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Research Article

A Quasi-ARX Model for Multivariable Decoupling Control of Nonlinear MIMO System

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This paper proposes a multiinput and multioutput (MIMO) quasi-autoregressive eXogenous (ARX) model and a multivariable-decoupling proportional integral differential (PID) controller for MIMO nonlinear systems based on the proposed model. The proposed MIMO quasi-ARX model improves the performance of ordinary quasi-ARX model. The proposed controller consists of a traditional PID controller with a decoupling compensator and a feed-forward compensator for the nonlinear dynamics based on the MIMO quasi-ARX model. Then an adaptive control algorithm is presented using the MIMO quasi-ARX radial basis function network (RBFN) prediction model and some stability analysis of control system is shown. Simulation results show the effectiveness of the proposed control method.

1. Introduction

Nonlinear system control has become a considerable topic in the field of control engineering [1, 2]. Many control results have been obtained for nonlinear single-input and single-output (SISO) systems based on the black box models, such as neural networks (NNs), wavelet networks (WNs), neurofuzzy networks (NFNs), and radial basis function networks (RBFNs), because of their abilities to approximate arbitrary mapping to any desired accuracy [3–9]. These black box models have been directly used to identify and control nonlinear dynamical systems.

Due to the complexity of nonlinear Multi-Input and Multi-Output (MIMO) systems, most of the control techniques developed for SISO systems cannot be extended directly for MIMO systems. One of the main difficulties in MIMO nonlinear system control is coupling problem. As such, it is important to investigate the realization of decoupling control. Many adaptive decoupling control algorithms have been proposed to deal with coupling in nonlinear system based on linear methods and nonlinear networks [10–14]. Some decoupling control methods of them are difficult not only to achieve accurate requirement and stability but also to be implemented in industrial applications. On the other hand, PID controller has been widely applied in controlling the SISO system because of its simple structure and relatively easy industrial application [15, 16]. However, PID controller cannot be directly used for MIMO model. Lang et al. [17] proposed a multivariable decoupling PID controller for MIMO linear systems based on the linear PID control and generalized minimum variance control law. What's more, Zhai & Chai [18] presented a multivariable PID control method using neural network to deal with nonlinear multivariable processes. In this control system, the nonlinear unmodeled part estimated by neural network is considered as a black box. The initial weights of neural network, local minima, and overfitting are the problems which need to be resolved.

In our previous work, a quasi-autoregressive exogenous (ARX) model with an ARX-like macromodel part and a kernel part was proposed, and a controller was designed for SISO systems [4, 19–21]. The kernel part is an ordinary network model, but it is used to parameterize the nonlinear coefficients of macromodel. As we know, RBFNs have played an important role in control engineering, especially in nonlinear system control because of their simple topological structure and precision in nonlinear approximation [22, 23]. Especially, RBFNs can be regarded as nonlinear models which are linear in parameters when fixing the nonlinear parameters by *a priori* knowledge [24, 25]. Incorporating the network models with this property, the quasi-ARX models become linear in parameters. Therefore, the RBFNs are chosen to replace the NNs as in [4].

The SISO model and control methods based on quasi-ARX model cannot directly be applied to MIMO nonlinear systems. Motivated by the above discussions, an MIMO quasi-ARX model is first proposed for MIMO nonlinear systems and then a nonlinear multivariable decoupling PID controller is proposed based on the MIMO quasi-ARX model, which consists of a traditional PID controller with a decoupling compensator and a feed-forward compensator for the nonlinear dynamics based on the MIMO quasi-ARX model. Then an adaptive controller is presented using the MIMO quasi-ARX RBFN prediction model. The parameters of such controller are selected based on the generalized minimum control variance. In this paper, quasi-ARX RBFN model is divided into two parts: the linear part is used to guarantee the stability and decoupling, and the nonlinear part is used to improve the accuracy.

The paper is organized as follows: in Section 2 the nonlinear MIMO system considered is first described, and then a hybrid system expression is obtained and an MIMO quasi-ARX RBFN model is proposed. In Section 3, a multivariable decoupling PID controller is developed based on the proposed model and generalized minimum variance control law. Then an adaptive control algorithm is presented using the MIMO quasi-ARX RBFN prediction model, and the corresponding parameter estimation methods are proposed in Section 4. Section 5 carries out numerical simulations to show the effectiveness of the proposed control method. Finally, Section 6 presents the conclusions.

2. An MIMO Quasi-ARX Model

2.1. Systems

Consider an MIMO nonlinear dynamical system with input-output relation as

$$\mathbf{y}(t+d) = \mathbf{f}(\varphi(t)),$$
$$\varphi(t) = \left[\mathbf{y}(t+d-1)^T, \ldots, \mathbf{y}(t+d-n_y)^T, \mathbf{u}(t)^T, \ldots, \mathbf{u}(t-n_u+1)^T\right]^T, \tag{2.1}$$

where $\mathbf{y} = [y_1, \ldots, y_n]^T \in R^n$ and $\mathbf{u} = [u_1, \ldots, u_n]^T \in R^n$ are system input and output vectors, respectively, d the known integer time delay, $\varphi(t)$ the regression vector, and n_y, n_u the system orders. $\mathbf{f}(\cdot) = [f_1(\cdot), \ldots, f_n(\cdot)]^T$ is a vector-valued nonlinear function, and, at a small region around $\varphi(t) = \mathbf{0}(\mathbf{0} = [0, \ldots, 0]^T)$, they are C^∞ continuous. The origin is an equilibrium point, then $\mathbf{f}(\mathbf{0}) = \mathbf{0}$. The system is controllable, in which a reasonable unknown controller may be expressed by $\mathbf{u}(t) = \rho(\xi(t))$, where $\xi(t)$ is defined in Section 2.4.

2.2. ARX-Like Expression

Under the continuous condition, the unknown nonlinear function $f_k(\varphi(t)), (i = 1, \ldots, n)$ can be performed Taylor expansion on a small region around $\varphi(t) = \mathbf{0}$:

$$y_k(t+d) = f_k'(0)\varphi(t) + \frac{1}{2}\varphi^T(t) f_k''(0)\varphi(t) + \cdots, \tag{2.2}$$

where the prime denotes differentiation with respect to $\varphi(t)$. Then the following notations are introduced:

$$\left(f_k'(0) + \frac{1}{2}\varphi^T(t) f_k''(0) + \cdots\right)^T = \left[a_{1,t}^{1,k} \cdots a_{n_y,t}^{1,k} \cdots a_{n_y,t}^{n,k}\, b_{1,t}^{1,k} \cdots b_{n_u,t}^{1,k} \cdots b_{n_u,t}^{n,k}\right]^T, \tag{2.3}$$

where $a_{i,t}^{l,k} = a_i^{l,k}(\varphi(t))(i = 1, \ldots, n_y)$ and $b_{j,t}^{l,k} = b_j^{l,k}(\varphi(t))(j = 0, \ldots, n_u - 1)$ are nonlinear functions of $\varphi(t)$.

However, we need to get $\mathbf{y}(t+d)$ by using the input-output data up to time t in a model. The coefficients $a_{i,t}^{l,k}$ and $b_{j,t}^{l,k}$ need to be calculable using the input-output data up to time t. To do so, let us iteratively replace $y(t+l)$ in the expressions of $a_{i,t}^{l,k}$ and $b_{j,t}^{l,k}$ with functions:

$$\mathbf{y}(t+s) \Longrightarrow \mathbf{g}(\tilde{\varphi}(t+s)), \quad s = 1, \ldots, d-1, \tag{2.4}$$

where $\tilde{\varphi}(t+s)$ is $\varphi(t+s)$ whose elements $\mathbf{y}(t+m), s+1 < m \le d-s$ are replaced by (2.4), and define the new expressions of the coefficients by

$$a_{i,t}^{l,k} = \tilde{a}_{i,t}^{l,k} = \tilde{a}_i^{l,k}(\phi(t)), \qquad b_{j,t}^{l,k} = \tilde{b}_{j,t}^{l,k} = \tilde{b}_j^{l,k}(\phi(t)), \tag{2.5}$$

where $\phi(t)$ is a vector:

$$\phi(t) = \left[\mathbf{y}(t)^T \cdots \mathbf{y}(t-n_y+1)^T \mathbf{u}(t)^T \cdots \mathbf{u}(t-n_u-d+2)^T\right]^T. \tag{2.6}$$

Now, introduce two polynomial matrices $\mathbf{A}(q^{-1},\phi(t))$ and $\mathbf{B}(q^{-1},\phi(t))$ based on the coefficients, defined by

$$\begin{aligned} \mathbf{A}\left(q^{-1},\phi(t)\right) &= \mathbf{I} - \mathbf{a}_{1,t}q^{-1} - \cdots - \mathbf{a}_{n_y,t}q^{-n_y}, \\ \mathbf{B}\left(q^{-1},\phi(t)\right) &= \mathbf{b}_{0,t} + \cdots + \mathbf{b}_{n_u-1,t}q^{-n_u+1}, \end{aligned} \tag{2.7}$$

where $\mathbf{a}_{i,t} = (a_{i,t}^{l,k})_{N\times N}$, $i = 1,\ldots,n_y$ and $\mathbf{b}_{j,t} = (b_{j,t}^{l,k})_{N\times N}$, $j = 1,\ldots,n_u$. Then, the nonlinear system (2.1) can be equivalently represented as the following ARX-like expression:

$$\mathbf{A}\left(q^{-1},\phi(t)\right)\mathbf{y}(t+d) = \mathbf{B}\left(q^{-1},\phi(t)\right)\mathbf{u}(t). \tag{2.8}$$

By (2.8), let $\mathbf{y}(t+d)$ satisfies the following equation:

$$\mathbf{y}(t+d) = \mathcal{A}\left(q^{-1},\phi(t)\right)\mathbf{y}(t) + \mathcal{B}\left(q^{-1},\phi(t)\right)\mathbf{u}(t), \tag{2.9}$$

where

$$\begin{aligned} \mathcal{A}\left(q^{-1},\phi(t)\right) &= A_{0,t} + A_{1,t}q^{-1} + \cdots + A_{n_y-1,t}q^{-n_y+1}, \\ \mathcal{B}\left(q^{-1},\phi(t)\right) &= \mathbf{F}\left(q^{-1},\phi(t)\right)\mathbf{B}\left(q^{-1},\phi(t)\right), \\ &= B_{0,t} + B_{1,t}q^{-1} + \cdots + B_{n_u+d-2,t}q^{-n_u-d+2}, \end{aligned} \tag{2.10}$$

$A_{i,t}(i = 0,\ldots,n_y-1)$ and $B_{j,t}(j = 0,\ldots,n_u+d-2)$ are coefficient matrices. And $\mathbf{G}(q^{-1},\phi(t))$, $\mathbf{F}(q^{-1},\phi(t))$ are unique polynomials satisfying

$$\mathbf{F}\left(q^{-1},\phi(t)\right)\mathbf{A}\left(q^{-1},\phi(t)\right) = \mathbf{I} - A\left(q^{-1},\phi(t)\right)q^{-d}. \tag{2.11}$$

2.3. Hybrid Expression

The coefficients matrices $A_{i,t}(i = 0,\ldots,n_y-1)$ and $B_{j,t}(j = 0,\ldots,n_u+d-2)$ can be considered as a summation of two parts: the constant part A_i^l and B_j^l and the nonlinear function part on $\phi(t)$ which are denoted $A_{i,t}^n$ and $B_{i,t}^n$. Then, the expression of system in the predictor form (2.9) can be described by

$$\mathbf{y}(t+d) = \mathcal{A}^l\left(q^{-1}\right)\mathbf{y}(t) + \mathcal{B}^l\left(q^{-1}\right)\mathbf{u}(t) + \mathcal{A}^n\left(q^{-1},\phi(t)\right)\mathbf{y}(t) + \mathcal{B}^n\left(q^{-1},\phi(t)\right)\mathbf{u}(t), \tag{2.12}$$

where

$$\begin{aligned}
\mathcal{A}^l\left(q^{-1}\right) &= A_0^l + A_1^l q^{-1} + \cdots + A_{n_y-1}^l q^{-n_y+1},\\
\mathcal{A}^n\left(q^{-1},\phi(t)\right) &= A_{0,t}^l + A_{1,t}^l q^{-1} + \cdots + A_{n_y-1,t}^l q^{-n_y+1},\\
\mathcal{B}^l\left(q^{-1}\right) &= B_0^l + B_1^l q^{-1} + \cdots + B_{n_y-d+2}^l q^{-n_u+d-2},\\
\mathcal{B}^n\left(q^{-1},\phi(t)\right) &= B_{0,t}^l + B_{1,t}^l q^{-1} + \cdots + B_{n_y-d+2,t}^l q^{-n_u+d-2}.
\end{aligned} \tag{2.13}$$

Similar with [18], the linear polynomial matrix $\mathcal{B}^l(q^{-1})$ can be expressed as $\mathcal{B}^l(q^{-1}) = \overline{\mathcal{B}}^l(q^{-1}) + \overline{\overline{\mathcal{B}}}^l(q^{-1})$ with $\overline{\mathcal{B}}^l(q^{-1})$ being diagonal and $\overline{\overline{\mathcal{B}}}^l(q^{-1})$ being a polynomial matrix with zero diagonal elements.

Then, the linear and nonlinear expression of system (2.12) can be obtained as

$$\begin{aligned}
\mathbf{y}(t+d) = {}& \mathcal{A}^l\left(q^{-1}\right)\mathbf{y}(t) + \overline{\mathcal{B}}^l\left(q^{-1}\right)\mathbf{u}(t) + \overline{\overline{\mathcal{B}}}^l\left(q^{-1}\right)\mathbf{u}(t)\\
& + \mathcal{A}^n\left(q^{-1},\phi(t)\right)\mathbf{y}(t) + \mathcal{B}^n\left(q^{-1},\phi(t)\right)\mathbf{u}(t).
\end{aligned} \tag{2.14}$$

2.4. Quasi-ARX RBFN Model

Now, we will propose an MIMO quasi-ARX RBFN model. However, the $\mathbf{v}(\phi(t))$ are based on $\Psi(t)$ whose elements contain $\mathbf{u}(t)$. To solve this problem, an *extravariable* $\mathbf{x}(t)$ Obviously, in a control system, the reference signal $\mathbf{y}^*(t+d)$ can be used as the extra variable $\mathbf{x}(t+d)$, is introduced, and an unknown nonlinear function $\rho(\xi(t))$ is used to replace the variable $\mathbf{u}(t)$ in $\phi(t)$. Under the assumption of the system is controllable in Section 2.1, the function $\rho(\xi(t))$ exists. Define

$$\xi(t) = \left[\mathbf{y}(t)^T \cdots \mathbf{y}(t-n_1)^T \mathbf{x}(t+d)^T \cdots \mathbf{x}(t-n_3+d)^T \mathbf{u}(t-1)^T \cdots \mathbf{u}(t-n_2)^T\right]^T, \tag{2.15}$$

including the extra variable $\mathbf{x}(t+d)$ as an element. A typical choice for n_1, n_2, and n_3 in $\xi(t)$ is $n_1 = n_y - 1$, $n_2 = n_u + d - 2$, and $n_3 = 0$. We can express (2.14) by

$$\mathbf{y}(t+d) = \varphi^T(t)\Omega_0 + (t)\theta_\xi^n, \tag{2.16}$$

where $\varphi^T(t) = \varphi(t-d)$. The elements of θ_ξ^n are unknown nonlinear function of $\xi(t)$, which can be parameterized by NN or RBFN. In this paper, the RBFN is used which has local property:

$$\theta_\xi^n = \sum_{j=1}^{M} \Omega_j \mathcal{R}_j\left(\mathbf{p}_j, \xi(t)\right), \tag{2.17}$$

where M is the number of RBFs. $\Omega_j = [\Omega_{j,1}, \ldots, \Omega_{j,n}]$ is the coefficient matrix with $\Omega_{j,i} = [\omega_{j,i}^1, \ldots, \omega_{j,i}^N]^T$, $j = 1, \ldots, M$. And $\mathcal{R}_j(\xi(t), \Omega_j)$ the RBFs defined by

$$\mathcal{R}_j(\mathbf{p}_j, \xi(t)) = e^{-\lambda_j \|\xi(t) - \mathbf{Z}_j\|^2}, \quad j = 1, 2, \ldots, M, \tag{2.18}$$

where $\mathbf{p}_j = \{\lambda_j, \mathbf{Z}_j\}$ is the parameters set of the RBFN; Z_j is the center vector of RBF and λ_j are the scaling parameters; $\| \bullet \|_2$ denotes the vector two norm. Then we can express the quasi-ARX RBFN prediction model for (2.16) in a form of

$$\mathbf{y}(t+d) = \varphi^T(t)\Omega_0 + \xi^T(t) \sum_{j=1}^{M} \Omega_j \mathcal{R}_j(\mathbf{p}_j, \xi(t)). \tag{2.19}$$

3. Controller Design

3.1. Nonlinear Multivariable Decoupling PID Controller

Introduce the following performance index:

$$M(t+d) = \left\| \mathbf{y}(t+d) - \mathbf{R}\left(q^{-1}\right)\mathbf{y}^*(t+d) + \mathbf{S}\left(q^{-1}\right)\mathbf{u}(t) + \mathbf{Q}\left(q^{-1}\right)\mathbf{u}(t) \right\|, \tag{3.1}$$

where $\mathbf{R}$ and $\mathbf{S}$ are the diagonal weighting polynomial matrices, and $\mathbf{Q}$ is a weighting polynomial matrix with diagonal elements.

The optimal control law minimizing (3.1) is

$$\mathbf{y}(t+d) - \mathbf{R}\left(q^{-1}\right)\mathbf{y}^*(t+d) + \mathbf{S}\left(q^{-1}\right)\mathbf{u}(t) + \mathbf{Q}\left(q^{-1}\right)\mathbf{u}(t) = 0. \tag{3.2}$$

Substituting (2.14) into (3.2), the following equation is obtained:

$$\begin{aligned} \left(\overline{\mathcal{B}}^l\left(q^{-1}\right) + \mathbf{Q}\left(q^{-1}\right)\right)\mathbf{u}(t) &= \mathbf{R}\left(q^{-1}\right)\mathbf{y}^*(t+d) - \mathcal{A}^l\left(q^{-1}\right)\mathbf{y}(t) - \left(\overline{\overline{\mathcal{B}}}^l\left(q^{-1}\right) + \mathbf{S}\left(q^{-1}\right)\right)\mathbf{u}(\mathrm{t}) \\ &\quad - \left(\mathcal{B}^n\left(q^{-1}, \phi(t)\right)\mathbf{u}(t) + \mathcal{A}^n\left(q^{-1}, \phi(t)\right)\mathbf{y}(t)\right), \end{aligned} \tag{3.3}$$

where $\overline{\mathcal{B}}^l(q^{-1}) + \mathbf{Q}(q^{-1}) = \lambda^{-1}\overline{\mathbf{H}}(q^{-1})$, with $\lambda = \mathbf{diag}\{\lambda_1, \ldots \lambda_n\}$ and $\overline{\mathbf{H}}(q^{-1}) = (1 - q^{-1}) \cdot \mathbf{I}$. By introducing $\mathbf{R}(q^{-1}) = \mathcal{A}^l(q^{-1})$ and $\overline{\mathcal{B}}^l(q^{-1})\mathbf{S}(q^{-1}) = \mathbf{Q}(q^{-1})\overline{\overline{\mathcal{B}}}^l(q^{-1})$, when $n_y - 1 \leq 2$, a nonlinear decoupling PID controller is obtained, similar to a traditional PID controller:

$$\overline{\mathbf{H}}\left(q^{-1}\right)\mathbf{u}(t) = \lambda \mathcal{A}^l\left(q^{-1}\right)\mathbf{e}(t) - \overline{\overline{\mathbf{H}}}\left(q^{-1}\right)\mathbf{u}(t) - \mathbf{v}(\phi(t)), \tag{3.4}$$

where $\overline{\overline{\mathbf{H}}}(q^{-1}) = \lambda(\overline{\overline{\mathcal{B}}}^l(q^{-1}) + \mathbf{S}(q^{-1}))$ and $\mathbf{v}(\phi(t)) = \lambda(\mathcal{B}^n(q^{-1}, \phi(t))\mathbf{u}(t) + \mathcal{A}^n(q^{-1}, \phi(t))\mathbf{y}(t))$. $\mathbf{e}(t) = \mathbf{y}^*(t+d) - \mathbf{y}(t)$.

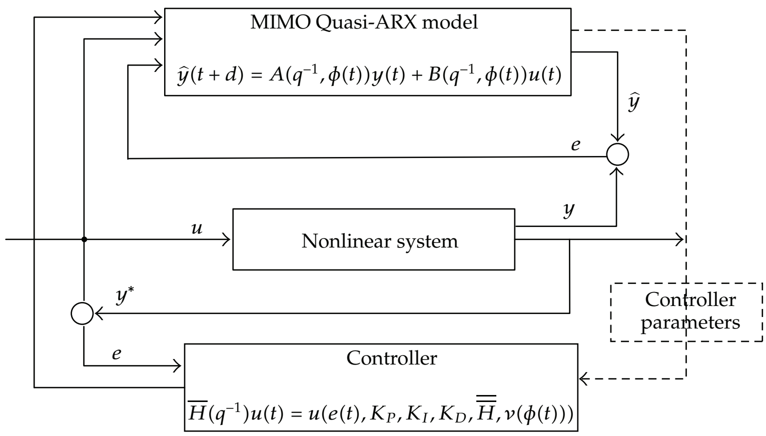


Figure 1: The multivariable decoupling PID control system based on MIMO quasi-ARX model.

The controller (3.4) is substituted into the system (3.2), the obtained closed-loop system which is shown in Figure 1 will be stable, and the decoupling control effect and tracking errors can be eliminated.

A velocity-type form of the PID controller is given:

$$\begin{aligned}\overline{\mathbf{H}}\left(q^{-1}\right)\mathbf{u}(t) = {} & \mathbf{K}_p(\mathbf{e}(t) - \mathbf{e}(t-1)) + \mathbf{K}_I\mathbf{e}(t) + \mathbf{K}_D(\mathbf{e}(t) - 2\mathbf{e}(t-1) + \mathbf{e}(t-2)) \\ & - \overline{\overline{\mathbf{H}}}\left(q^{-1}\right)\mathbf{u}(t) - \mathbf{v}(\phi(t)).\end{aligned} \tag{3.5}$$

The gain can be selected as

$$\begin{aligned}\mathbf{K}_p &= -\lambda(2A_2 + A_1), \\ \mathbf{K}_I &= \lambda(A_0 + A_1 + A_2), \\ \mathbf{K}_D &= \lambda A_2,\end{aligned} \tag{3.6}$$

where when $n_y = 1$, $A_1 = A_2 = 0$ and when $n_y = 2$, $A_2 = 0$.

3.2. Parameter Estimation

3.2.1. A Simple Strategy for Determining $\mathbf{p}_j$

Now let us initialize $\mathbf{p}_j$, denoted as follows:

$$\mathbf{p}_j = \left[\overline{z}_1^j \overline{z}_2^j \cdots \overline{z}_N^j, \lambda_j\right]^T \quad (j = 1, \ldots, M), \tag{3.7}$$

where $N = \dim(\xi(t))$. Since $\mathbf{p}_j$ is associated with partition of $\xi(t)$, the bounds of $\xi(t)$ can be used to determine a fairly good initial value. It will not be discussed here, and the interested readers are referred to [26].

3.2.2. Estimation of Parameter Vector Ω_0

If the process is known, Ω_0 is obtained by using Taylor expansion at its equilibrium; otherwise, it will be replaced by its estimation $\widehat{\Omega}_0$.

3.2.3. Estimation of Parameter Vector Ω_j

Parameter vector $\Omega_j (j = 1, \ldots, M)$ can be estimated by a simplified multivariable least-squares algorithm as in [27]. By introducing the notations:

$$\Omega = \left[\Omega_1^T, \ldots, \Omega_M^T\right]^T, \qquad \Phi(t) = \left[\xi(t)^T \otimes \Psi_{\mathcal{R}}^T(t)\right]^T, \tag{3.8}$$

where the symbol $\otimes$ denotes Kronecker production, then $\Psi_{\mathcal{R}}^T(t) = [\mathcal{R}_j(\mathbf{p}_j, \xi(t)), j = 1, \ldots, M]$, the MIMO quasi-ARX model (2.12) can be expressed in a regression form:

$$\mathbf{y}(t+d) = \varphi^T(t)\Omega_0 + \Phi^T(t)\Omega. \tag{3.9}$$

The parameter Ω is updated by an LS algorithm while fixing $\mathbf{p}_j$ and Ω_0:

$$\widehat{\Omega}(t) = \widehat{\Omega}(t-d) + \frac{p(\mathrm{t})\Phi(t-d)\mathbf{e}(t)}{1 + \Phi(t-d)^T p(\mathrm{t})\Phi(t-d)}, \tag{3.10}$$

where $\widehat{\Omega}(t)$ is the estimate of Ω at time instant t. $\mathbf{e}(t)$ is the error vector of MIMO quasi-ARX model, defined by

$$\begin{aligned} \mathbf{e}(t) &= \mathbf{y}(t) - \varphi^T(t)\Omega_0 - \Phi(t-d)^T \widehat{\Omega}(t-d), \\ P(t) &= \frac{P(t-d) - P^T(t-d)\Phi(t-d)^T \Phi(t-d) P(t-d)}{1 + \Phi(t-d)^T P(t)\Phi(t-d)}. \end{aligned} \tag{3.11}$$

Remark 3.1. Comparing with [18], there are three improvements: the unmodeled part is modeled in this paper by quasi-ARX model, RBFN is used to replace NN, and some priori knowledge can be used to determine the parameters.

4. Stability Analysis

There are some assumption made.

Assumption 1. (i) $y^*(t)$ is a bounded deterministic sequence; (ii) $v(\phi(t))$ is globally bounded, $|v(\phi(t))| \leq \Delta$, where the boundary Δ is known; (iii) the choices of λ and $\mathbf{S}(q^{-1})$ are such that $\det\{\widetilde{\mathbf{H}}(q^{-1})\mathbf{A}(q^{-1}) + q^{-d}\widetilde{\mathbf{B}}(q^{-1})\lambda\mathcal{A}^I(q^{-1})\} \neq 0$.

Theorem 4.1. *For the MIMO nonlinear* (2.1) *with the controller* (3.5)*, together with the parameters of the controller selected by* Section 3.2*, all the signals in the closed-loop system described above can be* *bounded, and the tracking error can be made less than any specified constant δ over a compact set by properly choosing the structures and parameters of quasi-ARX RBFN model, that is,* $\lim_{t\to\infty}\|\mathbf{y}(t+d)-\mathbf{y}^*(t+d)\|\leq\varepsilon$.

Proof. The nonlinear part estimation error vector can be described by

$$\varepsilon(t)=\mathbf{v}(\phi(t+d))-\xi^T(t+d)\sum_{j=1}^{M}\widehat{\Omega}(t+d)_j\mathcal{R}_j(\mathbf{p}_j,\xi(t+d)). \tag{4.1}$$

We can see that, if the nonlinear decoupling PID controller (3.5) is used to the system (2.14), the following input-output dynamics are obtained as in [18]:

$$\begin{aligned}&\left(\widetilde{\mathbf{H}}\left(q^{-1}\right)\mathbf{A}\left(q^{-1}\right)+q^{-d}\widetilde{\mathbf{B}}\left(q^{-1}\right)\lambda\mathcal{A}^l\left(q^{-1}\right)\right)\mathbf{y}(t+d)\\&\quad=\widetilde{\mathbf{B}}\left(q^{-1}\right)\lambda\mathcal{A}^l\left(q^{-1}\right)\mathbf{y}^*(t+d)+\widetilde{\mathbf{H}}\left(q^{-1}\right)\mathbf{v}(\phi(t+d))-\widetilde{\mathbf{B}}\left(q^{-1}\right)\widehat{\mathbf{v}}(\phi(t+d)),\\&\left(\mathbf{A}\left(q^{-1}\right)\mathbf{H}\left(q^{-1}\right)+q^{-d}\lambda\mathbf{A}\left(q^{-1}\right)\mathcal{A}^l\left(q^{-1}\right)\right)\mathbf{u}(t+d)\\&\quad=\widetilde{\mathbf{A}}\left(q^{-1}\right)\lambda\mathcal{A}^l\left(q^{-1}\right)\mathbf{y}^*(t+d)-q^{-d}\lambda\mathcal{A}^l\left(q^{-1}\right)\mathbf{v}(\phi(t+d))-\mathbf{A}\left(q^{-1}\right)\widehat{\mathbf{v}}(\phi(t+d)).\end{aligned} \tag{4.2}$$

Substitute (4.1) into (4.2), the equations are given as follows:

$$\begin{aligned}&\left(\widetilde{\mathbf{H}}\left(q^{-1}\right)\mathbf{A}\left(q^{-1}\right)+q^{-d}\widetilde{\mathbf{B}}\left(q^{-1}\right)\lambda G\left(q^{-1}\right)\right)\mathbf{y}(t+d)\\&\quad=\widetilde{\mathbf{B}}\left(q^{-1}\right)\lambda G\left(q^{-1}\right)\mathbf{y}^*(t+d)+\left(\widetilde{\mathbf{H}}\left(q^{-1}\right)-\widetilde{\mathbf{B}}\left(q^{-1}\right)\right)\mathbf{v}(\phi(t+d))+\widetilde{\mathbf{B}}\left(q^{-1}\right)\varepsilon(t),\\&\left(\mathbf{A}\left(q^{-1}\right)\mathbf{H}\left(q^{-1}\right)+q^{-d}\lambda\mathbf{A}\left(q^{-1}\right)\mathcal{A}^l\left(q^{-1}\right)\right)\mathbf{u}(t+d)\\&\quad=\widetilde{\mathbf{A}}\left(q^{-1}\right)\lambda\mathcal{A}^l\left(q^{-1}\right)\mathbf{y}^*(t+d)-\left(q^{-d}\lambda\mathcal{A}^l\left(\mathrm{q}^{-1}\right)+\mathbf{A}\left(q^{-1}\right)\right)\mathbf{v}(\phi(t+d))-\mathbf{A}\left(q^{-1}\right)\varepsilon(t).\end{aligned} \tag{4.3}$$

From (4.3) and Assumption 1, there exist constants C_1, C_2, C_3, C_4 satisfying

$$\begin{aligned}\|\mathbf{y}(t+d)\|&\leq C_1+C_2\max_{0\leq\tau\leq t}\|\varepsilon(t)\|,\\\|\mathbf{u}(t)\|&\leq C_3+C_4\max_{0\leq\tau\leq t}\|\varepsilon(t)\|.\end{aligned} \tag{4.4}$$

Because of the universal approximations of the RBFNs, the estimation error $\varepsilon(t)$ can be achieved less than any constant ζ over a compact set by properly choosing their structures and parameters. It can be got that

$$\left\|\varphi(t+d)\right\|\leq C_5+C_6\max_{0\leq\tau\leq t}\|\varepsilon(t)\|\leq C_7+C_8\zeta\leq C_9. \tag{4.5}$$

where C_5, C_6, C_7, C_8, C_9 are constants.

Then, the boundness of all the signals in the closed-loop system is got. □

The tracking error of the system is obtained as

$$e = \lim_{t\to\infty} \|\mathbf{y}(t+d) - \mathbf{y}^*(t+d)\| \le C, \tag{4.6}$$

where $C > 0$ is a constant.

5. Numerical Simulations

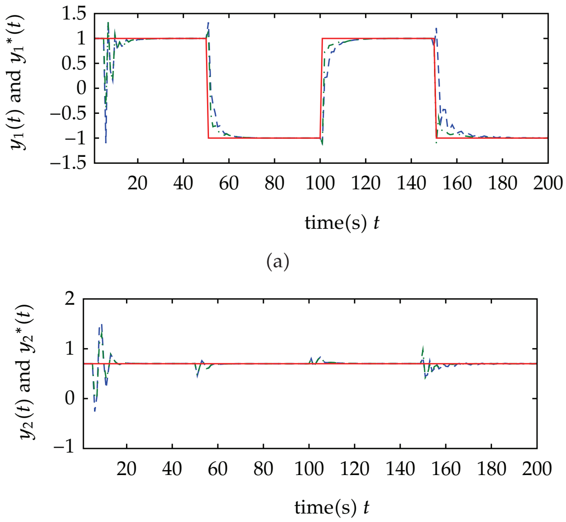
In order to study the behavior of the proposed control method, a numerical simulation is described in this section. The MIMO nonlinear system to be controlled is described by

$$\begin{aligned}
y_1(t+1) &= 0.9y_1(t) - \frac{0.3y_1(t-1)}{1+y_2^2(t-1)} + 0.4\sin(u_1(t)) + 0.7u_1(t-1) + 0.3u_2(t) - 0.5u_2(t-1),\\
y_2(t+1) &= -0.4\sin\left(y_2^2(t)\right) - 0.1y_2(t-1) + u_2(t-1) - 0.3\sin(u_1(t))\\
&\quad + 0.2u_1(t-1+0.8\sin(u_2(t))) + 0.5u_2^2(t-1), \quad \text{for } t \in [0,150),\\
y_1(t+1) &= 0.6y_1(t) - \frac{0.4y_1(t-1)}{1+y_2^2(t-1)} + 0.4\sin(u_1(t)) + 0.6u_1(t-1) + 0.4\mathrm{u}_2(t) - 0.5u_2(t-1),\\
y_2(t+1) &= -0.5\sin\left(y_2^2(t)\right) - 0.1y_2(t-1) + u_2(t-1) - 0.3\sin(u_1(t)) + 0.3u_1(t-1)\\
&\quad + 0.9\sin(u_2(t)) + 0.5u_2^2(t-1), \quad \text{for } t \in [150,\infty).
\end{aligned} \tag{5.1}$$

In this example, a system disturbance appears when $t = 150$. The desired output of system is given $y_1^*(t) = \mathrm{sign}(\sin(\pi t/50))$ and $y_2^*(t) = 0.7$.

In this example, the proposed control method in Sections 3 and 4 is illustrated effective in the control stability and robustness. The order is chosen as $n_y = n_u = 2$, and time delay $d = 1$. The regression $\varphi(t) = [y_1(t-1)y_2(t-1)y_1(t-2)y_2(t-2)u_1(t-1)u_2(t-1)u_1(t-2)u_2(t-2)]^T$ and $\xi(t) = [y_1(t-1)y_2(t-1)y_1(t-2)y_2(t-2)y_1^*(t)y_1^*(t)y_2^*(t)u_1(t-2)u_2(t-2)]^T$. Based on the priori acknowledge, we choose $\mathbf{Z}_{\max} = [2\,2\,2\,2\,4\,1\,4\,1]$ and $\mathbf{Z}_{\min} = [-2-2-2-2-4-1-4-1]$. The parameters $\mathbf{p}_j$ can be determined by the proposed method in Section 3.2.

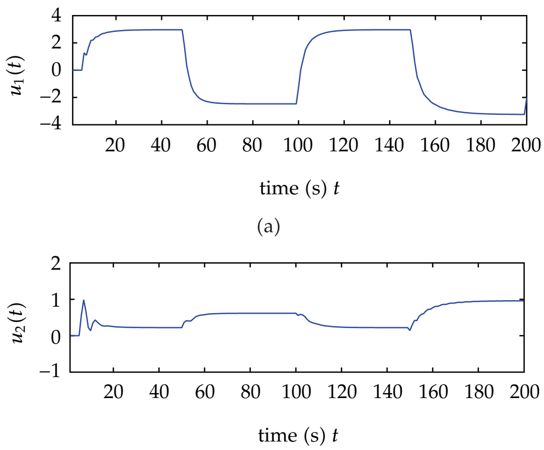
Under the same simulation conditions and with the same parameters value, the control output results by a typical PID controller are given for comparison, where the PID controller has neither the decoupling compensator nor the nonlinear part. The control outputs are shown in Figure 2, the solid red line is the desired outputs, the dashed blue line is the typical PID control outputs, and the dotted green line is the proposed method control outputs. The corresponding control inputs $u_1(t)$ and $u_2(t)$ are given in Figures 3 and 4. We can see that our proposed method is nearly consistent with the desired output at most of the time which is better than typical PID control method when $t \in [0,150)$. Obviously, the control performance of our proposed method is much better than typical PID control method when the system has disturbance when $t = 150$. The input signals have small fluctuation as shown in Figure 4.



(a)

(b)

Figure 2: Control outputs.



(a)

(b)

Figure 3: Corresponding control inputs of the PID control method.

Table 1 gives the comparison results of the errors. Obviously, the mean and variance of errors of the proposed method are smaller than the typical PID control method.

6. Conclusions

In this paper, an MIMO quasi-ARX model is first introduced, and a nonlinear multivariable decoupling PID controller is proposed based on the proposed model for MIMO nonlinear systems. The proposed controller consists of a traditional PID controller with a decoupling compensator and a feed-forward compensator for the nonlinear dynamics based on the

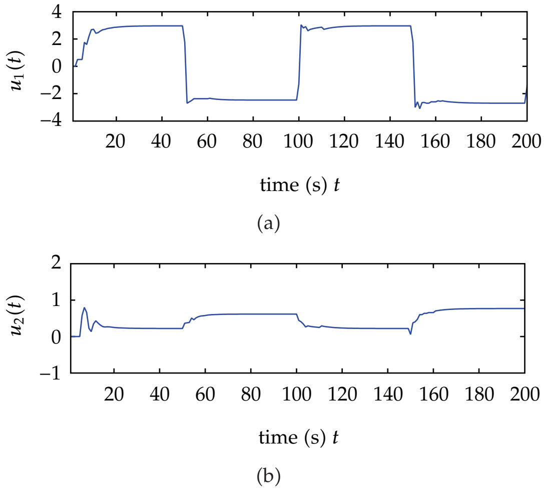


Figure 4: Corresponding control inputs of the proposed control method.

Table 1: Comparison results of errors based on two control method.

	Mean of errors	Variance of errors
$y_1(t)$ typical method	0.0132	0.1350
: proposed method	−0.0090	0.0668
$y_2(t)$ typical method	−0.0067	0.0157
: proposed method	−0.0039	0.0098

MIMO quasi-ARX model. And an adaptive control system is presented using the MIMO quasi-ARX RBFN prediction model. The parameters of such controller are selected based on the generalized minimum control variance. The proposed control method has more simplicity structures and better control performance. The nonlinear part is not a black box whose parameters can be determined by *a priori* acknowledge. Simulation results show the effectiveness of the proposed method on control accuracy and robustness when a disturbance appears in the system. Because the PID controller can be realized on standard DCS/PLC modules, the algorithm is more useful for industrial process control. Otherwise, because the parameters of controller are chosen from the generalized minimum variance control law, it is easier for engineers and process operators to relate the parameter settings.

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