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Research Article

Hyers-Ulam-Rassias RNS Approximation of Euler-Lagrange-Type Additive Mappings

H. Azadi Kenary,[1] H. Rezaei,[1] A. Ebadian,[2] and A. R. Zohdi[3]

[1] *Department of Mathematics, College of Sciences, Yasouj University, Yasouj 75914-353, Iran*
[2] *Department of Mathematics, Payame Noor University, Tehran, Iran*
[3] *Department of Mathematics, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran*

Correspondence should be addressed to A. Ebadian, ebadian.ali@gmail.com

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Recently the generalized Hyers-Ulam (or Hyers-Ulam-Rassias) stability of the following functional equation $\sum_{j=1}^{m} f(-r_j x_j + \sum_{1\le i\le m, i\ne j} r_i x_i) + 2\sum_{i=1}^{m} r_i f(x_i) = m f(\sum_{i=1}^{m} r_i x_i)$ where $r_1, \ldots, r_m \in \mathbb{R}$, proved in Banach modules over a unital C^*-algebra. It was shown that if $\sum_{i=1}^{m} r_i \ne 0$, $r_i, r_j \ne 0$ for some $1 \le i < j \le m$ and a mapping $f : X \to Y$ satisfies the above mentioned functional equation then the mapping $f : X \to Y$ is Cauchy additive. In this paper we prove the Hyers-Ulam-Rassias stability of the above mentioned functional equation in random normed spaces (briefly RNS).

1. Introduction and Preliminaries

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms. Hyers [2] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki [3] for additive mappings and by Rassias [4] for linear mappings by considering an unbounded Cauchy difference.

The paper of Rassias has provided a lot of influence in the development of what we call the *generalized Hyers-Ulam stability* of functional equations. In 1994, a generalization of Rassias' theorem was obtained by Găvruţa [5] by replacing the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$.

The functional equation:

$$f(x+y) + f(x-y) = 2f(x) + 2f(y), \tag{1.1}$$

is called a *quadratic functional equation*. In particular, every solution of the quadratic functional equation is said to be a *quadratic mapping*. The generalized Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [6] for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space. Cholewa [7] noticed that the theorem of Skof is still true if the relevant domain X is replaced by an Abelian group. Czerwik [8] proved the generalized Hyers-Ulam stability of the quadratic functional equation.

The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [2, 4, 5, 9–28]).

In the sequel, we will adopt the usual terminology, notions, and conventions of the theory of random normed spaces as in [29]. Throughout this paper, the spaces of all probability distribution functions are denoted by Δ^+. Elements of Δ^+ are functions $F : \mathbb{R} \cup \{-\infty, +\infty\} \to [0,1]$, such that F is left continuous and nondecreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1$. It's clear that the subset $D^+ = \{F \in \Delta^+ : l^- F(+\infty) = 1\}$, where $l^- f(x) = \lim_{t \to x^-} f(t)$, is a subset of Δ^+. The space Δ^+ is partially ordered by the usual point-wise ordering of functions, that is, for all $t \in R$, $F \le G$ if and only if $F(t) \le G(t)$. For every $a \ge 0$, $H_a(t)$ is the element of D^+ defined by

$$H_a(t) = \begin{cases} 0 & \text{if } t \le a \\ 1 & \text{if } t > a. \end{cases} \tag{1.2}$$

One can easily show that the maximal element for Δ^+ in this order is the distribution function $H_0(t)$.

Definition 1.1. A function $T : [0,1]^2 \to [0,1]$ is a continuous triangular norm (briefly a t-norm) if T satisfies the following conditions:

(i) T is commutative and associative;

(ii) T is continuous;

(iii) $T(x,1) = x$ for all $x \in [0,1]$;

(iv) $T(x,y) \le T(z,w)$ whenever $x \le z$ and $y \le w$ for all $x,y,z,w \in [0,1]$.

Three typical examples of continuous t-norms are $T_P(x,y) = xy$, $T_{\max}(x,y) = \max\{a + b - 1, 0\}$, and $T_M(x,y) = \min(a,b)$. Recall that, if T is a t-norm and $\{x_n\}$ is a given of numbers in $[0,1]$, $T_{i=1}^n x_i$ is defined recursively by $T_{i=1}^1 x_1$ and $T_{i=1}^n x_i = T(T_{i=1}^{n-1} x_i, x_n)$ for $n \ge 2$.

Definition 1.2. A random normed space (briefly RNS) is a triple (X, μ', T), where X is a vector space, T is a continuous t-norm, and $\mu' : X \to D^+$ is a mapping such that the following conditions hold.

(i) $\mu'_x(t) = H_0(t)$ for all $t > 0$ if and only if $x = 0$.

(ii) $\mu'_{\alpha x}(t) = \mu'_x(t/|\alpha|)$ for all $\alpha \in \mathbb{R}$, $\alpha \neq 0$, $x \in X$ and $t \ge 0$.

(iii) $\mu'_{x+y}(t+s) \ge T(\mu'_x(t), \mu'_y(s))$, for all $x, y \in X$ and $t, s \ge 0$.

Definition 1.3. Let (X, μ', T) be an RNS.

(i) A sequence $\{x_n\}$ in X is said to be convergent to $x \in X$ in X if for all $t > 0$, $\lim_{n \to \infty} \mu'_{x_n - x}(t) = 1$

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence in X if for all $t > 0$, $\lim_{n\to\infty}\mu'_{x_n - x_m}(t) = 1$.

(iii) The RN-space (X, μ', T) is said to be complete if every Cauchy sequence in X is convergent.

Theorem 1.4. *If (X, μ', T) is RNS and $\{x_n\}$ is a sequence such that $x_n \to x$, then $\lim_{n\to\infty}\mu'_{x_n}(t) = \mu'_x(t)$.*

In this paper, we investigate the generalized Hyers-Ulam stability of the following additive functional equation of Euler-Lagrange type:

$$\sum_{j=1}^{m} f\left(-r_j x_j + \sum_{1\le i\le m,\, i\neq j} r_i x_i\right) + 2\sum_{i=1}^{m} r_i f(x_i) = m f\left(\sum_{i=1}^{m} r_i x_i\right), \tag{1.3}$$

where $r_1, \ldots, r_n \in \mathbb{R}$, $\sum_{i=1}^{m} r_i \neq 0$, and $r_i, r_j \neq 0$ for some $1 \le i < j \le m$, in random normed spaces.

Every solution of the functional equation (1.3) is said to be a *generalized Euler-Lagrange type additive mapping.*

2. RNS Approximation of Functional Equation (1.3)

Remark 2.1. Throughout this paper, $r_1, \ldots, r_m$ will be real numbers such that $r_i, r_j \neq 0$ for fixed $1 \le i < j \le m$.

Theorem 2.2. *Let X be a real linear space, $(Z, \mu', \min)$ be an RN space, $\varphi : X^n \to Z$ be a function such that for some $0 < \alpha < 2$,*

$$\mu'_{\varphi(2x_1,\ldots,2x_m)}(t) \ge \mu'_{\alpha\varphi(x_1,\ldots,x_m)}(t) \quad \forall x_i \in X,\ t > 0. \tag{2.1}$$

$f(0) = 0$ and for all $x_i \in X$ and $t > 0$

$$\lim_{n\to\infty}\mu'_{(\varphi(2^n x_1,\ldots,2^n x_m)/2^n)}(t) = 1. \tag{2.2}$$

Let $(Y, \mu, \min)$ be a complete RN space. If $f : x \to Y$ is a mapping such that for all $x_i, x_j \in X$ and $t > 0$

$$\mu_{\sum_{j=1}^{m} f(-r_j x_j + \sum_{1\le i\le m, i\neq j} r_i x_i) + 2\sum_{i=1}^{m} r_i f(x_i) - m f(\sum_{i=1}^{m} r_i x_i)}(t) \ge \mu'_{\varphi(x_1,\ldots,x_m)}(t) \tag{2.3}$$

then there is a unique generalized Euler-Lagrange-type additive mapping $\mathrm{EL} : X \to Y$ *such that, for all* $x \in X$ *and all* $t > 0$

$$\mu_{\mathrm{EL}(x)-f(x)}(t) \geq T_M\Bigg(T_M\bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{(2-\alpha)t}{6}\right), \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{(2-\alpha)t}{6}\right), \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{(2-\alpha)t}{6}\right)\bigg), T_M\bigg(\mu'_{\varphi_{i,j}(x/r_i,\, x/r_j)}\left(\frac{(2-\alpha)t}{3}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{(2-\alpha)t}{3}\right), \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{(2-\alpha)t}{3}\right)\bigg)\Bigg). \tag{2.4}$$

Proof. For each $1 \leq k \leq m$ with $k \neq i, j$, let $x_k = 0$ in (2.3). Then we get the following inequality:

$$\mu_{\lambda(x_i,x_j)}(t) \geq \mu'_{\varphi_{i,j}(x_i,x_j)}(t), \tag{2.5}$$

for all $x_i, x_j \in X$, where

$$\varphi_{i,j}(x,y) := \varphi\Bigg(0,\ldots,0,\underbrace{x}_{i\text{th}},0,\ldots,0,\underbrace{y}_{j\text{th}},0,\ldots,0\Bigg), \tag{2.6}$$

for all $x, y \in X$ and all $1 \leq i < j \leq m$, and

$$\lambda(x_i,x_j) = f(-r_ix_i + r_jx_j) + f(r_ix_i - r_jx_j) - 2f(r_ix_i + r_jx_j) + 2r_if(x_i) + 2r_jf(x_j). \tag{2.7}$$

Letting $x_i = 0$ in (2.5), we get

$$\mu_{f(-r_jx_j)-f(r_jx_j)+2r_jf(x_j)}(t) \geq \mu'_{\varphi_{i,j}(0,x_j)}(t), \tag{2.8}$$

for all $x_j \in X$. Similarly, letting $x_j = 0$ in (2.5), we get

$$\mu_{f(-r_ix_i)-f(r_ix_i)+2r_if(x_i)}(t) \geq \mu'_{\varphi_{i,j}(x_i,0)}(t), \tag{2.9}$$

for all $x_i \in X$. It follows from (2.5), (2.8), and (2.9) that for all $x_i, x_j \in X$

$$\mu_{\lambda(x_i,x_j)-(f(-r_ix_i)-f(r_ix_i)+2r_if(x_i))-(f(-r_jx_j)-f(r_jx_j)+2r_jf(x_j))}(t) \geq T_M\left(\mu'_{\varphi_{i,j}(x_i,x_j)}\left(\frac{t}{3}\right), \mu'\varphi_{i,j}(x_i,0)\left(\frac{t}{3}\right), \mu'\varphi_{i,j}(0,x_j)\left(\frac{t}{3}\right)\right). \tag{2.10}$$

Replacing x_i and x_j by (x/r_i) and (y/r_j) in (2.10), we get that

$$\mu_{f(-x+y)+f(x-y)-2f(x+y)+f(x)+f(y)-f(-x)-f(-y)}(t) \geq T_M\left(\mu'_{\varphi_{i,j}(x/r_i,y/r_j)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(0,y/r_j)}\left(\frac{t}{3}\right)\right), \tag{2.11}$$

for all $x, y \in X$. Putting $y = x$ in (2.11), we get

$$\mu_{2f(x)-2f(-x)-2f(2x)}(t) \geq T_M\left(\mu'_{\varphi_{i,j}(x/r_i,\ x/r_j)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{t}{3}\right)\right), \tag{2.12}$$

for all $x \in X$. Replacing x and y by $(x/2)$ and $-(x/2)$ in (2.11), respectively, we get

$$\mu_{f(x)+f(-x)}(t) \geq T_M\left(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3}\right)\right), \tag{2.13}$$

for all $x \in X$. It follows from (2.12) and (2.13) that

$$\begin{aligned}
\mu_{f(2x)-2f(x)}(t) &= \mu_{f(x)+f(-x)+((2f(x)-2f(-x)-2f(2x))/2)}(t) \\
&\geq T_M\left(\mu_{f(x)+f(-x)}\left(\frac{t}{2}\right),\ \mu_{2f(x)-2f(-x)-2f(2x)}(t)\right) \\
&\geq T_M\left(T_M\left(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{6}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{6}\right),\ \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{6}\right)\right),\right. \\
&\qquad \left. T_M\left(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{t}{3}\right)\right)\right),
\end{aligned} \tag{2.14}$$

for all $x \in X$. So

$$\begin{aligned}
\mu_{(f(2x)/2)-f(x)}(t) \geq T_M\Bigg(&T_M\left(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3}\right),\right. \\
&\qquad \left.\mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3}\right)\right), \\
&T_M\left(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3}\right),\ \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3}\right)\right)\Bigg).
\end{aligned} \tag{2.15}$$

Replacing x by $2^n x$ in (2.15) and using (2.1), we get

$$\begin{aligned}
&\mu_{(f(2^{n+1}x)/2^{n+1})-(f(2^n x)/2^n)}(t) \\
&\quad\geq T_M\left(T_M\left(\mu'_{\varphi_{i,j}((2^n x/2r_i),-(2^n x/2r_j))}\left(\frac{2^n t}{3}\right), \mu'_{\varphi_{i,j}((2^n x/2r_i),0)}\left(\frac{2^n t}{3}\right), \mu'_{\varphi_{i,j}(0,-(2^n x/2r_j))}\left(\frac{2^n t}{3}\right)\right),\right. \\
&\qquad T_M\left(\mu'_{\varphi_{i,j}((2^n x/r_i),(2^n x/r_j))}\left(\frac{2^{n+1} t}{3}\right), \mu'_{\varphi_{i,j}((2^n x/r_i),0)}\left(\frac{2^{n+1} t}{3}\right),\right. \\
&\qquad\qquad \left.\left.\mu'_{\varphi_{i,j}(0,(2^n x/r_j))}\left(\frac{2^{n+1} t}{3}\right)\right)\right) \\
&\quad\geq T_M\left(T_M\left(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{2^n t}{3\alpha^n}\right), \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{2^n t}{3\alpha^n}\right), \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{2^n t}{3\alpha^n}\right)\right),\right. \\
&\qquad\quad \left.T_M\left(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2^{n+1} t}{3\alpha^n}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2^{n+1} t}{3\alpha^n}\right), \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2^{n+1} t}{3\alpha^n}\right)\right)\right),
\end{aligned} \tag{2.16}$$

for all $x \in X$ and all $n \in \mathbb{N}$. Therefore, we have

$$\begin{aligned}
&\mu_{(f(2^n x)/2^n)-f(x)}\left(\sum_{k=0}^{n-1}\frac{\alpha^k t}{2^k}\right) \\
&\quad= \mu_{\sum_{k=0}^{n-1}((f(2^{k+1}x)/2^{k+1})-(f(2^k x)/2^k))}\left(\sum_{k=0}^{n-1}\frac{\alpha^k t}{2^k}\right) \\
&\quad\geq T_{k=0}^{n-1}\left(\mu_{((f(2^{k+1}x)/2^{k+1})-(f(2^k x)/2^k))}\left(\frac{\alpha^k t}{2^k}\right)\right) \\
&\quad\geq T_{k=0}^{n-1}\left(T_M\left(T_M\left(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3}\right), \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3}\right), \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3}\right)\right),\right.\right. \\
&\qquad\qquad \left.\left.T_M\left(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3}\right), \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3}\right)\right)\right)\right) \\
&\quad= T_M\left(T_M\left(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3}\right), \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3}\right), \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3}\right)\right),\right. \\
&\qquad\quad \left.T_M\left(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3}\right), \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3}\right)\right)\right),
\end{aligned} \tag{2.17}$$

for all $x \in X$. This implies that

$$\begin{aligned}
&\mu_{(f(2^n x)/2^n)-f(x)}(t) \\
&\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3\sum_{k=0}^{n-1}\alpha^k/2^k}\right), \\
&\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right)\Bigg), \\
&\quad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \\
&\qquad \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right)\Bigg)\Bigg).
\end{aligned} \tag{2.18}$$

Replacing x by $2^p x$ in (2.18), we obtain

$$\begin{aligned}
&\mu_{(f(2^{n+p} x)/2^{n+p})-(f(2^p x)/2^p)}(t) \\
&\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3\sum_{k=p}^{p+n-1}(\alpha^k/2^k)}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3\sum_{k=p}^{p+n-1}(\alpha^k/2^k)}\right), \\
&\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3\sum_{k=p}^{p+n-1}(\alpha^k/2^k)}\right)\Bigg), \\
&\quad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3\sum_{k=p}^{p+n-1}(\alpha^k/2^k)}\right), \\
&\qquad \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3\sum_{k=p}^{p+n-1}(\alpha^k/2^k)}\right) \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3\sum_{k=p}^{p+n-1}(\alpha^k/2^k)}\right)\Bigg)\Bigg)
\end{aligned} \tag{2.19}$$

Since the right-hand side of the above inequality tends to 1, when $p, n \to \infty$, then the sequence $\{f(2^k x)/2^k\}_{n=1}^{+\infty}$ is a Cauchy sequence in complete RN space $(Y, \mu, \min)$, so there exists some point $\mathrm{EL}(x) \in Y$ such that

$$\mathrm{EL}(x) = \lim_{n\to\infty} \frac{f(2^k x)}{2^k}, \tag{2.20}$$

for all $x \in X$.

Fix $x \in X$ and put $P = 0$ in (2.19). Then we obtain

$$\begin{aligned}
&\mu_{(f(2^n x)/2^n)-f(x)}(t) \\
&\quad \geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \\
&\qquad\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right)\Bigg), \\
&\qquad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \\
&\qquad\qquad \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right)\Bigg)\Bigg),
\end{aligned} \tag{2.21}$$

and so, for every $\epsilon > 0$, we have

$$\begin{aligned}
\mu_{\mathrm{EL}(x)-f(x)}(t+\epsilon) &\geq T\left(\mu_{\mathrm{EL}(x)-(f(2^n x)/2^n)}(\epsilon), \mu_{(f(2^n x)/2^n)-f(x)}(t)\right) \\
&\geq T\Bigg(\mu_{\mathrm{EL}(x)-(f(2^n x)/2^n)}(\epsilon), T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \\
&\qquad \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right) \\
&\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right)\Bigg), \\
&\qquad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \\
&\qquad \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right), \\
&\qquad \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2t}{3\sum_{k=0}^{n-1}(\alpha^k/2^k)}\right)\Bigg)\Bigg)\Bigg).
\end{aligned} \tag{2.22}$$

Taking the limit as $n \to \infty$ and using (2.22), we get

$$\begin{aligned}
&\mu_{\mathrm{EL}(x)-f(x)}(t+\epsilon) \\
&\geq T_{\mathrm{M}}\Bigg(T_M\bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{(2-\alpha)t}{6}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{(2-\alpha)t}{6}\right), \\
&\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{(2-\alpha)t}{6}\right)\bigg),\ T_M\bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{(2-\alpha)t}{3}\right), \\
&\qquad \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{(2-\alpha)t}{3}\right), \\
&\qquad \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{(2-\alpha)t}{3}\right)\bigg)\Bigg).
\end{aligned} \tag{2.23}$$

Since ϵ was arbitrary by taking $\epsilon \to 0$ in (2.23), we get

$$\begin{aligned}
\mu_{\mathrm{EL}(x)-f(x)}(t) \geq T_M\Bigg(&T_M\bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{(2-\alpha)t}{6}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{(2-\alpha)t}{6}\right), \\
&\mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{(2-\alpha)t}{6}\right)\bigg), \\
&T_M\bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{(2-\alpha)t}{3}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{(2-\alpha)t}{3}\right), \\
&\mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{(2-\alpha)t}{3}\right)\bigg)\Bigg).
\end{aligned} \tag{2.24}$$

Replacing x_i by $2^n x_i$ for all $1 \leq i \leq m$, in (2.3), we get for all $x_i, x_j \in X$ and for all $t > 0$,

$$\mu_{\sum_{j=1}^{m} f(-2^n r_j x_j+\sum_{1\leq i\leq m, i\neq j} 2^n r_i x_i)+2\sum_{i=1}^{m} r_i f(2^n x_i)-m f(\sum_{i=1}^{m} 2^n r_i x_i)/2^n}(t) \geq \mu'_{\varphi(2^n x_1,\ldots,2^n x_m)/2^n}(t). \tag{2.25}$$

since

$$\lim_{n\to\infty}\mu'_{\varphi(2^n x_1,\ldots,2^n x_m)/2^n}(t) = 1, \tag{2.26}$$

We conclude that

$$\sum_{j=1}^{m}\mathrm{EL}\left(-r_j x_j + \sum_{1\leq i\leq m, i\neq j} r_i x_i\right) + 2\sum_{i=1}^{m} r_i \mathrm{EL}(x_i) - m\mathrm{EL}\left(\sum_{i=1}^{m} r_i x_i\right) = 0. \tag{2.27}$$

To prove the uniqueness of mapping EL, assume that there exists another mapping $A : X \to Y$ which satisfies (2.4). Fix $x \in X$, clearly $\mathrm{EL}(2^n x) = 2^n\ \mathrm{EL}(x)$ and $A(2^n x) = 2^n A(x)$, for all $n \in N$. Since $\mu_{\mathrm{EL}(x)-A(x)}\ (t) = \lim_{n\to\infty}\mu_{(\mathrm{EL}(2^n x)/2^n)-(A(2^n x)/2^n)}(t)$, so

$$\begin{aligned}\mu_{(\mathrm{EL}(2^n x)/2^n)-(A(2^n x)/2^n)}(t) &\geq \min\left\{\mu_{(\mathrm{EL}(2^n x)/2^n)-(f(2^n x)/2^n)}(t)\left(\frac{t}{2}\right), \mu_{(f(2^n x)/2^n)-(A(2^n x)/2^n)}(t)\left(\frac{t}{2}\right)\right\}\\ &\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{2^n(2-\alpha)t}{12\alpha^n}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{2^n(2-\alpha)t}{12\alpha^n}\right),\\ &\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{2^n(2-\alpha)t}{12\alpha^n}\right)\Bigg),\\ &\qquad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{2^n(2-\alpha)t}{6\alpha^n}\right), \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{2^n(2-\alpha)t}{6\alpha^n}\right),\\ &\qquad \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{2^n(2-\alpha)t}{6\alpha^n}\right)\Bigg)\Bigg).\end{aligned} \tag{2.28}$$

Since the right-hand side of the above inequality tends to 1, when $n \to \infty$, therefore, it follows that for all $t > 0$, $\mu_{\mathrm{EL}(x)-A(x)}(t) = 1$ and so $\mathrm{EL}(x) = A(x)$. This completes the proof. □

Corollary 2.3. *Let X be a real linear space, $(Z, \mu', \min)$ be an RN space, and $(Y, \mu, \min)$ a complete RN space. Let $0 < p < 1$, $z_0 \in Z$ and $f : X \to Y$ be a mapping with $f(0) = 0$ and satisfying*

$$\mu_{\sum_{j=1}^m f(-r_j x_j+\sum_{1\leq i\leq m, i\neq j} r_i x_i)+2\sum_{i=1}^m r_i f(x_i)-mf(\sum_{i=1}^m r_i x_i)}(t) \geq \mu'_{(\sum_{k=1}^m \|x_k\|^p)z_0}(t), \tag{2.29}$$

for all $x_i, x_j \in X$ and $t > 0$. Then the limit $\mathrm{EL}(x) = \lim_{n\to\infty} f(2^n x)/2^n$ exists for all $x \in X$ and defines a unique Euler-Lagrange additive mapping $\mathrm{EL} : X \to Y$ such that

$$\begin{aligned}\mu_{\mathrm{EL}(x)-f(x)}(t) \geq T_M\Bigg(&T_M\Bigg(\mu'_{\|x\|^p z_0}\left(\frac{2^p|r_i r_j|^p(2-2^p)t}{6(|r_i|^p+|r_j|^p)}\right),\ \mu'_{\|x\|^p z_0}\left(\frac{|2r_i|^p(2-2^p)t}{6}\right),\\ &\mu'_{\|x\|^p z_0}\left(\frac{|2r_j|^p(2-2^p)t}{6}\right)\Bigg),\\ &T_M\Bigg(\mu'_{\|x\|^p z_0}\left(\frac{|r_i r_j|^p(2-2^p)t}{3(|r_i|^p+|r_j|^p)}\right),\ \mu'_{\|x\|^p z_0}\left(\frac{|r_i|^p(2-2^p)t}{3}\right),\\ &\mu'_{\|x\|^p z_0}\left(\frac{|r_j|^p(2-2^p)t}{3}\right)\Bigg)\Bigg),\end{aligned} \tag{2.30}$$

for all $x \in X$ and $t > 0$.

Proof. Let $\alpha = 2^p$ and $\varphi : X^m \to Z$ be defined as $\varphi(x_1, \ldots, x_m) = (\sum_{k=1}^m \|x_i\|^p)z_0$. □

Corollary 2.4. *Let X be a real linear space, $(Z,\mu',\min)$ be an RN space, and $(Y,\mu,\min)$ a complete RN space. Let $z_0 \in Z$ and $f : X \to Y$ be a mapping with $f(0) = 0$ and satisfying*

$$\mu_{\sum_{j=1}^{m} f(-r_j x_j+\sum_{1\leq i\leq m, i\neq j} r_i x_i)+2\sum_{i=1}^{m} r_i f(x_i)-mf(\sum_{i=1}^{m} r_i x_i)}(t) \geq \mu'_{\delta z_0}(t), \tag{2.31}$$

for all $x_i \in X$ for all $1 \leq i \leq m$ and all $t > 0$. Then, the limit $C(x) = \lim_{n\to\infty}(f(2^n x)/2^n)$ exists for all $x \in X$ and defines a unique Euler-Lagrange additive mapping $\mathrm{EL} : X \to Y$ such that

$$\mu_{\mathrm{EL}(x)-f(x)}(t) \geq T_M\left(\mu'_{\delta z_0}\left(\frac{t}{6}\right), \mu'_{\delta z_0}\left(\frac{t}{3}\right)\right), \tag{2.32}$$

for all $x \in X$ and $t > 0$.

Proof. Let $\alpha = 1$ and $\varphi : X^m \to Z$ be defined as $\varphi(x_1,\ldots,x_m) = \delta z_0$. □

Theorem 2.5. *Let X be a real linear space, $(Z,\mu',\min)$ be an RN space, $\varphi : X^m \to Z$ be a function such that for some $0 < \alpha < 1/2$,*

$$\mu'_{\varphi(x_1/2,\ldots,x_m/2)}(t) \geq \mu'_{\alpha\varphi(x_1,\ldots,x_m)}(t) \quad \forall x_i \in X,\ t > 0, \tag{2.33}$$

$f(0) = 0$ and for all $x_i \in X$ and $t > 0$, $\lim_{n\to\infty}\mu'_{2^n\varphi(x_1/2^n,\ldots,x_m/2^n)}(t) = 1$. Let $(Y,\mu,\min)$ be a complete RN space. If $f : X \to Y$ is a mapping satisfying (2.3), then there is a unique generalized Euler-Lagrange-type additive mapping $\mathrm{EL} : X \to Y$ such that, for all $x \in X$

$$\begin{aligned}\mu_{\mathrm{EL}(x)-f(x)}(t) \geq T_M\Bigg(&T_M\left(\mu'_{\varphi_{i,j}(x/r_i,-x/r_j)}\left(\frac{(1-2\alpha)t}{6\alpha}\right), \mu'_{\varphi_{i,j}(x/r_j,0)}\left(\frac{(1-2\alpha)t}{6\alpha}\right),\right.\\ &\left.\mu'_{\varphi_{i,j}(0,-(x/r_j))}\left(\frac{(1-2\alpha)t}{6\alpha}\right)\right),\\ &T_M\left(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{(1-2\alpha)t}{3\alpha}\right),\right.\\ &\left.\mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{(1-2\alpha)t}{3\alpha}\right), \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{(1-2\alpha)t}{3\alpha}\right)\right)\Bigg),\end{aligned} \tag{2.34}$$

for all $x \in X$ and all $t > 0$.

Proof. Replacing x by $x/2^{n+1}$ in (2.14) and using (2.33), we obtain

$$\begin{aligned}
&\mu_{2^n f(x/2^n)-2^{n+1}f(x/2^{n+1})}(t)\\
&\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2^{n+2}r_i,-(x/2^{n+2}r_j))}\left(\frac{t}{2^n\cdot 6}\right),\ \mu'_{\varphi_{i,j}(x/2^{n+2}r_i,0)}\left(\frac{t}{2^n\cdot 6}\right),\\
&\qquad\qquad \mu'_{\varphi_{i,j}(0,-(x/2^{n+2}r_j))}\left(\frac{t}{2^n\cdot 6}\right)\Bigg),\\
&\qquad T_M\Bigg(\mu'_{\varphi_{i,j}(x/2^{n+1}r_i,x/2^{n+1}r_j)}\left(\frac{t}{2^n\cdot 3}\right),\mu'_{\varphi_{i,j}(x/2^{n+1}r_i,0)}\left(\frac{t}{2^n\cdot 3}\right),\mu'_{\varphi_{i,j}(0,x/2^{n+1}r_j)}\left(\frac{t}{2^n\cdot 3}\right)\Bigg)\Bigg)\\
&\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/2r_i,-(x/2r_j))}\left(\frac{t}{\alpha^{n+1}2^n\cdot 6}\right),\ \mu'_{\varphi_{i,j}(x/2r_i,0)}\left(\frac{t}{\alpha^{n+1}2^n\cdot 6}\right),\\
&\qquad\qquad \mu'_{\varphi_{i,j}(0,-(x/2r_j))}\left(\frac{t}{\alpha^{n+1}2^n\cdot 6}\right)\Bigg),\\
&\qquad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/j)}\left(\frac{t}{\alpha^{n+1}2^n\cdot 3}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{\alpha^{n+1}2^n\cdot 3}\right),\ \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{t}{\alpha^{n+1}2^n\cdot 3}\right)\Bigg)\Bigg).
\end{aligned}\tag{2.35}$$

So

$$\begin{aligned}
&\mu_{2^n f(x/2^n)\,-f(x)}\left(\sum_{i=1}^{n-1}2^k\alpha^{k+1}t\right)\\
&\quad\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,-(x/r_j))}\left(\frac{t}{6}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{6}\right),\ \mu'_{\varphi_{i,j}(0,-(x/r_j))}\left(\frac{t}{6}\right)\Bigg),\\
&\qquad\quad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{3}\right),\ \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{t}{3}\right)\Bigg)\Bigg),
\end{aligned}\tag{2.36}$$

for all $x\in X$. This implies that

$$\begin{aligned}
&\mu_{2^n f(x/2^n)-f(x)}(t)\\
&\geq T_M\Bigg(T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,-(x/r_j))}\left(\frac{t}{6\alpha\sum_{k=0}^{n-1}2^k\alpha^k}\right),\ \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{6\alpha\sum_{k=0}^{n-1}2^k\alpha^k}\right),\\
&\qquad\qquad \mu'_{\varphi_{i,j}(0,-(x/r_j))}\left(\frac{t}{6\alpha\sum_{k=0}^{n-1}2^k\alpha^k}\right)\Bigg),\\
&\qquad T_M\Bigg(\mu'_{\varphi_{i,j}(x/r_i,x/r_j)}\left(\frac{t}{3\alpha\sum_{k=0}^{n-1}2^k\alpha^k}\right),\\
&\qquad\qquad \mu'_{\varphi_{i,j}(x/r_i,0)}\left(\frac{t}{3\alpha\sum_{k=0}^{n-1}2^k\alpha^k}\right),\ \mu'_{\varphi_{i,j}(0,x/r_j)}\left(\frac{t}{3\alpha\sum_{k=0}^{n-1}2^k\alpha^k}\right)\Bigg)\Bigg).
\end{aligned}\tag{2.37}$$

Proceeding as in the proof of Theorem 2.2, one can easily show that the sequence $\{2^n f(x/2^n)\}_{n=1}^{+\infty}$ is a Cauchy sequence in complete RN space $(Y,\mu,\min)$, so there exists some point $\mathrm{EL}(x)\in Y$ such that

$$\mathrm{EL}(x)=\lim_{n\to\infty}2^n f\left(\frac{x}{2^n}\right), \tag{2.38}$$

for all $x\in X$.

Taking the limit $n\to\infty$ from both sides of the above inequality, we obtain (2.34).

The rest of the proof is similar to the proof of Theorem 2.2. □

Corollary 2.6. *Let X be a real linear space, $(Z,\mu',\min)$ be an RN space and $(Y,\mu,\min)$ a complete RN space. Let $p>1$, $z_0\in Z$ and $f:X\to Y$ be a mapping with $f(0)=0$ and satisfying*

$$\mu_{\sum_{j=1}^m f(-r_jx_j+\sum_{1\le i\le m, i\ne j} r_ix_i)+2\sum_{i=1}^m r_if(x_i)-mf(\sum_{i=1}^m r_ix_i)}(t)\ge\mu'_{(\sum_{k=1}^m\|x_i\|^p)z_0}(t), \tag{2.39}$$

for all $x_i\in X$ for all $1\le i\le m$ and all $t>0$. Then the limit $\mathrm{EL}(x)=\lim_{n\to\infty}2^nf(x/2^n)$ exists for all $x\in X$ and defines a unique Euler-Lagrange additive mapping $\mathrm{EL}:X\to Y$ such that

$$\begin{aligned}\mu_{\mathrm{EL}(x)-f(x)}(t)\ge T_M\Bigg(&T_M\left(\mu'_{\|x\|^pz_0}\left(\frac{2^p|r_ir_j|^p(2^p-2)t}{6(|r_i|^p+|r_j|^p)}\right),\ \mu'_{\|x\|^pz_0}\left(\frac{|2r_i|^p(2^p-2)t}{6}\right),\right.\\&\left.\mu'_{\|x\|^pz_0}\left(\frac{|2r_j|^p(2^p-2)t}{6}\right)\right),\\&T_M\left(\mu'_{\|x\|^pz_0}\left(\frac{|r_ir_j|^p(2^p-2)t}{3(|r_i|^p+|r_j|^p)}\right),\ \mu'_{\|x\|^pz_0}\left(\frac{|r_i|^p(2^p-2)t}{3}\right),\right.\\&\left.\mu'_{\|x\|^pz_0}\left(\frac{|r_j|^p(2^p-2)t}{3}\right)\right)\Bigg),\end{aligned} \tag{2.40}$$

for all $x\in X$ and $t>0$.

Proof. Let $\alpha=2^{-p}$ and $\varphi:X^m\to Z$ be defined as $\varphi(x_1,\ldots,x_m)=(\sum_{k=1}^m\|x_i\|^p)z_0$. □

Corollary 2.7. *Let X be a real linear space, $(Z,\mu',\min)$ be an RN space and $(Y,\mu,\min)$ a complete RN space. Let $z_0\in Z$ and $f:X\to Y$ be a mapping with $f(0)=0$ and satisfying*

$$\mu_{\sum_{j=1}^m f(-r_jx_j+\sum_{1\le i\le m, i\ne j} r_ix_i)+2\sum_{i=1}^m r_if(x_i)-mf(\sum_{i=1}^m r_ix_i)}(t)\ge\mu'_{\delta z_0}(t), \tag{2.41}$$

for all $x_i,x_j\in X$ and $t>0$. Then, the limit $\mathrm{EL}(x)=\lim_{n\to\infty}2^nf(x/2^n)$ exists for all $x\in X$ and defines a unique Euler-Lagrange additive mapping $\mathrm{EL}:X\to Y$ such that

$$\mu_{\mathrm{EL}(x)-f(x)}(t)\ge T_M\left(\mu'_{\delta z_0}\left(\frac{4t}{3}\right),\mu'_{\delta z_0}\left(\frac{2t}{3}\right)\right), \tag{2.42}$$

for all $x\in X$ and $t>0$.

Proof. Let $\alpha = 1/4$ and $\varphi : X^m \rightarrow Z$ be defined as $\varphi(x_1, \ldots, x_m) = \delta z_0$. □

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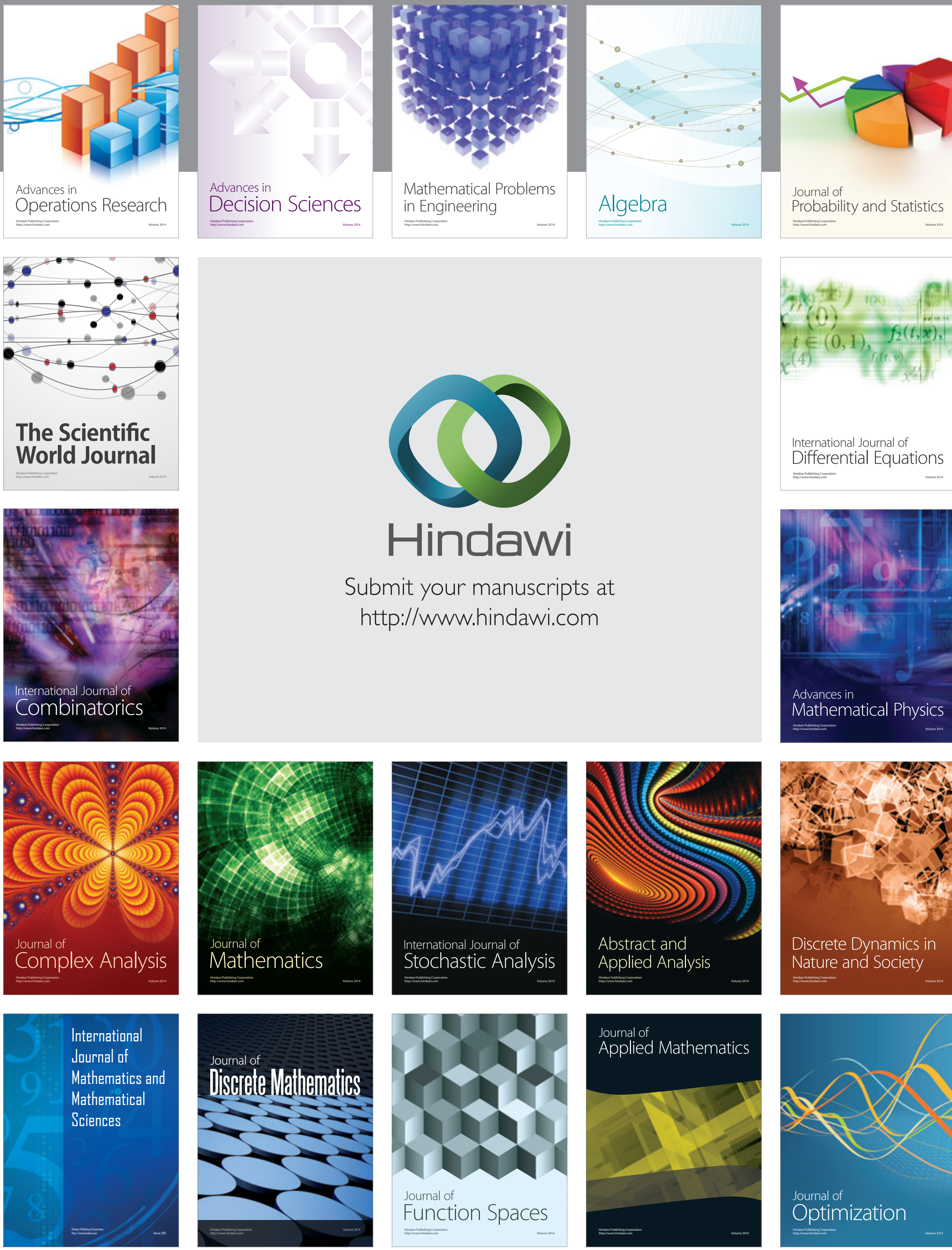
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