + e^T P (B - LE_1) w, \end{aligned} \tag{3.4}$$

where Q is given by (2.8). The second term in the right-hand side of (3.4) can be upper bounded as follows

$$\begin{aligned} 2\tilde{\phi}^T {B_\phi}^T P e &\leq 2 \left\| \tilde{\phi} \right\| \left\| {B_\phi}^T P e \right\| \\ &\leq 2\alpha \|e\| \left\| {B_\phi}^T P e \right\|. \end{aligned} \tag{3.5}$$

Using Lemma 1.1, we have

$$2\alpha \|e\| \left\| {B_\phi}^T P e \right\| \leq e^T \left(\beta \alpha^2 I + \frac{1}{\beta} P B_\phi {B_\phi}^T P \right) e, \tag{3.6}$$

where β is any positive real constant and hence from (3.4), we have

$$\dot{V} \leq e^T \tilde{Q} e + w^T (B - LE_1)^T P e + e^T P (B - LE_1) w, \tag{3.7}$$

where

$$\tilde{Q} = (A - LC_1)^T P + P(A - LC_1) + \beta \alpha^2 I + \frac{1}{\beta} P B_\phi {B_\phi}^T P. \tag{3.8}$$

Now, from (2.17), it is easy to show that

$$J < \int_0^\infty \left({e_f}^T e_f - \gamma^2 w^T w + \dot{V} \right) dt. \tag{3.9}$$

Therefore, a sufficient condition for $J < 0$ is that

$$\forall t \in [0, \infty), \quad {e_f}^T e_f - \gamma^2 w^T w + \dot{V} < 0. \tag{3.10}$$

But, from (3.7), we have

$$
\begin{aligned}
{e_f}^T e_f - \gamma^2 w^T w + \dot{V} &= (H_1 e + H_2 w)^T (H_1 e + H_2 w) - \gamma^2 w^T w + \dot{V} \\
&\leq e^T \widetilde{Q} e + w^T (B - LE_1)^T P e + e^T P (B - LE_1) w \\
&\quad + e^T {H_1}^T H_1 e + e^T {H_1}^T H_2 w + w^T {H_2}^T H_1 e \\
&\quad + w^T {H_2}^T H_2 w - \gamma^2 w^T w \\
&= \begin{pmatrix} e^T & w^T \end{pmatrix} M \begin{pmatrix} e \\ w \end{pmatrix},
\end{aligned}
\tag{3.11}
$$

where

$$
M = \begin{pmatrix} {H_1}^T H_1 + \widetilde{Q} & P(B - LE_1) + {H_1}^T H_2 \\ {H_2}^T H_1 + (B - LE_1)^T P & {H_2}^T H_2 - \gamma^2 I \end{pmatrix}. \tag{3.12}
$$

Thus a sufficient condition for $J < 0$ is that $M < 0$. Using Schur's complement lemma and the change of variable $Z = PL$, the inequality $M < 0$ can be replaced by (3.1) immediately. Therefore, if there exists scalars $\beta > 0$ and $\gamma > 0$ and matrices $P = P^T > 0$ and Z such that the LMIs in (3.1) have a solution, then $L = P^{-1} Z$. □

Using (2.5) and (2.11), the nonlinear dynamical system for sensor fault reconstruction is given by

$$
\overline{\Sigma} : \begin{cases} \dot{\widehat{x}} = A\widehat{x} + \Gamma(y, u, t) + B_\phi \phi(\widehat{x}, u, t) + L(y_1 - C_1 \widehat{x} - D_1 u), \\ \widehat{f} = K(y - C\widehat{x} - Du). \end{cases} \tag{3.13}
$$

Thanks to Theorem 3.1, Problem 1 can be solved efficiently using the following algorithm and by reducing γ iteratively, an optimal solution is approached.

Algorithm 1. Given plant (2.1) with Lipschitz constant $\alpha > 0$, construct the sensor fault signal by performing the following steps.

Step 1. Choose the weighting matrix K_1 and compute K using (2.12).

Step 2. Given $\gamma > 0$ and $\beta > 0$, obtain $P = P^T > 0$ and Z to the LMIs in (3.1).

Step 3. Compute the gain L using (3.3).

Step 4. Construct the nonlinear dynamical system $\overline{\Sigma}$ in (3.13).

This algorithm is constructive and can be implemented using standard scientific softwares such as Scilab [42] and Matlab [43].

Remark 3.2. Although the main objective of this paper is sensor fault reconstruction, but the proposed method has good potential to extended to the even more interesting case of the reconstruction of actuator fault situations. It can however be an interesting topic for future research and it is under investigation. A good staring point for this research can be motivated by the developments in [46] to transform the plant (2.1) into two subsystems with one of them decoupled from the actuator fault. Then, the nonlinear observer (2.5) could be designed to provide the estimation of unmeasurable state, which are used to construct actuator fault estimation algorithm. It is worth mentioning that a constructive algorithm based on mixed $\mathcal{H}_2/\mathcal{H}_\infty$ approach is also proposed in [25] for actuator fault reconstruction for Lipschitz nonlinear systems.

4. Numerical Examples

To illustrate the application of the results obtained in the paper, we consider two different examples of nonlinear systems.

Example 4.1. Consider the plant (2.1) with the following state space matrices for an aircraft model [47]

$$A = \begin{pmatrix} -1.05 & -2.55 & 0 & 0 & -169 & -0.0091 \\ 2.55 & -1.05 & 0 & 0 & 57.09 & 0.0017 \\ 0 & 0 & -77.53 & 39.57 & 0 & 0 \\ 0 & 0 & 0 & -20.2 & 0 & 0 \\ 0 & 0 & -8.8 & 0 & -20.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1 \end{pmatrix}, \qquad B_\phi = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$C = \begin{pmatrix} -0.01 & 0.09 & 0.07 & 0 & 0 & 0 \\ -0.48 & -0.59 & 0 & 0 & -49.51 & -0.0026 \\ 0.03 & 0.09 & -0.06 & 0 & 0 & 0 \\ 0.26 & -0.07 & 0.01 & 0 & 0 & 0 \end{pmatrix}, \qquad D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{4.1}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad E = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad F = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -4.49 \\ 0 \\ 0 \end{pmatrix} u,$$

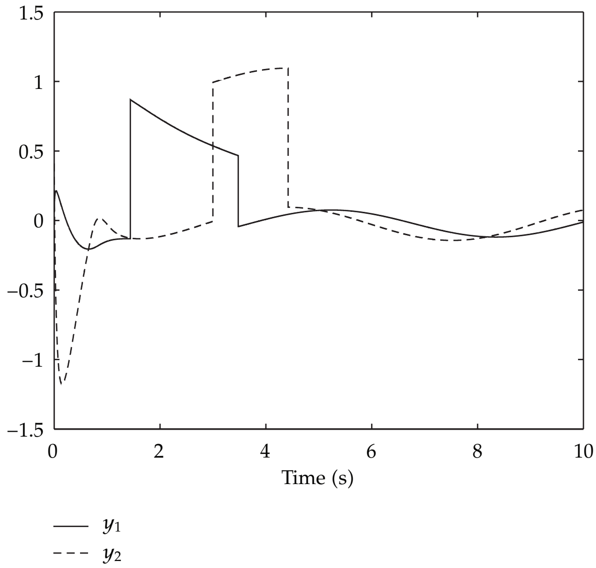

Figure 1: The faulty outputs y_1 and y_2.

and $\phi(x, u, t) = 0.5(|x_1(t) + 1| - |x_1(t) - 1|)$ with Lipschitz constant $\alpha = 1$. The sensor fault reconstruction is obtained by using Algorithm 1 where $\gamma = 0.3$ and $\beta = 0.01$. The LMI minimization has been performed using LMITOOL, a user-friendly Scilab package [42]. The simulation results are shown in Figures 1–3. The disturbance w_1 is set as [47]

$$w_1(t) = \frac{1}{1 + \sqrt{t}}, \quad t \geq 0. \tag{4.2}$$

And w_2 and w_3 are white noise processes which are assumed to be zero-mean white noise processes with variance 0.05. The control signal is assumed to be $u(t) = \sin(t)$. The faulty outputs y_1 and y_2 are shown in Figure 1. As shown in Figures 2-3, the fault reconstruction scheme reconstructs the faults perfectly when sensor faults are applied in the presence of disturbance and noises which justify the proposed scheme for fault tolerant control.

Here, a comparison of the estimation capabilities of the presented approach with the *descriptor system approach* for Lipschitz nonlinear systems as recently proposed in [10] can be performed. In this direction, the sensor fault model (2.1) with $D = 0$ can be denoted as

$$\begin{gathered} \overline{E}\dot{\overline{x}} = \overline{A}\overline{x} + \overline{\Gamma} + \Phi(x, u, t) + \overline{B}\omega, \\ y = \overline{C}\overline{x}, \end{gathered} \tag{4.3}$$

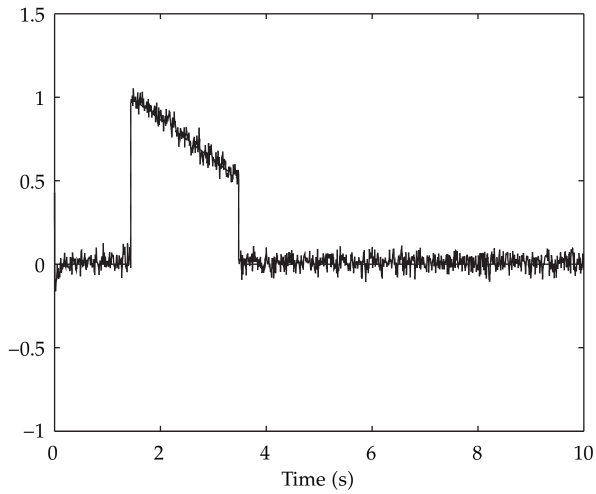

Figure 2: The reconstructed sensor fault $\widehat{f}_1$.

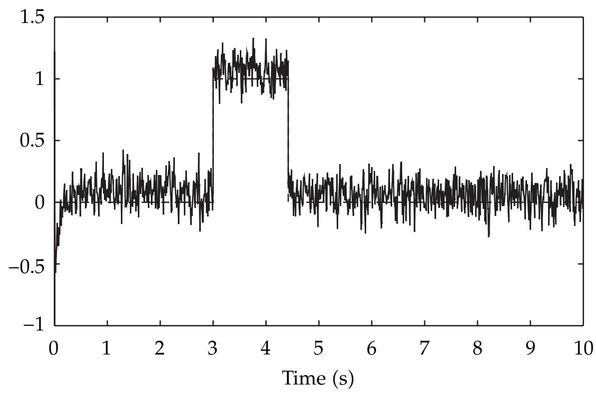

Figure 3: The reconstructed sensor fault $\widehat{f}_2$.

where

$$\overline{x} = \begin{pmatrix} x \\ x_{wf} \end{pmatrix}, \quad x_{wf} = E\omega + Ff,$$

$$\overline{E} = \begin{pmatrix} I_n & 0 \\ 0 & 0 \end{pmatrix}, \qquad \overline{A} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}, \qquad \overline{\Gamma} = \begin{pmatrix} \Gamma \\ 0 \end{pmatrix}, \tag{4.4}$$

$$\Phi(x,u,t) = \begin{pmatrix} B_\phi \phi(x,u,t) \\ 0 \end{pmatrix}, \qquad \overline{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \qquad \overline{C} = \begin{pmatrix} C & I_p \end{pmatrix}.$$

Following the approach presented in [10], a sensor fault estimator in the form

$$\dot{\hat{\overline{x}}} = M\hat{\overline{x}} + Ly + \overline{\Gamma} + \Phi(\hat{x}, u, t),$$
$$\hat{f} = N\hat{\overline{x}}, \tag{4.5}$$

can be constructed, where $\hat{x} = (I_n \ \ 0)\hat{\overline{x}}$ and the matrices M, L, and N can be obtained through satisfying an LMI as proposed in Theorem 2 in [10]. Using LMITOOL in Scilab package, it can be shown that for the aircraft example there does not exist any solution to this LMI for all $\alpha > 0$ and $\gamma > 0$, so the descriptor system approach as proposed in [10] is no longer applicable this exhibits the significance of our approach proposed in this paper.

Example 4.2. Consider the following nonlinear system [41]

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} x + \begin{pmatrix} x_1^3 \\ -6x_1^5 - 6x_1^2 x_2 - 2x_1^4 - 2x_1^2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} w,$$
$$y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} w + \begin{pmatrix} 0 \\ 1 \end{pmatrix} f, \tag{4.6}$$

where $w = (w_1 \ \ w_2)^T$, where w_1 is the disturbance and w_2 is the measurement noise which is assumed to be a zero-mean white noise process with unit covariance and f is the sensor fault. The fault reconstruction scheme is performed by using Algorithm 1 in the previous section with $\alpha = 1.17$ and $\beta = 3$. To be able to make a fair comparison between the fault reconstruction for different values of γ, the actual and estimated fault are displayed in Figures 4, 5, 6, and 7. As shown in these figures, in order to analyze the performance of the fault reconstruction, a sensor fault f with magnitude 1 and a disturbance w_1 are applied. Figures 4–7 clearly indicate that by reducing γ, the effect of w_1 on $\hat{f}$ can be made arbitrarily small and the sensor fault f can be effectively reconstructed. Also, as shown in Figure 7, when γ is reduced to optimal value 0.38, the effect of noise will increase. This clearly shows that there is a definite tradeoff between fault reconstruction, disturbance attenuation, and noise rejection.

5. Conclusion

In this paper, a robust sensor fault reconstruction method for a class of Lipschitz nonlinear systems is proposed through LMI optimization in the presence of disturbances and noises. The advantage of the fault reconstruction method is that it provides a good estimate of faults, thus providing useful information for fault tolerant controller design. As shown in simulation results, by adjusting a single parameter, it becomes possible to trade off between fault reconstruction, disturbance attenuation and noise rejection.

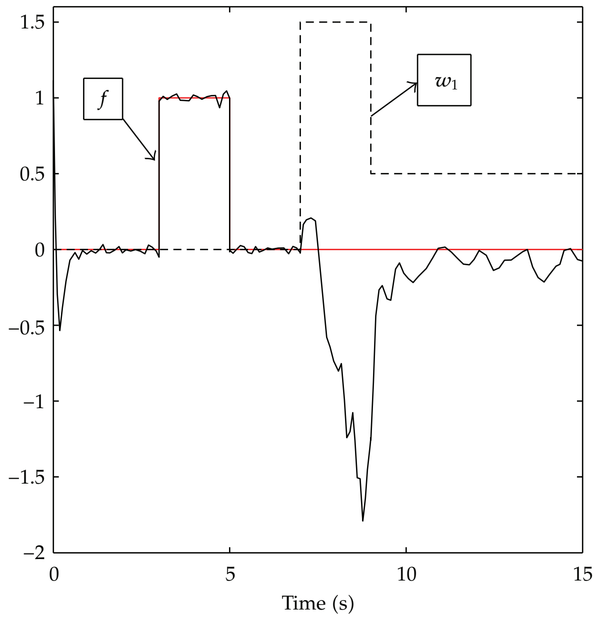

Figure 4: Fault reconstruction and disturbance attenuation for $\gamma = 1$.

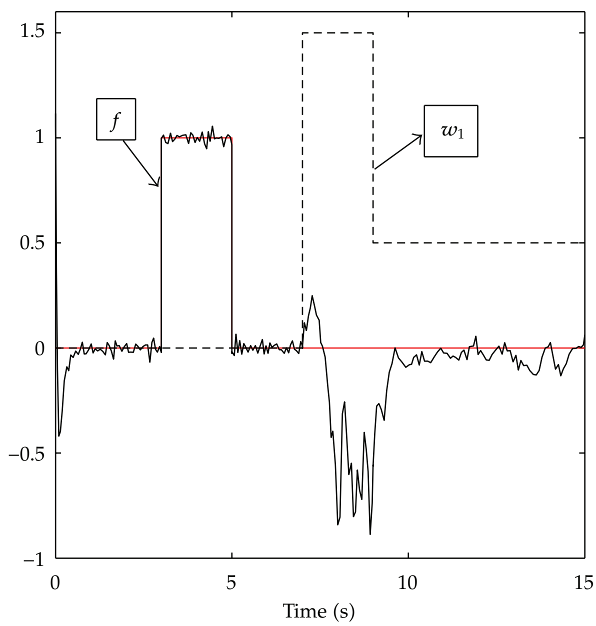

Figure 5: Fault Reconstruction and disturbance attenuation for $\gamma = 0.7$.

Further research work includes two aspects. The first one is that the proposed sensor fault reconstruction approach could be extended to nonlinear systems with arbitrarily large Lipschitz constant or *one-sided Lipschitz systems* as described in [48]. Possible extensions to

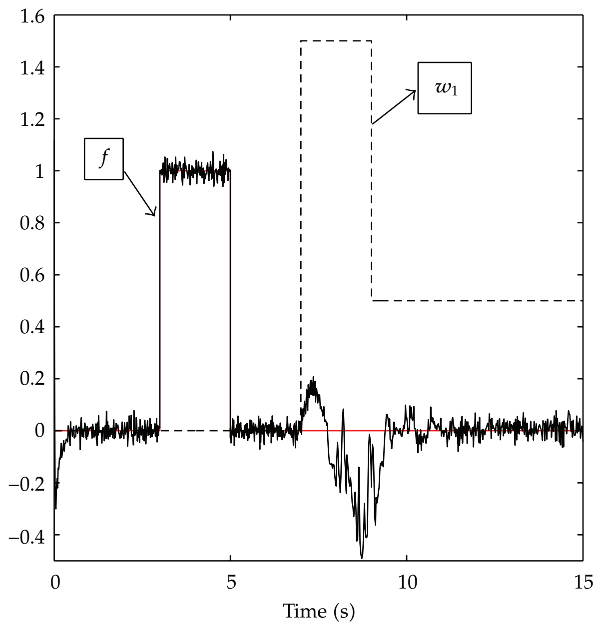

Figure 6: Fault Reconstruction and disturbance attenuation for $\gamma = 0.5$.

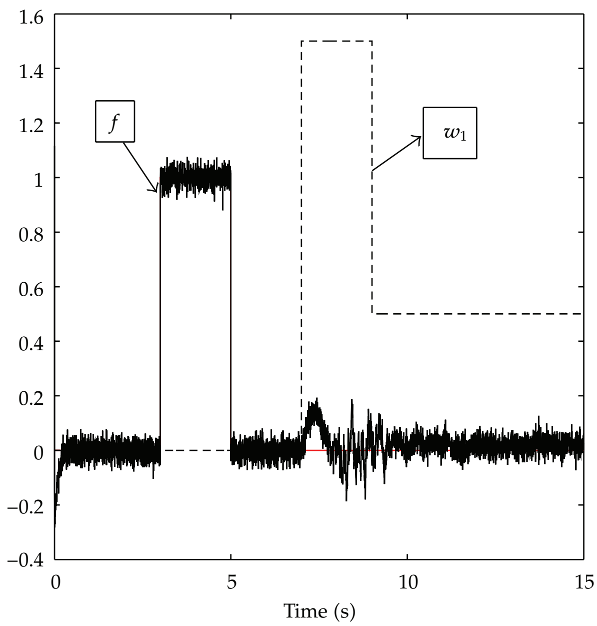

Figure 7: Fault Reconstruction and disturbance attenuation for $\gamma = 0.38$.

a large class of uncertain nonlinear systems as described in [49] with simultaneous actuator and sensor faults and implementation on an experimental setup similar to that in [19] could be another interesting issues.

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