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Research Article

H_∞ Filtering for a Class of Piecewise Homogeneous Markovian Jump Nonlinear Systems

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H_∞ filtering problem for a class of piecewise homogeneous Markovian jump nonlinear systems is investigated. The aim of this paper is to design a mode-dependent filter such that the filtering error system is stochastically stable and satisfies a prescribed H_∞ disturbance attenuation level. By using a new mode-dependent Lyapunov-Krasovskii functional, mixed mode-dependent sufficient conditions on stochastic stability are formulated in terms of linear matrix inequalities (LMIs). Based on this, the mode-dependent filter is obtained. A numerical example is given to illustrate the effectiveness of the proposed main results.

1. Introduction

The filtering problem has received significant attention in the past decade. Current efforts on this topic can be divided into two classes: the Kalman filtering approach and the H_∞ filtering approach. As we all know, Kalman filtering approach is based on the assumption that the system is exactly known, and its disturbances are stationary Gaussian noises with known statistics. These assumptions limit the application scope of the Kalman filtering technique when there are uncertainties in either the exogenous input signals or the system model [1]. To overcome the restriction described above, H_∞ filtering has been introduced as an alternative filtering technique [2–6].

On the other hand, Markovian jump systems are an active area of research. It switches from one mode to another in a random way, and the switching between the modes is governed by a Markovian process with discrete and finite state space. These models serve as convenient tools for analyzing plants that are subjected to random abrupt parameter changes due to, for instance, component and/or interconnection failures, sudden environmental changes, or change of the operating point of a linearized model of a nonlinear plant. A wide class of industrial system applications experience time delays due to various reasons including inherent physical properties (mass transport flow, recycling), data transmission delays or finite capabilities of information exchange [7]. When considering the continuous systems with time-varying delay, the systems can be clarified into two types, one is slow time-varying delay systems, that is, the derivative of the time delay is less than one, for example, [2, 8, 9], and the other is fast time-varying delay systems, that is, there are no constraints on the derivative of the time delay. Both Lyapunov-Krasovskii and Lyapunov-Razumikhin approaches are fundamental for time-delay systems, and some existing work usually do not require the derivative of the time delay to be less than one, see, for example, [10, 11]. Due to their extensive practical applications, considerable attention has been devoted to Markovian jump systems with time delays. The issues of stability and control have been well investigated; see, for example, [9, 11–29] and references therein. In [30–32], the sliding mode control of Markovian jump singular systems was studied, and, new integral-type sliding surface functions were designed. Moreover, strict LMI conditions of the stochastic stability were proposed in [30, 31], which are easy to be checked by Matlab LMI toolbox. In [32], a suitable switching surface function and a sliding mode control law were designed to ensure the attraction of the sliding surface when the system changes from one mode to another under Markovian switching, and the slack matrix approach was used to derive less conservative LMI conditions assuring stochastic admissibility. The filtering problem for Markovian jump time-delay systems was reported in [8, 33–40]. Many nonlinear physical systems can be represented as a connection of a linear dynamical system and a nonlinear element. Filtering for Markovian jump nonlinear system is an important research area that has attracted considerable interest [41–43]. It should be pointed out that the above-mentioned references assume that the Markovian processes are homogeneous, that is, the considered transition probabilities (TPs) in Markovian process are assumed to be time invariant. However, the assumption cannot always be satisfied in real applications, and the ideal assumption on TPs inevitably limits the applications of the established results to some extent [44]. Therefore, it is important and necessary to pay attention to the study of Markovian jump systems with time-varying TPs. Recently, the problem of H_∞ estimation for discrete-time Markovian jump linear system with time-varying TPs has been investigated in [44]. The H_∞ control problem has been conducted for a class of discrete-time Markovian jump systems with time-varying TPs in [45], where the average dwell-time switching is used to describe the variation among the TPs. The stochastic stability analysis of piecewise homogeneous Markovian jump neural networks with mixed time delays has been studied in [46]. But, the time-varying delays in [46] are independent of jump mode. To the best of our knowledge, no results have been given for piecewise homogeneous Markovian jump nonlinear systems with mode-dependent time-varying delays. With the appearance of time-varying TPs and mode-dependent time-varying discrete and distributed delays, the main difficulties are as follows: (1) the new Lyapunov functional should be constructed to deal with above problem; (2) since the system involves joint jump processes and mode-dependent time-varying delays, the calculation of derivative of the Lyapunov functional and the using of inequality techniques become more complicated. Moreover, the Lyapunov matrix P_i is assumed to be diagonal matrix in some existing literature, which leads to some conservativeness. Therefore, the key problems in this research are: (1) how to choose a Lyapunov function to derive a sufficient stochastic stability condition for the considered systems; (2) how to use the inequality techniques and calculate the parameters of the filter such that the resulting sufficient conditions are less conservative? Which has motivated this paper.

In this study, we are concerned to develop an efficient approach for H_∞ filtering problem of piecewise homogeneous Markovian jump system. The system under study involves mode-dependent time-varying discrete and distributed delays and inherent sector-like nonlinearities. By using a novel Lyapunov-Krasovskii functional, mixed mode-dependent sufficient condition on stochastic stability with an H_∞ performance is derived in terms of LMIs. Based on this, the existence condition of the desired filter which guarantees stochastic stability and an H_∞ performance of the corresponding filtering error system is presented. A numerical example is provided to show the effectiveness of the proposed results.

Notation. Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathcal{E}\{\cdot\}$ refers to the expectation operator with respect to some probability measure $\mathcal{P}$. We use $\operatorname{diag}\{\cdot,\cdot,\cdot\}$ as a block diagonal matrix. $A > 0$ (< 0) means A is a symmetric positive (negative) definite matrix. A^T denotes the transpose of matrix A, I is the identity matrix with compatible dimension.

2. System Description and Definitions

Fix a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following stochastic Markovian jump system with mode-dependent time-varying delays:

$$\begin{aligned}
\dot{x}(t) &= A(r_t)x(t) + A_1(r_t)x(t-\tau(t,r_t)) + A_2(r_t)\int_{t-\tau(t,r_t)}^{t} x(s)\mathrm{d}s + B(r_t)f(x(t)) + D_1(r_t)\omega(t),\\
y(t) &= C(r_t)x(t) + C_1(r_t)x(t-\tau(t,r_t)) + C_2(r_t)\int_{t-\tau(t,r_t)}^{t} x(s)\mathrm{d}s + E(r_t)g(x(t)) + D_2(r_t)\omega(t),\\
z(t) &= H(r_t)x(t),\\
x(t) &= \phi(t), \quad t \in [-\tau, 0],
\end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $\omega(t)$ is the exogenous disturbance input which belongs to $L_2[0\ \infty)$; $y(t) \in \mathbb{R}^p$ is the measured output; $z(t) \in \mathbb{R}^q$ is the signal to be estimated; $\phi(t)$ is a compatible vector-valued initial function defined on $[-\tau, 0]$; $A(r_t)$, $A_1(r_t)$, $A_2(r_t)$, $B(r_t)$, $D_1(r_t)$, $C(r_t)$, $C_1(r_t)$, $C_2(r_t)$, $E(r_t)$, $D_2(r_t)$ and $H(r_t)$ are real constant matrices with appropriate dimensions. $\tau(t,r_t)$ is the mode-dependent time-varying delay. The process $\{r_t, t \geqslant 0\}$ is described by a Markov chain with finite state space $S_1 = \{1,2,\ldots,N\}$, and its transition probability matrix, $\Pi^{(\sigma_{t+\Delta})} = [\pi_{il}^{(\sigma_{t+\Delta})}]_{N\times N}$ $(i, l \in S_1)$, is governed by

$$\Pr\{r_{t+\Delta} = l \mid r_t = i\} = \begin{cases} \pi_{il}^{(\sigma_{t+\Delta})}\Delta + o(\Delta), & l \neq i,\\ 1 + \pi_{ii}^{(\sigma_{t+\Delta})}\Delta + o(\Delta), & l = i, \end{cases} \tag{2.2}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$; $\pi_{il}^{(\sigma_{t+\Delta})} \geqslant 0$ for $l \neq i$ is the transition rate from mode i at time t to mode l at time $t+\Delta$ and $\pi_{ii}^{(\sigma_{t+\Delta})} = -\sum_{l=1, l\neq i}^{N} \pi_{il}^{(\sigma_{t+\Delta})}$. In this study, we assume that σ_t vary in another finite set $S_2 = \{1,2,\ldots,M\}$, and the variations are considered as the stochastic variation. The variation of σ_t is governed by a higher-level transition probability (HTP) matrix $\Lambda = [\lambda_{jk}]_{M\times M}$ $(j, k \in S_2)$ and the TPs of Markov chain satisfy

$$\Pr\{\sigma_{t+\Delta} = k \mid \sigma_t = j\} = \begin{cases} \lambda_{jk}\Delta + o(\Delta), & k \neq j, \\ 1 + \lambda_{jj}\Delta + o(\Delta), & k = j, \end{cases} \tag{2.3}$$

where $\lambda_{jk} \geqslant 0$ for $k \neq j$ is the transition rate from mode j at time t to mode k at time $t + \Delta$ and $\lambda_{jj} = -\sum_{k=1,k\neq j}^{M} \lambda_{jk}$. The stochastic processes r_t and σ_t are assumed to be independent throughout this paper. For vector-valued functions $f(x(t))$ and $g(x(t))$, we assume:

$$\begin{aligned} &[f(x) - f(y) - M_1(x-y)]^T [f(x) - f(y) - M_2(x-y)] \leqslant 0, \\ &[g(x) - g(y) - L_1(x-y)]^T [g(x) - g(y) - L_2(x-y)] \leqslant 0, \end{aligned} \tag{2.4}$$

where for all $x, y \in \mathbb{R}^n$ and $M_1, M_2, L_1, L_2 \in \mathbb{R}^{n\times n}$ are the known constant matrices. In what follows, for implicity of presentations and without loss of generality, we always assume that $f(0) = 0$ and $g(0) = 0$.

For simplicity, a matrix $R(r_t)$ will be denoted by R_i. For example, $A(r_t)$ is denoted by A_i, $A_1(r_t)$ is denoted by A_{1i} and $\tau(r_t, t)$ is denoted by $\tau_i(t)$, $(i \in S_1)$. When the mode is in $r_t = i$, the mode-dependent time-varying delay satisfies

$$0 \leqslant \tau_i(t) \leqslant \tau_i \leqslant \tau, \qquad \dot{\tau}_i(t) \leqslant \mu_i, \tag{2.5}$$

where $\tau = \max\{\tau_i\}$.

In this study, the following full-order linear filter is proposed to estimate the signal $z(t)$:

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{F_{ij}}\hat{x}(t)\mathrm{d}t + B_{F_{ij}}y(t), \\ \hat{z}(t) &= C_{F_{ij}}\hat{x}(t), \\ \hat{x}(0) &= 0, \end{aligned} \tag{2.6}$$

where $\hat{x}(t)$ is the filter state vector, and $(A_{F_{ij}} \;\; B_{F_{ij}} \;\; C_{F_{ij}})$ are appropriately dimensioned filter matrices to be determined.

Define the estimation error by $e(t) = z(t) - \hat{z}(t)$, we obtain the following filtering error system:

$$\begin{aligned} \dot{\xi}(t) &= \overline{A}_{ij}\xi(t) + \overline{A}_{1ij}K\xi(t-\tau_i(t)) + \overline{A}_{2ij}K\int_{t-\tau_i(t)}^{t}\xi(s)\mathrm{d}s + \overline{B}_{ij}f(K\xi(t)) + \overline{E}_{ij}g(K\xi(t)) + \overline{D}_{ij}\omega(t), \\ e(t) &= \overline{H}_{ij}\xi(t), \\ \tilde{x}(t) &= \tilde{\phi}(t), \quad \forall t \in [-\tau, 0], \end{aligned} \tag{2.7}$$

where $\xi(t) = [x^T(t) \;\; \hat{x}^T(t)]^T$, $\tilde{\phi}(t) = [\phi^T(t) \;\; 0^T]^T$, and

$$\begin{gathered}
\overline{A}_{ij} = \begin{bmatrix} A_i & 0 \\ B_{F_{ij}} C_i & A_{F_{ij}} \end{bmatrix}, \qquad \overline{A}_{1ij} = \begin{bmatrix} A_{1i} \\ B_{F_{ij}} C_{1i} \end{bmatrix}, \qquad \overline{A}_{2ij} = \begin{bmatrix} A_{2i} \\ B_{F_{ij}} C_{2i} \end{bmatrix}, \qquad \overline{B}_{ij} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\
\overline{E}_{ij} = \begin{bmatrix} 0 \\ B_{F_{ij}} E_i \end{bmatrix}, \qquad \overline{D}_{ij} = \begin{bmatrix} D_{1i} \\ B_{F_{ij}} D_{2i} \end{bmatrix}, \qquad \overline{H}_{ij} = \begin{bmatrix} H_i & -C_{F_{ij}} \end{bmatrix}, \quad K = \begin{bmatrix} I & 0 \end{bmatrix}.
\end{gathered} \tag{2.8}$$

Remark 2.1. According to the definitions of homogeneous Markovian chain and nonhomogeneous Markovian chain in [44, 47], one can see that the Markovian chain σ_t in this paper is homogeneous, while the Markovian chain r_t is neither homogeneous nor nonhomogeneous, but a state between them, which can be called the finite piecewise homogeneous Markovian chain.

Remark 2.2. In this paper, the nonlinear functions $f(x(t))$ and $g(x(t))$ are said to belong to sectors, which means that the nonlinearities are bounded by sectors. The nonlinear descriptions in (2.4) are quite general that include the usual Lipschitz conditions as a special case [2].

The following lemma and definitions are introduced, which will be used in the proof of the main results.

Lemma 2.3 (see [48]). *For any matrix $M > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left[\int_0^{\gamma} \omega^T(s) ds\right] M \left[\int_0^{\gamma} \omega(s) ds\right] \leqslant \gamma \int_0^{\gamma} \omega^T(s) M \omega(s) ds. \tag{2.9}$$

Definition 2.4. The filtering error system (2.7) with $\omega(t) = 0$ is said to be stochastically stable, if for any initial $\varphi(t) \in \mathbb{R}^n$ defined on $[-\tau, 0]$ and modes r_t and σ_t, the following relation holds: [7]

$$\lim_{T \to \infty} \mathcal{E}\left\{\int_0^T \left\|\xi(s, r_0, \sigma_0, \varphi)\right\|^2 \mathrm{d}s\right\} < \infty. \tag{2.10}$$

Definition 2.5. Given a scalar $\gamma > 0$, the filtering error system (2.7) is said to be stochastically stable with an H_∞ performance γ, if for every system mode r_t, the filtering error system (2.7) with $\omega(t) = 0$ is stochastically stable and, under zero initial condition, it satisfies $\|e\|_2 \leqslant \gamma \|\omega\|_2$ for any nonzero $\omega(t) \in L_2[0, \infty]$.

3. Main Results

In this section, we first propose a delay-dependent sufficient condition for stochastic stability with the H_∞ performance of filtering error system (2.7). Now, consider the following Lyapunov-Krasovskii functional for systems (2.7):

$$V(\xi_t, t, i, j) = \sum_{n=1}^{6} V_n(\xi_t, t, i, j), \tag{3.1}$$

where

$$\begin{aligned}
V_1(\xi_t, t, i, j) &= \xi^T(t) P(r_t, \sigma_t) \xi(t), \\
V_2(\xi_t, t, i, j) &= \int_{t-\tau(t, r_t)}^{t} \xi^T(s) K^T Q_1(r_t, \sigma_t) K \xi(s) \mathrm{d}s + \int_{t-\tau(r_t)}^{t} \xi^T(s) K^T Q_2(r_t, \sigma_t) K \xi(s) \mathrm{d}s, \\
V_3(\xi_t, t, i, j) &= \int_{t-\tau(t, r_t)}^{t} \left[\int_{\theta}^{t} \xi^T(s) K^T \mathrm{d}s\right] R_1(r_t, \sigma_t) \left[\int_{\theta}^{t} K \xi(s) \mathrm{d}s\right] \mathrm{d}\theta, \\
V_4(\xi_t, t, i, j) &= \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{\xi}^T(s) K^T Z K \dot{\xi}(s) \mathrm{d}s\, \mathrm{d}\theta, \\
V_5(\xi_t, t, i, j) &= \int_{-\tau}^{0} \int_{t+\theta}^{t} \xi^T(s) K^T R_2 K \xi(s) \mathrm{d}s \mathrm{d}\theta + \int_{-\tau(r_t)}^{0} \int_{t+\theta}^{t} \xi^T(s) K^T G_1(r_t) K \xi(s) \mathrm{d}s\, \mathrm{d}\theta, \\
V_6(\xi_t, t, i, j) &= \int_{0}^{\tau} \int_{-\theta}^{0} \int_{t+s}^{t} \xi^T(\alpha) K^T R_3 K \xi(\alpha) \mathrm{d}\alpha \mathrm{d}s \mathrm{d}\theta + \int_{0}^{\tau} \int_{t-\theta}^{t} (s - t + \theta) \xi^T(s) K^T G_2 K \xi(s) \mathrm{d}s\, \mathrm{d}\theta.
\end{aligned} \tag{3.2}$$

Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{\xi_t, r_t, \sigma_t\}$. Then, for each $i \in S_1$, $j \in S_2$, the stochastic differential of $V_1(\xi_t, t, i, j)$ along the trajectory of system (2.7) is given by

$$\begin{aligned}
\mathcal{L}V_1(\xi_t, t, i, j) &= 2\xi^T(t) P_{ij} \\
&\quad \times \left[\overline{A}_{ij} \xi(t) + \overline{A}_{1ij} K \xi(t - \tau_i(t)) + \overline{A}_{2ij} K \int_{t-\tau_i(t)}^{t} \xi(s) \mathrm{d}s + \overline{B}_{ij} f(K\xi(t))\right. \\
&\quad \left. + \overline{E}_{ij} g(K\xi(t)) + \overline{D}_{ij} \omega(t)\right] + \xi^T(t) \left[\sum_{l \in S_1} \pi_{il}^{(j)} P_{lj} + \sum_{k \in S_2} \lambda_{jk} P_{ik}\right] \xi(t), \\
\mathcal{L}V_2(\xi_t, t, i, j) &= \mathcal{L} \int_{t-\tau(t, r_t)}^{t} \xi^T(s) K^T Q_1(r_t, \sigma_t) K \xi(s) \mathrm{d}s + \mathcal{L} \int_{t-\tau(r_t)}^{t} \xi^T(s) K^T Q_2(r_t, \sigma_t) K \xi(s) \mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&= \lim_{\Delta \to 0^+} \frac{1}{\Delta} E\left\{ \int_{t+\Delta-\tau(r_{t+\Delta}, t+\Delta)}^{t+\Delta} \xi^T(s) K^T Q_1(r_{t+\Delta}, \sigma_{t+\Delta}) K\xi(s) \mathrm{d}s \right. \\
&\qquad \left. - \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T Q_{1ij} K\xi(s) \mathrm{d}s \right\} \\
&\quad + \lim_{\Delta \to 0^+} \frac{1}{\Delta} E\left\{ \int_{t+\Delta-\tau(r_{t+\Delta})}^{t+\Delta} \xi^T(s) K^T Q_2(r_{t+\Delta}, \sigma_{t+\Delta}) K\xi(s) \mathrm{d}s \right. \\
&\qquad \left. - \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T Q_{2ij} K\xi(s) \mathrm{d}s \right\} \\
&= \xi^T(t) K^T Q_{1ij} K\xi(t) - (1 - \dot{\tau}_i(t)) \xi^T(t - \tau_i(t)) K^T Q_{1ij} K\xi(t - \tau_i(t)) \\
&\quad + \xi^T(t) K^T Q_{2ij} K\xi(t) \\
&\quad - \xi^T(t - \tau_i) K^T Q_{2ij} K\xi(t - \tau_i) + \sum_{l \in S_1} \pi_{il}^{(j)} \int_{t-\tau_l(t)}^{t} \xi^T(s) K^T Q_{1lj} K\xi(s) \mathrm{d}s \\
&\quad + \sum_{k \in S_2} \lambda_{jk} \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T Q_{1ik} K\xi(s) \mathrm{d}s + \sum_{l \in S_1} \pi_{il}^{(j)} \int_{t-\tau_l}^{t} \xi^T(s) K^T Q_{2lj} K\xi(s) \mathrm{d}s \\
&\quad + \sum_{k \in S_2} \lambda_{jk} \int_{t-\tau_i}^{t} \xi^T(s) K^T Q_{2ik} K\xi(s) \mathrm{d}s \\
&\leqslant \xi^T(t) K^T Q_{1ij} K\xi(t) - (1 - \mu_i) \xi^T(t - \tau_i(t)) K^T Q_{1ij} K\xi(t - \tau_i(t)) \\
&\quad + \xi^T(t) K^T Q_{2ij} K\xi(t) \\
&\quad - \xi^T(t - \tau_i) K^T Q_{2ij} K\xi(t - \tau_i) + \pi_{ii}^{(j)} \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T Q_{1ij} K\xi(s) \mathrm{d}s \\
&\quad + \int_{t-\tau}^{t} \xi^T(s) K^T \left(\sum_{l \in S_1, l \neq i} \pi_{il}^{(j)} Q_{1lj} \right) K\xi(s) \mathrm{d}s \\
&\quad + \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T \left(\sum_{k \in S_2} \lambda_{jk} Q_{1ik} \right) K\xi(s) \mathrm{d}s \\
&\quad + \pi_{ii}^{(j)} \int_{t-\tau_i}^{t} \xi^T(s) K^T Q_{2ij} K\xi(s) \mathrm{d}s + \int_{t-\tau}^{t} \xi^T(s) K^T \left(\sum_{l \in S_1, l \neq i} \pi_{il}^{(j)} Q_{2lj} \right) K\xi(s) \mathrm{d}s \\
&\quad + \int_{t-\tau_i}^{t} \xi^T(s) K^T \left(\sum_{k \in S_2} \lambda_{jk} Q_{2ik} \right) K\xi(s) \mathrm{d}s.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V_3(\xi_t, t, i, j) = &- (1 - \dot{\tau}_i(t)) \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T \mathrm{d}s R_{1ij} \int_{t-\tau_i(t)}^{t} K\xi(s) \mathrm{d}s \\
&+ 2 \int_{t-\tau_i(t)}^{t} \xi^T(t) K^T R_{1ij} \int_{\theta}^{t} K\xi(s) \mathrm{d}s \mathrm{d}\theta
\end{aligned}
$$

$$+\sum_{l\in S_1}\pi_{il}^{(j)}\int_{t-\tau_l(t)}^{t}\left(\int_{\theta}^{t}\xi^T(s)K^T\mathrm{d}s\right)R_{1lj}\left(\int_{\theta}^{t}K\xi(s)\mathrm{d}s\right)\mathrm{d}\theta$$
$$+\sum_{k\in S_2}\lambda_{jk}\int_{t-\tau_i(t)}^{t}\int_{\theta}^{t}\xi^T(s)K^T R_{1ik}K\xi(s)\mathrm{d}s\mathrm{d}\theta. \tag{3.3}$$

Using Lemma 2.3 and considering (2.5), it can be deduced that

$$\begin{aligned}
\mathcal{L}V_3(\xi_t,t,i,j) \leqslant\; & -(1-\mu_i)\left(\int_{t-\tau_i(t)}^{t}\xi^T(s)K^T\mathrm{d}s\right)R_{1ij}\left(\int_{t-\tau_i(t)}^{t}K\xi(s)\mathrm{d}s\right)\\
& +\xi^T(t)K^T\left(\frac{1}{2}\tau_i^2R_{1ij}\right)K\xi(t)+\int_{t-\tau_i(t)}^{t}\int_{\theta}^{t}\xi^T(s)K^TR_{1ij}K\xi(s)\mathrm{d}s\,\mathrm{d}\theta\\
& +\sum_{l\neq i}\pi_{il}^{(j)}\int_{t-\tau_l(t)}^{t}(t-\theta)\int_{\theta}^{t}\xi^T(s)K^TR_{1lj}K\xi(s)\mathrm{d}s\,\mathrm{d}\theta\\
& +\sum_{k\neq j}\lambda_{jk}\int_{t-\tau_i(t)}^{t}(t-\theta)\int_{\theta}^{t}\xi^T(s)K^TR_{1ik}K\xi(s)\mathrm{d}s\mathrm{d}\theta\\
\leqslant\; & -(1-\mu_i)\left(\int_{t-\tau_i(t)}^{t}\xi^T(s)K^T\mathrm{d}s\right)R_{1ij}\left(\int_{t-\tau_i(t)}^{t}K\xi(s)\mathrm{d}s\right)\\
& +\xi^T(t)K^T\left(\frac{1}{2}\tau_i^2R_{1ij}\right)K\xi(t)+\int_{t-\tau_i(t)}^{t}\int_{\theta}^{t}\xi^T(s)K^TR_{1ij}K\xi(s)\mathrm{d}s\,\mathrm{d}\theta\\
& +\sum_{l\neq i}\pi_{il}^{(j)}\int_{t-\tau_l(t)}^{t}\xi^T(s)K^TR_{1lj}K\xi(s)\int_{t-\tau_l(t)}^{t}(t-\theta)\mathrm{d}\theta\,\mathrm{d}s\\
& +\sum_{k\neq j}\lambda_{jk}\int_{t-\tau_i(t)}^{t}\xi^T(s)K^TR_{1ik}K\xi(s)\int_{t-\tau_i(t)}^{t}(t-\theta)\mathrm{d}\theta\,\mathrm{d}s\\
\leqslant\; & -(1-\mu_i)\left(\int_{t-\tau_i(t)}^{t}\xi^T(s)K^T\mathrm{d}s\right)R_{1ij}\left(\int_{t-\tau_i(t)}^{t}K\xi(s)\mathrm{d}s\right)\\
& +\xi^T(t)K^T\left(\frac{1}{2}\tau_i^2R_{1ij}\right)K\xi(t)+\int_{t-\tau}^{t}\int_{\theta}^{t}\xi^T(s)K^TR_{1ij}K\xi(s)\mathrm{d}s\,\mathrm{d}\theta\\
& +\sum_{l\neq i}\frac{1}{2}\tau_l^2\pi_{il}^{(j)}\int_{t-\tau}^{t}\xi^T(s)K^TR_{1lj}K\xi(s)\mathrm{d}s\\
& +\frac{1}{2}\sum_{k\neq j}\tau_i^2\lambda_{jk}\int_{t-\tau_i(t)}^{t}\xi^T(s)K^TR_{1ik}K\xi(s)\mathrm{d}s.
\end{aligned} \tag{3.4}$$

In addition, it is not difficult to get

$$\mathcal{L}V_4(\xi_t, t, i, j) = \tau^2 \dot{\xi}^T(t) K^T Z K \dot{\xi}(t) - \tau \int_{t-\tau}^{t} \dot{\xi}^T(s) K^T Z K \dot{\xi}(s) \mathrm{d}s, \tag{3.5}$$

$$\begin{aligned}\mathcal{L}V_5(\xi_t, t, i, j) \leqslant{}& \tau \xi^T(t) K^T R_2 K \xi(t) - \int_{t-\tau}^{t} \xi^T(s) K^T R_2 K \xi(s) \mathrm{d}s \\ &+ \tau_i \xi^T(t) K^T G_{1i} K \xi(t) - \frac{1}{\tau} \int_{t-\tau_i(t)}^{t} \xi^T(s) K^T \mathrm{d}s G_{1i} \int_{t-\tau_i(t)}^{t} K \xi(s) \mathrm{d}s \\ &+ \int_{-\tau}^{0} \int_{t+\theta}^{t} \xi^T(s) K^T \left(\sum_{l \in S_1, l \neq i} \pi_{il}^{(j)} G_{1i} \right) K \xi(s) \mathrm{d}s \, \mathrm{d}\theta,\end{aligned} \tag{3.6}$$

$$\begin{aligned}\mathcal{L}V_6(\xi_t, t, i, j) ={}& \frac{1}{2} \tau^2 \xi^T(t) K^T R_3 K \xi(t) - \int_{t-\tau}^{t} \int_{\theta}^{t} \xi^T(s) K^T R_3 K \xi(s) \mathrm{d}s \, \mathrm{d}\theta \\ &+ \frac{1}{2} \tau^2 \xi^T(t) K^T G_2 K \xi(t) - \int_{-\tau}^{0} \int_{t+\theta}^{t} \xi^T(s) K^T G_2 K \xi(s) \mathrm{d}s \, \mathrm{d}\theta.\end{aligned} \tag{3.7}$$

Next, following a similar method of [46], to (3.5), denote

$$\delta_1(t) = \int_{t-\tau_i(t)}^{t} K \dot{\xi}(s) \mathrm{d}s, \qquad \delta_2(t) = \int_{t-\tau}^{t-\tau_i(t)} K \dot{\xi}(s) \mathrm{d}s. \tag{3.8}$$

When $0 < \tau_i(t) < \tau$, according to Jensen's inequality, we have that

$$\begin{aligned}\tau \int_{t-\tau}^{t} \dot{\xi}^T(s) K^T Z K \dot{\xi}(s) \mathrm{d}s &= \tau \int_{t-\tau_i(t)}^{t} \dot{\xi}^T(s) K^T Z K \dot{\xi}(s) \mathrm{d}s + \tau \int_{t-\tau}^{t-\tau_i(t)} \dot{\xi}^T(s) K^T Z K \dot{\xi}(s) \mathrm{d}s \\ &\geqslant \frac{\tau}{\tau_i(t)} \delta_1(t)^T Z \delta_1(t) + \frac{\tau}{\tau - \tau_i(t)} \delta_2(t)^T Z \delta_2(t) \\ &= \delta_1(t)^T Z \delta_1(t) + \frac{\tau - \tau_i(t)}{\tau_i(t)} \delta_1(t)^T Z \delta_1(t) \\ &\quad + \delta_2(t)^T Z \delta_2(t) + \frac{\tau_i(t)}{\tau - \tau_i(t)} \delta_2(t)^T Z \delta_2(t).\end{aligned} \tag{3.9}$$

It is clear that [49]

$$\begin{bmatrix} \sqrt{\dfrac{\tau - \tau_i(t)}{\tau_i(t)}} \delta_1(t) \\ -\sqrt{\dfrac{\tau_i(t)}{\tau - \tau_i(t)}} \delta_2(t) \end{bmatrix}^T \begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \begin{bmatrix} \sqrt{\dfrac{\tau - \tau_i(t)}{\tau_i(t)}} \delta_1(t) \\ -\sqrt{\dfrac{\tau_i(t)}{\tau - \tau_i(t)}} \delta_2(t) \end{bmatrix} \geqslant 0, \tag{3.10}$$

which implies

$$\frac{\tau-\tau_i(t)}{\tau_i(t)}\delta_1(t)^T Z\delta_1(t)+\frac{\tau_i(t)}{\tau-\tau_i(t)}\delta_2(t)^T Z\delta_2(t)\geqslant \delta_1(t)^T S\delta_2(t)+\delta_2(t)^T S^T\delta_1(t)^T. \tag{3.11}$$

Then, we can get from (3.9) and (3.11) that

$$\begin{aligned}\tau\int_{t-\tau}^{t}\dot{\xi}^T(s)K^T ZK\dot{\xi}(s)\mathrm{d}s &\geqslant \delta_1(t)^T Z\delta_1(t)+\delta_2(t)^T Z\delta_2(t)+\delta_1(t)^T S\delta_2(t)+\delta_2(t)^T S^T\delta_1(t)\\ &=\begin{bmatrix}\delta_1(t)\\ \delta_2(t)\end{bmatrix}^T\begin{bmatrix}Z & S\\ * & Z\end{bmatrix}\begin{bmatrix}\delta_1(t)\\ \delta_2(t)\end{bmatrix}.\end{aligned} \tag{3.12}$$

Note that when $\tau_i(t)=0$ or $\tau_i(t)=\tau$, we have $\delta_1(t)=0$ or $\delta_2(t)=0$, respectively. So relation (3.12) still holds. It is clear that (3.12) implies

$$-\tau\int_{t-\tau}^{t}\dot{\xi}^T(s)K^T ZK\dot{\xi}(s)\mathrm{d}s\leqslant \chi^T(t)\Omega\chi(t), \tag{3.13}$$

where $\chi(t)=[\xi^T(t)\quad \xi^T(t-\tau_i(t))K^T\quad \xi^T(t-\tau)K^T]^T$

$$\Omega=\begin{bmatrix}-K^T ZK & K^T(Z-S) & K^T S\\ * & -2Z+S+S^T & Z-S\\ * & * & -Z\end{bmatrix}. \tag{3.14}$$

The following equation is true for any matrix N with appropriate dimensions:

$$\begin{aligned}0=\;&2\dot{\xi}^T(t)K^T N\\ &\times\Bigg[-K\dot{\xi}(t)+K\overline{A}_{ij}\xi(t)+K\overline{A}_{1ij}K\xi(t-\tau_i(t))+K\overline{A}_{2ij}K\int_{t-\tau_i(t)}^{t}\xi(s)\mathrm{d}s+K\overline{B}_{ij}f(K\xi(t))\\ &\quad+K\overline{E}_{ij}g(K\xi(t))+K\overline{D}_{ij}w(t)\Bigg].\end{aligned} \tag{3.15}$$

From (2.4), it is clear that [2]

$$\begin{bmatrix}\xi(t)\\ f(K\xi(t))\end{bmatrix}^T\begin{bmatrix}K^T\widehat{M}_1K & K^T\widehat{M}_2\\ * & I\end{bmatrix}\begin{bmatrix}\xi(t)\\ f(K\xi(t))\end{bmatrix}\leqslant 0 \tag{3.16}$$

$$\begin{bmatrix}\xi(t)\\ g(K\xi(t))\end{bmatrix}^T\begin{bmatrix}K^T\widehat{L}_1K & K^T\widehat{L}_2\\ * & I\end{bmatrix}\begin{bmatrix}\xi(t)\\ g(K\xi(t))\end{bmatrix}\leqslant 0, \tag{3.17}$$

where $\widehat{M}_1 = (1/2)(M_1^T M_2 + M_2^T M_1)$, $\widehat{M}_2 = (-1/2)(M_1^T + M_2^T)$, $\widehat{L}_1 = (1/2)(L_1^T L_2 + L_2^T L_1)$, $\widehat{L}_2 = -(1/2)(L_1^T + L_2^T)$. It implies from (3.17) and (3.18) that there exist $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$-\varepsilon_1 \begin{bmatrix} \xi(t) \\ f(K\xi(t)) \end{bmatrix}^T \begin{bmatrix} K^T \widehat{M}_1 K & K^T \widehat{M}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \xi(t) \\ f(K\xi(t)) \end{bmatrix} \geqslant 0, \tag{3.18}$$

$$-\varepsilon_2 \begin{bmatrix} \xi(t) \\ g(K\xi(t)) \end{bmatrix}^T \begin{bmatrix} K^T \widehat{L}_1 K & K^T \widehat{L}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \xi(t) \\ g(K\xi(t)) \end{bmatrix} \geqslant 0. \tag{3.19}$$

We define

$$\begin{aligned} \eta_i(t) = \Big[& \xi^T(t) \quad \xi^T(t-\tau_i(t))K^T \quad \xi^T(t-\tau_i)K^T \quad \dot{\xi}^T(t)K^T \quad \xi^T(t-\tau)K^T \\ & \int_{t-\tau_i(t)}^{t} \xi^T(s)K^T \mathrm{d}s \quad f^T(K\xi(t)) \quad g^T(K\xi(t)) \Big]^T. \end{aligned} \tag{3.20}$$

From the above discussion, we have

$$\begin{aligned} \mathcal{L}V(\xi_t, i, j) \leqslant{} & \begin{bmatrix} \eta_i(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Sigma_{ij} & \Phi_{1ij} \\ * & 0 \end{bmatrix} \begin{bmatrix} \eta_i(t) \\ w(t) \end{bmatrix} \\ & + \int_{t-\tau_i(t)}^{t} \xi^T(s)K^T \left(\sum_{k\in S_2} \lambda_{jk} Q_{1ik} + \frac{\tau_i^2}{2} \sum_{k\neq j} \lambda_{jk} R_{1ik} + \pi_{ii}^{(j)} Q_{1ij} \right) K\xi(s)\mathrm{d}s \\ & + \int_{t-\tau}^{t} \xi^T(s)K^T \left[\sum_{l\in S_1, l\neq i} \pi_{il}^{(j)} \left(Q_{1lj} + Q_{2lj} + \frac{\tau_l^2}{2} R_{1lj} \right) - R_2 \right] K\xi(s)\mathrm{d}s \\ & + \int_{t-\tau_i}^{t} \xi^T(s)K^T \left(\sum_{k\in S_2} \lambda_{jk} Q_{2ik} + \pi_{ii}^{(j)} Q_{2ij} \right) K\xi(s)\mathrm{d}s \\ & + \int_{t-\tau}^{t} \int_{\theta}^{t} \xi^T(s)K^T (R_{1ij} - R_3) K\xi(s)\mathrm{d}s\mathrm{d}\theta \\ & + \int_{-\tau}^{0} \int_{t+\theta}^{t} \xi^T(s)K^T \left(\sum_{l\in S_1, l\neq i} \pi_{il}^{(j)} G_{1i} - G_2 \right) K\xi(s)\mathrm{d}s\mathrm{d}\theta, \end{aligned} \tag{3.21}$$

where

$$\Sigma_{ij}=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & 0 & \left(NK\overline{A}_{ij}\right)^T & K^TS & P_{ij}\overline{A}_{2ij} & \Sigma_{17} & \Sigma_{18}\\ * & \Sigma_{22} & 0 & \left(NK\overline{A}_{1ij}\right)^T & Z-S & 0 & 0 & \\ * & * & -Q_{2ij} & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Sigma_{44} & 0 & NK\overline{A}_{2ij} & NK\overline{B}_{ij} & NK\overline{E}_{ij}\\ * & * & * & * & -Z & 0 & 0 & 0\\ * & * & * & * & * & \Sigma_{66} & 0 & 0\\ * & * & * & * & * & * & -\varepsilon_1 I & 0\\ * & * & * & * & * & * & * & -\varepsilon_2 I\end{bmatrix},$$

$$\begin{aligned}\Sigma_{11}&=P_{ij}\overline{A}_i+\overline{A}_i^TP_{ij}+\sum_{l\in S_1}\pi_{il}^{(j)}P_{lj}+\sum_{k\in S_2}\lambda_{jk}P_{ik}\\&\quad+K^T\left(Q_{1ij}+Q_{2ij}+\tau R_2-Z+\frac{1}{2}\tau_i^2R_{1ij}+\frac{1}{2}\tau^2R_3+\tau_iG_{1i}+\frac{1}{2}\tau^2G_2\right)\\&\quad\times K-\varepsilon_1K^T\widehat{M}_1K-\varepsilon_2K^T\widehat{L}_1K,\\ \Sigma_{12}&=P_{ij}\overline{A}_{1ij}+K^T(Z-S),\qquad \Sigma_{17}=P_{ij}\overline{B}_{ij}-\varepsilon_1K^T\widehat{M}_2,\qquad \Sigma_{18}=P_{ij}\overline{E}_{ij}-\varepsilon_2K^T\widehat{L}_2,\\ \Sigma_{22}&=-(1-\mu_i)Q_{1ij}-2Z+S+S^T,\qquad \Sigma_{44}=\tau^2Z-N-N^T,\\ \Sigma_{66}&=-(1-\mu_i)R_{1ij}-\frac{1}{\tau}G_{1i},\qquad \Phi_{1ij}=\left[\overline{D}_{ij}^TP_{ij}\ \ 0\ \ 0\ \ \overline{D}_{ij}^TK^TN^T\ \ 0\ \ 0\ \ 0\ \ 0\right]^T.\end{aligned}\tag{3.22}$$

Therefore, we have the following result for the H_∞ performance analysis.

Theorem 3.1. *Given scalars τ, τ_i, and μ_i, the filtering error system* (2.7) *is stochastically stable with an H_∞ performance γ for any time delay $\tau_i(t)$ satisfying* (2.5)*, if there exist matrices $P_{ij}>0$, $Q_{1ij}>0$, $Q_{2ij}>0$, $R_{1ij}>0$, $R_2>0$, $R_3>0$, $Z>0$, $G_{1i}>0$, $G_2>0$, and matrices S, N such that for each $i\in S_1$, $j\in S_2$*

$$\begin{bmatrix}\Sigma_{ij} & \Phi_{1ij} & \Phi_{2ij}\\ * & -\gamma^2I & 0\\ * & * & -I\end{bmatrix}<0,\tag{3.23}$$

$$\sum_{k\in S_2}\lambda_{jk}Q_{1ik}+\frac{\tau_i^2}{2}\sum_{k\neq j}\lambda_{jk}R_{1ik}+\pi_{ii}^{(j)}Q_{1ij}<0,\tag{3.24}$$

$$\sum_{l\in S_1,l\neq i}\pi_{il}^{(j)}\left(Q_{1lj}+Q_{2lj}+\frac{\tau_l^2}{2}R_{1lj}\right)-R_2<0,\tag{3.25}$$

$$\sum_{k \in S_2} \lambda_{jk} Q_{2ik} + \pi_{ii}^{(j)} Q_{2ij} < 0, \tag{3.26}$$

$$R_{1ij} < R_3, \tag{3.27}$$

$$\sum_{l \in S_1, l \neq i} \pi_{il}^{(j)} G_{1i} < G_2, \tag{3.28}$$

where

$$\Phi_{2ij} = \begin{bmatrix} \overline{H}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \tag{3.29}$$

Proof. Using the Schur complement formula to (3.23), it can be seen that inequality (3.23) is equivalent to

$$\begin{bmatrix} \Sigma_{ij} + \Phi_{2ij}\Phi_{2ij}^T & \Phi_{1ij} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{3.30}$$

which implies $\Sigma_{ij} < 0$. Now, we show that the filtering error system (2.7) with $\omega(t) = 0$ is stochastically stable. If $\omega(t) = 0$, from (3.1), (3.21), (3.24)–(3.28), and $\Sigma_{ij} < 0$, there exists a scalar $\lambda > 0$ such that

$$\mathcal{L}V(\xi_t, t, i, j) \leqslant -\lambda \|x(t)\|^2. \tag{3.31}$$

Therefore, for any $T > 0$, by Dynkin's formula, we have

$$\mathcal{E}V(\xi_t, t, i, j) - \mathcal{E}V(\xi_0, 0, 0, 0) \leqslant -\lambda \mathcal{E} \int_0^T \|x(s)\|^2 \mathrm{d}s, \tag{3.32}$$

which yields

$$\mathcal{E} \int_0^T \|x(s)\|^2 \mathrm{d}s \leqslant \frac{1}{\lambda} \mathcal{E}V(\xi_0, 0, 0, 0) < \infty. \tag{3.33}$$

Thus, the filtering error system (2.7) with $\omega(t) = 0$ is stochastically stable by Definition 2.4.

In the sequel, we will deal with the H_∞ performance of the filtering error system (2.7). Using (3.30) and H_∞ performance, we have

$$\begin{aligned} &\mathcal{E}\left\{ \mathcal{L}V(\xi_t, t, i, j) + e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) \right\} \\ &\quad \leqslant \begin{bmatrix} \eta_i(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \Sigma_{ij} + \Phi_{2ij}\Phi_{2ij}^T & \Phi_{1ij} \\ * & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \eta_i(t) \\ \omega(t) \end{bmatrix} < 0. \end{aligned} \tag{3.34}$$

Noting that the zero initial condition, then it follows from (3.34) that

$$
\begin{aligned}
J &= \mathcal{E}\left\{\int_0^\infty \left[e^T(t)e(t) - \gamma^2\omega^T(t)\omega(t)\right]\mathrm{d}t\right\} \\
&\leqslant \mathcal{E}\left\{\int_0^\infty \left[e^T(t)e(t) - \gamma^2\omega^T(t)\omega(t) + \mathcal{L}V(\xi_t, t, i, j)\right]\mathrm{d}t\right\} < 0.
\end{aligned}
\tag{3.35}
$$

Hence, if (3.23)–(3.28) hold, $J < 0$ can be guaranteed. That is, $\|e\|_2 \leqslant \gamma\|\omega\|_2$ for all nonzero $\omega(t)$. Therefore, the filtering error system (2.7) is stochastically stable with the H_∞ performance γ by Definition 2.5. This completes the proof. □

Remark 3.2. A new stochastic stability criterion is obtained in Theorem 3.1 by constructing a novel mode-dependent Lyapunov functional. The Lyapunov functional in this paper uses all information about r_t, σ_t and $\tau(t, r_t)$. The Lyapunov matrices $P(r_t, \sigma_t)$, $Q_1(r_t, \sigma_t)$, $Q_2(r_t, \sigma_t)$, and $R_1(r_t, \sigma_t)$ depend on both the system mode r_t and the higher-level Markovian chain σ_t. Compared with the mode-independent Lyapunov matrices [40, 42], the mode-dependent Lyapunov matrices can reduce the conservativeness since they provide additional degrees of freedom which are very important for deriving LMIs solutions in general. Hence, the Lyapunov functional in this paper is more general and the condition on stability is more applicable.

Remark 3.3. It should be pointed out that the aim of the introduction of $V_3(\xi_t, t, i, j)$ is to propose a stability condition which depends not only on the delay upper bound τ, but also on the subsystems' delay upper bounds τ_i, in other words, if $V_3(\xi_t, t, i, j)$ is not considered, the obtained stability condition only depends on the delay upper bound τ. Hence, the introduction of $V_3(\xi_t, t, i, j)$ may reduce some conservativeness.

Based on Theorem 3.1, the H_∞ filter synthesis problem can be developed in terms of LMIs for the system (2.1) with higher-level Markovian chain.

Theorem 3.4. *Consider the systems* (2.1)*. Given scalars τ, τ_i and μ_i, the filtering error system* (2.7) *is stochastically stable with an H_∞ performance γ for any time delay $\tau_i(t)$ satisfying* (2.5)*, if there exist matrices $P_{1ij} > 0$, $Q_{1ij} > 0$, $Q_{2ij} > 0$, $R_{1ij} > 0$, $R_2 > 0$, $R_3 > 0$, $Z > 0$, $G_{1i} > 0$, $G_2 > 0$, $S_{ij} > 0$, $U_{lij} > 0$, $V_{kij} > 0$, S_{lij}, T_{kij}, $\overline{A}_{F_{ij}}$, $\overline{B}_{F_{ij}}$, $\overline{C}_{F_{ij}}$, and matrices S, N such that for each $i \in S_1$, $j \in S_2$*

$$
\begin{bmatrix}
\overline{\Sigma}_{11} & \overline{\Sigma}_{12} & \overline{\Sigma}_{13} & 0 & (NA_i)^T & S & \overline{\Sigma}_{14} & \overline{\Sigma}_{15} & \overline{\Sigma}_{16} & \overline{\Sigma}_{17} & H_i^T \\
* & \overline{\Sigma}_{22} & \overline{\Sigma}_{23} & 0 & 0 & 0 & \overline{\Sigma}_{24} & S_{ij}B_i & \overline{B}_{F_{ij}}E_i & \overline{\Sigma}_{25} & -\overline{C}_{F_{ij}}^T \\
* & * & \overline{\Sigma}_{33} & 0 & \left(N\overline{A}_{1i}\right)^T & Z-S & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_{2ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \overline{\Sigma}_{55} & 0 & NA_{2i} & NB_i & 0 & ND_{1i} & 0 \\
* & * & * & * & * & -Z & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \overline{\Sigma}_{77} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\
* & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0,
\tag{3.36}
$$

$$\sum_{k\in S_2}\lambda_{jk}Q_{1ik}+\frac{\tau_i^2}{2}\sum_{k\neq j}\lambda_{jk}R_{1ik}+\pi_{ii}^{(j)}Q_{1ij}<0, \tag{3.37}$$

$$\sum_{l\in S_1,l\neq i}\pi_{il}^{(j)}\left(Q_{1lj}+Q_{2lj}+\frac{\tau_l^2}{2}R_{1lj}\right)-R_2<0, \tag{3.38}$$

$$\sum_{k\in S_2}\lambda_{jk}Q_{2ik}+\pi_{ii}^{(j)}Q_{2ij}<0, \tag{3.39}$$

$$R_{1ij}<R_3, \tag{3.40}$$

$$\sum_{l\in S_1,l\neq i}\pi_{il}^{(j)}G_{1i}<G_2, \tag{3.41}$$

where

$$\begin{aligned}
\overline{\Sigma}_{11} =\ & P_{1ij}A_i+A_i^TP_{1ij}+\overline{B}_{F_{ij}}C_i+C_i^T\overline{B}_{F_{ij}}^T+\sum_{l\in S_1}\pi_{il}^{(j)}P_{1lj}+\sum_{k\in S_2}\lambda_{jk}P_{1ik}+Q_{1ij}+Q_{2ij}\\
&+\frac{1}{2}\tau_i^2R_{1ij}+\tau R_2+\frac{1}{2}\tau^2R_3-Z+\tau_iG_{1i}+\frac{1}{2}\tau^2G_2-\varepsilon_1\widehat{M}_1-\varepsilon_2\widehat{L}_1,\\
\overline{\Sigma}_{12} =\ & \overline{A}_{F_{ij}}+A_i^TS_{ij}^T+C_i^T\overline{B}_{F_{ij}}^T+\left(\pi_{ii}^{(j)}+\lambda_{jj}\right)S_{ij}+\sum_{l\neq i}\pi_{il}^{(j)}S_{lij}+\sum_{k\neq j}\lambda_{jk}T_{kij},\\
\overline{\Sigma}_{13} =\ & P_{1ij}A_{1i}+\overline{B}_{F_{ij}}C_{1i}+Z-S, \qquad \overline{\Sigma}_{14}=P_{1ij}A_{2i}+\overline{B}_{F_{ij}}C_{2i}, \qquad \overline{\Sigma}_{15}=P_{1ij}B_i-\varepsilon_1\widehat{M}_2,\\
\overline{\Sigma}_{16} =\ & \overline{B}_{F_{ij}}E_i-\varepsilon_2\widehat{L}_2, \qquad \overline{\Sigma}_{17}=P_{1ij}D_{1i}+\overline{B}_{F_{ij}}D_{2i},\\
\overline{\Sigma}_{22} =\ & \overline{A}_{F_{ij}}+\overline{A}_{F_{ij}}^T+\left(\pi_{ii}^{(j)}+\lambda_{jj}\right)S_{ij}+\sum_{l\in S_1,l\neq i}\pi_{il}^{(j)}U_{lij}+\sum_{k\in S_2,k\neq j}\lambda_{jk}V_{kij},\\
\overline{\Sigma}_{23} =\ & S_{ij}A_{1i}+\overline{B}_{F_{ij}}C_{1i}, \qquad \overline{\Sigma}_{24}=S_{ij}A_{2i}+\overline{B}_{F_{ij}}C_{2i}, \qquad \overline{\Sigma}_{25}=S_{ij}D_{1i}+\overline{B}_{F_{ij}}D_{2i},\\
\overline{\Sigma}_{33} =\ & -(1-\mu_i)Q_{1ij}-2Z+S+S^T, \qquad \overline{\Sigma}_{55}=\tau^2Z-N-N^T, \qquad \overline{\Sigma}_{77}=-(1-\mu_i)R_{1ij}-\frac{1}{\tau}G_{1i}.
\end{aligned} \tag{3.42}$$

In this case, the parameters of the desired filter can be chosen by

$$A_{F_{ij}}=S_{ij}^{-1}\overline{A}_{F_{ij}}, \qquad B_{F_{ij}}=S_{ij}^{-1}\overline{B}_{F_{ij}}, \qquad C_{F_{ij}}=\overline{C}_{F_{ij}}. \tag{3.43}$$

Proof. For each $r_t=i\in S_1$, $\sigma_t=j\in S_2$, we define a matrix $P_{ij}>0$ by $P_{ij}=\begin{bmatrix}P_{1ij} & P_{2ij}\\ * & P_{3ij}\end{bmatrix}$. By invoking a small perturbation, if necessary, we can assume that P_{2ij} and P_{3ij} are nonsingular. Thus, we can introduce the following invertible matrix $J=\begin{bmatrix}I & 0\\ 0 & P_{3ij}^{-1}P_{2ij}^T\end{bmatrix}$. Pre- and postmultiplying (3.23) by diag $[J^T, I, I, I, I, I, I, I, I, I]$ and its transpose, respectively. Then, we define

$$\begin{gathered}
S_{ij} = P_{2ij} P_{3ij}^{-1} P_{2ij}^T, \qquad \overline{A}_{F_{ij}} = P_{2ij} A_{F_{ij}} P_{3ij}^{-1} P_{2ij}^T, \qquad \overline{B}_{F_{ij}} = P_{2ij} B_{F_{ij}}, \qquad \overline{C}_{F_{ij}} = C_{F_{ij}} P_{3ij}^{-1} P_{2ij}^T, \\
S_{lij} = P_{2lj} P_{3ij}^{-1} P_{2ij}^T \quad (l \neq i), \qquad T_{kij} = P_{2ik} P_{3ij}^{-1} P_{2ij}^T \quad (k \neq j), \qquad U_{lij} = P_{2ij} P_{3ij}^{-T} P_{3lj} P_{3ij}^{-1} P_{2ij}^T \quad (l \neq i), \\
V_{kij} = P_{2ij} P_{3ij}^{-T} P_{3ik} P_{3ij}^{-1} P_{2ij}^T \quad (k \neq j).
\end{gathered} \tag{3.44}$$

It is easy to obtain (3.36).

On the other hand, according to (3.44), we have

$$A_{F_{ij}} = P_{2ij}^{-1} \overline{A}_{F_{ij}} P_{2ij}^{-T} P_{3ij}, \qquad B_{F_{ij}} = P_{2ij}^{-1} \overline{B}_{F_{ij}}, \qquad C_{F_{ij}} = \overline{C}_{F_{ij}} P_{2ij}^{-T} P_{3ij}. \tag{3.45}$$

From (2.6), the transfer function from measured output $y(t)$ to estimated signal $\hat{z}(t)$ can be described by

$$\begin{aligned}
T_{\hat{z}y} &= C_{F_{ij}} \left(sI - A_{F_{ij}} \right)^{-1} B_{F_{ij}} \\
&= \overline{C}_{F_{ij}} P_{2ij}^{-T} P_{3ij} \left(sI - P_{2ij}^{-1} \overline{A}_{F_{ij}} P_{2ij}^{-T} P_{3ij} \right)^{-1} P_{2ij}^{-1} \overline{B}_{F_{ij}} \\
&= \overline{C}_{F_{ij}} \left(sI - S_{ij}^{-1} \overline{A}_{F_{ij}} \right)^{-1} S_{ij}^{-1} \overline{B}_{F_{ij}}.
\end{aligned} \tag{3.46}$$

Therefore, we can conclude from (3.46) that the parameters of the filter in (2.6) can be constructed by (3.43). This completes the proof. □

Remark 3.5. It should be pointed out that some existing work in control and filter design of Markovian jump systems, the Lyapunov matrix P_i is assumed to be diagonal matrix, for example, see [42]. It is well known that such assumption can lead to much more conservative result. Although the first diagonal element Σ_{11} in (3.23) includes P_{ij}, P_{lj}, and P_{ik} in this paper, P_{ij} is not assumed to be diagonal matrices.

Remark 3.6. In [30–32, 38], the authors have achieved some excellent work of Markovian jump singular systems. Due to the presence of the singular matrix E, the issues of stability and control of such systems are more difficult and complicated. However, there is no results on piecewise homogeneous Markovian jump singular systems in the existing work, and the problem of control for such system is an interesting issue.

Remark 3.7. Theorem 3.4 solves the filtering problem of a class of piecewise homogeneous Markovian jump nonlinear systems. The obtained conditions are formulated in terms of LMIs, which could be easily checked by using the LMI toolbox in Matlab. The feasible solutions to the conditions presented in Theorem 3.4 will depend on both the mode r_t and the higher-level Markovian chain σ_t, which ensure that the error system is stochastically stable. A numerical example verifies the validity of the designed filter in Section 4.

4. A Numerical Example

In this section, a numerical example will be presented to show the validity of the main results derived above.

Example 4.1. Let us consider the stochastic system (2.1) with the following system of matrices:

$$\begin{gathered}
A(1) = \begin{bmatrix} -8 & 0 \\ 1 & -12 \end{bmatrix}, \qquad A_1(1) = \begin{bmatrix} 1 & 0.3 \\ 1 & 1 \end{bmatrix}, \qquad A_2(1) = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.4 \end{bmatrix}, \\
B(1) = \begin{bmatrix} 0.7 & 0.8 \\ 0.1 & 0.6 \end{bmatrix}, \qquad D_1(1) = \begin{bmatrix} 0.7 \\ 0.1 \end{bmatrix}, \qquad M_1 = L_1 = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.02 \end{bmatrix}, \\
M_2 = L_2 = \begin{bmatrix} -0.01 & 0.01 \\ -0.03 & -0.02 \end{bmatrix}, \qquad C(1) = \begin{bmatrix} 1.9 & -2.1 \end{bmatrix}, \qquad C_1(1) = \begin{bmatrix} 1.5 & 0 \end{bmatrix}, \\
C_2(1) = \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad E(1) = \begin{bmatrix} 0.8 & 0.6 \end{bmatrix}, \qquad D_2(1) = 0.8, \qquad H(1) = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}, \\
A(2) = \begin{bmatrix} -9 & 0 \\ 0 & -9 \end{bmatrix}, \qquad A_1(2) = \begin{bmatrix} 0.8 & 0.6 \\ 1 & 0.8 \end{bmatrix}, \qquad A_2(2) = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.6 \end{bmatrix}, \\
B(2) = \begin{bmatrix} 0.9 & 1 \\ 2.3 & 0.6 \end{bmatrix}, \qquad D_1(2) = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \qquad M_1 = L_1 = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.02 \end{bmatrix}, \\
M_2 = L_2 = \begin{bmatrix} -0.01 & 0.01 \\ -0.03 & -0.02 \end{bmatrix}, \qquad C(2) = \begin{bmatrix} 1.8 & -2.2 \end{bmatrix}, \qquad C_1(2) = \begin{bmatrix} 1.2 & 0.1 \end{bmatrix}, \\
C_2(2) = \begin{bmatrix} 1 & -0.8 \end{bmatrix}, \qquad E(2) = \begin{bmatrix} 0.8 & 0.6 \end{bmatrix}, \qquad D_2(2) = 1, \qquad H(2) = \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}.
\end{gathered} \tag{4.1}$$

The piecewise homogeneous TP matrices are given as

$$\Pi^1 = \begin{bmatrix} -0.5 & 0.5 \\ 0.6 & -0.6 \end{bmatrix}, \qquad \Pi^2 = \begin{bmatrix} -0.7 & 0.7 \\ 0.4 & -0.4 \end{bmatrix}, \qquad \Pi^3 = \begin{bmatrix} -0.8 & 0.8 \\ 0.6 & -0.6 \end{bmatrix}. \tag{4.2}$$

The HTP matrix for the Markovian chain is considered as follows:

$$\Lambda = \begin{bmatrix} -0.8 & 0.4 & 0.4 \\ 0.4 & -0.9 & 0.5 \\ 0.7 & 0.8 & -1.5 \end{bmatrix}. \tag{4.3}$$

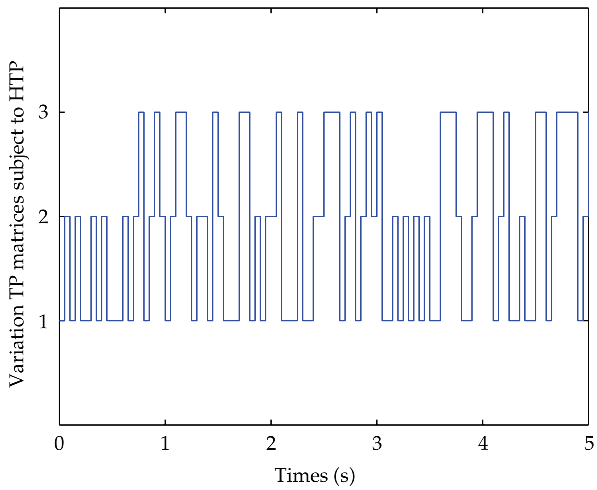

Figure 1: Variation of TP matrices subject to HTP.

In this example, we assume $\tau_1 = \tau_2 = 1$, $\mu_1 = 0.8$, $\mu_2 = 0.5$. For $\gamma = 1.2$. By solving LMIs (3.36)–(3.41), the filter matrices are obtained as

$$\begin{aligned}
A_{F_{11}} &= \begin{bmatrix} -7.6750 & -2.3822 \\ -3.7133 & -4.8336 \end{bmatrix}, & B_{F_{11}} &= \begin{bmatrix} 0.0496 \\ 0.3182 \end{bmatrix}, & C_{F_{11}} &= \begin{bmatrix} 0.0037 & 1.3061 \end{bmatrix}, \\
A_{F_{12}} &= \begin{bmatrix} -10.9509 & -3.4284 \\ -2.9982 & -4.2265 \end{bmatrix}, & B_{F_{12}} &= \begin{bmatrix} 0.7322 \\ 0.1760 \end{bmatrix}, & C_{F_{12}} &= \begin{bmatrix} -0.0053 & -0.0017 \end{bmatrix}, \\
A_{F_{13}} &= \begin{bmatrix} -11.2973 & -4.8499 \\ -5.0689 & -5.7131 \end{bmatrix}, & B_{F_{13}} &= \begin{bmatrix} 1.0209 \\ 0.7196 \end{bmatrix}, & C_{F_{13}} &= \begin{bmatrix} -0.0041 & -0.0029 \end{bmatrix}, \\
A_{F_{21}} &= \begin{bmatrix} -5.5850 & -6.2702 \\ -4.3917 & -15.3643 \end{bmatrix}, & B_{F_{21}} &= \begin{bmatrix} 0.4144 \\ 0.9799 \end{bmatrix}, & C_{F_{21}} &= \begin{bmatrix} 0.0002 & 1.2670 \end{bmatrix}, \\
A_{F_{22}} &= \begin{bmatrix} -6.2896 & -4.2309 \\ -4.7375 & -10.6039 \end{bmatrix}, & B_{F_{22}} &= \begin{bmatrix} -0.0779 \\ -0.3519 \end{bmatrix}, & C_{F_{22}} &= \begin{bmatrix} -0.0142 & -0.0107 \end{bmatrix} \\
A_{F_{23}} &= \begin{bmatrix} -7.9621 & -6.4413 \\ -6.1046 & -16.3128 \end{bmatrix}, & B_{F_{23}} &= \begin{bmatrix} -0.0841 \\ -0.3711 \end{bmatrix}, & C_{F_{23}} &= \begin{bmatrix} -0.0150 & -0.0117 \end{bmatrix}.
\end{aligned} \tag{4.4}$$

For simulation purposes, we assume the initial condition $x(0) = [0.1\ 0.1]^T$ and $\omega(t) = 1/(0.5+1.2t)$. The time delays are $\tau_1(t) = 0.2+0.8\sin(t)$, $\tau_2(t) = 0.5+0.5\cos(t)$. The nonlinear functions $f(x(t))$ and $g(x(t))$ are selected as

$$f(x(t)) = g(x(t)) = \begin{bmatrix} 0.02x_1(t)\sin^2(x_1(t)) - 0.01(x_1(t) - x_2(t)) \\ -0.01x_1(t) \end{bmatrix}, \tag{4.5}$$

Figures 1–5 illustrate the simulation results. A case for stochastic variation with HTP matrix is shown in Figure 1, and possible realizations of the Markov jumping mode of system

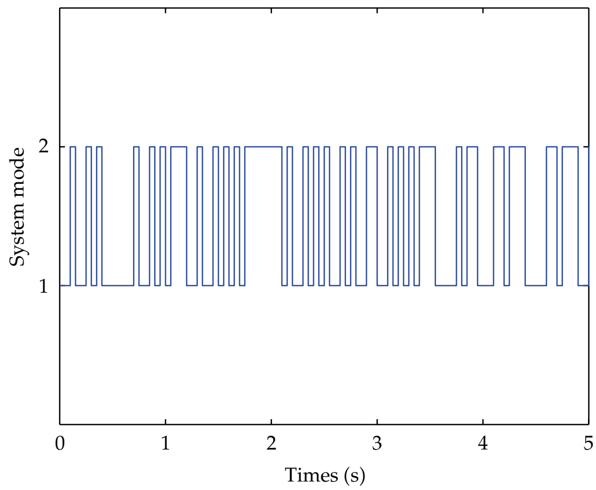

Figure 2: System jumping mode.

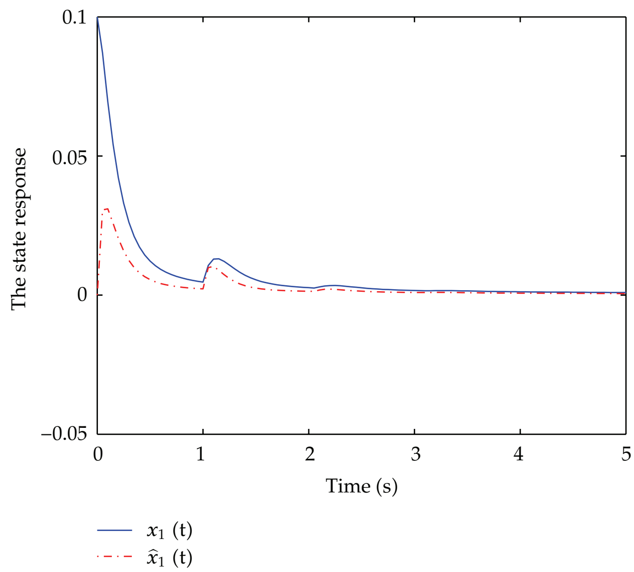

Figure 3: The state responses of $x_1(t)$ and $\hat{x}_1(t)$.

and delay are plotted in Figure 2, where the initial modes are assumed to be $r_0 = 1$ and $\sigma_0 = 1$. Figure 3 shows the state responses of real states $x_1(t)$ and its estimate $\hat{x}_1(t)$. Figure 4 shows the state responses of real states $x_2(t)$ and its estimate $\hat{x}_2(t)$. Figure 5 is the simulation result of the estimation error response of $e(t)$. The simulation results demonstrate that the designed H_∞ filters are feasible and effective.

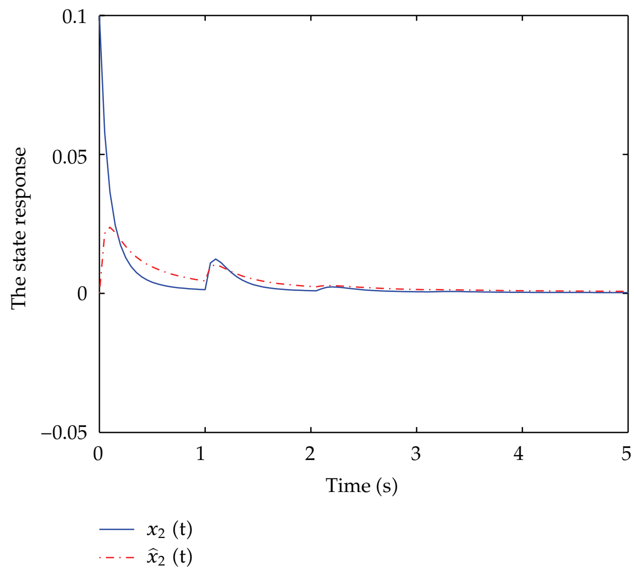

Figure 4: The state responses of $x_2(t)$ and $\widehat{x}_2(t)$.

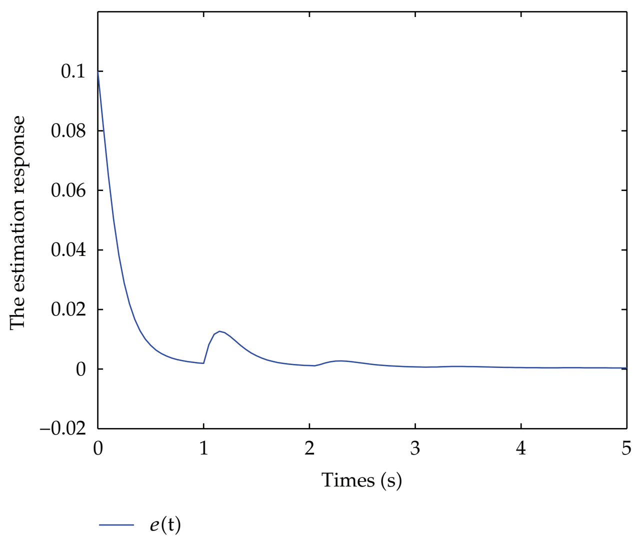

Figure 5: The estimation error response of $e(t)$.

5. Conclusion

The problem of H_∞ filtering for a class of Markovian jump nonlinear systems is investigated in this paper. The piecewise homogeneous Markovian chain and mode-dependent time-varying delays are considered in the model. By using the Lyapunov-Krasovskii functional, mixed mode-dependent sufficient conditions are developed to design stable filters. A numerical example demonstrates the effectiveness of the given method.

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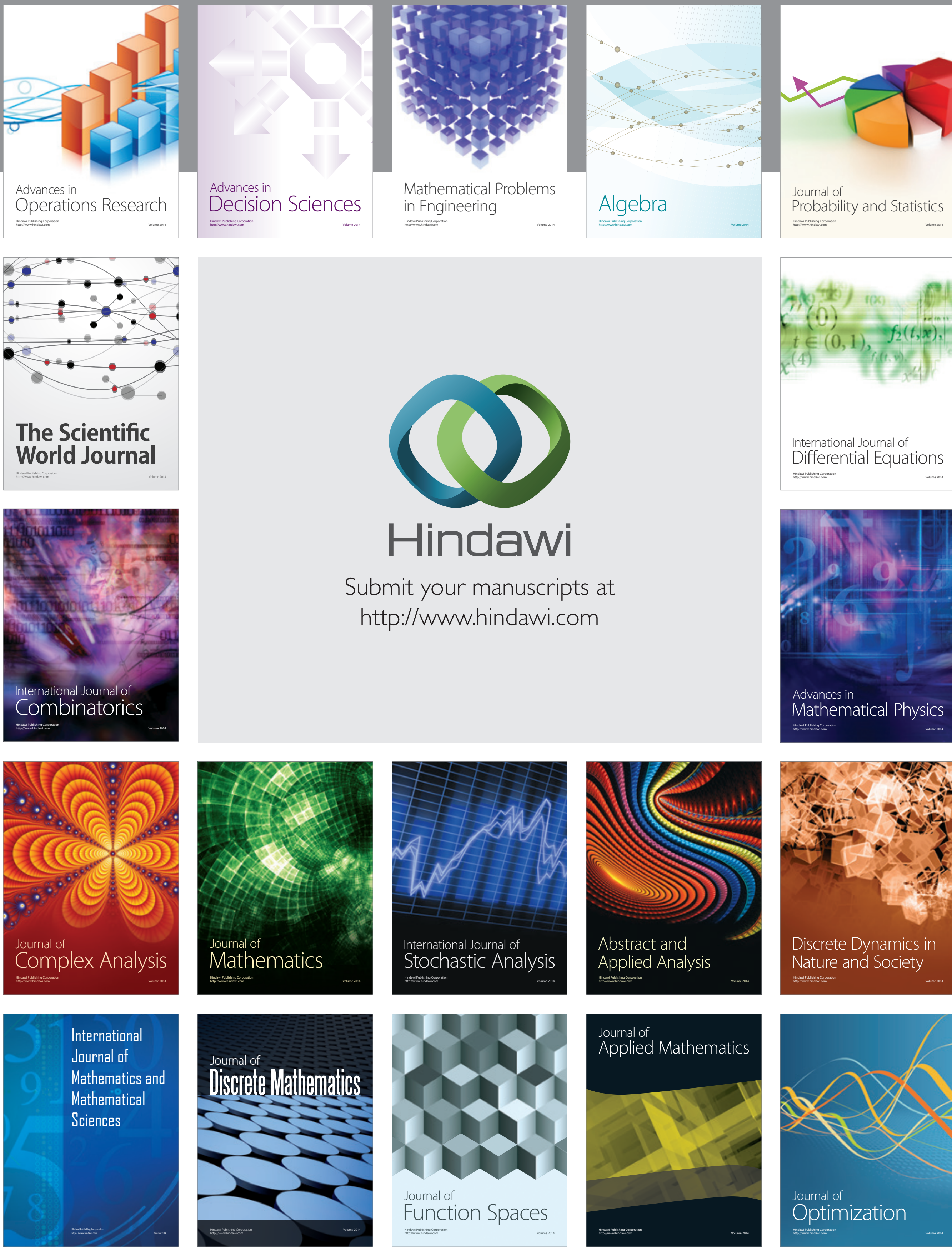
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