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Research Article

High-Order Stochastic Adaptive Controller Design with Application to Mechanical System

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The main purpose of this paper is to apply stochastic adaptive controller design to mechanical system. Firstly, by a series of coordinate transformations, the mechanical system can be transformed to a class of special high-order stochastic nonlinear system, based on which, a more general mathematical model is considered, and the smooth state-feedback controller is designed. At last, the simulation for the mechanical system is given to show the effectiveness of the design scheme.

1. Introduction

In recent years, the study for deterministic high-order nonlinear systems has achieved remarkable development, see, for example, [1–3] and references herein. Inspired by these interesting and important results, it is natural to generalize their results to the following stochastic high-order nonlinear systems which are neither necessarily feedback linearizable nor affine in the control input:

$$\begin{aligned} dz &= f_0(z, x_1)dt + g_0^T(z, x_1)d\omega, \\ dx_i &= \Big(d_i(\overline{x}_i, t)x_{i+1}^{p_i} + f_i(z, \overline{x}_i)\Big)dt + g_i^T(z, \overline{x}_i)d\omega, \quad i = 1, \dots, n-1, \\ dx_n &= \big(d_n(\overline{x}_n, t)u^{p_n} + f_n(z, \overline{x}_n)\big)dt + g_n^T(z, \overline{x}_n)d\omega, \end{aligned} \tag{1.1}$$

where $(z^T, x_1, \dots, x_n)^T \in \mathbb{R}^{m+n}$, and $u \in \mathbb{R}$ are the measurable state and the input of system, respectively, $\overline{x}_i = (x_1, \dots, x_i)^T$, $i = 1, \dots, n$, $z = (z_1, \dots, z_m)^T \in \mathbb{R}^m$ is referred to as the state of

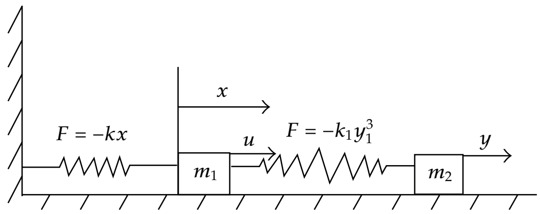

Figure 1: A mechanical system.

the stochastic inverse dynamics, ω is an r-dimensional standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$ with Ω being a sample space, $\mathcal{F}$ being a σ-algebra, and P being a probability measure, $p_i \geq 1$, $i = 1, \ldots, n$ are odd integers, and the functions $f_i(\cdot)$ and $g_i(\cdot)$, $i = 0, 1, \ldots, n$ are assumed to be smooth, vanishing at the origin $(z^T, \overline{x}_n^T) = (0_{1\times m}, 0_{1\times n})$.

For (1.1) with $d_i(\cdot) = 1$, Xie and Tian in [4] considered the state-feedback stabilization problem for the first time. After considering the stabilization of high-order stochastic nonlinear systems, [5] further addressed the problem of state-feedback inverse optimal stabilization in probability, that is, the designed stabilizing backstepping controller is also optimal with respect to meaningful cost functionals. When $d_i(\cdot) \neq 1$, [6] designed an adaptive state-feedback controller for a class of stochastic nonlinear uncertain systems with $0 < \lambda_i \leq d_i(\cdot) \leq \mu_i \leq \mu$, and [7] designed a smooth adaptive state-feedback controller for high-order stochastic systems with $\lambda_i(\overline{x}_i) \leq d_i(\cdot) \leq \overline{\mu}_i(\overline{x}_i, \theta)$ by using the parameter separation lemma and some flexible algebraic techniques. Recently, more excellent results [8–28] were achieved by Xie and his group.

However, all these theoretical results mentioned above are demonstrated only by some numerical simulation examples. Since many practical application systems in aerospace industry, industrial process control, and so forth, can be described by (or transformed to) stochastic high-order nonlinear systems, so it is very necessary to apply the control schemes to these systems. Based on this reason, we consider a practical example of mechanical movement in this paper. By a series of coordinate transformations, the mechanical system can be transformed to a high-order stochastic nonlinear system, based on which, we consider a more general mathematical model and design a smooth state-feedback control law. At last, the simulation for the mechanical system is given to show the effectiveness of the design scheme.

This paper is organized as follows. Section 2 gives a practical example. Section 3 provides preliminary knowledge and presents problem statement. Controller design and stability analysis are given in Section 4. The simulation for the practical example is provided to demonstrate the control scheme in Section 5. Section 6 gives some concluding remarks.

2. A Practical Example

Let us consider the following mechanical system which consists of two masses m_1 and m_2 on a horizontal smooth surface as shown in Figure 1. The mass m_1 is interconnected to the wall by a linear spring and to the mass m_2 by a nonlinear spring which has cubic force-deformation relation. Let x be the displacement of mass m_1 and y the displacement of mass m_2 such that at $x = 0$ and $y = 0$, that is, the springs are unstretched. A control force u acts on m_1.

Where the units of m_1, x, and u are "kg", "m", and "N", respectively, and $y_1 = x - y$. The equations of motion for the system are described by

$$\ddot{y} = \frac{k_1}{m_2}(x-y)^3,$$
$$\ddot{x} = -\frac{k}{m_1}x - \frac{k_1}{m_1}(x-y)^3 + \frac{u}{m_1}, \tag{2.1}$$

where k and k_1 are the spring coefficients, and their units are "N/m" and "N/m^3", respectively.

Introducing the smooth change of coordinates

$$x_1 = y, \qquad x_2 = \dot{x}_1 = \dot{y},$$
$$x_3 = (x-y)\sqrt[3]{\frac{k_1}{m_1}}, \qquad x_4 = \dot{x}_3, \tag{2.2}$$

one gets

$$y = x_1, \qquad \dot{y} = x_2,$$
$$x = \frac{x_3}{\sqrt[3]{k_1/m_1}} + y, \qquad x_4 = \frac{x_4}{\sqrt[3]{k_1/m_1}} + x_2. \tag{2.3}$$

The linear spring constant k has a specific nominal value $k_0 = 1.5$ which is considered uncertain, and $k \in [0.75, 2.25]$. Let $\Delta(t) = k(t) - k_0$. For all $t \geq 0$, $\Delta(t)$ is the Gaussian white noise process with $E\Delta(t) = 0$ and $E\Delta^2(t) = \sigma^2$. We can choose the value of parameter σ such that $k(t)$ obeys the bound $0.75 \leq k \leq 2.25$ with a sufficiently high probability. This model of spring rate variations leads to an uncertain stochastic system. By (2.2), one chooses the smooth state-feedback control

$$u = m_1\frac{v}{\sqrt[3]{k_1/m_1}} + \frac{m_1+m_2}{m_2}m_1 x_3^3, \tag{2.4}$$

which together with the property of $\Delta(t)$ leads to

$$dx_1 = x_2 dt,$$
$$dx_2 = \frac{m_1}{m_2}x_3^3 dt,$$
$$dx_3 = x_4 dt, \tag{2.5}$$
$$dx_4 = v\,dt + k_0 f(x)dt + \sigma f(x)d\omega,$$
$$y = x_1,$$

where $f(x) = -x_3/m_1 - \sqrt[3]{(k_1/m_1)}(x_1/m_1)$, and ω is standard Wiener process.

This stochastic high-order nonlinear systems can be generalized to a more general system which will be given in the following section.

3. Preliminary Knowledge and Problem Statement

3.1. Preliminary Knowledge

In this section, we will introduce the concept of input-to-state practical stability (ISpS) in probability.

Consider the following stochastic nonlinear system

$$dx = f(x,u)dt + g^T(x,u)d\omega, \quad x(0) = x_0 \in \mathbb{R}^n, \tag{3.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ are the state and the input of system, respectively. The Borel measurable functions $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}^{n\times r}$ are locally Lipschitz in x, and $\omega \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, P)$.

The following definitions and lemmas will be used throughout the paper.

Definition 3.1 (see [29]). For any given $V(x) \in C^2$, associated with stochastic system (3.1), the differential operator $\mathcal{L}$ is defined as follows:

$$\mathcal{L}V(x) = \frac{\partial V(x)}{\partial x} f(x,u) + \frac{1}{2}\operatorname{Tr}\left\{ g(x,u)\frac{\partial^2 V(x)}{\partial x^2} g^T(x,u)\right\}. \tag{3.2}$$

Definition 3.2 (see [30]). The stochastic system (3.1) is input-to-state practically stable (ISpS) in probability if for any $\varepsilon > 0$, there exist a class $\mathcal{KL}$-function $\beta(\cdot)$, a class $\mathcal{K}_\infty$-function $\gamma(\cdot)$, and a constant d_0 such that

$$P\left\{|x(t)| < \beta(|x_0|, t) + \gamma(|u_t|) + d_0\right\} \geq 1 - \varepsilon, \quad x_0 \in \mathbb{R}^n \setminus \{0\}. \tag{3.3}$$

Lemma 3.3 (see [30]). *For system* (3.1), *if there exist a C^2 function $V(x)$, class $\mathcal{K}_\infty$ functions α_1, α_2, χ, a class $\mathcal{K}$ function α, and a constant $\overline{d}$ such that*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \tag{3.4}$$

$$\mathcal{L}V(x) \leq -\alpha(|x|) + \chi(|u|) + \overline{d}, \tag{3.5}$$

then

(1) *There exists an almost surely unique solution on* $[0, \infty)$;

(2) *The system* (3.1) *is ISpS in probability.*

Lemma 3.4 (see [6]). *Let x and y be real variables. Then, for any positive integers m, n and any nonnegative smooth function $b(\cdot)$, the following inequality holds:*

$$\left|x^m y^n\right| \leq \frac{m}{m+n} b(\cdot)|x|^{m+n} + \frac{n}{m+n} b(\cdot)^{-m/n}\left|y\right|^{m+n}. \tag{3.6}$$

Lemma 3.5 (see [2]). *For real variables $x \geq 0$, $y > 0$, and real number $m \geq 1$, the following inequality holds:*

$$x \leq y + \left(\frac{x}{m}\right)^m \left(\frac{m-1}{y}\right)^{m-1}. \tag{3.7}$$

3.2. Problem Statement

From (2.5), we introduce a more general class of stochastic nonlinear systems as follows:

$$\begin{gathered} dx_i = d_i(x) x_{i+1}^{p_i} dt + f_i(\overline{x}_{i+1}) dt + g_i(\overline{x}_i)^T d\omega, \quad i = 1, \ldots, n-1, \\ dx_n = d_n(x) u^{p_n} dt + f_n(\overline{x}_n) dt + g_n(\overline{x}_n)^T d\omega, \\ y = x_1, \end{gathered} \tag{3.8}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $u, y \in \mathbb{R}$ are the state, the input, and the measurable output of system, respectively, $\overline{x}_i = (x_1, \ldots, x_i)^T$, p_i, $i = 1, \ldots, n$, are positive odd integers, $f_i(\cdot) : \mathbb{R}^{i+1} \to \mathbb{R}$ and $g_i(\cdot) : \mathbb{R}^i \to \mathbb{R} \times \mathbb{R}^r$ are smooth functions with $f_i(0) = 0$ and $g_i(0) = 0, d_i(x)$ is unknown control coefficient with known sign, and ω is an r-dimensional standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, P)$.

The following assumptions are made on system (3.8).

A1: for each $d_i(x)$, there exist unknown constant $\theta' > 0$ and known nonnegative smooth functions $b_i(\overline{x}_i)$ and $\overline{b}_i(\overline{x}_{i+1})$ such that

$$0 \leq b_i(\overline{x}_i) \leq d_i(x) \leq \theta' \overline{b}_i(\overline{x}_{i+1}). \tag{3.9}$$

A2: for functions $f_i(\cdot)$, $g_i(\cdot)$, $i = 1, 2, \ldots, n$, there exist known nonnegative smooth functions $\varphi_{ij}(\overline{x}_i)$ and $\overline{\varphi}_i(\overline{x}_i)$ such that

$$\begin{gathered} \left|f_i(\overline{x}_{i+1})\right| \leq \sum_{j=0}^{p_i-1} |x_{i+1}|^j \varphi_{ij}(\overline{x}_i), \\ \left|g_i(\overline{x}_i)\right| \leq \left(|x_1|^{(p_i+1)/2} + \cdots + |x_i|^{(p_i+1)/2}\right) \overline{\varphi}_i(\overline{x}_i). \end{gathered} \tag{3.10}$$

A3: the reference signal y_r and its derivative $\dot{y}_r$ are bounded.

The objective of this paper is to design an adaptive controller such that the closed-loop system is ISpS in probability and the tracking error $\xi_1 = y - y_r$ can be regulated to a neighborhood of the origin with radius as small as possible.

4. Controller Design and Stability Analysis

With the aid of Lemmas 3.3–3.5, we are ready to present the main results of this paper. In this section, we show that under A1–A3, it is possible to construct a globally stabilizing, state-feedback smooth controller for system (3.8). Introduce the odd positive integer $p = \max_{i=1,\ldots,n}\{p_i\}$, and the following coordinate change

$$\begin{aligned}
\xi_1 &= x_1 - y_r,\\
\xi_i &= x_i - x_i^*\left(\overline{x}_{i-1}, y_r, \widehat{\theta}\right), \quad i = 2,\ldots,n,
\end{aligned} \tag{4.1}$$

where $x_i^*(\overline{x}_{i-1}, y_r, \widehat{\theta})$, $i = 2,\ldots,n$, are virtual smooth controllers to be designed later, $\theta := \max\{\theta', \theta'^{(p+3)/(p-p_i+3)}\}$, and $\widehat{\theta}$ denotes the estimate of θ. Then, according to Itô differentiation rule, one has

$$\begin{aligned}
d\xi_1 &= d_1 x_2^{p_1} dt + f_1 dt + g_1^T d\omega - \dot{y}_r dt,\\
d\xi_i &= d_i x_{i+1}^{p_i} dt + f_i dt - \sum_{k=1}^{i-1} \frac{\partial x_i^*}{\partial x_k}\left(x_{k+1}^{p_k} + f_k\right)dt - \frac{1}{2}\sum_{j,k=1}^{i-1} \frac{\partial^2 x_i^*}{\partial x_j \partial x_k} g_j^T g_k dt\\
&\quad - \frac{\partial x_i^*}{\partial \widehat{\theta}}\dot{\widehat{\theta}}\, dt - \frac{\partial x_i^*}{\partial y_r}\dot{y}_r dt + \left(g_i^T - \sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k} g_k^T\right)d\omega, \quad i = 2,\ldots,n-1,\\
d\xi_n &= d_n u^{p_n} dt + f_n dt - \sum_{k=1}^{n-1} \frac{\partial x_n^*}{\partial x_k}\left(x_{k+1}^{p_k} + f_k\right)dt - \frac{1}{2}\sum_{j,k=1}^{n-1} \frac{\partial^2 x_n^*}{\partial x_j \partial x_k} g_j^T g_k dt\\
&\quad - \frac{\partial x_n^*}{\partial \widehat{\theta}}\dot{\widehat{\theta}}\, dt - \frac{\partial x_n^*}{\partial y_r}\dot{y}_r dt + \left(g_n^T - \sum_{k=1}^{n-1}\frac{\partial x_n^*}{\partial x_k} g_k^T\right)d\omega.
\end{aligned} \tag{4.2}$$

Let $G_i^T = g_i^T - \sum_{k=1}^{i-1}(\partial x_i^*/\partial x_k) g_k^T$, $i = 2,\ldots,n$. Next, we design the controller step by step by backstepping.

Step 1. Consider the 1st Lyapunov candidate function

$$V_1\left(\xi_1, \widetilde{\theta}\right) = \frac{1}{p - p_1 + 4}\xi_1^{p-p_1+4} + \frac{1}{2}\widetilde{\theta}^2, \tag{4.3}$$

where $\widetilde{\theta} = \theta - \widehat{\theta}$ is the parameter estimation error. In view of (3.2), (4.1), and (4.2), one has

$$\begin{aligned}
\mathcal{L}V_1\left(\xi_1, \widetilde{\theta}\right) &= \xi_1^{p-p_1+3}\left(d_1(x) x_2^{p_1} + f_1(\overline{x}_2) - \dot{y}_r\right) + \frac{1}{2}\operatorname{Tr}\left\{g_1(x_1)(p - p_1 + 3)\xi_1^{p-p_1+2} g_1^T(x_1)\right\} - \widetilde{\theta}\dot{\widehat{\theta}}\\
&\le d_1(x)\xi_1^{p-p_1+3} x_2^{p_1} + |\xi_1|^{p-p_1+3}\left|f_1(\overline{x}_2) - \dot{y}_r\right| + \frac{1}{2}(p - p_1 + 3)\xi_1^{p-p_1+2}\left|g_1(x_1)\right|^2 - \widetilde{\theta}\dot{\widehat{\theta}}.
\end{aligned} \tag{4.4}$$

By Lemma 3.4 and A2, there exist nonnegative smooth functions $\overline{\varphi}_1(x_1)$ and $\psi_1(x_1)$ such that

$$\begin{aligned}
\left|f_1(\overline{x}_2)\right| &\le \sum_{j=0}^{p_1-1}|x_2|^j\varphi_{1j}(x_1) = \sum_{j=0}^{p_1-1}|x_2|^j\left(\varphi_{1j}^{1/(p_1-j)}(x_1)\right)^{p_1-j} \\
&\le \sum_{j=0}^{p_1-1}\left(\frac{j}{p_1}\left(\frac{1}{2j}b_1(x_1)\right)|x_2|^{p_1} + \frac{p_1-j}{p_1}\left(\frac{2j}{b_1(x_1)}\right)^{j/(p_1-j)}\varphi_{1j}^{p_1/(p_1-j)}(x_1)\right) \\
&\le \frac{b_1(x_1)}{2}|x_2|^{p_1} + \overline{\varphi}_1(x_1), \\
\left|g_1(x_1)\right| &\le |\xi_1|^{(p_1+1)/2}\psi_1(x_1),
\end{aligned} \tag{4.5}$$

which together with the boundedness of $\dot{y}_r$ imply that

$$\left|f_1 - \dot{y}_r\right| \le \frac{b_1(x_1)}{2}|x_2|^{p_1} + \varphi_1'(x_1, y_r), \tag{4.6}$$

where $\varphi_1'(x_1, y_r)$ is a nonnegative smooth function, $\psi_1(x_1) = \overline{\varphi}_1(x_1)$. Then, for any real number $\delta_1 > 0$, choosing $a = |\xi_1^{p-p_1+3}|\varphi_1'(x_1, y_r)$, $b = \delta_1$, $m = (p+3)/(p-p_1+3)$, by Lemma 3.5, there is a smooth function $\phi_{11}(x_1, y_r)$ such that

$$\begin{aligned}
&|\xi_1|^{p-p_1+3}\left|f_1 - \dot{y}_r\right| \\
&\quad \le |\xi_1|^{p-p_1+3}\left(\frac{b_1(x_1)}{2}|x_2|^{p_1} + \varphi_1'(x_1, y_r)\right) \le |\xi_1|^{p-p_1+3}\frac{b_1(x_1)}{2}|x_2|^{p_1} \\
&\qquad + \delta_1 + \left(\frac{(p-p_1+3)|\xi_1|^{p-p_1+3}\varphi_1'(x_1, y_r)}{p+3}\right)^{(p+3)/(p-p_1+3)} \times \left(\frac{p_1}{\delta_1(p-p_1+3)}\right)^{p_1/(p-p_1+3)} \\
&\quad = |\xi_1|^{p-p_1+3}\frac{b_1(x_1)}{2}|x_2|^{p_1} + \xi_1^{p+3}\phi_{11}(x_1, y_r) + \delta_1,
\end{aligned} \tag{4.7}$$

where $\phi_{11}(x_1, y_r) = ((p-p_1+3)\varphi_1'(x_1, y_r)/(p+3))^{(p+3)/(p-p_1+3)}(p_1/\delta_1(p-p_1+3))^{p_1/(p-p_1+3)}$. Substituting (4.5) and (4.7) into (4.4), and adding and subtracting $(b_1(x_1)/2)\xi_1^{p-p_1+3}x_2^{*p_1}$ on the right-hand side of (4.4), we have

$$\begin{aligned}
\mathcal{L}V_1 &\le d_1(x)\xi_1^{p-p_1+3}x_2^{p_1} + \frac{b_1(x_1)}{2}\left|\xi_1^{p-p_1+3}x_2^{p_1}\right| + \xi_1^{p+3}\phi_{11}(x_1, y_r) + \delta_1 \\
&\quad + \frac{p-p_1+3}{2}\xi_1^{p-p_1+2}\xi_1^{p_1+1}\psi_1^2(x_1) - \tilde{\theta}\dot{\hat{\theta}} \\
&= d_1(x)\xi_1^{p-p_1+3}x_2^{p_1} + \frac{b_1(x_1)}{2}|\xi_1|^{p-p_1+3}|x_2|^{p_1} + \frac{b_1(x_1)}{2}\xi_1^{p-p_1+3}x_2^{*p_1} \\
&\quad - \frac{b_1(x_1)}{2}\xi_1^{p-p_1+3}x_2^{*p_1} + \xi_1^{p+3}\phi_{11}(x_1, y_r) + \xi_1^{p+3}\phi_{12}(x_1) + \delta_1 - \tilde{\theta}\dot{\hat{\theta}},
\end{aligned} \tag{4.8}$$

where $\phi_{12}(x_1) = ((p-p_1+3)/2)\xi_1^{p-p_1}\varphi_1^2(x_1)$. Suppose the virtual smooth controller $x_2^* = -\xi_1\beta_1(x_1, y_r, \widehat{\theta})$ with $\beta_1(x_1, y_r, \widehat{\theta}) > 0$, which together with A1 lead to

$$
\begin{aligned}
0 &\leq -b_1(x_1)\xi_1^{p-p_1+3}x_2^{*p_1} \leq -d_1(x)\xi_1^{p-p_1+3}x_2^{*p_1}, \\
-\frac{b_1(x_1)}{2}\xi_1^{p-p_1+3}x_2^{*p_1} &+ \frac{b_1(x_1)}{2}\left|\xi_1^{p-p_1+3}x_2^{*p_1}\right| \leq -d_1(x)\xi_1^{p-p_1+3}x_2^{*p_1}, \\
-\frac{b_1(x_1)}{2}\xi_1^{p-p_1+3}x_2^{*p_1} &\leq -d_1(x)\xi_1^{p-p_1+3}x_2^{*p_1} - \frac{b_1(x_1)}{2}\left|\xi_1^{p-p_1+3}x_2^{*p_1}\right|.
\end{aligned}
\tag{4.9}
$$

Substituting (4.9) into (4.8), one can obtain

$$
\begin{aligned}
\mathcal{L}V_1 \leq{}& d_1(x)\xi_1^{p-p_1+3}x_2^{p_1} + \frac{b_1(x_1)}{2}\left|\xi_1^{p-p_1+3}x_2^{p_1}\right| + \frac{b_1(x_1)}{2}\xi_1^{p-p_1+3}x_2^{*p_1} \\
&- d_1(x)\xi_1^{p-p_1+3}x_2^{*p_1} - \frac{b_1(x_1)}{2}\left|\xi_1^{p-p_1+3}x_2^{*p_1}\right| + \delta_1 - \widetilde{\theta}\dot{\widehat{\theta}} \\
&- \overline{c}_1\theta\xi_1^{p+3} + \overline{c}_1\theta\xi_1^{p+3} + \xi_1^{p+3}\phi_{11}(x_1, y_r) + \xi_1^{p+3}\phi_{12}(x_1) \\
\leq{}& -\overline{c}_1\theta\xi_1^{p+3} + \left(\theta\overline{b}_1(\overline{x}_2) + \frac{b_1(x_1)}{2}\right)|\xi_1|^{p-p_1+3}\left|x_2^{p_1} - x_2^{*p_1}\right| + \frac{b_1(x_1)}{2}\xi_1^{p-p_1+3}x_2^{*p_1} \\
&+ \overline{c}_1\widehat{\theta}\xi_1^{p+3} + \xi_1^{p+3}\phi_{11}(x_1, y_r) + \xi_1^{p+3}\phi_{12}(x_1) + \delta_1 + \widetilde{\theta}\left(\tau_1 - \dot{\widehat{\theta}}\right),
\end{aligned}
\tag{4.10}
$$

where $\tau_1 = \overline{c}_1\xi_1^{p+3}$ is a nonnegative smooth function. Choose x_2^* as follows:

$$
\begin{aligned}
x_2^*\left(x_1, y_r, \widehat{\theta}\right) &= -\xi_1\beta_1\left(x_1, y_r, \widehat{\theta}\right), \\
\beta_1\left(x_1, y_r, \widehat{\theta}\right) &= \left(\frac{2}{b_1(x_1)}\left(c_1 + \phi_{11}(x_1, y_r) + \phi_{12}(x_1) + \overline{c}_1\sqrt{1+\widehat{\theta}^2}\right)\right)^{1/p_1},
\end{aligned}
\tag{4.11}
$$

where $\beta_1(x_1, y_r, \widehat{\theta}) \geq 0$ is a smooth function. Then,

$$
\mathcal{L}V_1 \leq -c_1\xi_1^{p+3} - \overline{c}_1\theta\xi_1^{p+3} + \left(\theta\overline{b}_1(\overline{x}_2) + \frac{b_1(x_1)}{2}\right)|\xi_1|^{p-p_1+3}\left|x_2^{p_1} - x_2^{*p_1}\right| + \delta_1 + \widetilde{\theta}\left(\tau_1 - \dot{\widehat{\theta}}\right). \tag{4.12}
$$

Step i. $2 \leq i \leq n$: Assume that at Step $i-1$, there exists a smooth state-feedback virtual control

$$
x_i^*\left(\overline{x}_{i-1}, y_r, \widehat{\theta}\right) = -\beta_{i-1}\left(\overline{x}_{i-1}, y_r, \widehat{\theta}\right)\xi_{i-1}, \tag{4.13}
$$

such that

$$\begin{aligned}\mathcal{L}V_{i-1} \le & -\sum_{j=1}^{i-1}\left(c_j - \sum_{k=j+1}^{i-1} c_{kj}\right)\xi_j^{p+3} - \sum_{j=1}^{i-1}\left(\overline{c}_j - \sum_{k=j+1}^{i-1}\overline{c}_{kj}\right)\theta\xi_j^{p+3} \\ & + \left(\theta\overline{b}_{i-1} + \frac{b_{i-1}}{2}\right)|\xi_{i-1}|^{p-p_{i-1}+3}\left|x_i^{p_{i-1}} - x_i^{*p_{i-1}}\right| + \sum_{j=1}^{i-1}\delta_j + \left(\widetilde{\theta} + \sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}\right)\left(\tau_{i-1} - \dot{\widehat{\theta}}\right),\end{aligned} \tag{4.14}$$

where $\beta_{i-1} > 0$ is a smooth function, and $V_{i-1} = (1/4)\sum_{k=1}^{i-1}\xi_k^{p-p_k+4} + (1/2)\widetilde{\theta}^2$. We will prove that (4.14) still holds for Step i.

Define the ith Lyapunov candidate function

$$V_i = V_{i-1} + \frac{1}{4}\xi_i^{p-p_i+4}. \tag{4.15}$$

From (4.2) and (4.14), it follows that

$$\begin{aligned}\mathcal{L}V_i \le & -\sum_{j=1}^{i-1}\left(c_j - \sum_{k=j+1}^{i-1} c_{kj}\right)\xi_j^{p+3} - \sum_{j=1}^{i-1}\left(\overline{c}_j - \sum_{k=j+1}^{i-1}\overline{c}_{kj}\right)\theta\xi_j^{p+3} \\ & + \left(\theta\overline{b}_{i-1} + \frac{b_{i-1}}{2}\right)|\xi_{i-1}|^{p-p_{i-1}+3}\left|x_i^{p_{i-1}} - x_i^{*p_{i-1}}\right| + \sum_{j=1}^{i-1}\delta_j \\ & + \left(\widetilde{\theta} + \sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}\right)\left(\tau_{i-1} - \dot{\widehat{\theta}}\right) + \xi_i^{p-p_i+3}d_i(x)x_{i+1}^{p_i} + \xi_i^{p-p_i+3} \\ & \times\left(f_i - \sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}\left(d_k(x)x_k^{p_k} + f_k\right) - \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 x_i^*}{\partial x_j\partial x_k}g_j^T g_k - \frac{\partial x_i^*}{\partial y_r}\dot{y}_r - \frac{\partial x_i^*}{\partial\widehat{\theta}}\dot{\widehat{\theta}}\right) \\ & + \frac{1}{2}\operatorname{Tr}\left\{G_i(p-p_i+3)\xi_i^{p-p_i+2}G_i^T\right\}.\end{aligned} \tag{4.16}$$

By A2 and Lemma 3.4, there is a smooth nonnegative function $\overline{\varphi}_i(\overline{x}_i)$ such that

$$\left|f_i(\overline{x}_{i+1})\right| \le \sum_{j=0}^{p_i-1}|x_{i+1}|^j\varphi_{ij}(\overline{x}_i) \le \frac{b_i(\overline{x}_i)|x_{i+1}^{p_i}}{2} + \overline{\varphi}_i(\overline{x}_i), \tag{4.17}$$

then,

$$\left|f_i - \sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k}f_k - \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 x_i^*}{\partial x_j\partial x_k}g_j^T g_k - \frac{\partial x_i^*}{\partial y_r}\dot{y}_r\right| \le \frac{b_i(\overline{x}_i)|x_{i+1}^{p_i}}{2} + \varphi_i\left(\overline{x}_i, y_r, \widehat{\theta}\right), \tag{4.18}$$

where $\varphi_i(\overline{x}_i, y_r, \widehat{\theta})$ is a smooth function. By A2, (4.1) and (4.13), there exists a nonnegative smooth function $\varphi_i'(\overline{x}_i, y_r, \widehat{\theta})$ such that

$$|G_i(\overline{x}_i)| \leq \left(|\xi_1|^{(p_i+1)/2} + \cdots + |\xi_i|^{(p_i+1)/2}\right)\varphi_i'\left(\overline{x}_i, y_r, \widehat{\theta}\right). \tag{4.19}$$

By (4.13), we have

$$\begin{aligned}
&\left(\theta\overline{b}_{i-1}(\overline{x}_i) + \frac{b_{i-1}(x_{i-1})}{2}\right)|\xi_{i-1}|^{p-p_{i-1}+3}\left|x_i^{p_{i-1}} - x_i^{*p_{i-1}}\right| \\
&\quad = \left(\theta\overline{b}_{i-1}(\overline{x}_i) + \frac{b_{i-1}(x_{i-1})}{2}\right)\sum_{k=1}^{p_{i-1}} C_{p_{i-1}}^k |\xi_i|^k |\xi_1|^{p-k+3}\beta_1^{p_{i-1}-k} \\
&\quad \leq \sum_{k=1}^{i-1} c_{ik1}\xi_k^{p+3} + \sum_{k=1}^{i-1} \overline{c}_{ik}\theta\xi_k^{p+3} + \varphi_{i1}\left(\overline{x}_i, y_r, \widehat{\theta}\right)\xi_i^{p+3} + \theta\varphi_{i2}\xi_i^{p+3},
\end{aligned} \tag{4.20}$$

where $\varphi_{i1}(\overline{x}_i, y_r, \widehat{\theta})$ and $\varphi_{i2}(\overline{x}_i, y_r, \widehat{\theta})$ are two smooth functions. From A1, (4.1), and (4.13), it follows that

$$\begin{aligned}
&\left|-\xi_i^{p-p_i+3}\sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k} d_k(x) x_k^{p_k}\right| \\
&\qquad \leq \theta' |\xi_i|^{p-p_i+3}\sum_{k=1}^{i-1} b_{k-1}(\overline{x}_k)\left|\frac{\partial x_i^*}{\partial x_k}\right| \left|\xi_k + x_k^*\right|^{p_k} \\
&\qquad \leq \theta'^{(p+3)/(p-p_i+3)}\xi_i^{p+3}\varphi_{i3}\left(\overline{x}_i, y_r, \widehat{\theta}\right) + \delta_{i1} \\
&\qquad \leq \theta\xi_i^{p+3}\varphi_{i3}\left(\overline{x}_i, y_r, \widehat{\theta}\right) + \delta_{i1},
\end{aligned} \tag{4.21}$$

$$\begin{aligned}
&|\xi_i|^{p-p_i+3}\left| f_i - \sum_{k=1}^{i-1}\frac{\partial x_i^*}{\partial x_k} f_k - \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2 x_i^*}{\partial x_j \partial x_k} g_j^T g_k - \frac{\partial x_i^*}{\partial y_r}\dot{y}_r \right| \\
&\qquad \leq |\xi_i|^{p-p_i+3}\left(\frac{b_i(\overline{x}_i)|x_{i+1}|^{p_i}}{2} + \varphi_i'\left(\overline{x}_i, y_r, \widehat{\theta}\right)\right) \\
&\qquad \leq \frac{b_i(\overline{x}_i)}{2}|\xi_i|^{p-p_i+3}|x_{i+1}|^{p_i} + \varphi_{i4}\left(\overline{x}_i, y_r, \widehat{\theta}\right)\xi_i^{p+3} + \delta_{i2},
\end{aligned} \tag{4.22}$$

where $\varphi_{i3}(\overline{x}_i, y_r, \widehat{\theta})$ and $\varphi_{i4}(\overline{x}_i, y_r, \widehat{\theta})$ are two smooth functions. From (4.19), one can obtain

$$\frac{1}{2}\operatorname{Tr}\left\{G_i(p - p_i + 3)\xi_i^{p-p_i+2}G_i^T\right\}$$

$$\leq \frac{p-p_i+3}{2}\xi_2^{p-p_i+2}\left(|\xi_1|^{(p_i+1)/2}+\cdots+|\xi_i|^{(p_i+1)/2}\right)^2\varphi_i'^2\left(\overline{x}_i,y_r,\widehat{\theta}\right)$$
$$\leq \sum_{k=1}^{i-1}c_{ik2}\xi_k^{p+3}+\varphi_{i5}\left(\overline{x}_i,y_r,\widehat{\theta}\right)\xi_i^{p+3}, \tag{4.23}$$

where $\varphi_{i5}(\overline{x}_i,y_r,\widehat{\theta})$ is a smooth nonnegative function. Substituting (4.20)–(4.23) into (4.16), one gets

$$\begin{aligned}\mathcal{L}V_i \leq &-\sum_{j=1}^{i-1}\left(c_j-\sum_{k=j+1}^{i-1}c_{kj}\right)\xi_j^{p+3}-\sum_{j=1}^{i}\left(\overline{c}_j-\sum_{k=j+1}^{i-1}\overline{c}_{kj}\right)\theta\xi_j^{p+3}+\sum_{j=1}^{i-1}c_{ij}'\xi_j^{p+3}\\&+\sum_{j=1}^{i-1}\delta_j+\theta\sum_{j=1}^{i-1}\overline{c}_{ij}\xi_j^{p+3}+h_{i1}'\xi_i^{p+3}+\theta h_{i2}\xi_i^{p+3}-\overline{c}_i\theta\xi_i^{p+3}+\overline{c}_i\theta\xi_i^{p+3}\\&+\xi_i^{p-p_i+3}d_i(x)x_{i+1}^{p_i}+\frac{b_i(\overline{x}_i)}{2}|\xi_i|^{p-p_i+3}|x_{i+1}|^{p_i}+\frac{b_i(\overline{x}_i)}{2}\xi_i^{p-p_i+3}x_3^{*p_i}\\&-\frac{b_i(\overline{x}_i)}{2}\xi_i^{p-p_i+3}x_3^{*p_i}+\left(\widetilde{\theta}+\sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}\right)\left(\tau_{i-1}-\dot{\widehat{\theta}}\right)-\xi_i^{p-p_i+3}\frac{\partial x_i^*}{\partial\widehat{\theta}}\dot{\widehat{\theta}},\end{aligned} \tag{4.24}$$

where

$$\begin{aligned}c_{ij}'&=c_{ij1}+c_{ij2},\quad j=1,\ldots,i-1,\\h_{i1}'&=\varphi_{i1}+\varphi_{i4}+\varphi_{i5},\qquad h_{i2}=\varphi_{i2}+\varphi_{i3}.\end{aligned} \tag{4.25}$$

Suppose the virtual smooth controller $x_{i+1}^*=-\xi_i\beta_i(\overline{x}_i,y_r,\widehat{\theta})$ with $\beta_i(\overline{x}_i,y_r,\widehat{\theta})>0$, which together with A2 render

$$-\frac{b_i(\overline{x}_i)}{2}\xi_i^{p-p_i+3}x_{i+1}^{*p_i}\leq -d_i(x)\xi_i^{p-p_i+3}x_{i+1}^{*p_i}-\frac{b_i(\overline{x}_i)}{2}\left|\xi_i^{p-p_i+3}x_{i+1}^{*p_i}\right|. \tag{4.26}$$

Substituting (4.26) into (4.24) leads to

$$\begin{aligned}\mathcal{L}V_i \leq &-\sum_{j=1}^{i-1}\left(c_j-\sum_{k=j+1}^{i-1}c_{kj}\right)\xi_j^{p+3}-\sum_{j=1}^{i}\left(\overline{c}_j-\sum_{k=j+1}^{i}\overline{c}_{kj}\right)\theta\xi_j^{p+3}+\sum_{j=1}^{i-1}c_{ij}'\xi_j^{p+3}\\&+h_{i1}'\xi_i^{p+3}+\left(\widehat{\theta}+\widetilde{\theta}\right)h_{i2}\xi_i^{p+3}+\overline{c}_i\left(\widehat{\theta}+\widetilde{\theta}\right)\xi_i^{p+3}+\xi_i^{p-p_i+3}d_i(x)x_{i+1}^{p_i}\\&+\frac{b_i(\overline{x}_i)}{2}|\xi_i|^{p-p_i+3}|x_{i+1}|^{p_i}+\frac{b_i(\overline{x}_i)}{2}\xi_i^{p-p_i+3}x_3^{*p_i}-d_i(x)\xi_i^{p-p_i+3}x_{i+1}^{*p_2}\end{aligned}$$

$$
\begin{aligned}
&- \frac{b_i(\overline{x}_i)}{2}\left|\xi_i^{p-p_i+3}x_{i+1}^{*p_i}\right| + \left(\widetilde{\theta} + \sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}\right)\left(\tau_{i-1} - \dot{\widehat{\theta}}\right) - \xi_i^{p-p_i+3}\frac{\partial x_i^*}{\partial\widehat{\theta}}\dot{\widehat{\theta}} \\
&+ \sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}(h_{i2} + \overline{c}_i)\xi_i^{p+3} - \sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}(h_{i2} + \overline{c}_i)\xi_i^{p+3} \\
&+ \xi_i^{p-p_i+3}\frac{\partial x_i^*}{\partial\widehat{\theta}}\tau_i - \xi_i^{p-p_i+3}\frac{\partial x_i^*}{\partial\widehat{\theta}}\tau_i,
\end{aligned}
\tag{4.27}
$$

where $\tau_i = \tau_{i-1} + (h_{i2} + \overline{c}_i)\xi_i^{p+3}$. For (4.27), we have

$$
\begin{aligned}
\left|-\sum_{k=2}^{i-1}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}(h_{i2} + \overline{c}_i)\xi_i^{p+3}\right| &\le \varphi_{i6}\left(\overline{x}_i, y_r, \widehat{\theta}\right)\xi_i^{p+3}, \\
\left|-\xi_i^{p-p_i+3}\frac{\partial x_i^*}{\partial\widehat{\theta}}\tau_i\right| &\le \sum_{k=1}^{i-1}c_{ik3}\xi_k^{p+3} + \varphi_{i7}\left(\overline{x}_i, y_r, \widehat{\theta}\right)\xi_i^{p+3},
\end{aligned}
\tag{4.28}
$$

where c_{ik3} is a design parameter, $\varphi_{i6}(\overline{x}_i, y_r, \widehat{\theta})$ and $\varphi_{i7}(\overline{x}_i, y_r, \widehat{\theta})$ are the smooth functions. Let $c_{ij} = c'_{ij} + c_{ij3}$, $h_{i1} = h'_{i1} + \varphi_{i6} + \varphi_{i7}$. (4.27) becomes

$$
\begin{aligned}
\mathcal{L}V_i \le& -\sum_{j=1}^{i-1}\left(c_j - \sum_{k=j+1}^{i-1}c_{kj}\right)\xi_j^{p+3} - \sum_{j=1}^{i}\left(\overline{c}_j - \sum_{k=j+1}^{i}\overline{c}_{kj}\right)\theta\xi_j^{p+3} + \sum_{j=1}^{i-1}c_{ij}\xi_j^{p+3} \\
&+ h'_{i1}\xi_i^{p+3} + \widehat{\theta}h_{i2}\xi_i^{p+3} + \left(\theta\overline{b}_i(\overline{x}_{i+1}) + \frac{b_i(x_i)}{2}\right)|\xi_i|^{p-p_i+3}\left|x_{i+1}^{p_i} - x_{i+1}^{*p_i}\right| \\
&+ \frac{b_i(\overline{x}_i)}{2}\xi_i^{p-p_i+3}x_{i+1}^{*p_i} + \left(\widetilde{\theta} + \sum_{k=2}^{i}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}\right)\left(\tau_i - \dot{\widehat{\theta}}\right) + \sum_{j=1}^{i}\delta_j \\
\le& -\sum_{j=1}^{i}\left(c_j - \sum_{k=j+1}^{i}c_{kj}\right)\xi_j^{p+3} - \sum_{j=1}^{i}\left(\overline{c}_j - \sum_{k=j+1}^{i}\overline{c}_{kj}\right)\theta\xi_j^{p+3} \\
&+ \left(\theta\overline{b}_i(\overline{x}_{i+1}) + \frac{b_i(x_i)}{2}\right)|\xi_i|^{p-p_i+3}\left|x_{i+1}^{p_i} - x_{i+1}^{*p_i}\right| + \sum_{j=1}^{i}\delta_j + \left(\widetilde{\theta} + \sum_{k=2}^{i}\xi_k^{p-p_k+3}\frac{\partial x_k^*}{\partial\widehat{\theta}}\right)\left(\tau_i - \dot{\widehat{\theta}}\right),
\end{aligned}
\tag{4.29}
$$

by choosing

$$
\begin{aligned}
x_{i+1}^*\left(\overline{x}_i, y_r, \widehat{\theta}\right) &= -\xi_i\beta_i\left(\overline{x}_i, y_r, \widehat{\theta}\right), \\
\beta_i\left(\overline{x}_i, y_r, \widehat{\theta}\right) &= \left(\frac{2}{b_i(\overline{x}_i)}\left(c_i + h_{i1} + (h_{i2} + \overline{c}_i)\sqrt{1 + \widehat{\theta}^2}\right)\right)^{1/p_i},
\end{aligned}
\tag{4.30}
$$

where $\beta_i(\overline{x}_i, y_r, \widehat{\theta}) \ge 0$ is a smooth function.

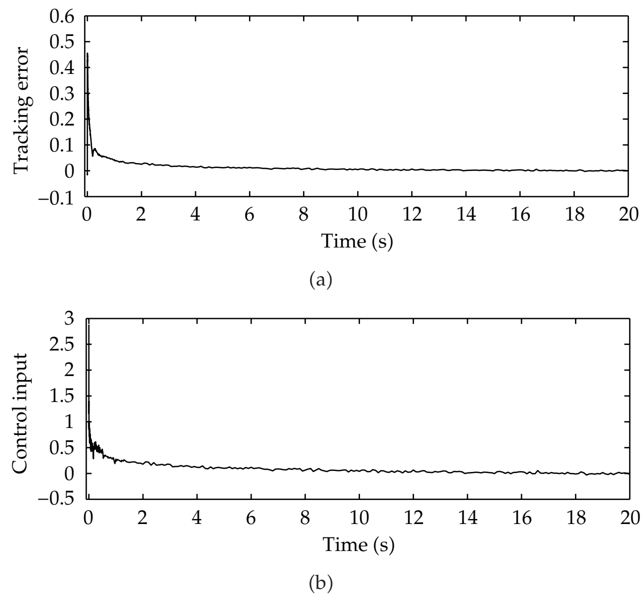

Figure 2: Gives the response of the closed-loop system, from which, the effectiveness of the controller is demonstrated.

Finally, when $i = n,\ x_{n+1} = x_{n+1}^{*} = u$ is the actual control. By choosing the actual control law and the adaptive law:

$$u\left(\overline{x}_n, y_r, \widehat{\theta}\right) = -\beta_n\left(\overline{x}_n, y_r, \widehat{\theta}\right)\xi_n, \qquad \dot{\widehat{\theta}} = \tau_n = \sum_{k=1}^{n} H_{k2}\xi_k^{p+3}, \tag{4.31}$$

where $\beta_n \geq 0$ and $H_{12}, \ldots, H_{n2}$ are smooth functions, one gets

$$\mathcal{L}V_n \leq -\sum_{j=1}^{n}\left(c_j - \sum_{k=j+1}^{n} c_{kj}\right)\xi_j^{p+3} - \sum_{j=1}^{n}\left(\overline{c}_j - \sum_{k=j+1}^{n} \overline{c}_{kj}\right)\theta\xi_j^{p+3} + \sum_{j=1}^{n}\delta_j. \tag{4.32}$$

Theorem 4.1. *If A1–A3 hold for the high-order stochastic nonlinear system* (3.8)*, under the smooth adaptive state-feedback controller* (4.32)*, the closed-loop system is ISpS in probability, and the tracking error $\xi_1 = y - y_r$ can be regulated to a neighborhood of the origin in probability with radius as small as possible (Figure 2).*

Proof. For $V_n = \sum_{i=1}^{n}(1/4)\xi_i^{p-p_i+4} + (1/2)\widetilde{\theta}^2$, it is obvious that V_n satisfies (3.4). Choosing all the design parameters c_j and $\overline{c}_j$ to satisfy

$$c_j > \sum_{k=j+1}^{n} c_{kj}, \qquad \overline{c}_j > \sum_{k=j+1}^{n} \overline{c}_{kj}, \quad j = 1, \ldots, n, \tag{4.33}$$

such that (3.5) holds, and then using Lemma 3.3, one can prove Theorem 4.1. □

5. Simulation

Now, we apply the control scheme to the mechanical system (2.5). Let $\xi_1 = x_1 - y_r$ be the tracking error, where $y_r = \sin t$ is a bounded smooth reference signal. For (2.5), $d_i(\cdot) = 1$, and $p = \max\{1, 3\} = 3$.

Choose $V_1(\xi_1) = (1/(p - p_1 + 4))\xi_1^{p-p_1+4} = \xi_1^6/6$. Then,

$$\mathcal{L}V_1(\xi_1) = \xi_1^5(x_2 - \dot{y}_r). \tag{5.1}$$

The smooth virtual controller can be chosen as $x_2^*(x_1, y_r) = -c_1\xi_1 + \dot{y}_r$, which renders

$$\mathcal{L}V_1(\xi_1) = -c_1\xi_1^6 + \xi_1^5(x_2 - x_2^*). \tag{5.2}$$

Next, defining $V_2(\xi_1, \xi_2) = V_1 + (1/(p - p_2 + 4))\xi_2^{p-p_2+4} = \xi_1^6/6 + \xi_2^4/4$, a direct calculation gives

$$\mathcal{L}V_2 = -c_1\xi_1^6 + \xi_1^5\xi_2 + \xi_2^3\left(x_3^3 - \frac{\partial x_2^*}{\partial x_1}x_2 - \frac{\partial x_2^*}{\partial y_r}\dot{y}_r\right) = -c_1\xi_1^6 + \xi_1^5\xi_2 + \xi_2^3\left(x_3^3 - h_2\right), \tag{5.3}$$

where $\xi_2 = x_2 - x_2^*$. By Lemma 3.5, choosing $m = 3/2$, one can obtain that for any constant $\delta_2 > 0$,

$$\left|\xi_2^4 h_2\right| \leq \delta_2 + \left(\frac{2\xi_2^4 h_2}{3}\right)^{3/2}\left(\frac{1}{2\delta_2}\right)^{1/2} \leq \delta_2 + \xi_2^6\varphi_2(\overline{x}_2). \tag{5.4}$$

Then, by (5.4) and (5.5), it is easy to see that

$$\mathcal{L}V(\xi_1, \xi_2) = -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 + \xi_2^3\left(x_3^3 - x_3^{*3}\right) + \delta_2, \tag{5.5}$$

by choosing $x_3^*(x_1, x_2, y_r) = -\xi_2(c_2 + d_2 + \varphi_2)^{1/3}$.

Defining $\xi_3 = x_3 - x_3^*$ and the Lyapunov function $V_3(\xi_1, \xi_2, \xi_3) = V_2(\xi_1, \xi_2) + (1/6)\xi_3^6$, one gets

$$\begin{aligned}\mathcal{L}V_3 &\leq -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 + \xi_2^3\left(x_3^3 - x_3^{*3}\right) + \delta_2 + \xi_3^5\left(x_4 - \frac{\partial x_3^*}{\partial x_1}x_2 - \frac{\partial x_3^*}{\partial x_2}x_3^3 - \frac{\partial x_3^*}{\partial y_r}\dot{y}_r\right)\\ &\leq -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 + \delta_{31} + \xi_3^6\varphi_{31} + \delta_{32} + \xi_3^6\varphi_{32} + \xi_3^5(x_4 - x_4^*) + \xi_3^5 x_4^*\\ &= -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 - c_3\xi_3^6 + \xi_3^5(x_4 - x_4^*) + \delta_2 + \delta_3,\end{aligned} \tag{5.6}$$

by choosing $x_4^*(x_1, x_2, x_3, y_r) = -\xi_3(c_3 + \varphi_{31} + \varphi_{32})$. At last, choosing $\xi_4 = x_4 - x_4^*$, $V_4(\xi_1, \xi_2, \xi_3, \xi_4) = V_3(\xi_1, \xi_2, \xi_3) + (1/6)\xi_4^6$, a direct calculation gives

$$\begin{aligned}
V_4 &\leq -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 - c_3\xi_3^6 + \xi_3^5(x_4 - x_4^*) + \delta_2 + \delta_3 \\
&\quad + \xi_4^5\left(v + k_0 f - \frac{\partial x_4^*}{\partial x_1}x_2 - \frac{\partial x_4^*}{\partial x_2}x_3^3 - \frac{\partial x_4^*}{\partial x_3}x_4 - \frac{\partial x_4^*}{\partial y_r}\dot{y}_r\right) + 5\xi_4^4\sigma^2 f^2 \\
&\leq -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 - c_3\xi_3^6 - c_4\xi_4^6 + \delta_2 + \delta_3 + \delta_{41} + \xi^6\varphi_{41} + \delta_{42} + \xi^6\varphi_{42} + \xi_4^5 v \\
&= -(c_1 - c_{21})\xi_1^6 - c_2\xi_2^6 - c_3\xi_3^6 - c_4\xi_4^6 + \delta_2 + \delta_3 + \delta_4,
\end{aligned} \tag{5.7}$$

by choosing

$$v = -\xi_4(c_4 + \varphi_{41} + \varphi_{42}). \tag{5.8}$$

Choose the design parameters $\sigma = 0.125$, $\delta_2 = 0.01$, $\delta_3 = 0.01$, and $\delta_4 = 0.01$. Moreover, to satisfy (5.3), we choose $c_1 = 1 > c_{21} = 5/6$, $c_2 = 1.5$, $c_3 = 0.5$ and $c_4 = 0.5$. Choose the initial values $x_1(0) = 0.45$, $x_2(0) = 0.5$, $x_3(0) = 0.5$, and $x_4(0) = 0.5$.

6. Concluding Remarks

In this paper, a mechanical system is firstly introduced. Then, by a series of coordinate transformations, the mechanical system can be transformed to a class of high-order stochastic nonlinear system, based on which, a more general mathematical model is considered and the smooth state-feedback controller is designed which guarantees that the tracking error $\xi_1 = y - y_r$ can be regulated to a neighborhood of the origin in probability with radius as small as possible. At last, the simulation is given to show the effectiveness of the design scheme.

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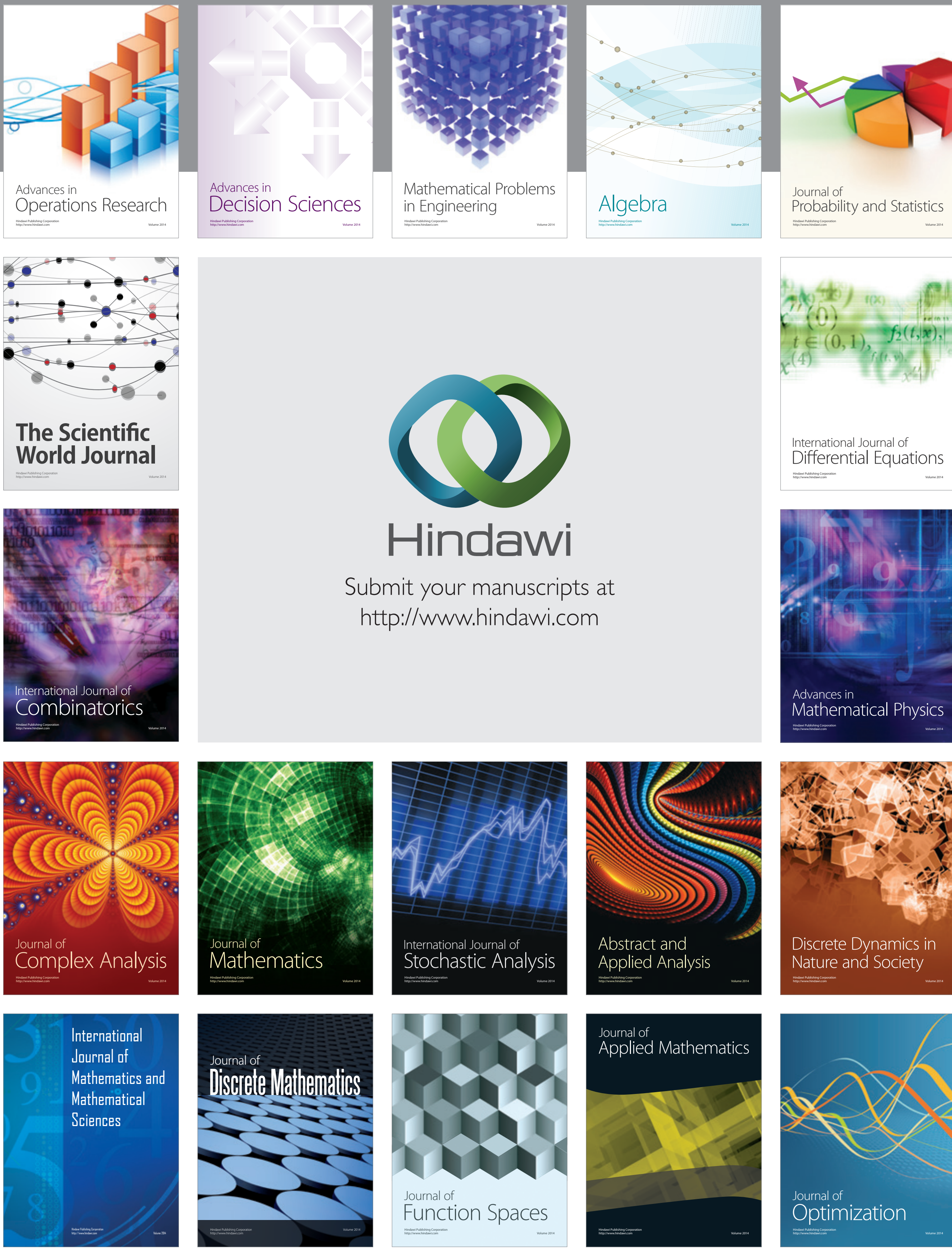
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