Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2011, Article ID 791052, 11 pages
doi:10.1155/2011/791052

Research Article

Collective Coordination of High-Order Dynamic Multiagent Systems Guided by Multiple Leaders

Xin-Lei Feng, Ting-Zhu Huang, and Jin-Liang Shao

School of Mathematical Sciences/Institute of Computational Science, University of Electronic Science and Technology of China, Sichuan, Chengdu 611731, China

Correspondence should be addressed to Xin-Lei Feng, xinleifeng@uestc.edu.cn

Received 6 December 2010; Revised 20 March 2011; Accepted 17 April 2011

Academic Editor: John Burns

For second-order and high-order dynamic multiagent systems with multiple leaders, the coordination schemes that all the follower agents flock to the polytope region formed by multiple leaders are considered. Necessary and sufficient conditions which the follower agents can enter the polytope region by the leaders are obtained. Finally, numerical examples are given to illustrate our theoretical results.

1. Introduction

Recently, collective coordinations of multiagent systems have received significant attention due to their potential impact in numerous civilian, homeland security, and military applications, and so forth. For example, Wei et al. [1] described a multiagent recommender system in which the agents form a marketplace and compete to provide the best recommendation for a given user. De Meo et al. [2] presented an XML-based multiagent system for supporting e-recruitment services, in which the various agents collaborate to extract data and rank them according to user queries and needs.

Consensus plays an important role in achieving distributed coordination. The basic idea of consensus is that a team of vehicles reaches an agreement on a common value by negotiating with their neighbors. Consensus algorithms are studied for both first-order dynamics [3–5] and high-order dynamics [6–10].

Formal study of consensus problems in groups of experts originated in management science and statistics in 1960s. Distributed computation over networks has a tradition in systems and control theory starting with the pioneering work of Borkar and Varaiya [11] and Tsitsiklis [12] in 1980s. In 1995, Vicsek et al. [13] provided a formal analysis of emergence of alignment in the simplified model of flocking. Due to providing Vicsek Model, [13] has an important influence on the development of the multiagent systems consensus theory. On the study of consensus of continuous-time system, the classical model of consensus is provided by Olfati-Saber and Murray [14] in 2004.

In multiagent coordination, leader-follower is an important architecture. Hu and Yuan [15] presented a first-order dynamic collective coordination algorithm of multiagent systems guided by multiple leaders, which make all the follower agents flock to the polytope region formed by the leaders. In this paper, we consider the second-order and high-order dynamic collective coordination algorithms of multiagent systems guided by multiple leaders.

2. Preliminaries

A directed graph (digraph) $G = (V, E)$ of order n consists of a set of nodes $V = \{1, \ldots, n\}$ and a set of edges $E = V \times V$. (j, i) is an edge of G if and only if $(j, i) \in G$. Accordingly, node j is a neighbor of node i. The set of neighbors of node i is denoted by $\mathcal{N}_i(t)$. Suppose that there are n nodes in the graph. The weighted adjacency matrix $A \in \mathbb{R}^{n\times n}$ is defined as $a_{ii} = 0, a_{ij} \geq 0$, and $a_{ij} > 0$ if and only if $(j, i) \in E$. A graph with the property that $(i, j) \in E$ implies $(j, i) \in E$ is said to be undirected. The Laplacian matrix $L \in \mathbb{R}^{n\times n}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}, l_{ij} = -a_{ij}$, for $i \neq j$. Moreover, matrix L is symmetric if an undirected graph has symmetric weights, that is, $a_{ij} = a_{ji}$.

In this paper, we consider a system consisting of n follower-agents and k leaders, and the interconnection topology among them can be described by an undirected graph $\overline{G}$. Where each follower-agent (or leader) is regarded as a node in a graph $\overline{G} = (\overline{V}, \overline{E})$, and each available information channel between the follower-agent (or leader) i and the follower-agent (or leader) j corresponds to a couple of edges $(i, j), (j, i) \in \overline{E}$. $(i, j) \in \overline{E}$ is said that i and j is connected. Moreover, the interconnection topology among follower-agents can be described by an undirected graph G. The undirected graph $\overline{G}$ is connected; we mean that at least one node in each component of G is connected to the nodes who are leaders. A diagonal matrix B to be a leader adjacency matrix associate with $\overline{G}$ with diagonal elements $b_i (i \in \{1, \ldots, n\})$ such that each b_i is some positive number if agent i is connected to the leader node and 0 otherwise.

Let $S \subset \mathbb{R}^m$, S is said to be convex if $(1 - \gamma)x + \gamma y \in S$ whenever $x \in S, y \in S$ and $0 < \gamma < 1$. A vector sum $\gamma_1 x_1 + \gamma_2 x_2 + \cdots + \gamma_n x_n$ is called a convex combination of $x_1, \ldots, x_n$, if the coefficients γ_i are all nonnegative and $\gamma_1 + \cdots + \gamma_n = 1$. The intersection of all convex sets containing S is the convex hull of S. The convex hull of a finite set of points $x_1, \ldots, x_n \in \mathbb{R}^m$ is a polytope [16].

The Kronecker product of $A = [a_{ij}] \in M_{m,n}(F)$ and $B = [b_{ij}] \in M_{p,q}(F)$ is denoted by $A \otimes B$ and is defined to be the block matrix [17]

Lemma 2.1 (see [4]). *(i) All the eigenvalues of Laplacian matrix L have nonnegative real parts; (ii) Zero is an eigenvalue of L with 1_n (where 1_n is the $n \times 1$ column vector of all ones) as the corresponding right eigenvector. Furthermore, zero is a simple eigenvalue of L if and only if graph G has a directed spanning tree.*

3. Coordination Algorithms That All the Follower-Agents Flock to the Polytope Region Formed by the Leaders

A continuous-time second-order dynamics of n follower-agents is described as follows:

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= u_i, \end{aligned} \tag{3.1}$$

where $x_i, v_i \in \mathbb{R}^m$ are the position and velocity of follower-agent i. We consider the following dynamical protocol:

$$u_i = \sum_{j\in\mathcal{N}_i} a_{ij}\left(x_j - x_i\right) + \sum_{q=1}^{k} b_i^q\left(x_0^q - x_i\right) + \sum_{j\in\mathcal{N}_i} a_{ij}\left(v_j - v_i\right) + \sum_{q=1}^{k} b_i^q\left(v_0^q - v_i\right), \tag{3.2}$$

where x_0^j, v_0^j $(j = 1,\ldots,k)$ are the position and velocity of the leader j, nonnegative constant $b_i^q > 0$ if and only if follower-agent i is connected to leader q $(q = 1,\ldots,k)$. The objective of this paper is to lead all the follower-agents to enter the polytope region formed by the leaders, namely, x_i, $i = 1,\ldots,n$, will be contained in a convex hull of x_0^q, $q = 1,\ldots,k$, as $t \to \infty$.

Furthermore, (3.1) and (3.2) can be rewritten as vector form:

$$\begin{gathered} \dot{x} = v, \\ \dot{v} = -(H\otimes I_m)x + [B(I_k\otimes 1_n)]\otimes I_m x_0 - (H\otimes I_m)v + [B(I_k\otimes 1_n)]\otimes I_m v_0, \end{gathered} \tag{3.3}$$

where $H = L + B(1_k \otimes I_n)$, $B^q \in \mathbb{R}^{n\times n}$ is a diagonal matrix with diagonal entry b_i^q, and $B = [B^1 \cdots B^k] \in \mathbb{R}^{n\times nk}$.

Theorem 3.1. *For the multiagent systems given by* (3.1) *and* (3.2)*, the follower-agents can enter the polytope region formed by the leaders if and only if* $\overline{G}$ *is connected.*

Proof. Sufficiency: Let

$$\begin{gathered} \overline{x} = x - \left[H^{-1}B(I_k\otimes 1_n)\right]\otimes I_m x_0, \\ \overline{v} = v - \left[H^{-1}B(I_k\otimes 1_n)\right]\otimes I_m v_0. \end{gathered} \tag{3.4}$$

Then, (3.3) can be rewritten as

$$\begin{gathered} \dot{\overline{x}} = \overline{v}, \\ \dot{\overline{v}} = -(H\otimes I_m)\overline{x} - (H\otimes I_m)\overline{v}. \end{gathered} \tag{3.5}$$

Further, (3.5) can be rewritten as

$$\begin{pmatrix} \dot{\overline{x}} \\ \dot{\overline{v}} \end{pmatrix} = \begin{pmatrix} 0_{mn\times mn} & I_{mn} \\ -(H\otimes I_m) & -(H\otimes I_m) \end{pmatrix} \begin{pmatrix} \overline{x} \\ \overline{v} \end{pmatrix}. \tag{3.6}$$

Let λ be an eigenvalue of $\Gamma = \left(\begin{smallmatrix} 0_{mn\times mn} & I_{mn} \\ -H\otimes I_m & -H\otimes I_m \end{smallmatrix}\right)$, and $\left(\begin{smallmatrix} f \\ g \end{smallmatrix}\right)$ be the corresponding eigenvector of Γ. Then we get

$$\begin{pmatrix} 0_{mn\times mn} & I_{mn} \\ -H\otimes I_m & -H\otimes I_m \end{pmatrix} x = \lambda x, \tag{3.7}$$

which implies

$$g = \lambda f, \qquad -(H \otimes I_m) f - (H \otimes I_m) g = \lambda g. \tag{3.8}$$

Let $\mu_i,\ i = 1, \ldots, n$ be the eigenvalues of $H \otimes I_m$. Thus,

$$-(H \otimes I_m) f - (H \otimes I_m) \lambda f = \lambda^2 f. \tag{3.9}$$

That is, each eigenvalue of $H \otimes I_m$, $\mu_i > 0$ ([11, Lemma 1]), corresponds to two eigenvalues of Γ, denoted by

$$\lambda_{2i-1,2i} = \frac{-\mu_i \pm \sqrt{\mu_i^2 - 4\mu_i}}{2}. \tag{3.10}$$

From (3.10), we can obtain that all the eigenvalues of Γ have negative real parts. By (3.6), we can get

$$\begin{pmatrix} \overline{x} \\ \overline{v} \end{pmatrix} = e^{\Gamma t} \begin{pmatrix} \overline{x}(0) \\ \overline{v}(0) \end{pmatrix}, \tag{3.11}$$

which implies that $\overline{x} \rightarrow 0$ and $\overline{v} \rightarrow 0$ when $t \rightarrow \infty$. Therefore,

$$x = \left[H^{-1} B (I_k \otimes 1_n)\right] \otimes I_m x_0, \qquad v = \left[H^{-1} B (I_k \otimes 1_n)\right] \otimes I_m v_0. \tag{3.12}$$

We also know that $[H^{-1} B (I_k \otimes 1_n)] \otimes I_m$ is a row-stochastic matrix which is a nonnegative matrix and the sum of the entries in every row equals 1 [15]. So the follower-agents can enter the polytope region formed by the leaders.

Necessity: If $\overline{G}$ is not connected, by the definition of connectivity, then some follower-agents, without loss of generality, are denoted by $x_i,\ i = 1, \ldots, l (0 < l < n)$, will not get information from the leaders, and the other follower-agents $x_j,\ j = l+1, \ldots, n$. Then these follower-agents $x_i,\ i = 1, \ldots, l$ will not get any position information about leaders. Therefore, follower-agents $x_i,\ i = 1, \ldots, l$ can not enter the polytope region formed by the leaders. So $\overline{G}$ must be connected. □

Remark 3.2. (1) For switching network topologies of multiagents systems, if topologies are finite, and the shift is made in turn, then the system given by (3.1) and (3.2) is still asymptotically convergence.

(2) The system given by (3.1) can solve formation control of network. This can be made by the following protocol.

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij} (x_j - x_i - f_{ij}) + \sum_{q=1}^{k} b_i^q \left(x_0^q - x_i\right) + \sum_{j \in \mathcal{N}_i} a_{ij} (v_j - v_i) + \sum_{q=1}^{k} b_i^q \left(v_0^q - v_i\right), \tag{3.13}$$

where f_{ij} can be decomposed into $f_{ij} = f_i - f_j$ for any $i, j = 1, \dots, n$, f_i and f_j are some specified constants. Let

$$\widetilde{x} = x_i - f_i, \qquad v_0^i = v_0^j, \quad i, j = 1, \dots, m. \tag{3.14}$$

Then Protocol (3.13) has the same form as (3.2), and the system will asymptotically converge. By choosing suitable f_i and f_j, we can obtain on appropriate formation of network.

In the following, a continuous-time high-order dynamics of n follower-agents system are considered.

Consider multiagent systems with l-th ($l \geq 3$) order dynamics given by

$$\begin{aligned} \dot{x}_i^{(0)} &= x_i^{(1)}, \\ &\vdots \\ \dot{x}_i^{(l-2)} &= x_i^{(l-1)}, \\ \dot{x}_i^{(l-1)} &= u_i, \end{aligned} \tag{3.15}$$

where $x_i^{(d)} \in \mathbb{R}^m$, $d = 0, \dots, l-1$ are the states of follower-agents, $u_i \in \mathbb{R}^m$ is the control input, and $x_i^{(d)}$ denotes the k-th derivation of x_i, with $x_i^{(0)} = x_i$, $i = 1, \dots, n$. We consider the following dynamical protocol:

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij} \left[\sum_{d=o}^{l-1} \left(x_j^{(d)} - x_i^{(d)} \right) \right] + \sum_{q=1}^{k} b_i^q \sum_{d=0}^{l-1} \left(x_{q0}^{(d)} - x_i^{(d)} \right), \tag{3.16}$$

where $x_{q0}^{(d)}$, $q = 1, \dots, k$, are the states of the leaders, $x_{q0}^{(d)}$ denotes the d-th derivation of x_{q0}.

Furthermore, the above equations can be rewritten as vector form:

$$\begin{aligned} \dot{x}^{(0)} &= x^{(1)}, \\ &\vdots \\ \dot{x}^{(l-2)} &= x^{(l-1)}, \end{aligned} \tag{3.17}$$

$$\dot{x}^{(l-1)} = -\sum_{d=o}^{l-1} (H \otimes I_m) x^{(d)} + \sum_{d=o}^{l-1} [B(I_k \otimes I_n)] \otimes I_m x_0^{(d)}, \tag{3.18}$$

where $H = L + B(1_k \otimes I_n)$, $x = [x_1^T \cdots x_n^T]^T$ and $x_0 = [x_{10}^T \cdots x_{k0}^T]^T$.

Denote $\overline{x} = [\overline{x}_1^T \cdots \overline{x}_l^T]^T$ and $\overline{x}_0 = [\overline{x}_{01}^T \cdots \overline{x}_{0l}^T]^T$ as the stacked vector of the follower-agents' states and the leaders' states, respectively,

$$\overline{x}_s = \left[\left(x_1^{(s-1)}\right)^T \cdots \left(x_n^{(s-1)}\right)^T\right]^T, \qquad \overline{x}_{0s} = \left[\left(x_{10}^{(s-1)}\right)^T \cdots \left(x_{k0}^{(s-1)}\right)^T\right]^T, \quad s = 1, \dots, l, \tag{3.19}$$

and $x_0 = \overline{x}_{01}$. Then the above equations can be rewritten as

$$\overline{x} = \Gamma\overline{x} + \Upsilon\overline{x}_0, \tag{3.20}$$

where

$$\Gamma = \begin{pmatrix} 0_{mn\times mn} & I_{mn} & 0_{mn\times mn} & \cdots & 0_{mn\times mn} \\ 0_{mn\times mn} & 0_{mn\times mn} & I_{mn} & \cdots & 0_{mn\times mn} \\ 0_{mn\times mn} & 0_{mn\times mn} & 0_{mn\times mn} & \cdots & 0_{mn\times mn} \\ \vdots & \vdots & \ddots & \vdots & \\ -H\otimes I_m & -H\otimes I_m & -H\otimes I_m & \cdots & -H\otimes I_m \end{pmatrix}, \tag{3.21}$$

$$\Upsilon = \begin{pmatrix} 0_{nm\times km} & 0_{nm\times km} & 0_{nm\times km} & \cdots & 0_{nm\times km} \\ 0_{nm\times km} & 0_{nm\times km} & 0_{nm\times km} & \cdots & 0_{nm\times km} \\ 0_{nm\times km} & 0_{nm\times km} & 0_{nm\times km} & \cdots & 0_{nm\times km} \\ \vdots & \vdots & \ddots & \vdots & \\ \Lambda & \Lambda & \Lambda & \cdots & \Lambda \end{pmatrix}, \quad \Lambda = [B(I_k \otimes 1_n)] \otimes I_m. \tag{3.22}$$

Let $\tilde{x} = \overline{x} - I_l \otimes \{[H^{-1}B(I_k \otimes 1_n)] \otimes I_m\}\overline{x}_0$. Then (3.20) can be written as

$$\dot{\tilde{x}} = \Gamma\tilde{x}. \tag{3.23}$$

For high-order dynamic systems (3.23), we have the following theorem.

Theorem 3.3. *For the multiagent system* (3.23), *the follower-agents can enter the polytope region formed by the leaders, if and only if $\overline{G}$ is connected and $\lambda^l + \mu_i\lambda^{l-1} + \cdots + \mu_i$ is Hurwitz stable, where μ_i, $i = 1, \dots, mn$ are eigenvalues of $H \otimes I_m$.*

Proof. Sufficiency. Let λ be an eigenvalue of Γ, and $\tilde{x} = [\tilde{x}_1^T \cdots \tilde{x}_l^T]^T$ be the corresponding eigenvector of Γ. Then we get

$$\begin{gathered} \tilde{x}_2 = \lambda\tilde{x}_1, \\ \tilde{x}_3 = \lambda\tilde{x}_2, \\ \cdots \\ \tilde{x}_l = \lambda\tilde{x}_{i-1} \\ -H\otimes I_m\tilde{x}_1 - H\otimes I_m\tilde{x}_2 - \cdots - H\otimes I_m\tilde{x}_l = \lambda\tilde{x}_l. \end{gathered} \tag{3.24}$$

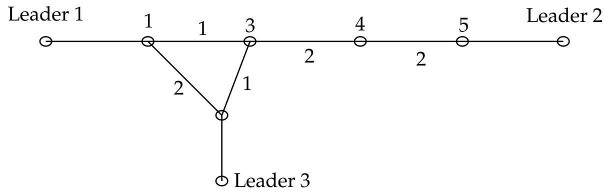

Figure 1: Network topology $\overline{G}_1$ of a multiagent system.

Furthermore, we have

$$-H \otimes I_m \tilde{x}_1 - \lambda H \otimes I_m \tilde{x}_1 - \cdots - \lambda^{l-1} H \otimes I_m \tilde{x}_l = \lambda^l \tilde{x}_l. \tag{3.25}$$

Let μ_i, $i = 1, \ldots, n$ be the eigenvalues of $H \otimes I_m$. Then we get characteristic equation of Γ

$$\lambda^l + \lambda^{l-1}\mu_i + \cdots + \mu_i = 0. \tag{3.26}$$

By (3.23), we get

$$\tilde{x} = e^{\Gamma t}\tilde{x}(0). \tag{3.27}$$

If $\lambda^l + \lambda^{l-1}\mu_i + \cdots + \mu_i$ is Hurwitz stable, then all the eigenvalues of Γ have negative real parts. Therefore, $\tilde{x} \to 0$, when $t \to \infty$. So

$$\overline{x} = I_l \otimes \left\{ \left[H^{-1}B(I_k \otimes 1_n) \right] \otimes I_m \right\} \overline{x}_0. \tag{3.28}$$

Thus,

$$x = \left[H^{-1}B(I_k \otimes 1_n) \right] \otimes I_m x_0, \ldots, \; x^{(l)} = \left[H^{-1}B(I_k \otimes 1_n) \right] \otimes I_m \overline{x}_{0l}. \tag{3.29}$$

$[H^{-1}B(I_k \otimes 1_n)] \otimes I_m$ is a row stochastic matrix which is a nonnegative matrix and the sum of the entries in every row equals 1, so the follower-agents can enter the region formed by the leaders.

Necessity: Similar to the Proof of Theorem 3.1. □

4. Simulation

In this section, simulation examples are presented to illustrate the proposed algorithms introduced in Section 3.

Example 4.1. We consider a system of five follower-agents guarded by three leaders with the topology $\overline{G}_1$ in Figure 1. The corresponding weights of edges of $\overline{G}_1$ are shown in Figure 1.

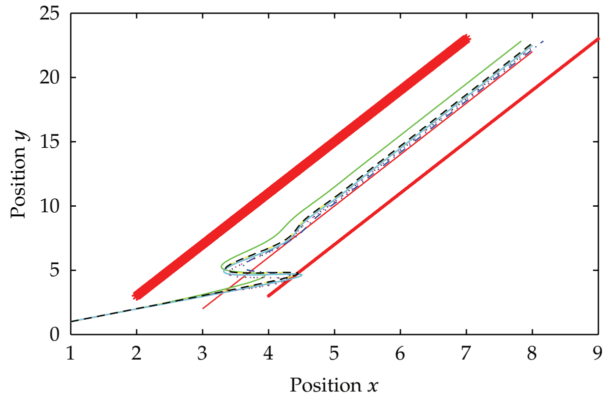

Figure 2: Trajectories of the agents in the multiagent system with topology $\overline{G}_1$ under the condition that the leaders have the same velocity.

Moreover, the initial positions and velocities of follower-agents and leaders are given as follows:

$$x_1(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad x_2(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad x_3(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad x_4(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad x_5(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix},$$

$$v_1(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad v_2(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad v_3(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad v_4(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad v_5(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix},$$

$$x_0^1(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad x_0^2(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \quad x_0^3(0) = \begin{pmatrix} 4 \\ 3 \end{pmatrix},$$

$$v_0^1 = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix}, \quad v_0^2 = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix}, \quad v_0^3 = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.1}$$

Figure 2 is position trajectories of the agents. In Figure 2, the red lines are the trajectories of the three leaders, and the others are the trajectories of the five follower-agents. From Figure 2, we can obtain that the five follower-agents can enter the polytope region formed by three leaders as the time t gradually increasing.

The velocities of leaders have no effect on follower-agents' flocking to the polytope region formed by leaders. We consider the following simulation for the same multiagent system as the above example. The network topology of multiagents is still $\overline{G}_1$ in Figure 1.

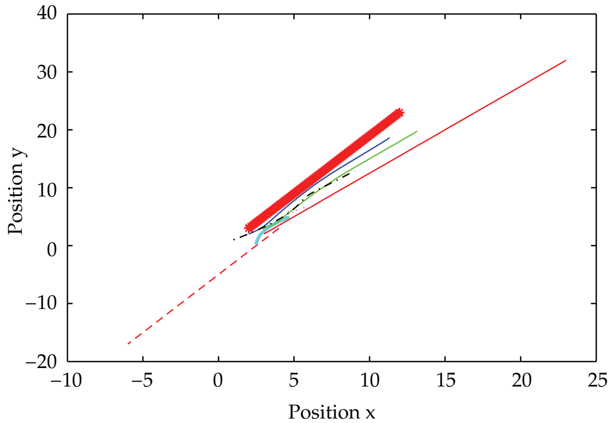

Figure 3: Trajectories of the agents in the multiagent system with topology $\overline{G}_1$ under the condition that the leaders have different velocities.

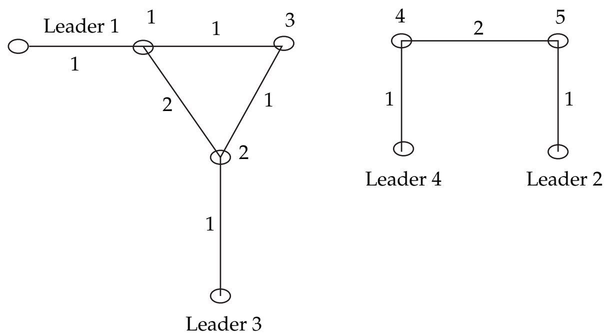

Figure 4: Network topology $\overline{G}_2$ of a multiagent system.

The initial positions and velocities of follower-agents, the initial positions of leaders and the corresponding weights of edges of $\overline{G}_1$ are the same as the above example, and only the velocities of leaders are changed as

$$v_0^1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad v_0^2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad v_0^3 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \tag{4.2}$$

Figure 3 is trajectories of the agents. The red lines are the trajectories of the three leaders, and the others are the trajectories of the five follower-agents. From Figure 3, though the velocities of leaders are different, we can still obtain that the five follower-agents can enter the polytope region formed by three leaders as the time t gradually increasing. The final states of the follower-agents are consistent with Theorem 3.1.

Example 4.2. We consider a system of five follower-agents guarded by four leaders with the topology $\overline{G}_2$ in Figure 4. The corresponding weights of edges of $\overline{G}_2$ are shown in Figure 4.

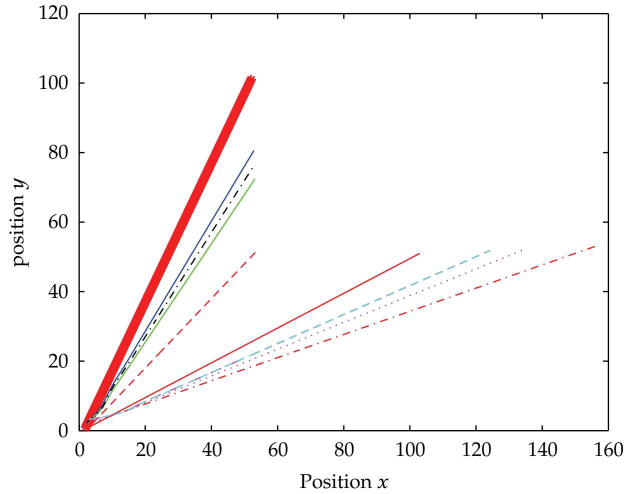

Figure 5: Trajectories of the agents in the multiagent system with topology $\overline{G}_2$.

Moreover, the initial positions and velocities of leaders are given as follows:

$$x_0^1(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad x_0^2(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \quad x_0^3(0) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad x_0^4(0) = \begin{pmatrix} 6 \\ 3 \end{pmatrix},$$
$$v_0^1(0) = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix}, \quad v_0^2(0) = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix}, \quad v_0^3(0) = \begin{pmatrix} 0.1 \\ 0.4 \end{pmatrix}, \quad v_0^4(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \tag{4.3}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.4}$$

Likewise, the initial positions and velocities of follower-agents are the same as those in Example 4.1. The topology G_2 of follower-agents is not connected, according to the definition in Preliminaries, but $\overline{G}_2$ is connected.

In Figure 5, the red lines are the trajectories of the four leaders, and the others are the trajectories of the five follower-agents. From Figure 5, we can obtain that the five follower-agents can enter the polytope region formed by four leaders as the time t gradually increasing.

5. Conclusion

In this paper, we consider the second order and high-order dynamic collective coordination algorithms of multiagent systems guided by multiple leaders. We give the necessary and sufficient conditions which follower-agents can enter the polytope region formed by leaders. Numerical examples are given to illustrate our theoretical results.

Acknowledgments

The authors would like to express their great gratitude to the referees and the Editor Professor John Burns for their constructive comments and suggestions that lead to the enhancement of this paper. This research was supported by 973 Program (2007CB311002), Sichuan Province Sci. & Tech. Research Project (2009GZ0004, 2009HH0025).

References

[1] Y. Z. Wei, L. Moreau, and N. R. Jennings, "A market-based approach to recommender systems," *ACM Transactions on Information and System Security*, vol. 23, no. 3, pp. 227–266, 2005.

[2] P. de Meo, G. Quattrone, G. Terracina, and D. Ursino, "An XML-based multiagent system for supporting online recruitment services," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 37, no. 4, pp. 464–480, 2007.

[3] N. A. Lynch, *Distributed Algorithms*, The Morgan Kaufmann Series in Data Management Systems, Morgan Kaufmann, San Francisco, Calif, USA, 1997.

[4] Y. P. Tian and C. L. Liu, "Consensus of multi-agent systems with diverse input and communication delays," *IIEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2122–2128, 2008.

[5] W. Ren, "Collective motion from consensus with cartesian coordinate coupling. Part I: single-integrator kinematics," in *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 1006–1011, Cancun, Mexico, December 2008.

[6] S. C. Weller and N. C. Mann, "Assessing rater performance without a F gold standard using consensus theory," *Medical Decision Making*, vol. 17, no. 1, pp. 71–79, 1997.

[7] J. Zhu, Y. P. Tian, and J. Kuang, "On the general consensus protocol of multi-agent systems with double integrator dynamics," *Linear Algebra and Its Applications*, vol. 431, no. 5–7, pp. 701–715, 2009.

[8] W. Zhang, D. Zeng, and S. Qu, "Dynamic feedback consensus control of a class of high-order multi-agent systems," *IET Control Theory & Applications*, vol. 4, no. 10, pp. 2219–2222, 2010.

[9] W. Ren, "Collective motion from consensus with cartesian coordinate coupling. Part II: double-integrator dynamics," in *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 1012–1017, Cancun, Mexico, December 2008.

[10] H. Su and X. Wang, "Second-order consensus of multiple agents with coupling delay," in *Proceedings of the 7th World Congress on Intelligent Control and Automation*, pp. 7181–7186, Chongqing, China, June 2008.

[11] V. Borkar and P. P. Varaiya, "Asymptotic agreement in distributed estimation," *IEEE Transactions on Automatic Control*, vol. 27, no. 3, pp. 650–655, 1982.

[12] J. N. Tsitsiklis, *Problems in decentralized decision making and computation*, Ph.D. thesis, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, Mass, USA, November 1984.

[13] T. Vicsek, A. Czirook, E. Ben-Jacob, I. Cohen, and O. Shochet, "Novel type of phase transition in a system of self-deriven particles," *Physical Review Letters*, vol. 75, no. 6, pp. 1226–1229, 1995.

[14] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[15] J. P. Hu and H. W. Yuan, "Collective coordination of multi-agent systems guided by multiple leaders," *Chinese Physics B*, vol. 18, no. 9, pp. 3771–3776, 2009.

[16] R. T. Rockafellar, *Convex Analysis*, Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, USA, 1997.

[17] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, UK, 1985.