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Research Article

Rayleigh Waves in Generalized Magneto-Thermo-Viscoelastic Granular Medium under the Influence of Rotation, Gravity Field, and Initial Stress

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The surface waves propagation in generalized magneto-thermo-viscoelastic granular medium subjected to continuous boundary conditions has been investigated. In addition, it is also subjected to thermal boundary conditions. The solution of the more general equations are obtained for thermoelastic coupling. The frequency equation of Rayleigh waves is obtained in the form of a determinant containing a term involving the coefficient of friction of a granular media which determines Rayleigh waves velocity as a real part and the attenuation coefficient as an imaginary part, and the effects of rotation, magnetic field, initial stress, viscosity, and gravity field on Rayleigh waves velocity and attenuation coefficient of surface waves have been studied in detail. Dispersion curves are computed numerically for a specific model and presented graphically. Some special cases have also been deduced. The results indicate that the effect of rotation, magnetic field, initial stress, and gravity field is very pronounced.

1. Introduction

The dynamical problem in granular media of generalized magneto-thermoelastic waves has been studied in recent times, necessitated by its possible applications in soil mechanics, earthquake science, geophysics, mining engineering, and plasma physics, and so forth. The granular medium under consideration is a discontinuous one and is composed of numerous large or small grains. Unlike a continuous body each element or grain cannot only translate but also rotate about its center of gravity. This motion is the characteristic of the medium and has an important effect upon the equations of motion to produce internal friction. It was assumed that the medium contains so many grains that they will never be separated from each other during the deformation and that each grain has perfect thermoelasticity. The effect of the granular media on dynamics was pointed out by Oshima [1]. The dynamical problem of a generalized thermoelastic granular infinite cylinder under initial stress has been illustrated by El-Naggar [2]. Rayleigh wave propagation of thermoelasticity or generalized thermoelasticity was pointed out by Dawan and Chakraporty [3]. Rayleigh waves in a magnetoelastic material under the influence of initial stress and a gravity field were discussed by Abd-Alla et al. [4] and El-Naggar et al. [5].

Rayleigh waves in a thermoelastic granular medium under initial stress on the propagation of waves in granular medium are discussed by Ahmed [6]. Abd-Alla and Ahmed [7] discussed the problem of Rayleigh wave propagation in an orthotropic medium under gravity and initial stress. Magneto-thermoelastic problem in rotating nonhomogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model is discussed by Abd-Alla and Mahmoud [8]. Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section is discussed by Venkatesan and Ponnusamy [9]. Some problems discussed the effect of rotation of different materials. Thermoelastic wave propagation in a rotating elastic medium without energy dissipation was studied by Roychoudhuri and Bandyopadhyay [10]. Sharma and Grover [11] studied the body wave propagation in rotating thermoelastic media. Thermal stresses in a rotating nonhomogeneous orthotropic hollow cylinder were discussed by El-Naggar et al. [12]. Abd-El-Salam et al. [13] investigated the numerical solution of magneto-thermoelastic problem nonhomogeneous isotropic material.

In this paper, the effect of magnetic field, rotation, thermal relaxation time, gravity field, viscosity, and initial stress on propagation of Rayleigh waves in a thermoelastic granular medium is discussed. General solution is obtained by using Lame's potential. The frequency equation of Rayleigh waves is obtained in the form of a determinant. Some special cases have also been deduced. Dispersion curves are computed numerically for a specific model and presented graphically. The results indicate that the effect of rotation, magnetic field, initial stress, and gravity field are very pronounced.

2. Formulation of the Problem

Let us consider a system of orthogonal Cartesian axes, Oxyz, with the interface and the free surface of the granular layer resting on the granular half space of different materials being the planes $z = K$ and $z = 0$, respectively. The origin O is any point on the free surface, the z-axis is positive along the direction towards the exterior of the half space, and the x-axis is positive along the direction of Rayleigh waves propagation. Both media are under initial compression stress P along the x-direction and the primary magnetic field $\overrightarrow{H_0}$ acting on y-axis, as well as the gravity field and incremental thermal stresses, as shown in Figure 1. The state of deformation in the granular medium is described by the displacement vector $\overrightarrow{U}(u, o, w)$ of the center of gravity of a grain and the rotation vector $\overrightarrow{\xi}(\xi, \eta, \zeta)$ of the grain about its center of gravity. The elastic medium is rotating uniformly with an angular velocity $\underline{\Omega} = \Omega\underline{n}$, where $\underline{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame has two additional terms, $\underline{\Omega} \times (\underline{\Omega} \times \underline{u})$

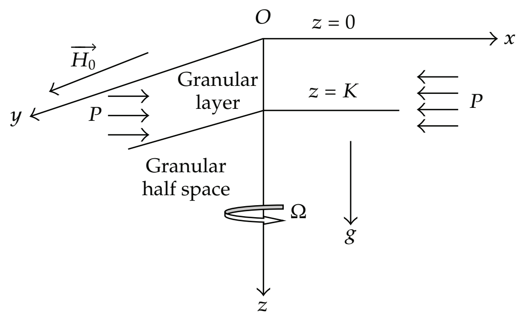

Figure 1: Depiction of the problem.

is the centripetal acceleration due to time varying motion only, and $2\overrightarrow{\Omega} \times \overrightarrow{\dot{u}}$ is the Coriolis acceleration, and $\underline{\Omega} = (0, \Omega, 0)$.

The electromagnetic field is governed by Maxwell equations, under the consideration that the medium is a perfect electric conductor taking into account the absence of the displacement current (SI) (see the work of Mukhopadhyay [14]):

$$\begin{gathered}\overrightarrow{J} = \text{curl } \overrightarrow{h}, \\ -\mu_e \frac{\partial \overrightarrow{h}}{\partial t} = \text{curl } \overrightarrow{E}, \\ \text{div } \overrightarrow{h} = 0, \\ \text{div } \overrightarrow{E} = 0, \\ \overrightarrow{E} = -\mu_e \left(\frac{\partial \overrightarrow{u}}{\partial t} \times \overrightarrow{H} \right),\end{gathered} \tag{2.1}$$

where

$$\overrightarrow{h} = \text{curl}\left(\overrightarrow{u} \times \overrightarrow{H_0}\right), \qquad \overrightarrow{H} = \overrightarrow{H_0} + \overrightarrow{h}, \quad \overrightarrow{H_0} = (0, H_0, 0), \tag{2.2}$$

where $\overrightarrow{h}$ is the perturbed magnetic field over the primary magnetic field vector, $\overrightarrow{E}$ is the electric intensity, $\overrightarrow{J}$ is the electric current density, μ_e is the magnetic permeability, $\overrightarrow{H_0}$ is the constant primary magnetic field vector, and $\overrightarrow{u}$ is the displacement vector.

The stress and stress couple may be taken to be nonsymmetric, that is, $\tau_{ij} \neq \tau_{ji}$, $M_{ij} \neq M_{ji}$. The stress tensor τ_{ij} can be expressed as the sum of symmetric and antisymmetric tensors

$$\tau_{ij} = \sigma_{ij} + \sigma'_{ij}, \tag{2.3}$$

where

$$\sigma_{ij} = \frac{1}{2}(\tau_{ij} + \tau_{ji}), \qquad \sigma'_{ij} = \frac{1}{2}(\tau_{ij} - \tau_{ji}). \tag{2.4}$$

The symmetric tensor $\sigma_{ij} = \sigma_{ji}$ is related to the symmetric strain tensor

$$e_{ij} = e_{ji} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{2.5}$$

The antisymmetric stress σ'_{ij} are given by

$$\sigma'_{23} = -F\frac{\partial \xi}{\partial t}, \qquad \sigma'_{31} = -F\frac{\partial \eta}{\partial t}, \qquad \sigma'_{12} = -F\frac{\partial \zeta}{\partial t}, \qquad \sigma'_{11} = \sigma'_{22} = \sigma'_{33} = 0, \tag{2.6}$$

where F is the coefficient of friction between the individual grains. The stress couple M_{ij} is given by

$$M_{ij} = M\nu_{ij}, \tag{2.7}$$

where, M is the third elastic constant, $M_{11},\ M_{13},\ M_{33}$, and so forth, are the components of the resultant acting on a surface.

The non-symmetric strain tensor ν_{ij} is defined as

$$\begin{aligned} &\nu_{11} = \frac{\partial \xi}{\partial x}, \qquad \nu_{31} = \frac{\partial \xi}{\partial z}, \qquad \nu_{33} = \frac{\partial \zeta}{\partial z}, \quad \nu_{21} = \nu_{22} = \nu_{23} = 0, \\ &\nu_{12} = \frac{\partial}{\partial x}(\omega_2 + \eta), \qquad \nu_{32} = \frac{\partial}{\partial z}(\omega_2 + \eta), \qquad \nu_{13} = \frac{\partial \zeta}{\partial x}, \end{aligned} \tag{2.8}$$

where $\omega_2 = (1/2)((\partial u/\partial z) - (\partial w/\partial x))$.

The dynamic equation of motion, if the magnetic field and rotation are added, can be written as [15]

$$\tau_{ji,j} + F_i = \rho\left[\ddot{u}_i + \left\{\vec{\Omega} \times \left(\vec{\Omega} \times \vec{u}\right)\right\}_i + \left(2\vec{\Omega} \times \dot{\vec{u}}\right)_i\right], \quad i, j = 1, 2, 3. \tag{2.9}$$

The heat conduction equation is given by [16]

$$K\nabla^2 T = \rho s \frac{\partial}{\partial t}\left(1 + \tau_2 \frac{\partial}{\partial t}\right)T + \gamma T_0 \frac{\partial}{\partial t}\left(1 + \tau_2 \delta \frac{\partial}{\partial t}\right)\underline{\nabla} \cdot \vec{u}, \tag{2.10}$$

where ρ is density of the material, K is thermal conductivity, s is specific heat of the material per unit mass, τ_1, τ_2 are thermal relaxation parameter, α_t is coefficient of linear thermal expansion, λ and μ are Lame's elastic constants, θ is the absolute temperature, $\gamma = \alpha_t(3\lambda+2\mu)$,

T_0 is reference temperature solid, T is temperature difference $(\theta - T_0)$, τ_0 is the mechanical relaxation time due to the viscosity, and $\tau_m = (1 + \tau_0(\partial/\partial t))$.

The components of stress in generalized thermoelastic medium are given by

$$\sigma_{11} = \left[\tau_m(\lambda + 2\mu) + p\right]\frac{\partial u}{\partial x} + (\tau_m\lambda + P)\frac{\partial w}{\partial z} - \gamma\left(1 + \tau_1\frac{\partial}{\partial t}\right)T,$$
$$\sigma_{33} = \tau_m\lambda\frac{\partial u}{\partial x} + \tau_m(\lambda + 2\mu)\frac{\partial w}{\partial z} - \gamma\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{2.11}$$
$$\sigma_{13} = \tau_m\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right).$$

If we neglect the thermal relaxation time, then (2.11) tends to Nowacki [17] and Biot [18].

The Maxwell's electro-magnetic stress tensor $\overline{\tau}_{ij}$ is given by

$$\overline{\tau}_{ij} = \mu_e\left[H_i h_j + H_j h_i - (H_k \cdot h_k)\delta_{ij}\right], \quad i, j = 1, 2, 3, \tag{2.12}$$

which takes the form

$$\overline{\tau}_{11} = -\mu_e H_0^2 \nabla^2\phi, \quad \overline{\tau}_{13} = \overline{\tau}_{23} = 0, \quad \overline{\tau}_{33} = \mu_e H_0^2 \nabla^2\phi, \quad \nabla^2\phi = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}. \tag{2.13}$$

The dynamic equations of motion are

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{31}}{\partial z} + \frac{P}{2}\frac{\partial \omega_2}{\partial z} - \rho g\frac{\partial w}{\partial x} + F_x = \rho\left[\frac{\partial^2 u}{\partial t^2} + 2\Omega\frac{\partial w}{\partial t} - \Omega^2 u\right],$$
$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{32}}{\partial z} + F_y = 0, \tag{2.14}$$
$$\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} + \frac{P}{2}\frac{\partial \omega_2}{\partial x} + \rho g\frac{\partial w}{\partial x} + F_z = \rho\left[\frac{\partial^2 w}{\partial t^2} - 2\Omega\frac{\partial u}{\partial t} - \Omega^2 w\right],$$

where g is the Earth's gravity and

$$\vec{F} = \left(-\mu_e H_0^2 \nabla^2\phi, 0, \mu_e H_0^2 \nabla^2\phi\right), \tag{2.15}$$

$$\tau_{23} - \tau_{32} + \frac{\partial M_{11}}{\partial x} + \frac{\partial M_{31}}{\partial z} = 0,$$
$$\tau_{31} - \tau_{13} + \frac{\partial M_{12}}{\partial x} + \frac{\partial M_{32}}{\partial z} = 0, \tag{2.16}$$
$$\tau_{12} - \tau_{21} + \frac{\partial M_{13}}{\partial x} + \frac{\partial M_{33}}{\partial z} = 0.$$

From (2.3)–(2.8) and (2.11), we have

$$\tau_{11} = \left[\tau_m(\lambda+2\mu)+p\right]\frac{\partial u}{\partial x} + (\tau_m\lambda + P)\frac{\partial w}{\partial z} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T,$$
$$\tau_{33} = \tau_m\lambda\frac{\partial u}{\partial x} + \tau_m(\lambda+2\mu)\frac{\partial w}{\partial z} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)T,$$
$$\tau_{13} = \tau_m\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) + F\frac{\partial \eta}{\partial t},$$
$$\tau_{12} = -F\frac{\partial \zeta}{\partial t}, \tag{2.17}$$
$$\tau_{23} = -F\frac{\partial \xi}{\partial t},$$
$$M_{11} = M\frac{\partial \xi}{\partial x}, \qquad M_{31} = M\frac{\partial \xi}{\partial z}, \qquad M_{33} = M\frac{\partial \zeta}{\partial z}, \qquad M_{21} = M_{22} = M_{23} = 0,$$
$$M_{12} = M\frac{\partial}{\partial x}(\omega_2+\eta), \qquad M_{32} = M\frac{\partial}{\partial z}(\omega_2+\eta), \qquad M_{13} = M\frac{\partial \zeta}{\partial x}.$$

Substituting (2.17) into (2.14) and (2.16) tends to

$$\left[\tau_m(\lambda+2\mu)+P\right]\frac{\partial^2 u}{\partial x^2} + (\tau_m\lambda+P)\frac{\partial^2 w}{\partial x\,\partial z} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} + \tau_m\mu\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\,\partial z}\right)$$
$$+\frac{P}{2}\left(\frac{\partial^2 u}{\partial z^2}-\frac{\partial^2 w}{\partial x\,\partial z}\right) - \rho g\frac{\partial w}{\partial x} + F\frac{\partial^2 \eta}{\partial z\,\partial t} + \mu_e H_0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\,\partial z}\right) = \rho\left[\frac{\partial^2 u}{\partial t^2}+2\Omega\frac{\partial w}{\partial t}-\Omega^2 u\right], \tag{2.18}$$

then

$$\left[\tau_m(\lambda+2\mu)+P+\mu_e H_0^2\right]\frac{\partial^2 u}{\partial x^2} + \left[\tau_m(\lambda+\mu)+\frac{P}{2}+\mu_e H_0^2\right]\frac{\partial^2 w}{\partial x\,\partial z} + \left(\tau_m\mu+\frac{P}{2}\right)\frac{\partial^2 u}{\partial z^2}$$
$$-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} - \rho g\frac{\partial w}{\partial x} + F\frac{\partial^2 \eta}{\partial z\,\partial t} = \rho\left[\frac{\partial^2 u}{\partial t^2}+2\Omega\frac{\partial w}{\partial t}-\Omega^2 u\right]. \tag{2.19}$$

Also,

$$\frac{\partial}{\partial t}\left(\frac{\partial \zeta}{\partial x}-\frac{\partial \xi}{\partial z}\right)=0, \tag{2.20}$$

$$\begin{aligned}
&\tau_m\mu\left(\frac{\partial^2 u}{\partial x\,\partial z}+\frac{\partial^2 w}{\partial x^2}\right)-F\frac{\partial^2\eta}{\partial x\,\partial t}+\tau_m\lambda\frac{\partial^2 u}{\partial x\,\partial z}+\tau_m(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}\\
&\quad+\frac{P}{2}\left(\frac{\partial^2 u}{\partial x\,\partial z}-\frac{\partial^2 w}{\partial x^2}\right)+\rho g\frac{\partial u}{\partial x}+\mu_e H_0^2\left(\frac{\partial^2 u}{\partial x\,\partial z}+\frac{\partial^2 w}{\partial z^2}\right)=\rho\left[\frac{\partial^2 w}{\partial t^2}-2\Omega\frac{\partial u}{\partial t}-\Omega^2 w\right],
\end{aligned} \tag{2.21}$$

then

$$\begin{aligned}
&\left[\tau_m(\lambda+\mu)+\frac{P}{2}+\mu_e H_0^2\right]\frac{\partial^2 u}{\partial x\,\partial z}+\left(\tau_m\mu-\frac{P}{2}\right)\frac{\partial^2 w}{\partial x^2}+\left[\tau_m(\lambda+2\mu)+\mu_e H_0^2\right]\frac{\partial^2 w}{\partial z^2}\\
&\quad-\gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}+\rho g\frac{\partial u}{\partial x}-F\frac{\partial^2\eta}{\partial x\,\partial t}=\rho\left[\frac{\partial^2 w}{\partial t^2}-2\Omega\frac{\partial u}{\partial t}-\Omega^2 w\right],
\end{aligned} \tag{2.22}$$

and, from (2.16), we have

$$\nabla^2\xi-s_2\frac{\partial\xi}{\partial t}=0, \tag{2.23}$$

$$\nabla^2(\omega_2+\eta)-s_2\frac{\partial\eta}{\partial t}=0, \tag{2.24}$$

$$\nabla^2\zeta-s_2\frac{\partial\zeta}{\partial t}=0, \tag{2.25}$$

where

$$s_2=\frac{2F}{M}. \tag{2.26}$$

3. Solution of the Problem

By Helmholtz's theorem [19], the displacement vector $\vec{u}$ can be written in the displacement potentials ϕ and ψ form, as

$$\vec{u}=\operatorname{grad}\phi+\operatorname{curl}\vec{\psi},\quad \vec{\psi}=(0,\psi,0), \tag{3.1}$$

which reduces to

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},\quad w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}. \tag{3.2}$$

Substituting (3.2) into (2.19), (2.22), and (2.24), the wave equations tend to

$$\alpha^2\nabla^2\phi - \frac{\gamma}{\rho}\left(1+\tau_1\frac{\partial}{\partial t}\right)T - g\frac{\partial\psi}{\partial x} = \frac{\partial^2\phi}{\partial t^2} + 2\Omega\frac{\partial\psi}{\partial t} - \Omega^2\phi, \tag{3.3}$$

$$\beta^2\nabla^2\psi - s_1\frac{\partial\eta}{\partial t} + g\frac{\partial\phi}{\partial x} = \frac{\partial^2\psi}{\partial t^2} - 2\Omega\frac{\partial\phi}{\partial t} - \Omega^2\psi, \tag{3.4}$$

$$\nabla^2\eta - s_2\frac{\partial\eta}{\partial t} - \nabla^4\psi = 0, \tag{3.5}$$

where

$$s_1 = \frac{F}{\rho}, \qquad \alpha^2 = \frac{\tau_m(\lambda+2\mu) + P + \mu_e H_0^2}{\rho}, \qquad \beta^2 = \frac{2\tau_m\mu - P}{2\rho}. \tag{3.6}$$

Substituting (3.2) into (2.10), we obtain

$$K\nabla^2 T = \rho s\frac{\partial}{\partial t}\left(1+\tau_2\frac{\partial}{\partial t}\right)T + \gamma T_0\frac{\partial}{\partial t}\left(1+\tau_2\delta\frac{\partial}{\partial t}\right)\nabla^2\phi. \tag{3.7}$$

From (3.3) and (3.7), by eliminating T, we obtain

$$\begin{aligned}&\left[\nabla^2 - \frac{1}{\chi}\frac{\partial}{\partial t}\left(1+\tau_2\frac{\partial}{\partial t}\right)\right]\left[\alpha^2\nabla^2\phi - g\frac{\partial\psi}{\partial x} - \frac{\partial^2\phi}{\partial t^2} - 2\Omega\frac{\partial\psi}{\partial t} + \Omega^2\phi\right]\\ &\quad - \varepsilon\frac{\partial}{\partial t}\left(1+\tau_1\frac{\partial}{\partial t}\right)\left(1+\tau_2\delta\frac{\partial}{\partial t}\right)\nabla^2\phi = 0,\end{aligned} \tag{3.8}$$

where

$$\chi = \frac{K}{\rho s}, \qquad \varepsilon = \frac{\gamma^2 T_0}{\rho K}. \tag{3.9}$$

From (3.4) and (3.5) by eliminating η, we obtain

$$\left(\nabla^2 - s_2\frac{\partial}{\partial t}\right)\left(\beta^2\nabla^2\psi - \frac{\partial^2\psi}{\partial t^2} + g\frac{\partial\phi}{\partial x} + 2\Omega\frac{\partial\phi}{\partial t} + \Omega^2\psi\right) - s_1\nabla^4\frac{\partial\psi}{\partial t} = 0. \tag{3.10}$$

For a plane harmonic wave propagation in the x-direction, we assume

$$\phi = \phi_1 e^{ik(x-ct)}, \qquad \psi = \psi_1 e^{ik(x-ct)}, \tag{3.11}$$

$$(\xi,\eta,\zeta) = (\xi_1,\eta_1,\zeta_1)e^{ik(x-ct)}. \tag{3.12}$$

From (3.12) into (2.20), (2.23), and (2.25), we get

$$D\xi_1 - ik\zeta_1 = 0, \tag{3.13}$$

$$D^2\xi_1 + q^2\xi_1 = 0, \tag{3.14}$$

$$D^2\zeta_1 + q^2\zeta_1 = 0, \tag{3.15}$$

where

$$q^2 = ikcs_2 - k^2, \qquad D \equiv \frac{d}{dz}. \tag{3.16}$$

The solution of (3.14) and (3.15) takes the form

$$\xi_1 = A_1 e^{iqz} + A_2 e^{-iqz}, \qquad \zeta_1 = B_1 e^{iqz} + B_2 e^{-iqz}, \tag{3.17}$$

where A_1, A_2, B_1, and B_2 are arbitrary constants.
From (3.13) and (3.17), we obtain

$$q\left(A_1 e^{iqz} - A_2 e^{-iqz}\right) - k\left(B_1 e^{iqz} + B_2 e^{-iqz}\right) = 0, \tag{3.18}$$

then

$$\left.\begin{aligned} qA_1 - kB_1 &= 0, \\ qA_2 - kB_2 &= 0 \end{aligned}\right\} \Longrightarrow A_j = \frac{(-1)^{j-1}k}{q}B_j, \quad j = 1, 2. \tag{3.19}$$

Substituting (3.11) into (3.8) and (3.10), we obtain

$$\begin{gathered} \left[\alpha_*^2 D^4 + G_1 D^2 + G_2\right]\phi_1 - \left[G_3 D^2 + G_4\right]\psi_1 = 0, \\ \left[R_1 D^4 + R_2 D^2 + R_3\right]\psi_1 + \left[R_4 D^2 + R_5\right]\phi_1 = 0, \end{gathered} \tag{3.20}$$

where

$$\Gamma_0 = 1 - ikc\tau_0, \qquad \Gamma_1 = 1 - ikc\tau_1, \qquad \Gamma_2 = 1 - ikc\tau_2, \qquad \Gamma_3 = 1 - ikc\tau_2\delta,$$

$$\alpha_*^2 = \frac{\Gamma_0(\lambda + 2\mu) + P + \mu_e H_0^2}{\rho}, \qquad \beta_*^2 = \frac{2\Gamma_0\mu - P}{2\rho}.$$

$$G_1 = k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2,$$

$$G_2 = k^4\left(\alpha_*^2 - c^2\right) + \frac{ikc\Gamma_2}{\chi}\left(k^2\left(1-\alpha_*^2\right)+\Omega^2\right) - k^2\left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right),$$

$$G_3 = ik(g-2\Omega c), \qquad G_4 = -(g-2\Omega c)\left(ik^3 + \frac{k^2 c\Gamma_2}{\chi}\right),$$

$$R_1 = \beta_*^2 + ikcs_1, \qquad R_2 = k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2 s_1\right) + \Omega^2,$$

$$R_3 = k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4 s_1\right),$$

$$R_4 = ik(g-2\Omega c), \qquad R_5 = (2\Omega c - g)\left(ik^3 - k^2 c s_2\right). \tag{3.21}$$

The solution of (3.20) takes the form

$$\phi_1 = \sum_{j=1}^{4}\left[C_j e^{ikN_j z} + D_j e^{-ikN_j z}\right],$$
$$\psi_1 = \sum_{j=1}^{4}\left[E_j e^{ikN_j z} + F_j e^{-ikN_j z}\right], \tag{3.22}$$

where the constants E_j and F_j are related to the constants C_j and D_j in the form

$$E_j = m_j C_j, \quad F_j = m_j D_j, \qquad j = 1,2,3,4,$$

$$m_j = \frac{1}{(g-2\Omega c)\left(ikN_j^2 - ik - (\varepsilon\Gamma_2/\chi)\right)} \times \left\{\alpha_*^2 k^2 N_j^4 - \left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2\right]N_j^2 + \frac{ikc\Gamma_2}{\chi}\left(1 - \alpha_*^2 + \frac{\Omega^2}{k^2}\right) - \left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right)\right\}. \tag{3.23}$$

Substituting (3.22) into (3.11), we obtain

$$\phi = \sum_{j=1}^{4}\left[C_j e^{ikN_j z} + D_j e^{-ikN_j z}\right]e^{ik(x-ct)},$$
$$\psi = \sum_{j=1}^{4}\left[E_j e^{ikN_j z} + F_j e^{-ikN_j z}\right]e^{ik(x-ct)}, \tag{3.24}$$

and values of displacement components u and w are

$$u = ik\sum_{j=1}^{4}\left[(1-N_jm_j)C_je^{ikN_jz} + (1+N_jm_j)D_je^{-ikN_jz}\right]e^{ik(x-ct)},$$
$$w = ik\sum_{j=1}^{4}\left[(N_j+m_j)C_je^{ikN_jz} + (m_j-N_j)D_je^{-ikN_jz}\right]e^{ik(x-ct)}, \tag{3.25}$$

where N_1, N_2, N_3, and N_4 are taken to be the complex roots of the following equation

$$N^8 + t_1N^6 + t_2N^4 + t_3N^2 + t_4 = 0, \tag{3.26}$$

where

$$t_1 = \frac{k^2}{\alpha_*^2}\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\alpha_*^2\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2 + \frac{1}{\beta_*^2 + ikcs_1} \times\left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2s_1\right) + \Omega^2\right], \tag{3.27}$$

$$\begin{aligned} t_2 = {} & \frac{1}{\alpha_*^2}\left[k^4\left(\alpha_*^2 - c^2\right) + \frac{ikc\Gamma_2}{\chi}\left(k^2\left(1-\alpha_*^2\right) + \Omega^2\right) - k^2\left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right)\right] \\ & + \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)}\left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2s_1\right) + \Omega^2\right] \\ & \times\left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2\right] \\ & + \frac{1}{(\beta_*^2 + ikcs_1)}\left[k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4s_1\right)\right] \\ & - \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)}\left[k^2(g - 2\Omega c)^2\right], \end{aligned} \tag{3.28}$$

$$\begin{aligned} t_3 = {} & \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)} \\ & \times\left\{\left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2s_1\right) + \Omega^2\right]\right. \\ & \quad\times\left[k^4\left(\alpha_*^2 - c^2\right) + \frac{ikc\Gamma_2}{\chi}\left(k^2\left(1-\alpha_*^2\right) + \Omega^2\right) - k^2\left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right)\right] \end{aligned}$$

$$
\begin{aligned}
&+\left[k^2\left(k^2-ikcs_2\right)\left(\beta_*^2-c^2\right)+ikc\left(s_2\Omega^2+k^4s_1\right)\right]\\
&\times\left[k^2\left(c^2-2\alpha_*^2\right)+\frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2+\chi\varepsilon\Gamma_1\Gamma_3\right)+\Omega^2\right]\\
&-\left[ik(g-2\Omega c)^2\left(ik^3+\frac{k^2c\Gamma_2}{\chi}\right)\right]-ik^3\left[(g-2\Omega c)^2(ik-cs_2)\right]\Bigg\},
\end{aligned}
\tag{3.29}
$$

$$
\begin{aligned}
t_4=&\frac{1}{\alpha_*^2(\beta_*^2+ikcs)}\\
&\times\left\{\left[k^2\left(k^2-ikcs_2\right)\left(\beta_*^2-c^2\right)+ikc\left(s_2\Omega^2+k^4s_1\right)\right]\times\left[ik(g-2\Omega c)\right]\right.\\
&\left.+\left[(2\Omega c-g)^2\left(ik^3-k^2cs_2\right)\left(ik^3+\frac{k^2c\Gamma_2}{\chi}\right)\right]\right\}.
\end{aligned}
\tag{3.30}
$$

From (3.4), (3.11), (3.12), (3.22), and (3.23), we obtain

$$
\eta_1=\sum_{j=1}^{4}\frac{1}{ikcs_1}\left\{k^2\beta_*^2m_j\left(1+N_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right\}\times\left[C_je^{ikN_jz}+D_je^{-ikN_jz}\right].
\tag{3.31}
$$

Using (3.22) and (3.11) into (3.3), we obtain

$$
T=\frac{\rho}{\gamma\Gamma_1}\sum_{j=1}^{4}\left[-\alpha_*^2k^2\left(1+N_j^2\right)+k^2c^2-ikgm_j\right]\left(C_je^{ikN_jz}+D_je^{-ikN_jz}\right)e^{ik(x-ct)}.
\tag{3.32}
$$

With the lower medium, we use the symbols with primes, for $\xi_1,\zeta_1,\eta_1,T,\phi,\varphi$, and q, for $z>K$,

$$
\xi_1'=-\frac{k}{q'}B_2'e^{-iq'z},\qquad \zeta_1'=B_2'e^{-iq'z},
$$

$$
\eta_1'=\sum_{j=1}^{4}\frac{1}{ikcs_1'}\left\{k^2\beta_*'^2m_j'\left(1+N_j'^2\right)-m_j'\left(k^2c^2+\Omega'^2\right)+ik(2\Omega'c-g)\right\}D_j'e^{-ikN_j'z},
$$

$$
T'=\frac{\rho'}{\gamma'\Gamma_1}\sum_{j=1}^{4}\left[-\alpha_*'^2k^2\left(1+N_j'^2\right)+k^2c^2-ikgm_j'\right]D_j'e^{-ikN_j'z}e^{ik(x-ct)},
$$

$$\phi' = \sum_{j=1}^{4} D'_j e^{-ikN'_j z} e^{ik(x-ct)},$$
$$\psi' = \sum_{j=1}^{4} F'_j e^{-ikN'_j z} e^{ik(x-ct)}. \tag{3.33}$$

4. Boundary Conditions and Frequency Equation

In this section, we obtain the frequency equation for the boundary conditions which are specific to the interface $z = K$, that is,

(i) $u = u'$,

(ii) $w = w'$,

(iii) $\xi = \xi'$,

(iv) $\eta = \eta'$,

(v) $\zeta = \zeta'$,

(vi) $M_{33} = M'_{33}$,

(vii) $M_{31} = M'_{31}$,

(viii) $M_{32} = M'_{32}$,

(ix) $\tau_{33} + \overline{\tau}_{33} = \tau'_{33} + \overline{\tau}'_{33}$

(x) $\tau_{31} + \overline{\tau}_{31} = \tau'_{31} + \overline{\tau}'_{31}$,

(xi) $\tau_{32} + \overline{\tau}_{32} = \tau'_{32} + \overline{\tau}'_{32}$,

(xii) $T = T'$,

(xiii) $(\partial T/\partial z) + \theta T = (\partial T'/\partial z) + \theta T'$.

The boundary conditions on the free surface $z = 0$ are

(xiv) $M_{33} = 0$,

(xv) $M_{31} = 0$,

(xvi) $M_{32} = 0$,

(xvii) $\tau_{33} + \overline{\tau}_{33} = 0$,

(xviii) $\tau_{31} + \overline{\tau}_{31} = 0$,

(xix) $\tau_{32} + \overline{\tau}_{32} = 0$,

(xx) $(\partial T/\partial z) + \theta T = 0$.

From conditions (iii), (v), (vi), and (vii), we obtain

$$
\begin{aligned}
B_1 e^{iqK} - B_2 e^{-iqK} &= -B_2' e^{-iq'K}, \\
B_1 e^{iqK} + B_2 e^{-iqK} &= B_2' e^{-iq'K}, \\
M\left(B_1 e^{iqK} - B_2 e^{-iqK}\right) &= -M' B_2' e^{-iq'K}, \\
M\left(B_1 e^{iqK} + B_2 e^{-iqK}\right) &= -M' B_2' e^{-iq'K}.
\end{aligned} \tag{4.1}
$$

Hence,

$$
B_1 = B_2 = B_2' = 0, \qquad \xi = \zeta = \xi' = \zeta' = 0. \tag{4.2}
$$

The other significant boundary conditions are responsible for the following relations:

(i)

$$
\sum_{j=1}^{4} (1 - N_j m_j) C_j e^{ikN_j K} + (1 + N_j m_j) D_j e^{-ikN_j K} - \left(1 + N_j' m_j'\right) D_j' e^{-ikN_j' K} = 0, \tag{4.3}
$$

(ii)

$$
\sum_{j=1}^{4} (N_j + m_j) C_j e^{ikN_j K} + (m_j - N_j) D_j e^{-ikN_j K} - \left(m_j' - N_j'\right) D_j' e^{-ikN_j' K} = 0, \tag{4.4}
$$

(iv)

$$
\begin{aligned}
&\sum_{j=1}^{4} \frac{1}{c s_1} \left\{ k^2 \beta_*^2 m_j \left(1 + N_j^2\right) - m_j \left(k^2 c^2 + \Omega^2\right) + ik(2\Omega c - g) \right\} \times \left[C_j e^{ikN_j K} + D_j e^{-ikN_j K} \right] \\
&\quad - \sum_{j=1}^{4} \frac{1}{c s_1'} \left\{ k^2 \beta_*'^2 m_j' \left(1 + N_j'^2\right) - m_j' \left(k^2 c^2 + \Omega'^2\right) + ik(2\Omega' c - g) \right\} D_j' e^{-ikN_j' K} = 0,
\end{aligned} \tag{4.5}
$$

(viii)

$$\begin{aligned}
&MN_j \sum_{j=1}^{4}\left\{k^2 m_j\left(N_j^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2 m_j\left(1+N_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik\left(2\Omega c-g\right)\right)\right\}\\
&\quad\times\left[C_j e^{ikN_jK}-D_j e^{-ikN_jK}\right]\\
&\quad+M'N_j'\sum_{j=1}^{4}\left\{k^2 m_j'\left(N_j^{'2}+1\right)+\frac{1}{ikcs_1'}\right.\\
&\qquad\left.\times\left(k^2\beta_*^{'2}m_j'\left(1+N_j^{'2}\right)-m_j'\left(k^2c^2+\Omega^{'2}\right)+ik\left(2\Omega' c-g\right)\right)\right\}D_j' e^{-ikN_j'K}=0,
\end{aligned}\tag{4.6}$$

(ix)

$$\begin{aligned}
&\sum_{j=1}^{4}\left\{\left[\left(\Gamma_0\lambda+\mu_e H_0^2\right)\left(1-N_j m_j\right)+\left(\Gamma_0\left(\lambda+2\mu\right)+\mu_e H_0^2\right)\left(N_j^2+m_jN_j\right)\right]\right.\\
&\quad\times C_j e^{ikN_jK}+\left[\left(\Gamma_0\lambda+\mu_e H_0^2\right)\left(1+N_j m_j\right)+\left(\Gamma_0\left(\lambda+2\mu\right)+\mu_e H_0^2\right)\left(N_j^2-m_jN_j\right)\right]\\
&\quad\times D_j e^{-ikN_jK}+\rho\left[-\alpha_*^2\left(1+N_j^2\right)+c^2-\frac{ig}{k}m_j\right]\left[C_j e^{ikN_jK}+D_j e^{-ikN_jK}\right]\\
&\quad-\left[\left(\Gamma_0'\lambda'+\mu_e' H_0^2\right)\left(1+N_j'm_j'\right)+\left(\Gamma_0'\left(\lambda'+2\mu'\right)+\mu_e' H_0^2\right)\left(N_j^{'2}-m_j'N_j'\right)\right]D_j' e^{-ikN_j'K}\\
&\quad\left.-\rho'\left[-\alpha_*^{'2}\left(1+N_j^{'2}\right)+c^2-\frac{ig}{k}m_j'\right]D_j' e^{-ikN_j'K}\right\}=0,
\end{aligned}\tag{4.7}$$

(x)

$$\begin{aligned}
&\sum_{j=1}^{4}\left\{-2k^2\Gamma_0\mu N_j\left[C_j e^{ikN_jK}-D_j e^{-ikN_jK}\right]\right.\\
&\quad+\left[-k^2\Gamma_0\mu m_j\left(1-N_j^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2 m_j\left(1+N_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik\left(2\Omega c-g\right)\right\}\right]\\
&\quad\times\left[C_j e^{ikN_jK}+D_j e^{-ikN_jK}\right]-2k^2\Gamma_0'\mu' N_j' D_j' e^{-ikN_j'K}\\
&\quad-\left[-k^2\Gamma_0'\mu' m_j'\left(1-N_j^{'2}\right)+\frac{F}{s_1'}\left\{k^2\beta_*^{'2} m_j'\left(1+N_j^{'2}\right)-m_j'\left(k^2c^2+\Omega^{'2}\right)+ik\left(2\Omega' c-g\right)\right\}\right]\\
&\quad\left.\times D_j' e^{-ikN_j'K}\right\}=0,
\end{aligned}\tag{4.8}$$

(xii)

$$\sum_{j=1}^{4} \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(N_j^2+1\right)+k^2c^2-igkm_j\right]\left[C_j e^{ikN_jK}+D_j e^{-ikN_jK}\right]$$
$$-\frac{\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(N_j'^2+1\right)+k^2c^2-igkm_j'\right]D_j' e^{-ikN_j'K}=0, \tag{4.9}$$

(xiii)

$$\sum_{j=1}^{4} \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(N_j^2+1\right)+k^2c^2-igkm_j\right]\left[(\theta+ikN_j)C_j e^{ikN_jK}+(\theta-ikN_j)D_j e^{-ikN_jK}\right]$$
$$-\frac{\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(N_j'^2+1\right)+k^2c^2-igkm_j'\right]\times\left(\theta-ikN_j'\right)D_j' e^{-ikN_j'K}=0, \tag{4.10}$$

(xvi)

$$MN_j\sum_{j=1}^{4}\left\{k^2m_j\left(N_j^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_j\left(1+N_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right)\right\}$$
$$\times\left[C_j-D_j\right]=0, \tag{4.11}$$

(xvii)

$$\sum_{j=1}^{4}\left\{\left[\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1-N_jm_j)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(N_j^2+m_jN_j\right)\right]C_j\right.$$
$$+\left[\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1+N_jm_j)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(N_j^2-m_jN_j\right)\right]D_j$$
$$\left.+\rho\left[-\alpha_*^2\left(1+N_j^2\right)+c^2-\frac{ig}{k}m_j\right][C_j+D_j]\right\}=0, \tag{4.12}$$

(xviii)

$$\sum_{j=1}^{4}\left\{-2k^2\Gamma_0\mu N_j[C_j-D_j]\right.$$
$$+\left[-k^2\Gamma_0\mu m_j\left(1-N_j^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_j\left(1+N_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right\}\right]$$
$$\left.\times[C_j+D_j]\right\}=0, \tag{4.13}$$

(xx)

$$\sum_{j=1}^{4}\left[-\alpha_*^2k^2\left(N_j^2+1\right)+k^2c^2-igkm_j\right]\left[(\theta+ikN_j)C_j+(\theta-ikN_j)D_j\right]=0. \tag{4.14}$$

5. Special Cases and Discussion

5.1. The Magnetic Field, Initial Stress, and Thermal Relaxation Time Are Neglected

In this case (i.e., $H_0=0, p=0$, and $\tau_1=\tau_2=0$), (3.26) tends to

$$V^8+h_1V^6+h_2V^4+h_3V^2+h_4=0, \tag{5.1}$$

where

$$\begin{aligned}
\alpha_*^2 &= \frac{\Gamma_0(\lambda+2\mu)}{\rho}, \qquad \beta_*^2=\frac{\Gamma_0\mu}{\rho},\\
m_j &= \frac{1}{(g-2\Omega c)\left(ikV_j^2-ik-(\varepsilon/\chi)\right)}\\
&\quad\times\left\{\alpha_*^2k^2V_j^4-\left[k^2\left(c^2-2\alpha_*^2\right)+\frac{ikc}{\chi}\left(\alpha_*^2+\chi\varepsilon\right)+\Omega^2\right]V_j^2+\frac{ikc}{\chi}\left(1-\alpha_*^2+\frac{\Omega^2}{k^2}\right)-\left(\Omega^2+ik\varepsilon c\right)\right\},\\
h_1 &= \frac{k^2}{\alpha_*^2}\left(c^2-2\alpha_*^2\right)+\frac{ikc}{\alpha_*^2\chi}\left(\alpha_*^2+\chi\varepsilon\right)+\Omega^2+\frac{1}{\beta_*^2+ikcs_1}\\
&\quad\times\left[k^2\left(c^2-2\beta_*^2\right)+ikc\left(s_2\beta_*^2-2k^2s_1\right)+\Omega^2\right],\\
h_2 &= \frac{1}{\alpha_*^2}\left[k^4\left(\alpha_*^2-c^2\right)+\frac{ikc}{\chi}\left(k^2\left(1-\alpha_*^2\right)+\Omega^2\right)-k^2\left(\Omega^2+ik\varepsilon c\right)\right]\\
&\quad+\frac{1}{\alpha_*^2(\beta_*^2+ikcs_1)}\left[k^2\left(c^2-2\beta_*^2\right)+ikc\left(s_2\beta_*^2-2k^2s_1\right)+\Omega^2\right]\\
&\quad\times\left[k^2\left(c^2-2\alpha_*^2\right)+\frac{ikc}{\chi}\left(\alpha_*^2+\chi\varepsilon\right)+\Omega^2\right]\\
&\quad+\frac{1}{(\beta_*^2+ikcs_1)}\left[k^2\left(k^2-ikcs_2\right)\left(\beta_*^2-c^2\right)+ikc\left(s_2\Omega^2+k^4s_1\right)\right]\\
&\quad-\frac{1}{\alpha_*^2(\beta_*^2+ikcs_1)}\left[k^2(g-2\Omega c)^2\right],
\end{aligned}$$

$$
\begin{aligned}
h_3 = \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)} \Bigg\{ & \left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2 s_1\right) + \Omega^2\right] \\
& \times \left[k^4\left(\alpha_*^2 - c^2\right) + \frac{ikc}{\chi}\left(k^2\left(1 - \alpha_*^2\right) + \Omega^2\right) - k^2\left(\Omega^2 + ik\varepsilon c\right)\right] \\
& + \left[k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4 s_1\right)\right] \\
& \times \left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2 + \chi\varepsilon\right) + \Omega^2\right] \\
& - \left[ik(g - 2\Omega c)^2\left(ik^3 + \frac{k^2 c}{\chi}\right)\right] - ik^3\left[(g - 2\Omega c)^2(ik - cs_2)\right] \Bigg\}, \\
h_4 = \frac{1}{\alpha_*^2(\beta_*^2 + ikcs)} \Bigg\{ & \left[k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4 s_1\right)\right] \times \left[ik(g - 2\Omega c)\right] \\
& + \left[(2\Omega c - g)^2\left(ik^3 - k^2 cs_2\right)\left(ik^3 + \frac{k^2 c}{\chi}\right)\right] \Bigg\}.
\end{aligned} \tag{5.2}
$$

Also,

$$
\begin{gathered}
\eta_1 = \sum_{j=1}^{4} \frac{1}{ikcs_1}\left\{k^2\beta_*^2 m_j\left(1 + V_j^2\right) - m_j\left(k^2c^2 + \Omega^2\right) + ik(2\Omega c - g)\right\} \times \left[C_j e^{ikV_j z} + D_j e^{-ikV_j z}\right]. \\
T = \frac{\rho}{\gamma} \sum_{j=1}^{4}\left[-\alpha_*^2 k^2\left(1 + V_j^2\right) + k^2c^2 - ikgm_j\right]\left(C_j e^{ikV_j z} + D_j e^{-ikV_j z}\right) e^{ik(x-ct)}, \\
\xi_1' = -\frac{k}{q'} B_2' e^{-iq'z}, \qquad \zeta_1' = B_2' e^{-iq'z}, \\
\eta_1' = \sum_{j=1}^{4} \frac{1}{ikcs_1'}\left\{k^2\beta_*'^2 m_j'\left(1 + V_j'^2\right) - m_j'\left(k^2c^2 + \Omega'^2\right) + ik(2\Omega' c - g)\right\} D_j' e^{-ikV_j' z}, \\
T' = \frac{\rho'}{\gamma'} \sum_{j=1}^{4}\left[-\alpha_*'^2 k^2\left(1 + V_j'^2\right) + k^2c^2 - ikgm_j'\right] D_j' e^{-ikV_j' z} e^{ik(x-ct)}, \\
\phi_1' = \sum_{j=1}^{4} D_j' e^{-ikV_j' z}, \\
\varphi_1' = \sum_{j=1}^{4} F_j' e^{-ikV_j' z},
\end{gathered} \tag{5.3}
$$

Using the boundary conditions, we obtain

$$\begin{bmatrix} d_{11} & d_{12} & \cdots & d_{18} & d'_{15} & d'_{16} & \cdots & d'_{18} \\ d_{21} & d_{22} & \cdots & d_{28} & d'_{25} & d'_{26} & \cdots & d'_{28} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{121} & d_{122} & \cdots & d_{128} & d'_{125} & d'_{126} & \cdots & d'_{128} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D'_1 \\ D'_2 \\ D'_3 \\ D'_4 \end{bmatrix} = [0], \tag{5.4}$$

where

$$\begin{aligned}
d_{1j} &= (1 - V_j m_j) C_j e^{ikV_j \mathrm{K}} + (1 + V_j m_j) D_j e^{-ikV_j \mathrm{K}}, \qquad d'_{1j} = \left(1 + V'_j m'_j\right) D'_j e^{-ikV'_j \mathrm{K}}, \\
d_{2j} &= (V_j + m_j) C_j e^{ikV_j \mathrm{K}} + (m_j - V_j) D_j e^{-ikV_j \mathrm{K}}, \qquad d'_{2j} = \left(m'_j - V'_j\right) D'_j e^{-ikV'_j \mathrm{K}}, \\
d_{3j} &= \frac{1}{cs_1}\left\{k^2\beta_*^2 m_j\left(1 + V_j^2\right) - m_j\left(k^2c^2 + \Omega^2\right) + ik(2\Omega c - g)\right\}\left[C_j e^{ikV_j z} + D_j e^{-ikV_j z}\right], \\
d'_{3j} &= \frac{1}{cs'_1}\left\{k^2\beta_*'^2 m'_j\left(1 + V_j'^2\right) - m'_j\left(k^2c^2 + \Omega^2\right) + ik(2\Omega c - g)\right\} D'_j e^{-ikV'_j z}, \\
d_{4j} &= MV_j\left\{k^2 m_j\left(V_j^2 + 1\right) + \frac{1}{ikcs_1}\left(k^2\beta_*^2 m_j\left(1 + V_j^2\right) - m_j\left(k^2c^2 + \Omega^2\right) + ik(2\Omega c - g)\right)\right. \\
&\quad \times \left[C_j e^{ikV_j K} - D_j e^{-ikV_j K}\right], \\
d'_{4j} &= M'V'_j\left\{k^2 m'_j\left(V_j'^2 + 1\right) + \frac{1}{ikcs'_1}\left(k^2\beta_*'^2 m'_j\left(1 + V_j'^2\right) - m'_j\left(k^2c^2 + \Omega^2\right) + ik(2\Omega c - g)\right)\right\} \\
&\quad \times D'_j e^{-ikV'_j K}, \\
d_{5j} &= \left[\Gamma_0\lambda(1 - V_j m_j) + \Gamma_0(\lambda + 2\mu)\left(V_j^2 + m_j V_j\right)\right] C_j e^{ikV_j K} \\
&\quad + \left[\Gamma_0\lambda(1 + V_j m_j) + \Gamma_0(\lambda + 2\mu)\left(V_j^2 - m_j V_j\right)\right] D_j e^{-ikV_j K} \\
&\quad + \rho\left[-\alpha_*^2\left(1 + V_j^2\right) + c^2 - \frac{ig}{k} m_j\right]\left[C_j e^{ikV_j K} + D_j e^{-ikV_j K}\right],
\end{aligned}$$

$$
\begin{aligned}
d'_{5j} &= \left[\Gamma'_0\lambda'\left(1+V'_j m'_j\right)+\Gamma'_0\left(\lambda'+2\mu'\right)\left(V_j'^2-m'_jV'_j\right)\right]D'_je^{-ikV'_jK}+\rho'\left[-\alpha_*'^2\left(1+V_j'^2\right)+c^2-\frac{ig}{k}m'_j\right] \\
&\quad \times D'_je^{-ikV'_jK}\Big\}, \\
d_{6j} &= -2k^2\Gamma_0\mu V_j\left[C_je^{ikV_jK}-D_je^{-ikV_jK}\right] \\
&\quad +\left[-k^2\Gamma_0\mu m_j\left(1-V_j^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_j\left(1+V_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right\}\right] \\
&\quad \times\left[C_je^{ikV_jK}+D_je^{-ikV_jK}\right], \\
d'_{6j} &= 2k^2\Gamma'_0\mu'\ V'_jD'_je^{-ikV'_jK} \\
&\quad +\left[-k^2\Gamma'_0\mu'm'_j\left(1-V_j'^2\right)+\frac{F}{s'_1}\left\{k^2\beta_*'^2m'_j\left(1+V_j'^2\right)-m'_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right\}\right] \\
&\quad \times D'_je^{-ikV'_jK}, \\
d_{7j} &= \frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(V_j^2+1\right)+k^2c^2-igkm_j\right]\left[C_je^{ikV_jK}+D_je^{-ikV_jK}\right], \\
d'_{7j} &= \frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(V_j'^2+1\right)+k^2c^2-igkm'_j\right]D'_je^{-ikV'_jK}, \\
d_{8j} &= \frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(V_j^2+1\right)+k^2c^2-igkm_j\right]\left[(\theta+ikV_j)C_je^{ikV_jK}+(\theta-ikV_j)D_je^{-ikV_jK}\right], \\
d'_{8j} &= \frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(V_j'^2+1\right)+k^2c^2-igkm'_j\right]\left(\theta-ikV'_j\right)D'_je^{-ikV'_jK}, \\
d_{9j} &= MV_j\left\{k^2m_j\left(V_j^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_j\left(1+V_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right)\right\} \\
&\quad \times[C_j-D_j], \\
d_{10j} &= \left[\Gamma_0\lambda(1-V_jm_j)+\Gamma_0(\lambda+2\mu)\left(V_j^2+m_jV_j\right)\right]C_j \\
&\quad +\left[\Gamma_0\lambda(1+V_jm_j)+\Gamma_0(\lambda+2\mu)\left(V_j^2-m_jV_j\right)\right]D_j \\
&\quad +\rho\left[-\alpha_*^2\left(1+V_j^2\right)+c^2-\frac{ig}{k}m_j\right][C_j+D_j], \\
d_{11j} &= -2k^2\Gamma_0\mu V_j[C_j-D_j] \\
&\quad +\left[-k^2\Gamma_0\mu m_j\left(1-V_j^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_j\left(1+V_j^2\right)-m_j\left(k^2c^2+\Omega^2\right)+ik(2\Omega c-g)\right\}\right] \\
&\quad \times[C_j+D_j], \\
d_{12j} &= \left[-\alpha_*^2k^2\left(V_j^2+1\right)+k^2c^2-igkm_j\right][(\theta+ikV_j)C_j+(\theta-ikV_j)D_j], \\
d'_{9j} &= d'_{10j}=d'_{11j}=d'_{12j}=0,\quad j=1,2,3,4.
\end{aligned}
\tag{5.5}
$$

5.2. The Magnetic Field, Initial Stress, Rotation, and Thermal Relaxation Time Are Neglected and in Viscoelastic Medium

In this case (i.e., $H_0 = 0$, $P = 0$, $\Omega = 0$, and $\tau_0 = \tau_1 = \tau_2 = 0$), the previous results obtained as in Abd-Alla et al. [20].

5.3. Absence of the Gravity Field

In this case, we put $g = 0$, then (3.20) becomes

$$\left[\alpha_*^2 D^4 + G_1 D^2 + G_2\right]\phi_1 - \left[G_3^* D^2 + G_4^*\right]\psi_1 = 0,$$
$$\left[R_1 D^4 + R_2 D^2 + R_3\right]\psi_1 + \left[R_4^* D^2 + R_5^*\right]\phi_1 = 0, \tag{5.6}$$

where

$$G_3^* = -2ik\Omega c, \qquad G_4^* = 2\Omega c\left(ik^3 + \frac{k^2 c\Gamma_2}{\chi}\right),$$
$$R_4^* = -2ik\Omega c, \qquad R_5^* = 2\Omega c\left(ik^3 - k^2 c s_2\right), \tag{5.7}$$

and G_1, G_2, R_1, R_2, and R_3 are as in (3).

The solution of (5.6) take the form

$$\phi = \sum_{j=1}^{4}\left[C_j^* e^{ikX_j z} + D_j^* e^{-ikX_j z}\right]e^{ik(x-ct)},$$
$$\psi = \sum_{j=1}^{4}\left[E_j^* e^{ikX_j z} + F_j^* e^{-ikX_j z}\right]e^{ik(x-ct)}, \tag{5.8}$$

where

$$E_j^* = m_j^* C_j^*, \quad F_j^* = m_j^* D_j^*, \quad j = 1,2,3,4, \tag{5.9}$$

$$m_j^* = \frac{1}{-2\Omega c\left(ikX_j^2 - ik - \left((\varepsilon\Gamma_2)/\chi\right)\right)}$$
$$\times\left\{\alpha_*^2 k^2 X_j^4 - \left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2\right]X_j^2\right.$$
$$\left. + \frac{ikc\Gamma_2}{\chi}\left(1 - \alpha_*^2 + \frac{\Omega^2}{k^2}\right) - \left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right)\right\}, \tag{5.10}$$

and X_1, X_2, X_3, and X_4 are taken to be the complex roots of equation

$$X^8 + t_1^* X^6 + t_2^* X^4 + t_3^* X^2 + t_4^* = 0, \tag{5.11}$$

where

$$\begin{aligned}
t_1^* &= \frac{k^2}{\alpha_*^2}\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\alpha_*^2\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2 + \frac{1}{\beta_*^2 + ikcs_1} \\
&\quad \times \left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2 s_1\right) + \Omega^2\right], \\
t_2^* &= \frac{1}{\alpha_*^2}\left[k^4\left(\alpha_*^2 - c^2\right) + \frac{ikc\Gamma_2}{\chi}\left(k^2\left(1 - \alpha_*^2\right) + \Omega^2\right) - k^2\left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right)\right] \\
&\quad + \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)}\left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2 s_1\right) + \Omega^2\right] \\
&\quad \times \left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2\right] \\
&\quad + \frac{1}{(\beta_*^2 + ikcs_1)}\left[k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4 s_1\right)\right] \\
&\quad - \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)}\left[4k^2\Omega^2 c^2\right], \\
t_3^* &= \frac{1}{\alpha_*^2(\beta_*^2 + ikcs_1)} \\
&\quad \times \Bigg\{\left[k^2\left(c^2 - 2\beta_*^2\right) + ikc\left(s_2\beta_*^2 - 2k^2 s_1\right) + \Omega^2\right] \\
&\qquad \times \left[k^4\left(\alpha_*^2 - c^2\right) + \frac{ikc\Gamma_2}{\chi}\left(k^2\left(1 - \alpha_*^2\right) + \Omega^2\right) - k^2\left(\Omega^2 + ik\varepsilon c\Gamma_1\Gamma_3\right)\right] \\
&\qquad + \left[k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4 s_1\right)\right] \\
&\qquad \times \left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\left(\alpha_*^2\Gamma_2 + \chi\varepsilon\Gamma_1\Gamma_3\right) + \Omega^2\right] \\
&\qquad - 4\Omega^2 c^2\left[ik\left(ik^3 + \frac{k^2 c\Gamma_2}{\chi}\right) + ik^3(ik - cs_2)\right]\Bigg\},
\end{aligned}$$

$$t_4^* = \frac{1}{\alpha_*^2(\beta_*^2 + ikcs)}\left\{\left[k^2\left(k^2 - ikcs_2\right)\left(\beta_*^2 - c^2\right) + ikc\left(s_2\Omega^2 + k^4 s_1\right)\right][-2ik\Omega c] + \left[4\Omega^2 c^2\left(ik^3 - k^2 cs_2\right)\left(ik^3 + \frac{k^2 c\Gamma_2}{\chi}\right)\right]\right\},$$

$$u = ik\sum_{j=1}^{4}\left[\left(1 - X_j m_j^*\right)C_j^* e^{ikX_j z} + \left(1 + X_j m_j^*\right)D_j^* e^{-ikX_j z}\right]e^{ik(x-ct)},$$

$$w = ik\sum_{j=1}^{4}\left[\left(X_j + m_j^*\right)C_j^* e^{ikX_j z} + \left(m_j^* - X_j\right)D_j^* e^{-ikX_j z}\right]e^{ik(x-ct)},$$

$$\eta_1 = \sum_{j=1}^{4}\frac{1}{ikcs_1}\left\{k^2\beta_*^2 m_j^*\left(1 + X_j^2\right) - m_j^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\} \times \left[C_j^* e^{ikX_j z} + D_j^* e^{-ikX_j z}\right],$$

$$T = \frac{\rho}{\gamma\Gamma_1}\sum_{j=1}^{4}\left[-\alpha_*^2 k^2\left(1 + X_j^2\right) + k^2c^2\right]\left(C_j^* e^{ikX_j z} + D_j^* e^{-ikX_j z}\right)e^{ik(x-ct)}. \tag{5.12}$$

With the lower medium, we use the symbols with primes, for $\xi_1, \zeta_1, \eta_1, T, \phi, \psi$, and q, for $z > K$,

$$\xi_1' = -\frac{k}{q'}B_2' e^{-iq'z}, \qquad \zeta_1' = B_2' e^{-iq'z},$$

$$\eta_1' = \sum_{j=1}^{4}\frac{1}{ikcs_1'}\left\{k^2\beta_*'^2 m_j'^*\left(1 + X_j'^2\right) - m_j'^*\left(k^2c^2 + \Omega'^2\right) + 2ik\Omega' c\right\}D_j'^* e^{-ikX_j' z},$$

$$T' = \frac{\rho'}{\gamma'\Gamma_1}\sum_{j=1}^{4}\left[-\alpha_*'^2 k^2\left(1 + X_j'^2\right) + k^2c^2\right]D_j'^* e^{-ikX_j' z}e^{ik(x-ct)}, \tag{5.13}$$

$$\phi' = \sum_{j=1}^{4} D_j' e^{-ikN_j' z}e^{ik(x-ct)},$$

$$\psi' = \sum_{j=1}^{4} F_j'^* e^{-ikX_j' z}e^{ik(x-ct)}.$$

From conditions (iii), (v), (vi), (vii), we get the same equations (4.1) and (4.2): the other significant boundary conditions are responsible for the following relations:

(i)

$$q_1 C_1^* e^{ikX_1K} + q_2 C_2^* e^{ikX_2K} + q_3 C_3^* e^{ikX_3K} + q_4 C_4^* e^{ikX_4K} + q_5 D_1^* e^{-ikX_1K} + q_6 D_2^* e^{-ikX_2K} + q_7 D_3^* e^{-ikX_3K} + q_8 D_4^* e^{-ikX_4K} = q_9 D_1'^* e^{-ikX_1'K} + q_{10} D_2'^* e^{-ikX_2'K} + q_{11} D_3'^* e^{-ikX_3'K} + q_{12} D_4'^* e^{-ikX_4'K}, \tag{5.14}$$

(ii)

$$q_{13} C_1^* e^{ikX_1\mathrm{K}} + q_{14} C_2^* e^{ikX_2\mathrm{K}} + q_{15} C_3^* e^{ikX_3\mathrm{K}} + q_{16} C_4^* e^{ikX_4\mathrm{K}} + q_{17} D_1^* e^{-ikX_1\mathrm{K}} + q_{18} D_2^* e^{-ikX_2\mathrm{K}} + q_{19} D_3^* e^{-ikX_3\mathrm{K}} + q_{20} D_4^* e^{-ikX_4\mathrm{K}} = q_{21} D_1'^* e^{-ikX_1'\mathrm{K}} + q_{22} D_2'^* e^{-ikX_2'\mathrm{K}} + q_{23} D_3'^* e^{-ikX_3'\mathrm{K}} + q_{24} D_4'^* e^{-ikX_4'\mathrm{K}}, \tag{5.15}$$

(iv)

$$q_{25} C_1^* e^{ikX_1\mathrm{K}} + q_{26} C_2^* e^{ikX_2\mathrm{K}} + q_{27} C_3^* e^{ikX_3\mathrm{K}} + q_{28} C_4^* e^{ikX_4\mathrm{K}} + q_{25} D_1^* e^{-ikX_1\mathrm{K}} + q_{26} D_2^* e^{-ikX_2\mathrm{K}} + q_{27} D_3^* e^{-ikX_3\mathrm{K}} + q_{28} D_4^* e^{-ikX_4\mathrm{K}} = q_{29} D_1'^* e^{-ikX_1'\mathrm{K}} + q_{30} D_2'^* e^{-ikX_2'\mathrm{K}} + q_{31} D_3'^* e^{-ikX_3'\mathrm{K}} + q_{32} D_4'^* e^{-ikX_4'\mathrm{K}}, \tag{5.16}$$

(viii)

$$q_{33} C_1^* e^{ikX_1\mathrm{K}} + q_{34} C_2^* e^{ikX_2\mathrm{K}} + q_{35} C_3^* e^{ikX_3\mathrm{K}} + q_{36} C_4^* e^{ikX_4\mathrm{K}} - q_{33} D_1^* e^{-ikX_1\mathrm{K}} - q_{34} D_2^* e^{-ikX_2\mathrm{K}} - q_{35} D_3^* e^{-ikX_3\mathrm{K}} - q_{36} D_4^* e^{-ikX_4\mathrm{K}} = -q_{37} D_1'^* e^{-ikX_1'\mathrm{K}} - q_{38} D_2'^* e^{-ikX_2'\mathrm{K}} - q_{39} D_3'^* e^{-ikX_3'\mathrm{K}} - q_{40} D_4'^* e^{-ikX_4'\mathrm{K}}, \tag{5.17}$$

(ix)

$$q_{41} C_1^* e^{ikX_1\mathrm{K}} + q_{42} C_2^* e^{ikX_2\mathrm{K}} + q_{43} C_3^* e^{ikX_3\mathrm{K}} + q_{44} C_4^* e^{ikX_4\mathrm{K}} + q_{45} D_1^* e^{-ikX_1\mathrm{K}} + q_{46} D_2^* e^{-ikX_2\mathrm{K}} + q_{47} D_3^* e^{-ikX_3\mathrm{K}} + q_{48} D_4^* e^{-ikX_4\mathrm{K}} = q_{49} D_1'^* e^{-ikX_1'\mathrm{K}} + q_{50} D_2'^* e^{-ikX_2'\mathrm{K}} + q_{51} D_3'^* e^{-ikX_3'\mathrm{K}} + q_{52} D_4'^* e^{-ikX_4'\mathrm{K}}, \tag{5.18}$$

(x)

$$q_{53} C_1^* e^{ikX_1\mathrm{K}} + q_{54} C_2^* e^{ikX_2\mathrm{K}} + q_{55} C_3^* e^{ikX_3\mathrm{K}} + q_{56} C_4^* e^{ikX_4\mathrm{K}} + q_{57} D_1^* e^{-ikX_1\mathrm{K}} + q_{58} D_2^* e^{-ikX_2\mathrm{K}} + q_{59} D_3^* e^{-ikX_3\mathrm{K}} + q_{60} D_4^* e^{-ikX_4\mathrm{K}} = q_{61} D_1'^* e^{-ikX_1'\mathrm{K}} + q_{62} D_2'^* e^{-ikX_2'\mathrm{K}} + q_{63} D_3'^* e^{-ikX_3'\mathrm{K}} + q_{64} D_4'^* e^{-ikX_4'\mathrm{K}}, \tag{5.19}$$

(xii)

$$\begin{aligned}
&q_{65}C_1^* e^{ikX_1\mathrm{K}} + q_{66}C_2^* e^{ikX_2\mathrm{K}} + q_{67}C_3^* e^{ikX_3\mathrm{K}} + q_{68}C_4^* e^{ikX_4\mathrm{K}} + q_{65}D_1^* e^{-ikX_1\mathrm{K}} \\
&\quad + q_{66}D_2^* e^{-ikX_2\mathrm{K}} + q_{67}D_3^* e^{-ikX_3\mathrm{K}} + q_{68}D_4^* e^{-ikX_4\mathrm{K}} \\
&\quad = q_{69}D_1^{'}{}^* e^{-ikX_1'\mathrm{K}} + q_{70}D_2'^* e^{-ikX_2'\mathrm{K}} + q_{71}D_3'^* e^{-ikX_3'\mathrm{K}} + q_{72}D_4'^* e^{-ikX_4'\mathrm{K}},
\end{aligned} \tag{5.20}$$

(xiii)

$$\begin{aligned}
&q_{73}C_1^* e^{ikX_1\mathrm{K}} + q_{74}C_2^* e^{ikX_2\mathrm{K}} + q_{75}C_3^* e^{ikX_3\mathrm{K}} + q_{76}C_4^* e^{ikX_4\mathrm{K}} + q_{77}D_1^* e^{-ikX_1\mathrm{K}} \\
&\quad + q_{78}D_2^* e^{-ikX_2\mathrm{K}} + q_{79}D_3^* e^{-ikX_3\mathrm{K}} + q_{80}D_4^* e^{-ikX_4\mathrm{K}} \\
&\quad = q_{81}D_1'^* e^{-ikX_1'\mathrm{K}} + q_{82}D_2'^* e^{-ikX_2'\mathrm{K}} + q_{83}D_3'^* e^{-ikX_3'\mathrm{K}} + q_{84}D_4'^* e^{-ikX_4'\mathrm{K}},
\end{aligned} \tag{5.21}$$

(xvi)

$$\begin{aligned}
&q_{85}C_1^* e^{ikX_1\mathrm{K}} + q_{86}C_2^* e^{ikX_2\mathrm{K}} + q_{87}C_3^* e^{ikX_3\mathrm{K}} + q_{88}C_4^* e^{ikX_4\mathrm{K}} \\
&\quad - \left[q_{85}D_1^* e^{-ikX_1\mathrm{K}} + q_{86}D_2^* e^{-ikX_2\mathrm{K}} + q_{87}D_3^* e^{-ikX_3\mathrm{K}} + q_{88}D_4^* e^{-ikX_4\mathrm{K}}\right] = 0,
\end{aligned} \tag{5.22}$$

(xvii)

$$\begin{aligned}
&q_{89}C_1^* e^{ikX_1\mathrm{K}} + q_{90}C_2^* e^{ikX_2\mathrm{K}} + q_{91}C_3^* e^{ikX_3\mathrm{K}} + q_{92}C_4^* e^{ikX_4\mathrm{K}} + q_{93}D_1^* e^{-ikX_1\mathrm{K}} \\
&\quad + q_{94}D_2^* e^{-ikX_2\mathrm{K}} + q_{95}D_3^* e^{-ikX_3\mathrm{K}} + q_{96}D_4^* e^{-ikX_4\mathrm{K}} = 0,
\end{aligned} \tag{5.23}$$

(xviii)

$$\begin{aligned}
&q_{97}C_1^* e^{ikX_1\mathrm{K}} + q_{98}C_2^* e^{ikX_2\mathrm{K}} + q_{99}C_3^* e^{ikX_3\mathrm{K}} + q_{100}C_4^* e^{ikX_4\mathrm{K}} + q_{101}D_1^* e^{-ikX_1\mathrm{K}} \\
&\quad + q_{102}D_2^* e^{-ikX_2\mathrm{K}} + q_{103}D_3^* e^{-ikX_3\mathrm{K}} + q_{104}D_4^* e^{-ikX_4\mathrm{K}} = 0,
\end{aligned} \tag{5.24}$$

(xx)

$$\begin{aligned}
&q_{105}C_1^* e^{ikX_1\mathrm{K}} + q_{106}C_2^* e^{ikX_2\mathrm{K}} + q_{107}C_3^* e^{ikX_3\mathrm{K}} + q_{108}C_4^* e^{ikX_4\mathrm{K}} + q_{109}D_1^* e^{-ikX_1\mathrm{K}} \\
&\quad + q_{110}D_2^* e^{-ikX_2\mathrm{K}} + q_{111}D_3^* e^{-ikX_3\mathrm{K}} + q_{112}D_4^* e^{-ikX_4\mathrm{K}} = 0,
\end{aligned} \tag{5.25}$$

where

$$q_1 = (1 - X_1 m_1^*),\quad q_2 = (1 - X_2 m_2^*),\quad q_3 = (1 - X_3 m_3^*),\quad q_4 = (1 - X_4 m_4^*),$$
$$q_5 = (1 + X_1 m_1^*),\quad q_6 = (1 + X_2 m_2^*),\quad q_7 = (1 + X_3 m_3^*),\quad q_8 = (1 + X_4 m_4^*),$$
$$q_9 = (1 + X_1' m_1'^*),\quad q_0 = (1 + X_2' m_2'^*),\quad q_{11} = (1 + X_3' m_3'^*),\quad q_{12} = (1 + X_4' m_4'^*),$$
$$q_{13} = (X_1 + m_1^*),\quad q_{14} = (X_2 + m_2^*),\quad q_{15} = (X_3 + m_3^*),\quad q_{16} = (X_4 + m_4^*),$$
$$q_{17} = (m_1^* - X_1),\quad q_{18} = (m_2^* - X_2),\quad q_{19} = (m_3^* - X_3),\quad q_{20} = (m_4^* - X_4),$$
$$q_{21} = (m_1'^* - X_1'),\quad q_{22} = ({m'}_2^* - {X'}_2),\quad q_{23} = ({m'}_3^* - {X'}_3),\quad q_{24} = ({m'}_4^* - {X'}_4),$$
$$q_{25} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{26} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_2^2\right) - m_2^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{27} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_3^*\left(1 + X_3^2\right) - m_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{28} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{29} = \frac{1}{cs_1'}\left\{k^2{\beta'}_*^2 {m'}_1^*\left(1 + {X'}_1^2\right) - {m'}_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{30} = \frac{1}{cs_1'}\left\{k^2{\beta'}_*^2 {m'}_2^*\left(1 + {X'}_2^2\right) - {m'}_2^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{31} = \frac{1}{cs_1'}\left\{k^2{\beta'}_*^2 {m'}_3^*\left(1 + {X'}_3^2\right) - {m'}_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{32} = \frac{1}{cs_1'}\left\{k^2{\beta'}_*^2 {m'}_4^*\left(1 + {X'}_4^2\right) - {m'}_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{33} = MX_1\left\{k^2 m_1^*\left(X_1^2 + 1\right) + \frac{1}{ikcs_1}\left(k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{34} = MX_2\left\{k^2 m_2^*\left(X_2^2 + 1\right) + \frac{1}{ikcs_1}\left(k^2\beta_*^2 m_2^*\left(1 + X_2^2\right) - m_2^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{35} = MX_3\left\{k^2 m_3^*\left(X_3^2 + 1\right) + \frac{1}{ikcs_1}\left(k^2\beta_*^2 m_3^*\left(1 + X_3^2\right) - m_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{36} = MX_4\left\{k^2 m_4^*\left(X_4^2 + 1\right) + \frac{1}{ikcs_1}\left(k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{37} = M'X_1'\left\{k^2 {m'}_1\left({X'}_1^2 + 1\right) + \frac{1}{ikcs_1'}\left(k^2{\beta'}_*^2 {m'}_1^*\left(1 + {X'}_1^2\right) - {m'}_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{38} = M'X_2'\left\{k^2 {m'}_2\left({X'}_2^2 + 1\right) + \frac{1}{ikcs_1'}\left(k^2{\beta'}_*^2 {m'}_2^*\left(1 + {X'}_2^2\ \right) - {m'}_2^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{39} = M'X_3'\left\{k^2 {m'}_3\left({X'}_3^2 + 1\right) + \frac{1}{ikc{s'}_1}\left(k^2{\beta'}_*^2 {m'}_3^*\left(1 + {X'}_3^2\right) - {m'}_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$
$$q_{40} = M'X_4'\left\{k^2 {m'}_4\left({X'}_4^2 + 1\right) + \frac{1}{ikc{s'}_1}\left(k^2{\beta'}_*^2 {m'}_4^*\left(1 + {X'}_1^2\right) - {m'}_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$

$$q_{41} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 - X_1 m_1^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_1^2 + m_1^* X_1\right) + \rho\left[-\alpha_*^2\left(1 + X_1^2\right) + c^2\right],$$

$$q_{42} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 - X_2 m_2^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_2^2 + m_2^* X_2\right) + \rho\left[-\alpha_*^2\left(1 + X_2^2\right) + c^2\right],$$

$$q_{43} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 - X_3 m_3^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_3^2 + m_3^* X_3\right) + \rho\left[-\alpha_*^2\left(1 + X_3^2\right) + c^2\right],$$

$$q_{64} = 2k^2\Gamma_0'\mu' X_4' - k^2\Gamma_0'\mu' m_4'^*\left(1 - X_4'^2\right) + \frac{F}{s_1'}\left\{k^2\beta_*'^2 m_4'^*\left(1 + X_4'^2\right) - m_4'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{44} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 - X_4 m_4^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_4^2 + m_4^* X_4\right) + \rho\left[-\alpha_*^2\left(1 + X_4^2\right) + c^2\right],$$

$$q_{45} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 + X_1 m_1^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_1^2 - m_1^* X_1\right) + \rho\left[-\alpha_*^2\left(1 + X_1^2\right) + c^2\right],$$

$$q_{46} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 + X_2 m_2^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_2^2 - m_2^* X_2\right) + \rho\left[-\alpha_*^2\left(1 + X_2^2\right) + c^2\right],$$

$$q_{47} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 + X_3 m_3^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_3^2 - m_3^* X_3\right) + \rho\left[-\alpha_*^2\left(1 + X_3^2\right) + c^2\right],$$

$$q_{48} = \left(\Gamma_0\lambda + \mu_e H_0^2\right)(1 + X_4 m_4^*) + \left(\Gamma_0(\lambda + 2\mu) + \mu_e H_0^2\right)\left(X_4^2 - m_4^* X_4\right) + \rho\left[-\alpha_*^2\left(1 + X_4^2\right) + c^2\right],$$

$$q_{49} = \left(\Gamma'_0\lambda' + \mu'_e H_0^2\right)(1 + X'_1 m_1'^*) + \left(\Gamma'_0(\lambda' + 2\mu') + \mu'_e H_0^2\right)\left(X_1'^2 - m_1'^* X'_1\right) + \rho'\left[-\alpha_*'^2\left(1 + X_1'^2\right) + c^2\right],$$

$$q_{50} = \left(\Gamma'_0\lambda' + \mu'_e H_0^2\right)(1 + X'_2 m_2'^*) + \left(\Gamma'_0(\lambda' + 2\mu') + \mu'_e H_0^2\right)\left(X_2'^2 - m_2'^* X'_2\right) + \rho'\left[-\alpha_*'^2\left(1 + X_2'^2\right) + c^2\right],$$

$$q_{51} = \left(\Gamma'_0\lambda' + \mu'_e H_0^2\right)(1 + X'_3 m_3'^*) + \left(\Gamma'_0(\lambda' + 2\mu') + \mu'_e H_0^2\right)\left(X_3'^2 - m_3'^* X'_3\right) + \rho'\left[-\alpha_*'^2\left(1 + X_3'^2\right) + c^2\right],$$

$$q_{52} = \left(\Gamma'_0\lambda' + \mu'_e H_0^2\right)(1 + X'_4 m_4'^*) + \left(\Gamma'_0(\lambda' + 2\mu') + \mu'_e H_0^2\right)\left(X_4'^2 - m_4'^* X'_4\right) + \rho'\left[-\alpha_*'^2\left(1 + X_4'^2\right) + c^2\right],$$

$$q_{53} = -2k^2\Gamma_0\mu X_1 - k^2\Gamma_0\mu m_1^*\left(1 - X_1^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{54} = -2k^2\Gamma_0\mu X_2 - k^2\Gamma_0\mu m_2^*\left(1 - X_2^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_2^*\left(1 + X_2^2\right) - m_2^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{55} = -2k^2\Gamma_0\mu X_3 - k^2\Gamma_0\mu m_3^*\left(1 - X_3^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_3^*\left(1 + X_3^2\right) - m_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{56} = -2k^2\Gamma_0\mu X_4 - k^2\Gamma_0\mu m_4^*\left(1 - X_4^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{57} = 2k^2\Gamma_0\mu X_1 - k^2\Gamma_0\mu m_1^*\left(1 - X_1^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{58}=2k^2\Gamma_0\mu X_2-k^2\Gamma_0\mu m_2^*\left(1-X_2^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{59}=2k^2\Gamma_0\mu X_3-k^2\Gamma_0\mu m_3^*\left(1-X_3^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{60}=2k^2\Gamma_0\mu X_4-k^2\Gamma_0\mu m_4^*\left(1-X_4^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{61}=2k^2\Gamma_0'\mu' X_1'-k^2\Gamma_0'\mu' m_1'^*\left(1-X_1'^2\right)+\frac{F}{s_1'}\left\{k^2\beta_*'^2m_1'^*\left(1+X_1'^2\right)-m_1'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{62}=2k^2\Gamma_0'\mu' X_2'-k^2\Gamma_0'\mu' m_2'^*\left(1-X_2'^2\right)+\frac{F}{s_1'}\left\{k^2\beta_*'^2m_2'^*\left(1+X_2'^2\right)-m_2'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{63}=2k^2\Gamma_0'\mu' X_3'-k^2\Gamma_0'\mu' m_3'^*\left(1-X_3'^2\right)+\frac{F}{s_1'}\left\{k^2\beta_*'^2m_3'^*\left(1+X_3'^2\right)-m_3'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{65}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_1^2+1\right)+k^2c^2\right],\quad q_{66}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_2^2+1\right)+k^2c^2\right],$$

$$q_{67}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_3^2+1\right)+k^2c^2\right],\quad q_{68}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_4^2+1\right)+k^2c^2\right],$$

$$q_{69}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_1'^2+1\right)+k^2c^2\right],\quad q_{70}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_2'^2+1\right)+k^2c^2\right],$$

$$q_{71}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_3'^2+1\right)+k^2c^2\right],\quad q_{72}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_4'^2+1\right)+k^2c^2\right],$$

$$q_{73}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_1^2+1\right)+k^2c^2\right](\theta+ikX_1),\quad q_{74}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_2^2+1\right)+k^2c^2\right](\theta+ikX_2),$$

$$q_{75}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_3^2+1\right)+k^2c^2\right](\theta+ikX_3),\quad q_{76}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_4^2+1\right)+k^2c^2\right](\theta+ikX_4),$$

$$q_{77}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_1^2+1\right)+k^2c^2\right](\theta-ikX_1),\quad q_{78}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_2^2+1\right)+k^2c^2\right](\theta-ikX_2),$$

$$q_{79}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_3^2+1\right)+k^2c^2\right](\theta-ikX_3),\quad q_{80}=\frac{\rho}{\gamma}\left[-\alpha_*^2k^2\left(X_4^2+1\right)+k^2c^2\right](\theta-ikX_4),$$

$$q_{81}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_1'^2+1\right)+k^2c^2\right](\theta-ikX'_1),\quad q_{82}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_2'^2+1\right)+k^2c^2\right](\theta-ikX'_2),$$

$$q_{83}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_3'^2+1\right)+k^2c^2\right](\theta-ikX'_3),\quad q_{84}=\frac{\rho'}{\gamma'}\left[-\alpha_*'^2k^2\left(X_3'^2+1\right)+k^2c^2\right](\theta-ikX'_3),$$

$$q_{85}=MX_1\left\{k^2m_1^*\left(X_1^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{86}=MX_2\left\{k^2m_j^*\left(X_2^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{87}=MX_3\left\{k^2m_j^*\left(X_3^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{88}=MX_4\left\{k^2m_4^*\left(X_4^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$
$$q_{89}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1-X_1m_1^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_1^2+m_1^*X_1\right)+\rho\left[-\alpha_*^2\left(1+X_1^2\right)+c^2\right],$$
$$q_{90}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1-X_2m_2^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_2^2+m_2^*X_2\right)+\rho\left[-\alpha_*^2\left(1+X_2^2\right)+c^2\right],$$
$$q_{91}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1-X_3m_3^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_3^2+m_3^*X_3\right)+\rho\left[-\alpha_*^2\left(1+X_3^2\right)+c^2\right],$$
$$q_{92}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1-X_4m_4^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_4^2+m_4^*X_4\right)+\rho\left[-\alpha_*^2\left(1+X_4^2\right)+c^2\right],$$
$$q_{93}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1+X_1m_1^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_1^2-m_1^*X_1\right)+\rho\left[-\alpha_*^2\left(1+X_1^2\right)+c^2\right],$$
$$q_{94}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1+X_2m_2^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_2^2-m_2^*X_2\right)+\rho\left[-\alpha_*^2\left(1+X_2^2\right)+c^2\right],$$
$$q_{95}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1+X_3m_3^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_3^2-m_3^*X_3\right)+\rho\left[-\alpha_*^2\left(1+X_3^2\right)+c^2\right],$$
$$q_{96}=\left(\Gamma_0\lambda+\mu_eH_0^2\right)(1+X_4m_4^*)+\left(\Gamma_0(\lambda+2\mu)+\mu_eH_0^2\right)\left(X_4^2-m_4^*X_4\right)+\rho\left[-\alpha_*^2\left(1+X_4^2\right)+c^2\right],$$
$$q_{97}=-2k^2\Gamma_0\mu X_1-k^2\Gamma_0\mu m_1^*\left(1-X_1^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{98}=-2k^2\Gamma_0\mu X_2-k^2\Gamma_0\mu m_2^*\left(1-X_j^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{99}=-2k^2\Gamma_0\mu X_3-k^2\Gamma_0\mu m_3^*\left(1-X_3^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{100}=-2k^2\Gamma_0\mu X_4-k^2\Gamma_0\mu m_4^*\left(1-X_4^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{101}=2k^2\Gamma_0\mu X_1-k^2\Gamma_0\mu m_1^*\left(1-X_1^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{102}=2k^2\Gamma_0\mu X_2-k^2\Gamma_0\mu m_2^*\left(1-X_2^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{103}=2k^2\Gamma_0\mu X_3-k^2\Gamma_0\mu m_3^*\left(1-X_3^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{104}=2k^2\Gamma_0\mu X_4-k^2\Gamma_0\mu m_4^*\left(1-X_4^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$
$$q_{105}=\left[-\alpha_*^2k^2\left(X_1^2+1\right)+k^2c^2\right](\theta+ikX_1),\quad q_{106}=\left[-\alpha_*^2k^2\left(X_2^2+1\right)+k^2c^2\right](\theta+ikX_2),$$
$$q_{107}=\left[-\alpha_*^2k^2\left(X_3^2+1\right)+k^2c^2\right](\theta+ikX_3),\quad q_{108}=\left[-\alpha_*^2k^2\left(X_4^2+1\right)+k^2c^2\right](\theta+ikX_4),$$
$$q_{109}=\left[-\alpha_*^2k^2\left(X_1^2+1\right)+k^2c^2\right](\theta-ikX_1),\quad q_{110}=\left[-\alpha_*^2k^2\left(X_2^2+1\right)+k^2c^2\right](\theta-ikX_2),$$
$$q_{111}=\left[-\alpha_*^2k^2\left(X_3^2+1\right)+k^2c^2\right](\theta-ikX_3),\quad q_{112}=\left[-\alpha_*^2k^2\left(X_4^2+1\right)+k^2c^2\right](\theta-ikX_4).\tag{5.26}$$

Elimination of C_j^*, D_j^*, and $D_j'^*$ gives the wave velocity equation in the determinant form

$$\det d_{ij} = 0. \tag{5.27}$$

This equation has complex roots: the real part (Re) gives the Rayleigh wave velocity, and the imaginary part (Im) gives the attenuation coefficient due to the friction of the granular nature of the medium, where the nonvanishing of the twelfth-order determinant of d_{ij} is given by

$$\begin{vmatrix} q_{85}e^{-ikX_1K} & q_{86}e^{-ikX_2K} & q_{87}e^{-ikX_3K} & q_{88}e^{-ikX_4K} & q_{85}e^{ikX_1K} & q_{86}e^{ikX_2K} & q_{87}e^{ikX_3K} & q_{88}e^{ikX_4K} & 0 & 0 & 0 & 0 \\ q_{89}e^{-ikX_1K} & q_{90}e^{-ikX_2K} & q_{91}e^{-ikX_3K} & q_{92}e^{-ikX_4K} & q_{93}e^{ikX_1K} & q_{94}e^{ikX_2K} & q_{95}e^{ikX_3K} & q_{96}e^{ikX_4K} & 0 & 0 & 0 & 0 \\ q_{97}e^{-ikX_1K} & q_{98}e^{-ikX_2K} & q_{99}e^{-ikX_3K} & q_{100}e^{-ikX_4K} & q_{101}e^{ikX_1K} & q_{102}e^{ikX_2K} & q_{103}e^{ikX_3K} & q_{104}e^{ikX_4K} & 0 & 0 & 0 & 0 \\ q_{105}e^{-ikX_1K} & q_{106}e^{-ikX_2K} & q_{107}e^{-ikX_3K} & q_{108}e^{-ikX_4K} & q_{109}e^{ikX_1K} & q_{110}e^{ikX_2K} & q_{111}e^{ikX_3K} & q_{112}e^{ikX_4K} & 0 & 0 & 0 & 0 \\ q_{13} & q_{14} & q_{15} & q_{16} & q_{17} & q_{18} & q_{19} & q_{20} & q_{21} & q_{22} & q_{23} & q_{24} \\ q_{25} & q_{26} & q_{27} & q_{28} & q_{25} & q_{26} & q_{27} & q_{28} & q_{29} & q_{30} & q_{31} & q_{32} \\ q_{33} & q_{34} & q_{35} & q_{36} & -q_{33} & -q_{34} & -q_{35} & -q_{36} & -q_{37} & -q_{38} & -q_{39} & -q_{40} \\ q_{41} & q_{42} & q_{43} & q_{44} & q_{45} & q_{46} & q_{47} & q_{48} & q_{49} & q_{50} & q_{51} & q_{52} \\ q_{53} & q_{54} & q_{55} & q_{56} & q_{57} & q_{58} & q_{59} & q_{60} & q_{61} & q_{62} & q_{63} & q_{64} \\ q_{65} & q_{66} & q_{67} & q_{68} & q_{65} & q_{66} & q_{67} & q_{68} & q_{69} & q_{70} & q_{71} & q_{72} \\ q_{73} & q_{74} & q_{75} & q_{76} & q_{77} & q_{78} & q_{79} & q_{80} & q_{81} & q_{82} & q_{83} & q_{84} \\ q_{1} & q_{2} & q_{3} & q_{4} & q_{5} & q_{6} & q_{7} & q_{8} & q_{9} & q_{10} & q_{11} & q_{12} \end{vmatrix} = 0. \tag{5.28}$$

5.4. The Gravity Field, Initial Stress, and Magnetic Field Are Neglected and There Is Uncoupling between the Temperature and Strain Field

In this case $g = 0$, $P = 0$, $H_0 = 0$, and $\theta = 0$, we obtain

$$\alpha_*^2 = \frac{\Gamma_0(\lambda + 2\mu)}{\rho}, \qquad \beta_*^2 = \frac{\Gamma_0\mu}{\rho},$$

$$\lim_{\varepsilon\to 0} m_j^* = \frac{1}{-2\Omega c\left(ikX_j^2 - ik\right)} \times \left\{\alpha_*^2k^2X_j^4 - \left[k^2\left(c^2 - 2\alpha_*^2\right) + \frac{ikc}{\chi}\alpha_*^2\Gamma_2 + \Omega^2\right]X_j^2 + \frac{ikc\Gamma_2}{\chi}\left(1 - \alpha_*^2 + \frac{\Omega^2}{k^2}\right) - \Omega^2\right\},$$

$$\lim_{\gamma\to 0}\begin{vmatrix} q_{22} & q_{23} & q_{24} \\ q_{30} & q_{31} & q_{32} \\ -q_{38} & -q_{39} & -q_{40} \\ q_{50} & q_{51} & q_{52} \\ q_{62} & q_{63} & q_{64} \end{vmatrix} = 0. \tag{5.29}$$

Multiplying the rows 10, 11, and 12 of the determinant $|d_{ij}|$ by γ and then taking $\lim_{\gamma\to 0}$, (5.28) reduces, after some computation, to the following ninth-order determinant equation:

$$\begin{vmatrix} q_{85}e^{-ikX_1K} & q_{86}e^{-ikX_2K} & q_{87}e^{-ikX_3K} & q_{88}e^{-ikX_4K} & q_{85}e^{ikX_1K} & q_{86}e^{ikX_2K} & q_{87}e^{ikX_3K} & q_{88}e^{ikX_4K} & 0 \\ q_{89}e^{-ikX_1K} & q_{90}e^{-ikX_2K} & q_{91}e^{-ikX_3K} & q_{92}e^{-ikX_4K} & q_{93}e^{ikX_1K} & q_{94}e^{ikX_2K} & q_{95}e^{ikX_3K} & q_{96}e^{ikX_4K} & 0 \\ q_{97}e^{-ikX_1K} & q_{98}e^{-ikX_2K} & q_{99}e^{-ikX_3K} & q_{100}e^{-ikX_4K} & q_{101}e^{ikX_1K} & q_{102}e^{ikX_2K} & q_{103}e^{ikX_3K} & q_{104}e^{ikX_4K} & 0 \\ q_{105}e^{-ikX_1K} & q_{106}e^{-ikX_2K} & q_{107}e^{-ikX_3K} & q_{108}e^{-ikX_4K} & q_{109}e^{ikX_1K} & q_{110}e^{ikX_2K} & q_{111}e^{ikX_3K} & q_{112}e^{ikX_4K} & 0 \\ q_{13} & q_{14} & q_{15} & q_{16} & q_{17} & q_{18} & q_{19} & q_{20} & q_{21} \\ q_{25} & q_{26} & q_{27} & q_{28} & q_{25} & q_{26} & q_{27} & q_{28} & q_{29} \\ q_{33} & q_{34} & q_{35} & q_{36} & -q_{33} & -q_{34} & -q_{35} & -q_{36} & -q_{37} \\ q_{41} & q_{42} & q_{43} & q_{44} & q_{45} & q_{46} & q_{47} & q_{48} & q_{49} \\ q_{53} & q_{54} & q_{55} & q_{56} & q_{57} & q_{58} & q_{59} & q_{60} & q_{61} \end{vmatrix} = 0, \tag{5.30}$$

where

$$q_1 = \left(1 - X_1 m_1^*\right), \qquad q_2 = \left(1 - X_2 m_2^*\right), \qquad q_3 = \left(1 - X_3 m_3^*\right), \qquad q_4 = \left(1 - X_4 m_4^*\right),$$
$$q_5 = \left(1 + X_1 m_1^*\right), \qquad q_6 = \left(1 + X_2 m_2^*\right), \qquad q_7 = \left(1 + X_3 m_3^*\right), \qquad q_8 = \left(1 + X_4 m_4^*\right),$$
$$q_9 = \left(1 + X_1' m_1'^*\right), \qquad q_{10} = \left(1 + X_2' m_2'^*\right), \qquad q_{11} = \left(1 + X_3' m_3'^*\right), \qquad q_{12} = \left(1 + X_4' m_4'^*\right),$$
$$q_{13} = \left(X_1 + m_1^*\right), \qquad q_{14} = \left(X_2 + m_2^*\right), \qquad q_{15} = \left(X_3 + m_3^*\right), \qquad q_{16} = \left(X_4 + m_4^*\right),$$
$$q_{17} = \left(m_1^* - X_1\right), \qquad q_{18} = \left(m_2^* - X_2\right), \qquad q_{19} = \left(m_3^* - X_3\right), \qquad q_{20} = \left(m_4^* - X_4\right),$$
$$q_{21} = \left(m_1'^* - X_1'\right), \qquad q_{22} = \left(m_2'^* - X_2'\right), \qquad q_{23} = \left(m_3'^* - X_3'\right), \qquad q_{24} = \left(m_4'^* - X_4'\right),$$
$$q_{25} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{26} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{27} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_3^*\left(1 + X_3^2\right) - m_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{28} = \frac{1}{cs_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{29} = \frac{1}{cs_1'}\left\{k^2\beta_*'^2 m_1'^*\left(1 + X_1^{2'}\right) - m_1'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{30} = \frac{1}{cs_1'}\left\{k^2\beta_*'^2 m_2'^*\left(1 + X_2^{2'}\right) - m_2'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{31} = \frac{1}{cs_1'}\left\{k^2\beta_*'^2 m_3'^*\left(1 + X_3^{2'}\right) - m_3'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{32} = \frac{1}{cs_1'}\left\{k^2\beta_*'^2 m_4'^*\left(1 + X_4^{2'}\right) - m_4'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$
$$q_{33} = MX_1\left\{k^2 m_1^*\left(X_1^2 + 1\right) + \frac{1}{ikcs_1}\left(k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right)\right\},$$

$$q_{34} = MX_2\left\{k^2 m_2^*\left(X_2^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2 m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{35} = MX_3\left\{k^2 m_3^*\left(X_3^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2 m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{36} = MX_4\left\{k^2 m_4^*\left(X_4^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2 m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{37} = M'X_1'\left\{k^2 m_1'\left(X_1'^2+1\right)+\frac{1}{ikcs_1'}\left(k^2\beta_*'^2 m_1'^*\left(1+X_1'^2\right)-m_1'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{38} = M'X_2'\left\{k^2 m_2'\left(X_2'^2+1\right)+\frac{1}{ikcs_1'}\left(k^2\beta_*'^2 m_2'^*\left(1+X_2'^2\right)-m_2'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{39} = M'X_3'\left\{k^2 m_3'\left(X_3'^2+1\right)+\frac{1}{ikcs_1'}\left(k^2\beta_*'^2 m_3'^*\left(1+X_3'^2\right)-m_3'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{40} = M'X_4'\left\{k^2 m_4'\left(X_4'^2+1\right)+\frac{1}{ikcs_1'}\left(k^2\beta_*'^2 m_4'^*\left(1+X_4'^2\right)-m_4'^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{41} = \Gamma_0\lambda(1-X_1m_1^*)+\Gamma_0(\lambda+2\mu)\left(X_1^2+m_1^*X_1\right)+\rho\left[-\alpha_*^2\left(1+X_1^2\right)+c^2\right],$$

$$q_{42} = \Gamma_0\lambda(1-X_2m_2^*)+\Gamma_0(\lambda+2\mu)\left(X_2^2+m_2^*X_2\right)+\rho\left[-\alpha_*^2\left(1+X_2^2\right)+c^2\right],$$

$$q_{43} = \Gamma_0\lambda(1-X_3m_3^*)+\Gamma_0(\lambda+2\mu)\left(X_3^2+m_3^*X_3\right)+\rho\left[-\alpha_*^2\left(1+X_3^2\right)+c^2\right],$$

$$q_{44} = \Gamma_0\lambda(1-X_4m_4^*)+\Gamma_0(\lambda+2\mu)\left(X_4^2+m_4^*X_4\right)+\rho\left[-\alpha_*^2\left(1+X_4^2\right)+c^2\right],$$

$$q_{45} = \Gamma_0\lambda(1+X_1m_1^*)+\Gamma_0(\lambda+2\mu)\left(X_1^2-m_1^*X_1\right)+\rho\left[-\alpha_*^2\left(1+X_1^2\right)+c^2\right],$$

$$q_{46} = \Gamma_0\lambda(1+X_2m_2^*)+\Gamma_0(\lambda+2\mu)\left(X_2^2-m_2^*X_2\right)+\rho\left[-\alpha_*^2\left(1+X_2^2\right)+c^2\right],$$

$$q_{47} = \Gamma_0\lambda(1+X_3m_3^*)+\Gamma_0(\lambda+2\mu)\left(X_3^2-m_3^*X_3\right)+\rho\left[-\alpha_*^2\left(1+X_3^2\right)+c^2\right],$$

$$q_{48} = \Gamma_0\lambda(1+X_4m_4^*)+\Gamma_0(\lambda+2\mu)\left(X_4^2-m_4^*X_4\right)+\rho\left[-\alpha_*^2\left(1+X_4^2\right)+c^2\right],$$

$$q_{49} = \Gamma_0'\lambda'(1+X_1'm_1'^*)+\Gamma_0'(\lambda'+2\mu')\left(X_1'^2-m_1'^*X_1'\right)+\rho'\left[-\alpha_*'^2\left(1+X_1'^2\right)+c^{2'}\right],$$

$$q_{50} = \Gamma_0'\lambda'(1+X_2'm_2'^*)+\Gamma_0'(\lambda'+2\mu')\left(X_2'^2-m_2'^*X_2'\right)+\rho'\left[-\alpha_*'^2\left(1+X_2'^2\right)+c^{2'}\right],$$

$$q_{51} = \Gamma_0'\lambda'(1+X_3'm_3'^*)+\Gamma_0'(\lambda'+2\mu')\left(X_3'^2-m_3'^*X_3'\right)+\rho'\left[-\alpha_*'^2\left(1+X_3'^2\right)+c^{2'}\right],$$

$$q_{52} = \Gamma_0'\lambda'(1+X_4'm_4'^*)+\Gamma_0'(\lambda'+2\mu')\left(X_4'^2-m_4'^*X_4'\right)+\rho'\left[-\alpha_*'^2\left(1+X_4'^2\right)+c^{2'}\right],$$

$$q_{53} = -2k^2\Gamma_0\mu X_1-k^2\Gamma_0\mu m_1^*\left(1-X_1^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2 m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{54} = -2k^2\Gamma_0\mu X_2-k^2\Gamma_0\mu m_2^*\left(1-X_2^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2 m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{55} = -2k^2\Gamma_0\mu X_3 - k^2\Gamma_0\mu m_3^*\left(1 - X_3^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_3^*\left(1 + X_3^2\right) - m_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{56} = -2k^2\Gamma_0\mu X_4 - k^2\Gamma_0\mu m_4^*\left(1 - X_4^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{57} = 2k^2\Gamma_0\mu X_1 - k^2\Gamma_0\mu m_1^*\left(1 - X_1^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_1^*\left(1 + X_1^2\right) - m_1^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{58} = 2k^2\Gamma_0 X_2 - k^2\Gamma_0\mu m_2^*\left(1 - X_2^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_2^*\left(1 + X_2^2\right) - m_2^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{59} = 2k^2\Gamma_0\mu X_3 - k^2\Gamma_0\mu m_3^*\left(1 - X_3^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_3^*\left(1 + X_3^2\right) - m_3^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{60} = 2k^2\Gamma_0\mu X_4 - k^2\Gamma_0\mu m_4^*\left(1 - X_4^2\right) + \frac{F}{s_1}\left\{k^2\beta_*^2 m_4^*\left(1 + X_4^2\right) - m_4^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{61} = 2k^2\Gamma_0'\mu' X_1' - k^2\Gamma_0'\mu' m_1'^*\left(1 - X_1'^2\right) + \frac{F}{s_1'}\left\{k^2\beta_*'^2 m_1'^*\left(1 + X_1'^2\right) - m_1'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{62} = 2k^2\Gamma_0'\mu' X_2' - k^2\Gamma_0'\mu' m_2'^*\left(1 - X_2'^2\right) + \frac{F}{s_1'}\left\{k^2\beta_*'^2 m_2'^*\left(1 + X_2'^2\right) - m_2'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{63} = 2k^2\Gamma_0'\mu' X_3' - k^2\Gamma_0'\mu' m_3'^*\left(1 - X_3'^2\right) + \frac{F}{s_1'}\left\{k^2\beta_*'^2 m_3'^*\left(1 + X_3'^2\right) - m_3'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{64} = 2k^2\Gamma_0'\mu' X_4' - k^2\Gamma_0'\mu' m_4'^*\left(1 - X_4'^2\right) + \frac{F}{s_1'}\left\{k^2\beta_*'^2 m_4'^*\left(1 + X_4'^2\right) - m_4'^*\left(k^2c^2 + \Omega^2\right) + 2ik\Omega c\right\},$$

$$q_{65} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_1^2 + 1\right) + k^2c^2\right], \qquad q_{66} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_2^2 + 1\right) + k^2c^2\right],$$

$$q_{67} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_3^2 + 1\right) + k^2c^2\right], \qquad q_{68} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_4^2 + 1\right) + k^2c^2\right],$$

$$q_{69} = \frac{\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_1'^2 + 1\right) + k^2c^2\right], \qquad q_{70} = \frac{\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_2'^2 + 1\right) + k^2c^2\right],$$

$$q_{71} = \frac{\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_3'^2 + 1\right) + k^2c^2\right], \qquad q_{72} = \frac{\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_4'^2 + 1\right) + k^2c^2\right],$$

$$q_{73} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_1^2 + 1\right) + k^2c^2\right]ikX_1, \qquad q_{74} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_2^2 + 1\right) + k^2c^2\right]ikX_2,$$

$$q_{75} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_3^2 + 1\right) + k^2c^2\right]ikX_3, \qquad q_{76} = \frac{\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_4^2 + 1\right) + k^2c^2\right]ikX_4,$$

$$q_{77} = \frac{-\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_1^2 + 1\right) + k^2c^2\right]ikX_1, \qquad q_{78} = \frac{-\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_2^2 + 1\right) + k^2c^2\right]ikX_2,$$

$$q_{79} = \frac{-\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_3^2 + 1\right) + k^2c^2\right]ikX_3, \qquad q_{80} = \frac{-\rho}{\gamma}\left[-\alpha_*^2 k^2\left(X_4^2 + 1\right) + k^2c^2\right]ikX_4,$$

$$q_{81} = \frac{-\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_1'^2 + 1\right) + k^2c^2\right]ikX_1', \qquad q_{82} = \frac{-\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_2'^2 + 1\right) + k^2c^2\right]ikX_2',$$

$$q_{83} = \frac{-\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_3'^2+1\right)+k^2c^2\right]ikX_3', \qquad q_{84} = \frac{-\rho'}{\gamma'}\left[-\alpha_*'^2 k^2\left(X_3'^2+1\right)+k^2c^2\right]ikX_3',$$

$$q_{85} = MX_1\left\{k^2m_1^*\left(X_1^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{86} = MX_2\left\{k^2m_j^*\left(X_2^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{87} = MX_3\left\{k^2m_j^*\left(X_3^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{88} = MX_4\left\{k^2m_4^*\left(X_4^2+1\right)+\frac{1}{ikcs_1}\left(k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right)\right\},$$

$$q_{89} = \Gamma_0\lambda(1-X_1m_1^*)+\Gamma_0(\lambda+2\mu)\left(X_1^2+m_1^*X_1\right)+\rho\left[-\alpha_*^2\left(1+X_1^2\right)+c^2\right],$$

$$q_{90} = \Gamma_0\lambda(1-X_2m_2^*)+\Gamma_0(\lambda+2\mu)\left(X_2^2+m_2^*X_2\right)+\rho\left[-\alpha_*^2\left(1+X_2^2\right)+c^2\right],$$

$$q_{91} = \Gamma_0\lambda(1-X_3m_3^*)+\Gamma_0(\lambda+2\mu)\left(X_3^2+m_3^*X_3\right)+\rho\left[-\alpha_*^2\left(1+X_3^2\right)+c^2\right],$$

$$q_{92} = \Gamma_0\lambda(1-X_4m_4^*)+\Gamma_0(\lambda+2\mu)\left(X_4^2+m_4^*X_4\right)+\rho\left[-\alpha_*^2\left(1+X_4^2\right)+c^2\right],$$

$$q_{93} = \Gamma_0\lambda(1+X_1m_1^*)+\Gamma_0(\lambda+2\mu)\left(X_1^2-m_1^*X_1\right)+\rho\left[-\alpha_*^2\left(1+X_1^2\right)+c^2\right],$$

$$q_{94} = \Gamma_0\lambda(1+X_2m_2^*)+\Gamma_0(\lambda+2\mu)\left(X_2^2-m_2^*X_2\right)+\rho\left[-\alpha_*^2\left(1+X_2^2\right)+c^2\right],$$

$$q_{95} = \Gamma_0\lambda(1+X_3m_3^*)+\Gamma_0(\lambda+2\mu)\left(X_3^2-m_3^*X_3\right)+\rho\left[-\alpha_*^2\left(1+X_3^2\right)+c^2\right],$$

$$q_{96} = \Gamma_0\lambda(1+X_4m_4^*)+\Gamma_0(\lambda+2\mu)\left(X_4^2-m_4^*X_4\right)+\rho\left[-\alpha_*^2\left(1+X_4^2\right)+c^2\right],$$

$$q_{97} = -2k^2\Gamma_0\mu X_1-k^2\Gamma_0\mu m_1^*\left(1-X_1^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{98} = -2k^2\Gamma_0\mu X_2-k^2\Gamma_0\mu m_2^*\left(1-X_j^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{99} = -2k^2\Gamma_0\mu X_3-k^2\Gamma_0\mu m_3^*\left(1-X_3^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{100} = -2k^2\Gamma_0\mu X_4-k^2\Gamma_0\mu m_4^*\left(1-X_4^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{101} = 2k^2\Gamma_0\mu X_1-k^2\Gamma_0\mu m_1^*\left(1-X_1^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_1^*\left(1+X_1^2\right)-m_1^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{102} = 2k^2\Gamma_0\mu X_2-k^2\Gamma_0\mu m_2^*\left(1-X_2^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_2^*\left(1+X_2^2\right)-m_2^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{103} = 2k^2\Gamma_0\mu X_3-k^2\Gamma_0\mu m_3^*\left(1-X_3^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_3^*\left(1+X_3^2\right)-m_3^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{104} = 2k^2\Gamma_0\mu X_4-k^2\Gamma_0\mu m_4^*\left(1-X_4^2\right)+\frac{F}{s_1}\left\{k^2\beta_*^2m_4^*\left(1+X_4^2\right)-m_4^*\left(k^2c^2+\Omega^2\right)+2ik\Omega c\right\},$$

$$q_{105} = \left[-\alpha_*^2 k^2 \left(X_1^2 + 1\right) + k^2 c^2\right] ikX_1, \qquad q_{106} = \left[-\alpha_*^2 k^2 \left(X_2^2 + 1\right) + k^2 c^2\right] ikX_2,$$
$$q_{107} = \left[-\alpha_*^2 k^2 \left(X_3^2 + 1\right) + k^2 c^2\right] ikX_3, \qquad q_{108} = \left[-\alpha_*^2 k^2 \left(X_4^2 + 1\right) + k^2 c^2\right] ikX_4,$$
$$q_{109} = \left[\alpha_*^2 k^2 \left(X_1^2 + 1\right) - k^2 c^2\right] ikX_1, \qquad q_{110} = \left[\alpha_*^2 k^2 \left(X_2^2 + 1\right) - k^2 c^2\right] ikX_2,$$
$$q_{111} = \left[\alpha_*^2 k^2 \left(X_3^2 + 1\right) - k^2 c^2\right] ikX_3, \qquad q_{112} = \left[\alpha_*^2 k^2 \left(X_4^2 + 1\right) - k^2 c^2\right] ikX_4. \tag{5.31}$$

From (5.30), we can determine by numerical effects the initial stress, gravity field, friction coefficient, magnetic field, and rotation, for a computation using the maple program; we use sandstone as a granular medium and nephiline as a granular layer taking into consideration that the relaxation times $\tau_0 = 0.1, \tau_1 = 0.4$, and $\tau_2 = 0.5$, the friction coefficient $F = 0.4$, and the third elastic constant $M = 0.2$.

(i) Effects of the initial stress, gravity field, friction coefficient, magnetic field, relaxation time, and rotation are discussed in Figures 2 and 3.

(ii) From (5.30), if the initial stress are neglected, we can discuss the effects of the gravity field, friction coefficient, magnetic field, relaxation time, and rotation, and the discussion is clear up from Figure 4.

(iii) From (5.30), if the initial stress and magnetic field are neglected, we can discuss the effects of the gravity field, friction coefficient, relaxation time and rotation, and the discussion is clear up from Figure 5.

(iv) From (5.30), if the initial stress, magnetic field, and gravity field are neglected, we can discuss the effects of the friction coefficient, relaxation time, and rotation, and the discussion is clear up from Figure 6.

(v) From (5.30), if the initial stress, magnetic field, and gravity field are neglected and there is uncoupling between the temperature and strain field, we can discuss the effects the friction coefficient, relaxation time, rotation, and the discussion is clear up from Figure 7.

6. Numerical Results and Discussion

In order to illustrate theoretical results obtained in the proceeding section, we now present some numerical results. The material chosen for this purpose of Carbon steel, the physical data is given [21] as follows:

$$\rho = 2\,\text{kgm}^{-3}, \qquad \lambda = 9.3 \times 10^{10}\,\text{Nm}^{-1}, \qquad \mu = 8.4 \times 10^{10}\,\text{Nm}^{-1}, \qquad T_0 = 293.1\,\text{k},$$
$$K = 50\,\text{Wm}^{-1}\text{k}^{-1}, \quad s = 6.4 \times 10^2\,\text{JKg}^{-1}, \qquad \alpha_t = 13.2 \times 10^{-6}\,\text{deg}^{-1}. \tag{6.1}$$

6.1. Effects of the Initial Stress, Gravity Field, Friction Coefficient, Magnetic Field, Relaxation Time, and Rotation

Figure 2 shows the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) under the effect of gravity field, friction coefficient, magnetic field, relaxation time, and rotation with respect to the initial stress; we found that the velocity of Rayleigh waves (Re) and attenuation

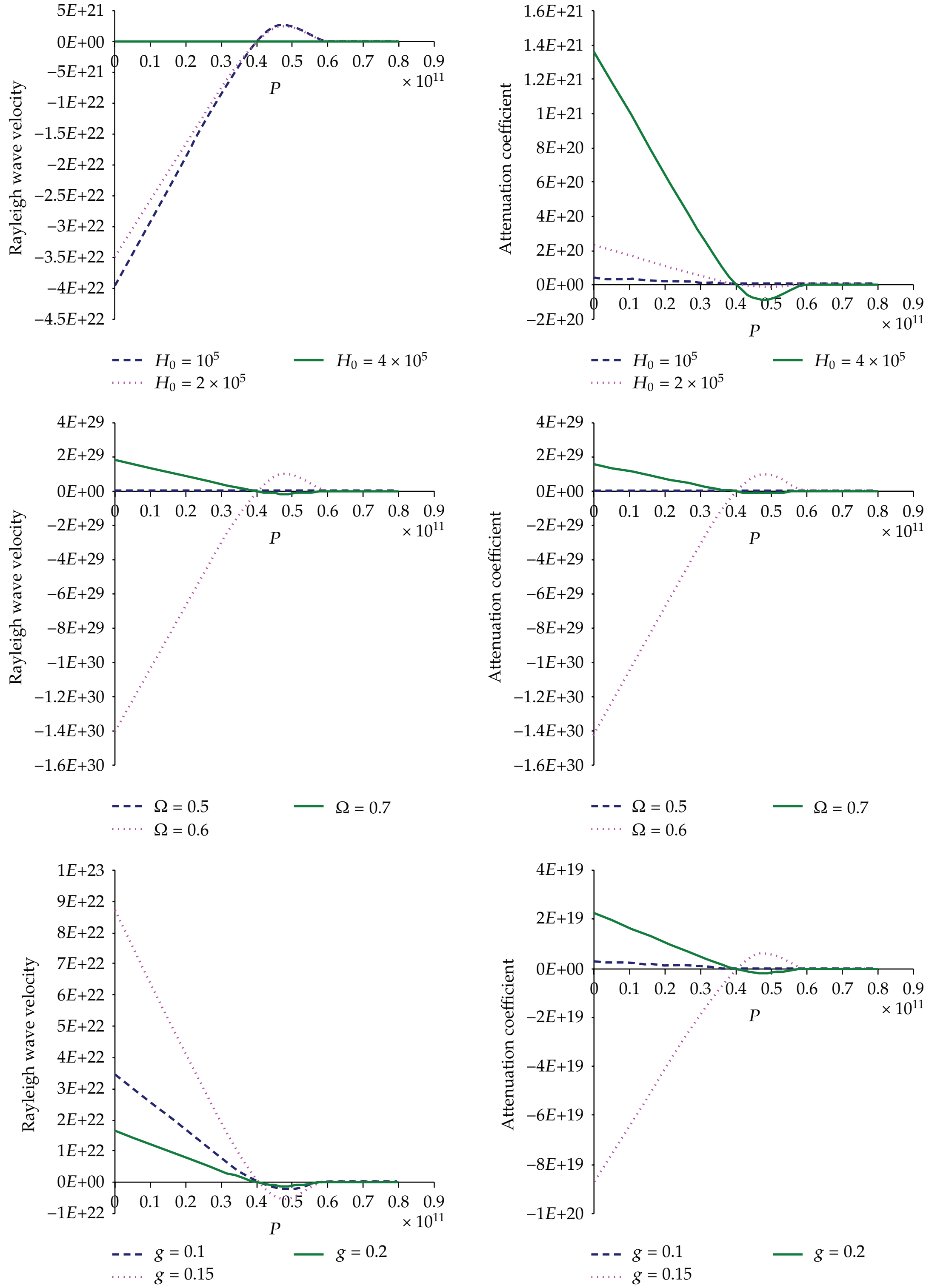

Figure 2: Continued.

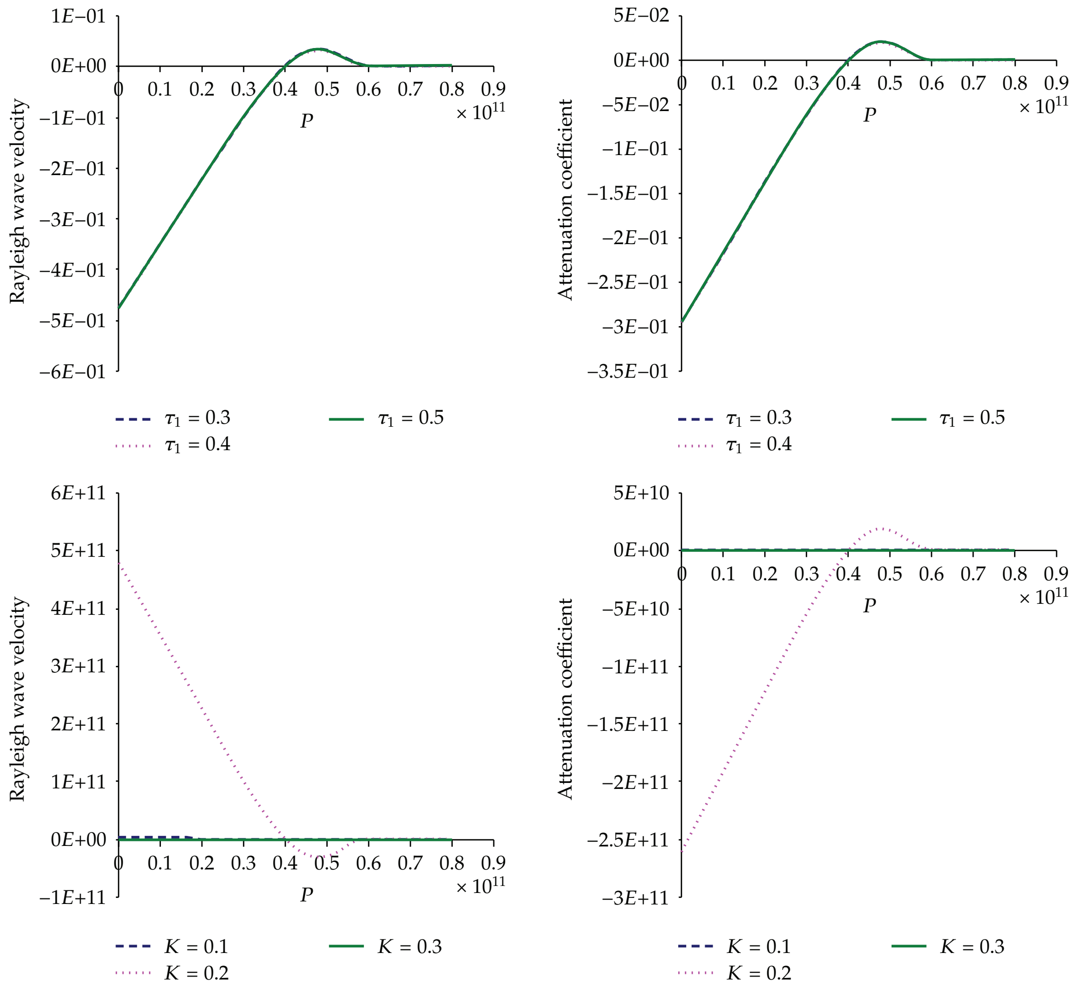

Figure 2: Effects of H_0, Ω, g, τ_1, and p on Rayleigh wave velocity and attenuation coefficient with respect to the initial stress.

coefficient (Im) increased with increasing values of p and H_0, and the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) decreased and increased with increasing values of g and K, respectively; while the values of (Re) and (Im) take one curve at another value of the relaxation time τ_1 increased with increasing values of initial stress P.

Figure 3 shows the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) under effect of initial stress, gravity field, friction coefficient, magnetic field, relaxation time and rotation with respect to the wave number, we find that the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) decreased and increased with increasing values of H_0, respectively, and the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) increased and decreased with increasing values of Ω and g, respectively; also, the values of (Re) and (Im) increased with increasing values of K, while the values of (Re) and (Im) take one curve at another value of the relaxation time τ_1, decreased with increasing values of wave number k.

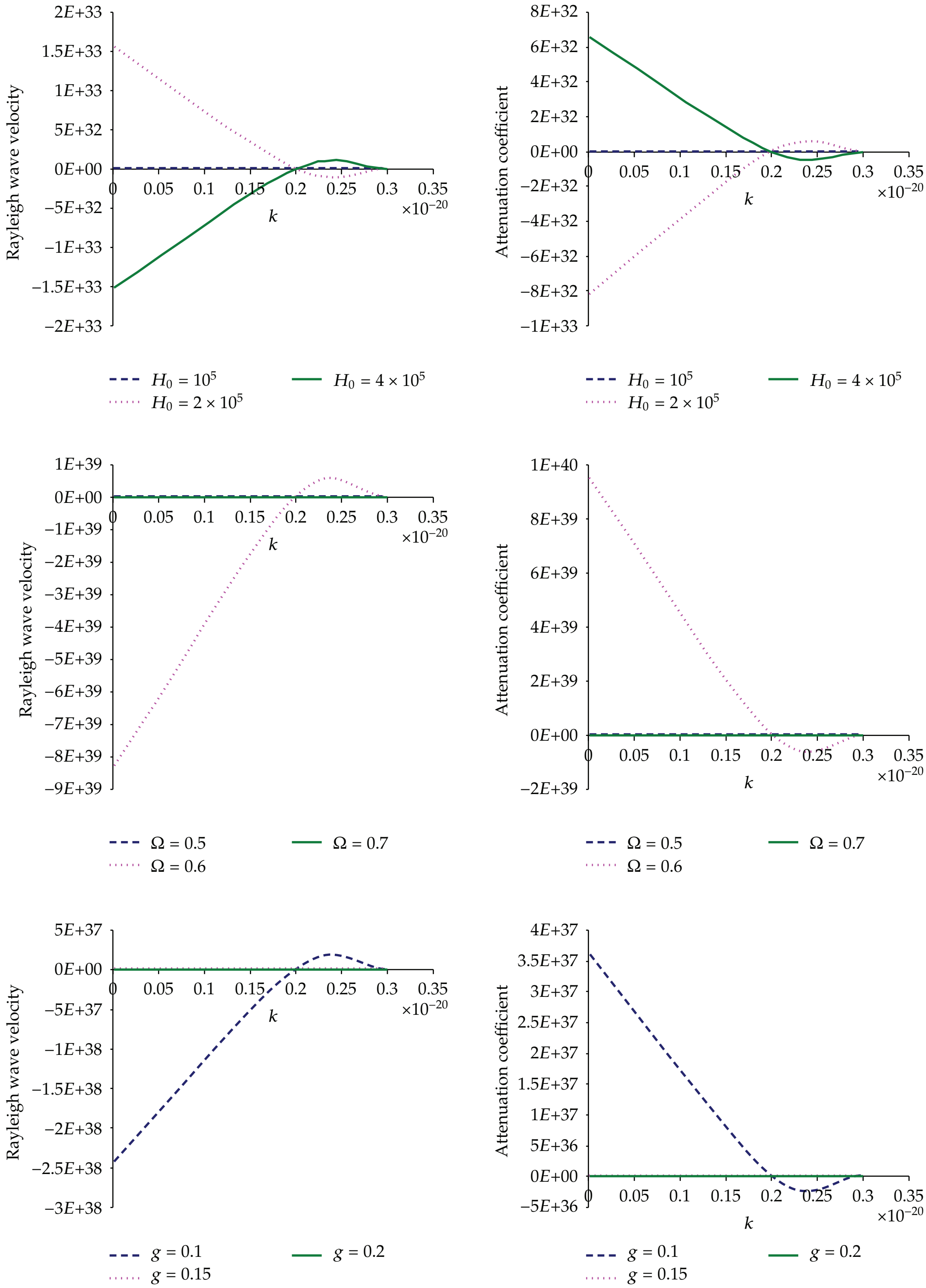

Figure 3: Continued.

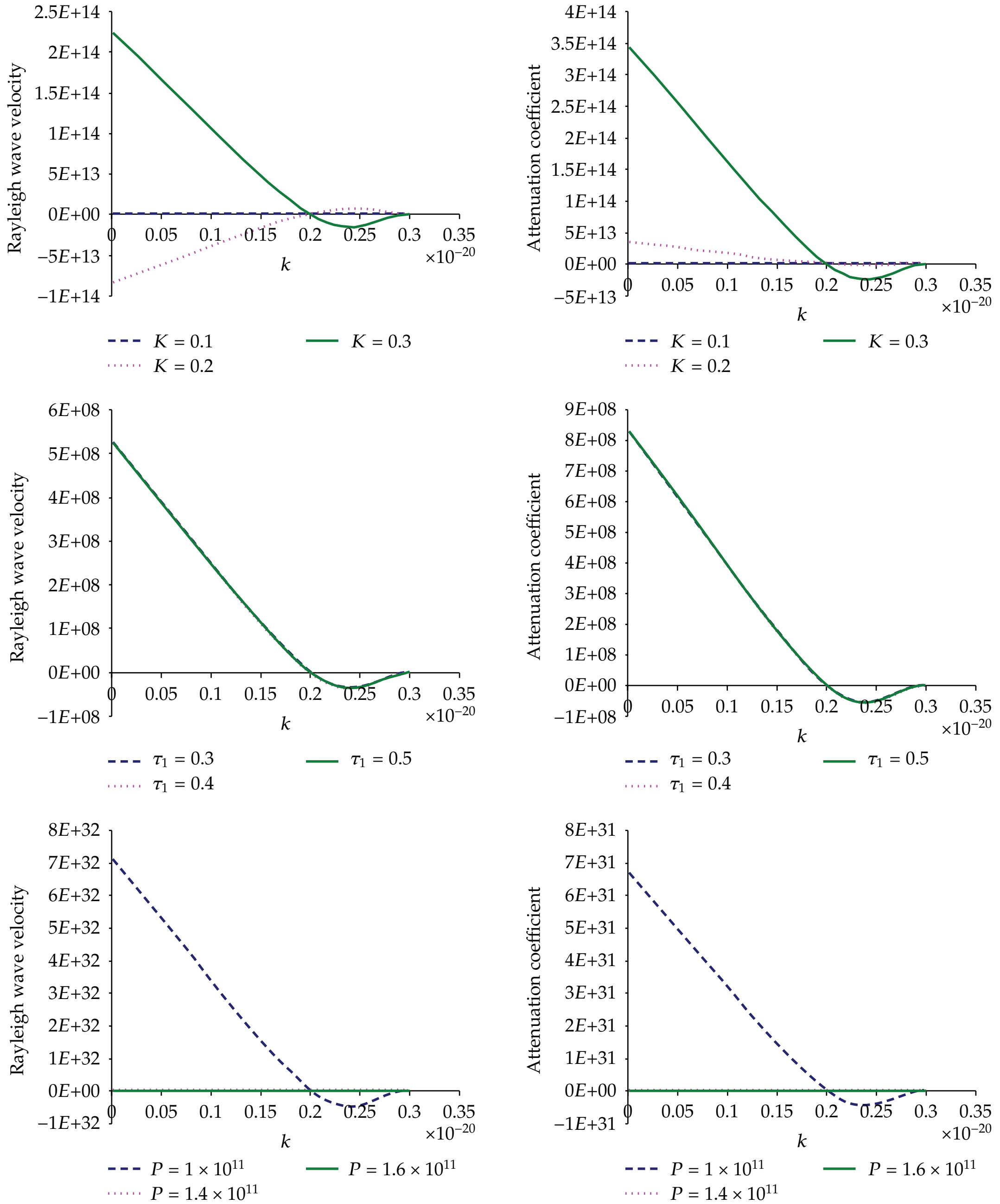

Figure 3: Effects of H_0, Ω, g, τ_1, and K, P on Rayleigh wave velocity and Attenuation coefficient with respect to the wave number.

6.2. If the Initial Stresses Are Neglected

Figure 4 shows the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) under effect of gravity field, friction coefficient, magnetic field, relaxation time, and rotation with respect to the wave number, we find that the velocity of Rayleigh waves (Re) and attenuation

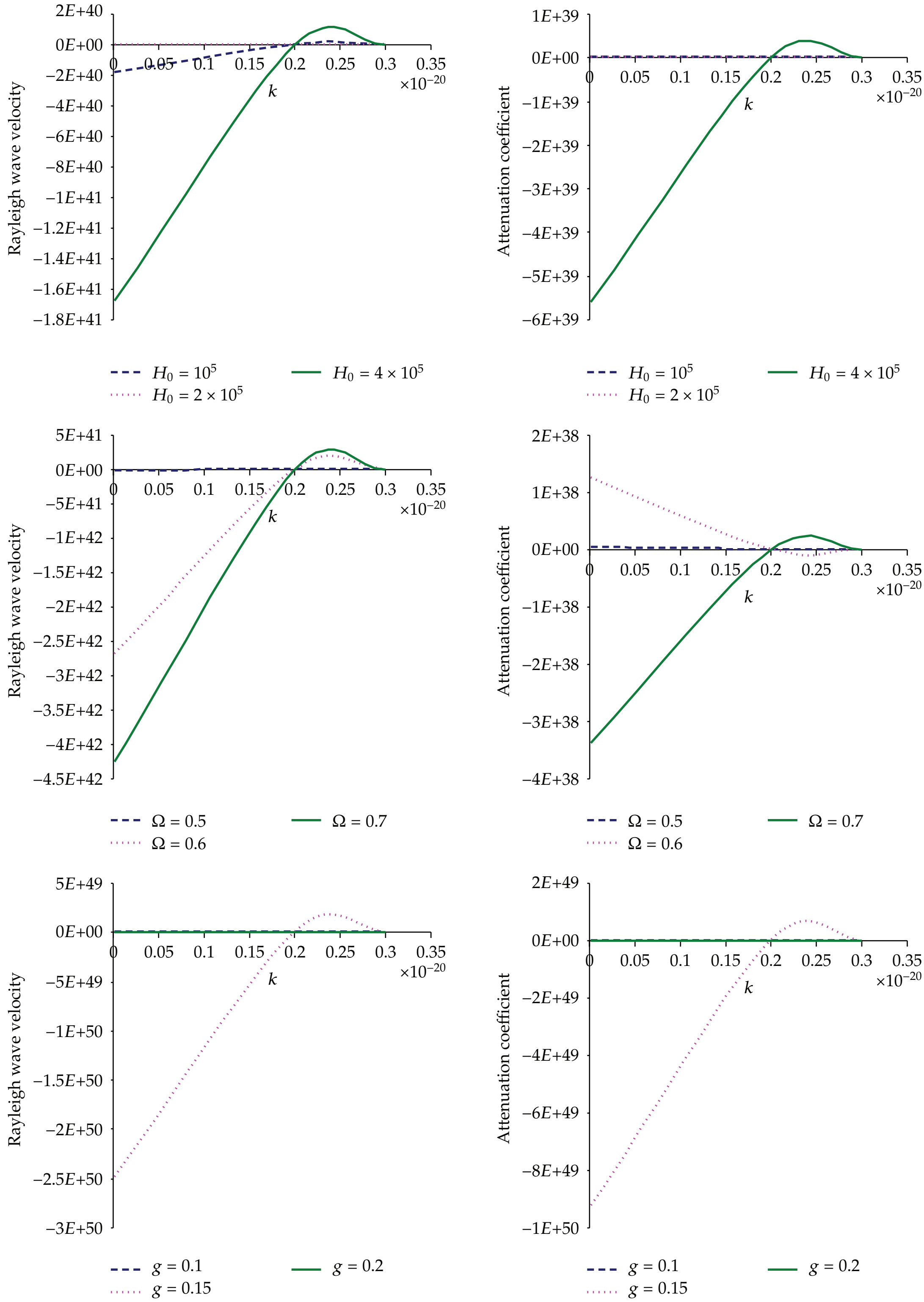

Figure 4: Continued.

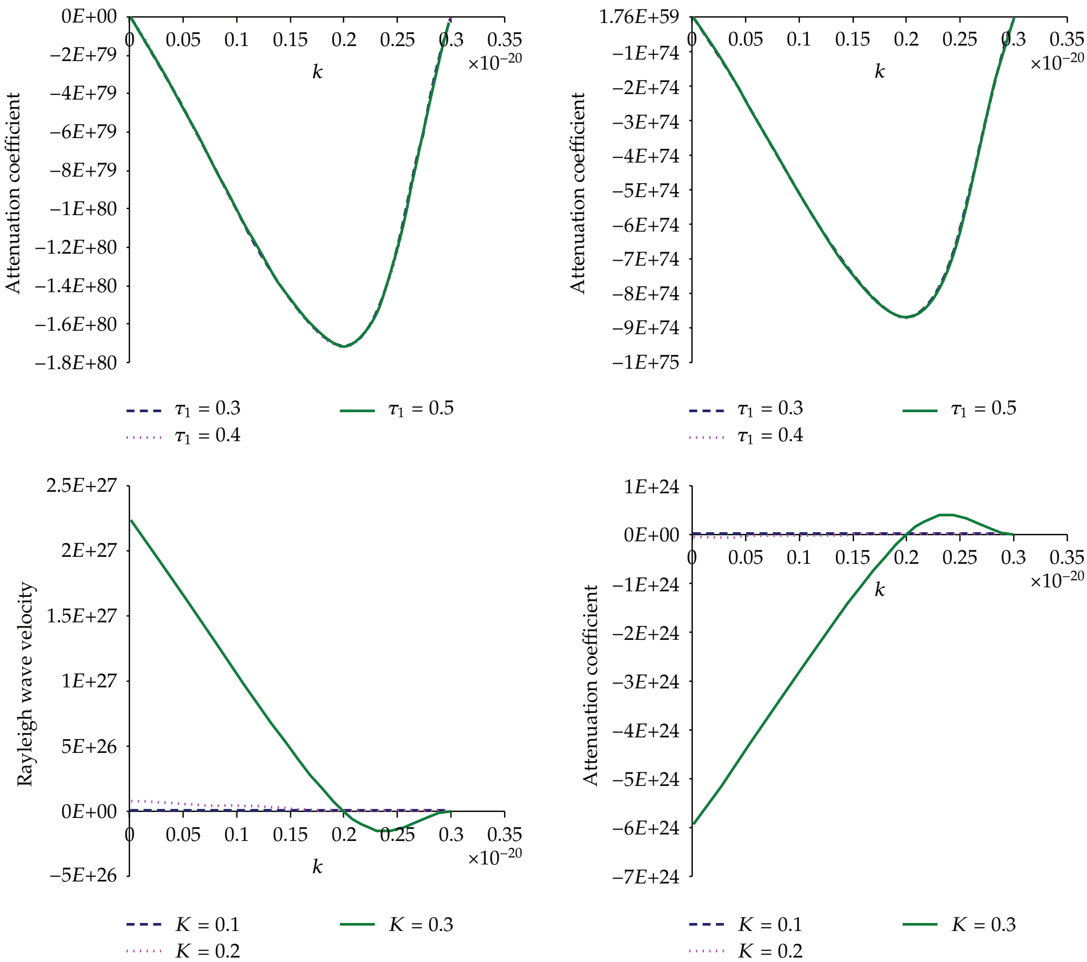

Figure 4: Effects of H_0, Ω, g, τ_1, and K on Rayleigh wave velocity and attenuation coefficient with respect to the wave number.

coefficient (Im) decreased with increasing values of H_0 and Ω, while that contrary with increasing values of g; also, the values of (Re) and (Im) increased and decreased with increasing values of K, respectively, while the values of (Re) and (Im) take one curve at another value of the relaxation time τ_1 decreased, then increased with increasing values of wave number k.

6.3. *If the Initial Stresses and Magnetic Field Are Neglected*

Figure 5 shows that the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) under effect of gravity field, friction coefficient, relaxation time, and rotation with respect to the wave number; we find that the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) decreased and increased with increasing values of Ω, and the values of (Re) and (Im) increased with increasing values of g, while that contrary with increasing values of K; also, the

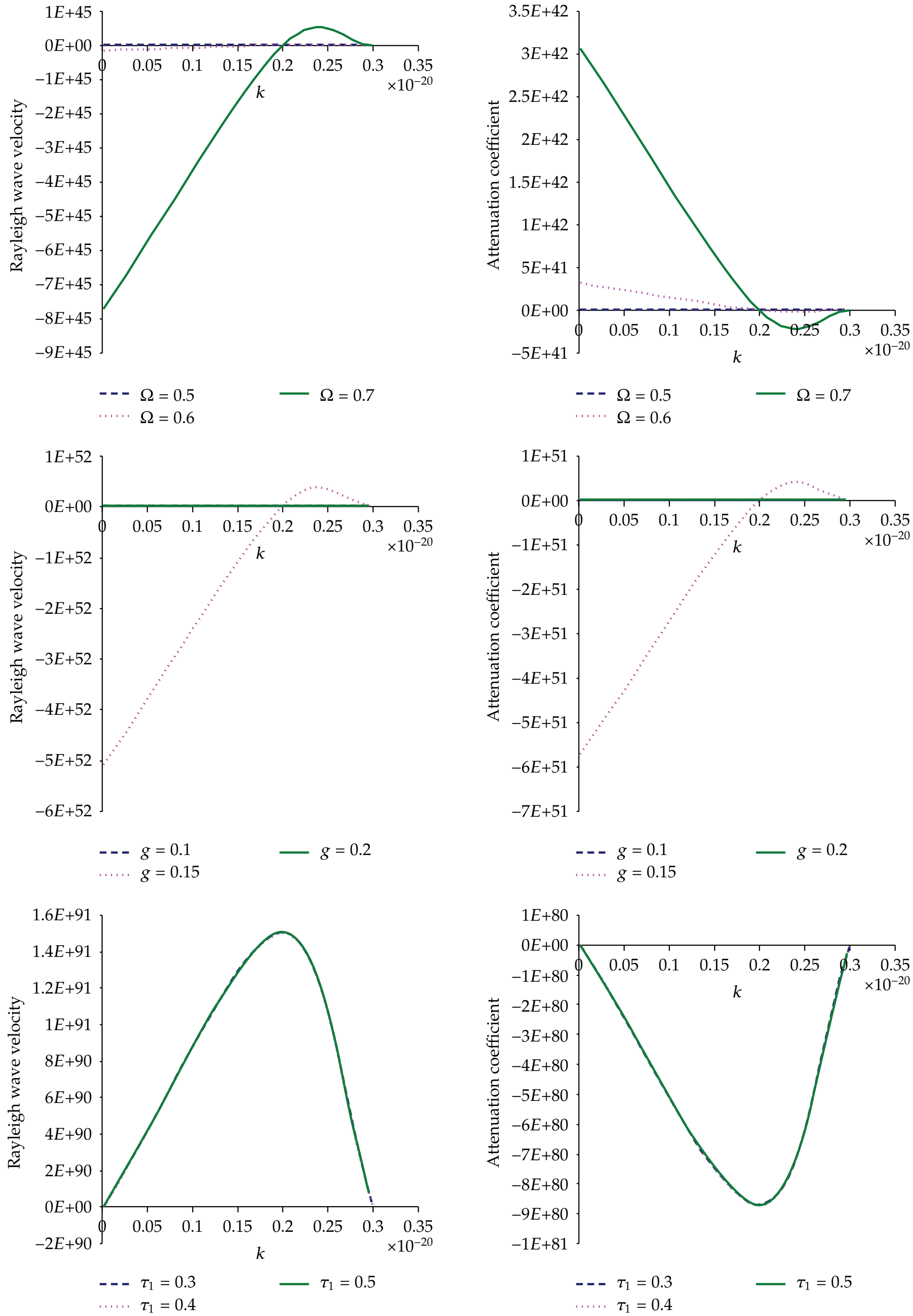

Figure 5: Continued.

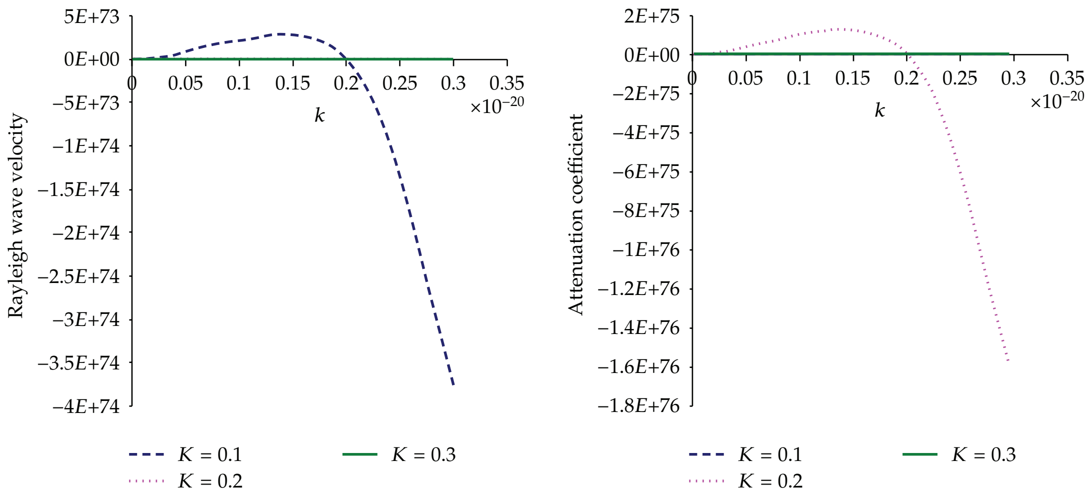

Figure 5: Effects of Ω, g, τ_1, and K on Rayleigh wave velocity and attenuation coefficient with respect to the wave number.

values of (Re) and (Im) take one curve at another value of the relaxation time τ_1, increased, then decreased with increasing values of wave number k.

6.4. If the Initial Stresses, Magnetic Field, and Gravity Field Are Neglected

Figure 6 shows the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) under the effect of friction coefficient, relaxation time, and rotation with respect to the wave number; we find that the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) decreased and increased with increasing Ω, and the values of (Re) and (Im) decreased with increasing values of K, while the values of (Re) and (Im) take one curve at another value of the relaxation time τ_1, decreased and increased with increasing values of the wave number k, respectively.

6.5. If the Initial Stresses, Magnetic Field, and Gravity Field Are Neglected and There Is Uncoupling between the Temperature and Strain Field

Figure 7 shows the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) under the effect of friction coefficient, relaxation time, and rotation with respect to the wave number; we find that the velocity of Rayleigh waves (Re) and attenuation coefficient (Im) increased and decreased with increasing of Ω, respectively, while that contrary with increasing values of K; finally, the values of (Re) and (Im) take one curve at another value of the relaxation time τ_1, decreased with increasing, the values of the wave number k.

7. Conclusions

The problem of the Rayleigh waves in generalized magneto-thermo-viscoelastic granular medium under the influence of rotation, gravity field, and initial stress is considered, and the frequency equation of the wave motion in the explicit form is derived, by considering

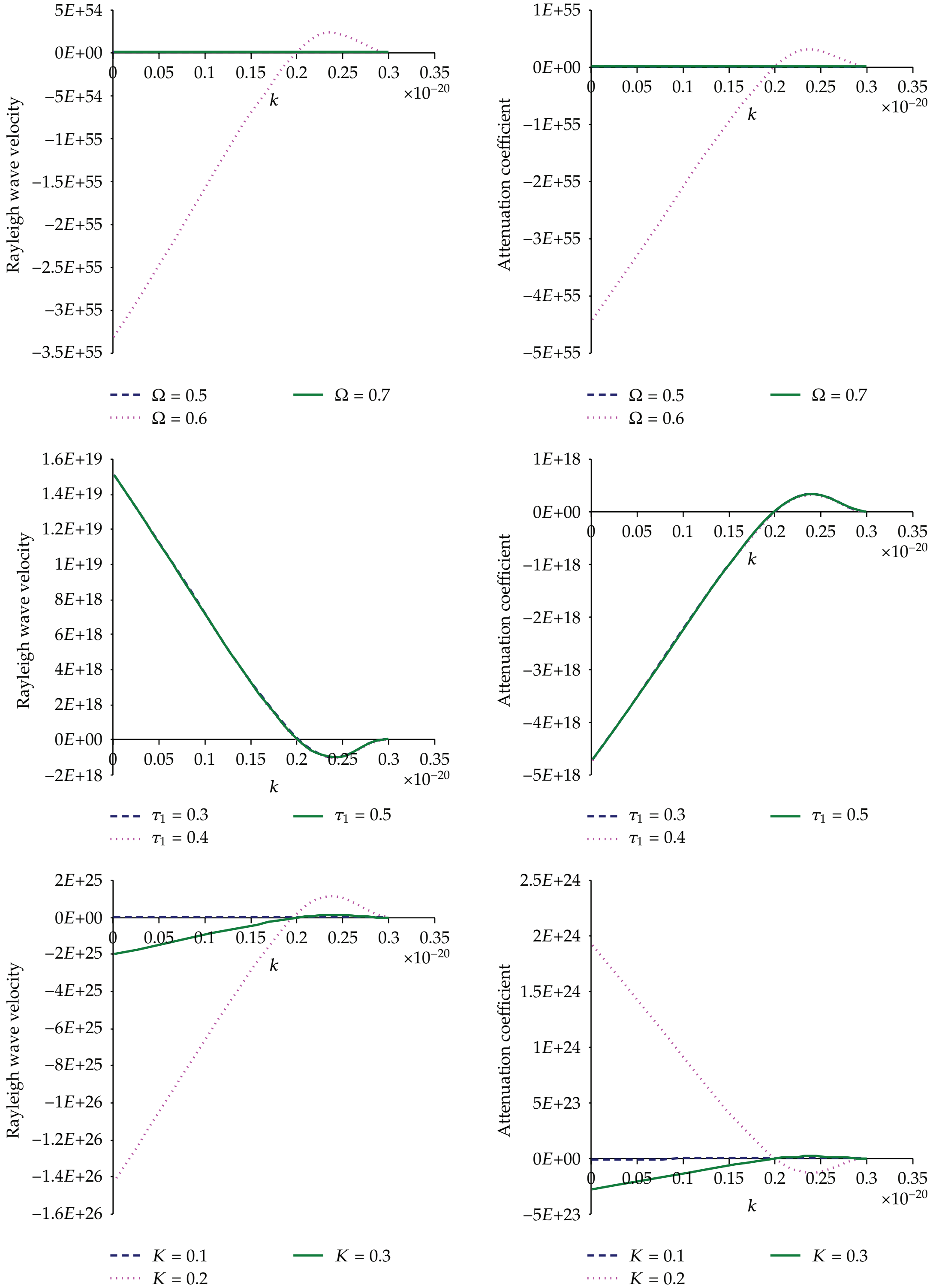

Figure 6: Effects of Ω, τ_1, and K on Rayleigh wave velocity and attenuation coefficient with respect to the wave number.

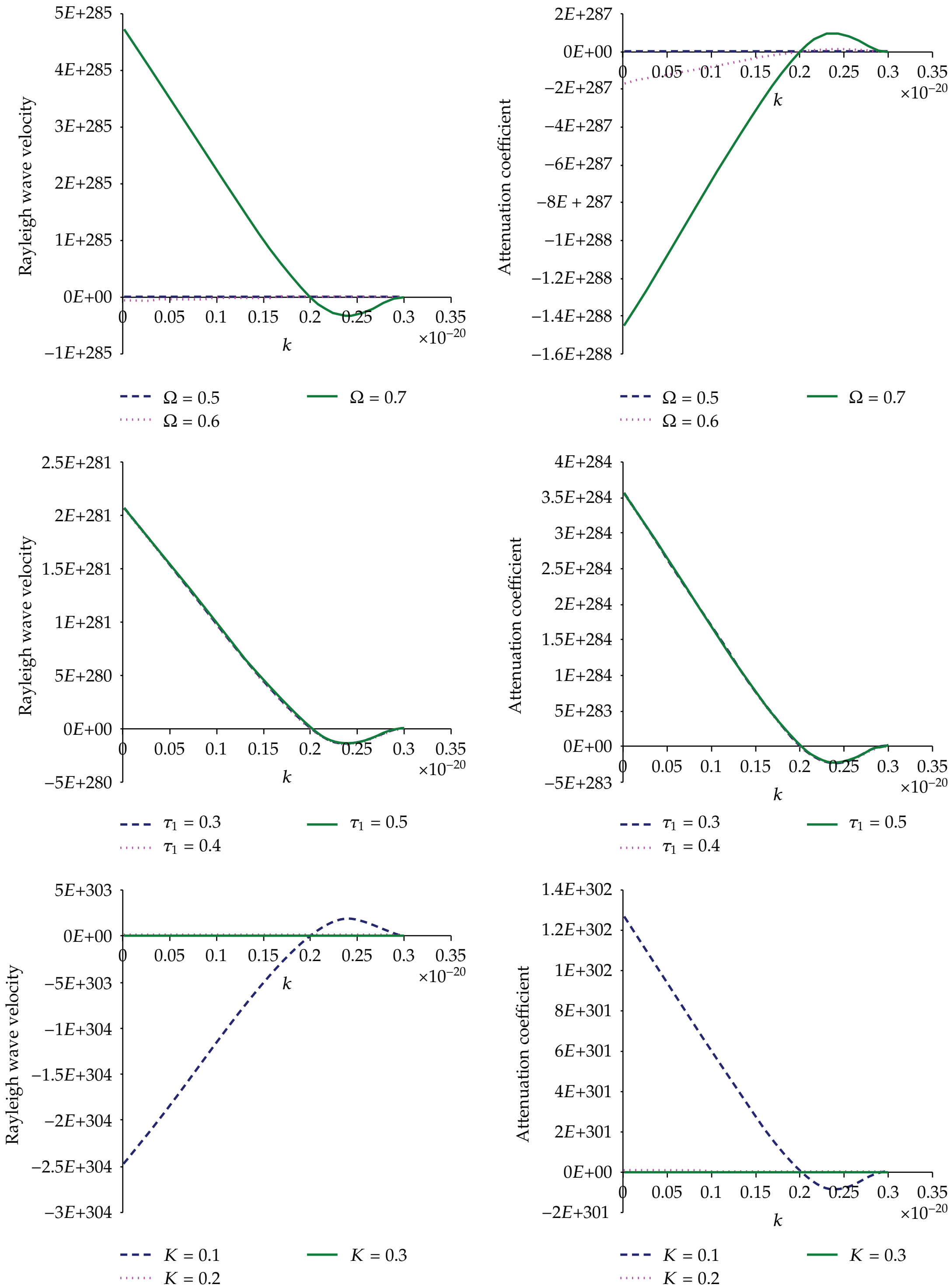

Figure 7: Effects of Ω, τ_1, and K on Rayleigh wave velocity and attenuation coefficient with respect to the wave number.

various special cases. The numerical results are obtained for carbon-steel material, although the effect of the rotation, magnetic field, relaxation times, initial stress, gravity field, and friction coefficient is observed to be quite large on wave propagation of Rayleigh wave velocity (Re) and attenuation coefficient (Im).

The problem of the Rayleigh waves in generalized magneto-thermo-viscoelastic granular medium under the influence of rotation, gravity field, and initial stress is considered, and the frequency equation of the wave motion in the explicit form is derived, by considering various special cases. The numerical results are obtained for carbon-steel material, although the effect of the rotation, magnetic field, relaxation times, initial stress, gravity field, and friction coefficient is observed to be quite large on wave propagation of Rayleigh wave velocity (Re) and attenuation coefficient (Im).

It is easy to see that the values of (Re) and (Im) with respect to the initial stress are increased with increasing values of Ω, while that contrary if the initial stress are neglected and with respect to the wave number; also, if the initial stress are constant with respect to the wave number the values of (Re) and (Im) increased and decreased with increasing values of Ω, respectively, and if the initial stress and the magnetic field are neglected, the values of (Re) and (Im) decreased and increased with increasing values of Ω, respectively, while that contrary if P, H_0, g, θ, and ε are neglected and with respect to the wave number; finally, if P, H_0, and g are neglected and with respect to the wave number, the values of (Re) and (Im) increased with increasing values of Ω.

It is easy to see that the values of (Re) and (Im) with respect to the initial stress are decreased and increased with increasing values of g, respectively, while that contrary if the initial stress are constant and with respect to the wave number; also, if the initial stress are neglected and if the initial stress and the magnetic field are neglected with respect to the wave number, the values of (Re) and (Im) increased with increasing values of g.

It is easy to see that the values of (Re) and (Im) with respect to the initial stress are decreased and increased with increasing values of K, respectively, while that contrary if the initial stress are neglected and with respect to the wave number; also, if the initial stress and the magnetic field are neglected and if P, H_0, and g are neglected with respect the wave number, the values of (Re) and (Im) decreased with increasing values of K; finally, if P, H_0, g, θ, and ε are neglected and with respect to the wave number, the values of (Re) and (Im) decreased and increased with increasing values of K.

Finally, the frequency equation has been discussed under effect of rotation, gravity field, and initial stress and in case of various classical and nonclassical theories of thermoelasticity. The results indicate that the effect of rotation, magnetic field, initial stress, and gravity field is very pronounced. The frequency equations derived in this paper may be useful in practical applications. It is concluded from the above analyses and results that the present solution is accurate and reliable and the method is simple and effective. So it may be as a reference to solve other problems of Rayleigh waves in generalized magneto-thermoelastic granular medium.

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