

## Research Article

# Robust Finite-Time $H_\infty$ Control for Impulsive Switched Nonlinear Systems with State Delay

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Received 20 June 2012; Revised 9 September 2012; Accepted 10 September 2012

Academic Editor: Zidong Wang

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This paper investigates robust finite-time  $H_\infty$  control for a class of impulsive switched nonlinear systems with time-delay. Firstly, using piecewise Lyapunov function, sufficient conditions ensuring finite-time boundedness of the impulsive switched system are derived. Then, finite-time  $H_\infty$  performance analysis for impulsive switched systems is developed, and a robust finite-time  $H_\infty$  state feedback controller is proposed to guarantee that the resulting closed-loop system is finite-time bounded with  $H_\infty$  disturbance attenuation. All the results are given in terms of linear matrix inequalities (LMIs). Finally, two numerical examples are provided to show the effectiveness of the proposed method.

## 1. Introduction

A switched system is a hybrid dynamical system consisting of a family of continuous-time or discrete-time subsystems and a switching law that orchestrates the switching between them [1]. In the last decades, in the stability analysis and stabilization for switched systems, lots of valuable results are established (see [2–5]). Most recently, on the basis of Lyapunov functions and other analysis tools, the stability problem of linear and nonlinear switched systems with time-delay has been further investigated (see [6–15]), and lots of valuable results are established for  $H_\infty$  control problems (see [16–22]).

It is well known that impulsive dynamical behaviors inevitably exist in some practical systems like physical, biological, engineering, and information science systems due to abrupt changes at certain instants during the dynamical process. Although hybrid system and switched system are important models for dealing with complex real systems, there is little work concerned with the above impulsive phenomena. Such a phenomenon can be modeled

as an impulsive switched system, it is characteristic that their states change during the switching because of the occurrence of impulses [23].

In recent years, the impulsive switched systems have drawn more and more attention and many useful conclusions have been obtained. Multiple Krasovskii-Lyapunov function approach is employed to study the problem of ISS stability of a class of impulsive switched systems with time-delay in [24]. By the Lyapunov-Razumikhin technique, a delay-independent criterion of the exponential stability is established on the minimum dwell time in [25]. The problem of robust  $H_\infty$  stabilization of nonlinear impulsive switched system with time-delays is studied in [23].

Usually, the stability of a system is defined over an infinite-time interval. But in many practical systems, we focus on the dynamical behavior of a system over a fixed finite-time interval. Based on this, finite-time stability is first proposed by Dorato in 1961 [26]. Compared with the classical Lyapunov stability, finite-time stability is proposed for the study of the transient performance of the system, which is a totally different concept. The so-called finite-time stability means the boundedness of the state of a system over a fixed finite-time interval. Finite-time stability problems can be found in [27–32]. The finite-time stability of linear impulsive systems is analyzed in [33], the finite-time stability and stabilization of impulsive dynamic systems are carried out in [34–36]. The finite-time stability and stabilization of switched systems are investigated in [37].

Recently, robust finite-time control of switched systems is studied in [38, 39]. However, to the best of our knowledge, there are very few results on finite-time boundedness and robust  $H_\infty$  control of the impulsive switched systems, which motivates the present study. The paper is organized as follows. In Section 2, problem formulation and some necessary lemmas are given. In Section 3, based on the dwell time approach, finite-time boundedness and finite-time  $H_\infty$  performance for switched impulsive systems are addressed, and sufficient conditions for the existence of a robust finite-time  $H_\infty$  state feedback controller are proposed in terms of a set of matrix inequalities. Numerical examples are provided to show the effectiveness of the proposed approach in Section 4. Concluding remarks are given in Section 5.

*Notations.* The notations used in this paper are standard. The notation  $P > 0$  means that  $P$  is a real positive definite matrix;  $\text{diag}\{\dots\}$  stands for a block-diagonal matrix;  $\lambda_{\max}(P)$  and  $\lambda_{\min}(P)$  denote the maximum and minimum eigenvalues of matrix  $P$ , respectively;  $\|x(t)\| = \sqrt{x^T(t)x(t)}$  and  $\|x(t)\|_2 = (\int_0^\infty \|x(t)\|^2 dt)^{1/2}$ .

## 2. Problem Formulation and Preliminaries

Consider the following impulsive switched system:

$$\dot{x}(t) = \hat{A}_{\sigma(t)}x(t) + \hat{A}_{d\sigma(t)}x(t-h) + \hat{B}_{1\sigma(t)}u_1(t) + f_{\sigma(t)}(x(t)) + B_{2\sigma(t)}w(t), \quad t \neq t_k \quad (2.1a)$$

$$\Delta x = E_{\sigma(t)}x(t) + u_2(t), \quad t = t_k, \quad k = 1, 2, 3, \dots \quad (2.1b)$$

$$z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u_1(t), \quad (2.1c)$$

$$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0], \quad (2.1d)$$

where  $x(t) \in R^n$  is the state vector,  $z(t) \in R^r$  is the controlled output,  $w(t) \in R^p$  is the disturbance input which belongs to  $L_2[0, \infty)$ ,  $u_1(t) \in R^m$ ,  $t \neq t_k$  is the switched control input,  $u_2(t_k) \in R^n$  is the impulsive control input at  $t_k$ , on the other hand,  $u_2(t) = 0$ ,  $t \neq t_k$ ,  $k = 1, 2, 3, \dots$ .  $\sigma(t) : [t_0, +\infty) \rightarrow \bar{N} = \{1, 2, \dots, N\}$  is a switching signal.  $t \in (t_k, t_{k+1}]$ ,  $\sigma(t) = i_k$ ,  $i_k \in \bar{N}$ ,  $k = 0, 1, 2, 3, \dots$ .  $\Delta x(t) = x(t^+) - x(t^-)$ ,  $x(t^+) = \lim_{h \rightarrow 0^+} x(t+h)$ ,  $x(t^-) = \lim_{h \rightarrow 0^+} x(t-h)$ .  $t_k$ ,  $k = 0, 1, 2, 3, \dots$  are the impulsive jumping points or switching points.  $t_0$  is the initial time,  $t_0 < t_1 < \dots < t_k < \dots$ , and  $\lim_{k \rightarrow \infty} t_k = +\infty$ .  $h > 0$  is the time-delay which is a positive constant.  $f_i(\cdot) : R^n \rightarrow R^n$ ,  $i \in \bar{N}$  is nonlinear vector-valued function.  $\varphi(t)$ ,  $t \in [t_0 - h, t_0]$  is a continuous vector-valued initial function.  $\hat{A}_i$ ,  $\hat{A}_{di}$ ,  $\hat{B}_{1i}$ ,  $i \in \bar{N}$  are uncertain real-valued matrices with appropriate dimensions,  $B_{2i}$ ,  $E_i$ ,  $C_i$ ,  $D_i$ ,  $i \in \bar{N}$  are known real constant matrices with appropriate dimensions.

*Assumption 2.1.* For each  $i \in \bar{N}$ ,  $\hat{A}_i$ ,  $\hat{A}_{di}$ ,  $\hat{B}_{1i}$  are uncertain real-valued matrices with appropriate dimensions. We assume that the uncertainties are of the form

$$\hat{A}_i = A_i + \Delta A_i, \quad \hat{A}_{di} = A_{di} + \Delta A_{di}, \quad \hat{B}_{1i} = B_{1i} + \Delta B_{1i}, \quad (2.2a)$$

$$[\Delta A_i \quad \Delta A_{di} \quad \Delta B_{1i}] = H_i F_i(t) [E_{Ai} \quad E_{A_{di}} \quad E_{Bi}], \quad (2.2b)$$

where  $A_i$ ,  $A_{di}$ ,  $B_{1i}$ ,  $H_i$ ,  $E_{Ai}$ ,  $E_{A_{di}}$ , and  $E_{Bi}$  are known real-valued constant matrices with appropriate dimensions,  $F_i(t)$  is the uncertain matrix satisfying

$$F_i^T(t) F_i(t) \leq I. \quad (2.3)$$

*Assumption 2.2.* For each  $i \in \bar{N}$ , nonlinear vector-valued function  $f_i$  satisfies Lipschitz condition

$$\|f_i(x(t))\| \leq \|U_i x(t)\|, \quad (2.4)$$

where  $U_i$  is the Lipschitz constant matrix.

*Assumption 2.3.* For a given time constant  $T_f > t_0$ , the external disturbance  $w(t)$  satisfies

$$\int_0^{T_f} w^T(t) w(t) dt \leq d^2. \quad (2.5)$$

*Assumption 2.4.* For system (2.1a)–(2.1d), the impulsive jump matrices  $E_i$  satisfy that  $(I + E_i)$  are invertible.

*Definition 2.5* (see [32]). For a given time constant  $T_f > t_0$ , impulsive switched system (2.1a), (2.1b), (2.1c) and (2.1d) with  $u_1(t) \equiv 0$ ,  $u_2(t) \equiv 0$ , and  $w(t) \equiv 0$ , is said to be finite-time stable with respect to  $(c_1^2, c_2^2, T_f, R, \sigma(t))$  if the following inequality holds:

$$\sup_{t_0-h \leq \tau \leq t_0} x^T(\tau) R x(\tau) \leq c_1^2 \implies x^T(t) R x(t) < c_2^2, \quad t \in (t_0, T_f], \quad (2.6)$$

where  $c_2 > c_1 > 0$ ,  $R$  is a positive definite matrix, and  $\sigma(t)$  is a switching signal.

*Remark 2.6.* Equation (2.6) stands for the boundedness of the state of a system over a fixed finite-time interval  $(t_0, T_f]$ , when the initial state is bounded.

*Definition 2.7* (see [40]). For a given time constant  $T_f$ , impulsive switched system (2.1a)–(2.1d) with  $u_1(t) \equiv 0$ ,  $u_2(t) \equiv 0$ , and  $w(t)$  satisfying (2.5), is said to be finite-time bounded with respect to  $(c_1^2, c_2^2, T_f, d^2, R, \sigma(t))$  if the condition (2.6) holds, where  $c_2 > c_1 > 0$ ,  $R$  is a positive definite matrix and  $\sigma(t)$  is a switching signal.

*Definition 2.8.* For any  $T_2 > T_1 > 0$ , let  $N_{\sigma(t)}(T_1, T_2)$  denote the switching number of  $\sigma(t)$  on an interval  $(T_1, T_2)$ . If  $N_{\sigma(t)}(T_1, T_2) \leq N_0 + (T_2 - T_1)/\tau_a$  holds for given  $N_0 \geq 0$ ,  $\tau_a > 0$ , then the constant  $\tau_a$  is called the average dwell time. In this paper we let  $N_0 = 0$ .

*Definition 2.9.* For a given time constant  $T_f$ , impulsive switched system (2.1a)–(2.1d) with  $u_1(t) \equiv 0$ ,  $u_2(t) \equiv 0$  is said to have finite-time  $H_\infty$  performance with respect to  $(0, c_2^2, T_f, d^2, \gamma, R, \sigma(t))$  if the system is finite-time bounded and the following inequality holds:

$$\|z(t)\|_2 \leq \gamma \|w(t)\|_2, \quad \forall w(t) \in L_2[0, \infty), \quad (2.7)$$

where  $c_2 > 0$ ,  $\gamma > 0$ ,  $R$  is a positive definite matrix and  $\sigma(t)$  is a switching signal.

*Definition 2.10.* For a given time constant  $T_f$ , impulsive switched system (2.1a)–(2.1d) is said to be robust finite-time stabilization with  $H_\infty$  disturbance attenuation level  $\gamma$ , if there exists a switched controller  $u_1(t) = K_{\sigma(t)}x(t)$ ,  $t \neq t_k$  and an impulsive controller  $u_2(t_k) = \bar{K}_{\sigma(t)}x(t_k)$ ,  $t = t_k$ , where  $t \in (t_0, T_f]$  such that

(i) the corresponding closed-loop system is finite-time bounded with respect to  $(0, c_2^2, T_f, d^2, R, \sigma(t))$ ;

(ii) under zero initial condition, inequality (2.7) holds for any  $w(t)$  satisfying (2.5).

**Lemma 2.11.** Let  $U$ ,  $V$ ,  $W$ , and  $X$  be real matrices of appropriate dimensions with  $X$  satisfying  $X = X^T$ , then for all  $V^T V \leq I$ ,

$$X + UVW + W^T V^T U^T < 0, \quad (2.8)$$

if and only if there exists a scalar  $\varepsilon > 0$  such that

$$X + \varepsilon U U^T + \varepsilon^{-1} W^T W < 0. \quad (2.9)$$

### 3. Main Results

#### 3.1. Finite-Time Boundedness Analysis

In this subsection, we focus on the finite-time boundedness of the following impulsive switched system:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-h) + f_{\sigma(t)}(x(t)) + B_{2\sigma(t)}w(t), \quad t \neq t_k \quad (3.1a)$$

$$\Delta x = E_{\sigma(t)}x(t), \quad t = t_k, \quad k = 1, 2, 3, \dots \quad (3.1b)$$

$$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0]. \quad (3.1c)$$

Before proceeding to Lemma 3.2, we first introduce a function  $v(t)$ . For given positive definite matrices  $Q_{i_k}, i_k \in \bar{N}$ , by Assumption 2.4, there exists a real number  $\rho_{i_k} \geq 1$ ,  $\rho^* = \max\{\rho_{i_k}, i_k \in \bar{N}\}$  such that

$$Q_{i_{k-1}} \leq \rho_{i_k} (I + E_{i_{k-1}})^T Q_{i_k} (I + E_{i_{k-1}}). \quad (3.2)$$

Furthermore, we define the following function

$$v_k(t) = \rho_{i_k} - \frac{(t - t_k)^2}{(t_{k+1} - t_k)^2} (\rho_{i_k} - 1), \quad t \in (t_k, t_{k+1}]. \quad (3.3)$$

Finally, a piecewise continuous function  $v(t)$  is as follows:

$$v(t) = v_k(t), \quad t \in (t_k, t_{k+1}]. \quad (3.4)$$

Consider the function  $v(t)$ , for each interval  $(t_k, t_{k+1}]$ ,  $v(t_k^+) = \rho_{i_k}$ ,  $v(t_{k+1}) = 1$ , and  $v(t)$  is monotonically nonincreasing and bounded function,  $v(t_{k+1}) \leq v(t) \leq v(t_k^+)$ .

*Remark 3.1.* Note that the previous works require the condition  $Q_{i_{k-1}} \leq (I + E_{i_{k-1}})^T Q_{i_k} (I + E_{i_{k-1}})$  (see [23, 41]), which can be obtained by setting  $\rho_{i_k} = 1$  in (3.2). Thus, the proposed approach may provide more relaxed conditions.

**Lemma 3.2.** Consider the following Lyapunov functional candidate:

$$V(t) = x^T(t)P_{\sigma(t)}x(t) + \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \quad (3.5)$$

for system (3.1a), (3.1b), and (3.1c), where  $P_i$  and  $Q_i$ ,  $i \in \bar{N}$  are symmetric positive definite matrices with appropriate dimensions.

The following inequality is derived:

$$\begin{aligned} \dot{V}(t) &\leq 2x^T(t)P_{i_k}\dot{x}(t) + \alpha \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \\ &\quad + v(t)x^T(t)Q_{i_k}x(t) - v(t-h)x^T(t-h)Q_{i_{k-m}}x(t-h)e^{ah} \\ t &\in (t_k, t_{k+1}], \quad t-h \in (t_{k-m}, t_{k-m+1}], \quad m \in \{0, 1, 2, 3, \dots\}. \end{aligned} \quad (3.6)$$

*Proof.* (i) When  $t_k + h \geq t_{k+1}$ ,

$$\begin{aligned} V(t) &= x^T(t)P_{\sigma(t)}x(t) + \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \\ &= x^T(t)P_{i_k}x(t) + \int_{t-h}^{t_{k-m+1}} v(s)x^T(s)e^{\alpha(t-s)}Q_{i_{k-m}}x(s)ds \\ &\quad + \int_{t_{k-m+1}}^{t_{k-m+2}} v(s)x^T(s)e^{\alpha(t-s)}Q_{i_{k-m+1}}x(s)ds \cdots + \int_{t_k}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{i_k}x(s)ds, \\ \dot{V}(t) &= 2x^T(t)P_{i_k}\dot{x}(t) + \alpha \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \\ &\quad + v(t_{k-m+1})x^T(t_{k-m+1})e^{\alpha(t-t_{k-m+1})}Q_{i_{k-m}}x(t_{k-m+1}) \\ &\quad - v(t-h)x^T(t-h)e^{ah}Q_{i_{k-m}}x(t-h) \\ &\quad + v(t_{k-m+2})x^T(t_{k-m+2})e^{\alpha(t-t_{k-m+2})}Q_{i_{k-m+1}}x(t_{k-m+2}) \\ &\quad - v(t_{k-m+1}^+)x^T(t_{k-m+1}^+)e^{\alpha(t-t_{k-m+1})}Q_{i_{k-m+1}}x(t_{k-m+1}^+) \cdots + v(t_k)x^T(t_k)e^{\alpha(t-t_k)}Q_{i_{k-1}}x(t_k) \\ &\quad - v(t_{k-1}^+)x^T(t_{k-1}^+)e^{\alpha(t-t_k)}Q_{i_{k-1}}x(t_{k-1}^+) + v(t)x^T(t)Q_{i_k}x(t) \\ &\quad - v(t_k^+)x^T(t_k^+)e^{\alpha(t-t_k)}Q_{i_k}x(t_k^+), \\ \dot{V}(t) &= 2x^T(t)P_{i_k}\dot{x}(t) + \alpha \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \\ &\quad + v(t)x^T(t)Q_{i_k}x(t) - v(t-h)x^T(t-h)e^{ah}Q_{i_{k-m}}x(t-h) \\ &\quad + x^T(t_{k-m+1})e^{\alpha(t-t_{k-m+1})} \left[ Q_{i_{k-m}} - \rho_{i_{k-m+1}}(I + E_{i_{k-m}})^T Q_{i_{k-m+1}}(I + E_{i_{k-m}}) \right] x(t_{k-m+1}) \cdots \\ &\quad + x^T(t_k)e^{\alpha(t-t_k)} \left[ Q_{i_{k-1}} - \rho_{i_k}(I + E_{i_{k-1}})^T Q_{i_k}(I + E_{i_{k-1}}) \right] x(t_k). \end{aligned} \quad (3.7)$$

From (3.2), we can obtain that

$$\begin{aligned} Q_{i_{k-m}} - \rho_{i_{k-m+1}}(I + E_{i_{k-m}})^T Q_{i_{k-m+1}}(I + E_{i_{k-m}}) &\leq 0 \\ &\vdots \\ Q_{i_{k-1}} - \rho_{i_k}(I + E_{i_{k-1}})^T Q_{i_k}(I + E_{i_{k-1}}) &\leq 0. \end{aligned} \quad (3.8)$$

Combining (3.7) and (3.8), (3.6) is obtained.

(ii) When  $t_k + h < t_{k+1}$ ,

(1)  $t \in (t_k, t_k + h]$ , the proof is similar to the proof line in the situation (i).

(2)  $t \in (t_k + h, t_{k+1}]$ ,

$$\begin{aligned} V(t) &= x^T(t)P_{\sigma(t)}x(t) + \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \\ &= x^T(t)P_{i_k}x(t) + \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{i_k}x(s)ds. \end{aligned} \quad (3.9)$$

The proof for this situation is omitted.

The proof is completed.  $\square$

**Lemma 3.3.** Consider the following Lyapunov function:

$$V(t) = x^T(t)P_{\sigma(t)}x(t) + \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}Q_{\sigma(s)}x(s)ds \quad (3.10)$$

for system (3.1a), (3.1b), and (3.1c), where  $P_i$  and  $Q_i$ ,  $i \in \{1, 2, \dots, N\}$  are symmetric positive definite matrices with appropriate dimensions. Under the condition

$$\begin{bmatrix} -e^{\alpha h} \rho^* P_j & I + E_j^T & E_i^T \\ * & -P_i^{-1} & 0 \\ * & * & -e^{-\alpha h} (\rho^*)^{-1} Q_i^{-1} \end{bmatrix} < 0, \quad \forall i, j \in \overline{N}, \quad (3.11)$$

we have

$$V(t_k^+) < e^{\alpha h} \rho^* V(t_k), \quad (3.12)$$

where  $\rho^* = \max\{\rho_{i_k}, i_k \in \overline{N}\}$ .

*Proof.* Without loss of generality, let  $\sigma(t_k^+) = i$ ,  $\sigma(t_k) = j$ . Then, we have

$$\begin{aligned} V(t_k^+) &= x^T(t_k^+)P_{\sigma(t_k^+)}x(t_k^+) + \int_{t_k^+-h}^{t_k^+} v(s)x^T(s)e^{\alpha(t_k^+-s)}Q_{\sigma(s)}x(s)ds \\ &\leq x^T(t_k^+)P_{\sigma(t_k^+)}x(t_k^+) + e^{\alpha h}\rho^* \int_{t_k^+-h}^{t_k^+} x^T(s)Q_{\sigma(s)}x(s)ds \end{aligned} \quad (3.13)$$

$$\begin{aligned} &\leq x^T(t_k)(I + E_j)^T P_i(I + E_j)x(t_k) + e^{\alpha h}\rho^* x^T(t_k)E_j^T Q_i E_j x(t_k) \\ &\quad + e^{\alpha h}\rho^* \int_{t_k-h}^{t_k} v(s)x^T(s)e^{\alpha(t_k-s)}Q_{\sigma(s)}x(s)ds, \\ V(t_k) &= x^T(t_k)P_j x(t_k) + \int_{t_k-h}^{t_k} v(s)x^T(s)e^{\alpha(t_k-s)}Q_{\sigma(s)}x(s)ds. \end{aligned} \quad (3.14)$$

Combining (3.13) with (3.14), we have

$$\begin{aligned} V(t_k^+) - e^{\alpha h}\rho^* V(t_k) &\leq x^T(t_k)(I + E_j)^T P_i(I + E_j)x(t_k) \\ &\quad + e^{\alpha h}\rho^* x^T(t_k)E_j^T Q_i E_j x(t_k) - e^{\alpha h}\rho^* x^T(t_k)P_j x(t_k) \\ &= x^T(t_k)\Sigma_{ij}x(t_k), \end{aligned} \quad (3.15)$$

where

$$\Sigma_{ij} = (I + E_j)^T P_i(I + E_j) + e^{\alpha h}\rho^* E_j^T Q_i E_j - e^{\alpha h}\rho^* P_j. \quad (3.16)$$

Using Schur complement, (3.11) is equivalent to

$$\Sigma_{ij} < 0 \quad \text{or} \quad V(t_k^+) - e^{\alpha h}\rho^* V(t_k) < 0. \quad (3.17)$$

The proof is completed. □



**Theorem 3.4.**  $R$  is a positive definite matrix. Let  $\tilde{P}_i = R^{-1/2}P_iR^{-1/2}$ ,  $\tilde{Q}_i = R^{-1/2}Q_iR^{-1/2}$ , For all  $i \in \bar{N}$ , if there exist positive scalars  $\rho_i \geq 1$ ,  $i \in \bar{N}$ ,  $\rho^* = \max\{\rho_i, i \in \bar{N}\}$ ,  $\alpha$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  and symmetric positive matrices  $P_i, P_j, Q_i, T_i$ ,  $i, j \in \bar{N}$  such that

$$\frac{1}{\rho^*}(I + E_i)^{-1}\tilde{Q}_j(I + E_i)^{-T} - \tilde{Q}_i \leq 0, \quad \forall i, j \in \bar{N} \quad (3.18)$$

$$\begin{bmatrix} \tilde{P}_i A_i^T + A_i \tilde{P}_i - \alpha \tilde{P}_i + I & A_{di} \tilde{Q}_i & B_{2i} & \tilde{P}_i \\ * & -e^{ah} \tilde{Q}_j & 0 & 0 \\ * & * & -T_i & 0 \\ * & * & * & -(\rho^{*-1} \tilde{Q}_i + U_{i_k}^T U_{i_k}) \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \quad (3.19)$$

$$\begin{bmatrix} -e^{ah} \rho^* \tilde{P}_j & \tilde{P}_j (I + E_j^T) & \tilde{P}_j E_j^T \\ * & -\tilde{P}_i & 0 \\ * & * & -e^{-ah} (\rho^*)^{-1} \tilde{Q}_i \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \quad (3.20)$$

$$\lambda_1 R^{-1} < \tilde{P}_i < R^{-1}, \quad \lambda_2 R^{-1} < \tilde{Q}_i, \quad T_i < \lambda_3 I, \quad \forall i \in \bar{N} \quad (3.21)$$

$$\begin{bmatrix} -c_2^2 e^{-aT_f} + d^2 \lambda_3 & c_1 & c_1 \\ * & -\lambda_1 & 0 \\ * & * & -\frac{1}{\rho^* h} e^{-ah} \lambda_2 \end{bmatrix} < 0 \quad (3.22)$$

hold, under the average dwell time scheme

$$\tau_a > \tau_a^* = \frac{T_f(\alpha h + \ln \rho^*)}{\ln(c_2^2 e^{-aT_f}) - \ln[(1/\lambda_1 + h\rho^* e^{ah}/\lambda_2)c_1^2 + d^2 \lambda_3]}, \quad (3.23)$$

system (3.1a)–(3.1c) is finite-time bounded with respect to  $(c_1^2, c_2^2, T_f, d^2, R, \sigma(t))$ .

*Proof.* Assuming that when  $t \in (t_k, t_{k+1}]$ ,  $\sigma(t) = i_k$ ,  $i_k \in \bar{N}$ ,  $k = 0, 1, 2, 3, \dots$

Choose the following Lyapunov functional candidate:

$$V(t) = x^T(t) \tilde{P}_{\sigma(t)}^{-1} x(t) + \int_{t-h}^t v(s) x^T(s) e^{\alpha(t-s)} \tilde{Q}_{\sigma(s)}^{-1} x(s) ds. \quad (3.24)$$

When  $t \in (t_k, t_{k+1}]$ , according to (3.18) and Lemma 3.2, we have

$$\begin{aligned}
\dot{V}(t) &\leq 2x^T(t)\tilde{P}_{i_k}^{-1}\dot{x}(t) + \alpha \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}\tilde{Q}_{\sigma(s)}^{-1}x(s)ds \\
&\quad + v(t)x^T(t)\tilde{Q}_{i_k}^{-1}x(t) - v(t-h)x^T(t-h)\tilde{Q}_{i_{k-m}}^{-1}x(t-h)e^{\alpha h}, \\
\dot{V}(x(t)) - \alpha V(x(t)) - w^T(t)T_{i_k}w(t) &\leq 2x^T(t)\tilde{P}_{i_k}^{-1}\dot{x}(t) + \alpha \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}\tilde{Q}_{\sigma(s)}^{-1}x(s)ds \\
&\quad + v(t)x^T(t)\tilde{Q}_{i_k}^{-1}x(t) - v(t-h)x^T(t-h)\tilde{Q}_{i_{k-m}}^{-1}x(t-h)e^{\alpha h} \\
&\quad - \alpha x^T(t)\tilde{P}_{i_k}^{-1}x(t) - \alpha \int_{t-h}^t v(s)x^T(s)e^{\alpha(t-s)}\tilde{Q}_{\sigma(s)}^{-1}x(s)ds \\
&\quad - w^T(t)T_{i_k}w(t) \\
&\leq 2x^T(t)\tilde{P}_{i_k}^{-1}\dot{x}(t) + \rho^*x^T(t)\tilde{Q}_{i_k}^{-1}x(t) \\
&\quad - x^T(t-h)\tilde{Q}_{i_{k-m}}^{-1}x(t-h)e^{\alpha h} - \alpha x^T(t)\tilde{P}_{i_k}^{-1}x(t) \\
&\quad - w^T(t)T_{i_k}w(t).
\end{aligned} \tag{3.25}$$

According to (3.1a)–(3.1c), and (3.25), Assumption 2.2, and the following inequality:

$$\begin{aligned}
2x^T(t)\tilde{P}_{i_k}^{-1}f(x(t)) &\leq f_{i_k}^T(x(t))f_{i_k}(x(t)) + x^T(t)\tilde{P}_{i_k}^{-1}\tilde{P}_{i_k}^{-1}x(t) \\
&\leq x^T(t)U_{i_k}^TU_{i_k}x(t) + x^T(t)\tilde{P}_{i_k}^{-1}\tilde{P}_{i_k}^{-1}x(t),
\end{aligned} \tag{3.26}$$

we have

$$\dot{V}(x(t)) - \alpha V(x(t)) - w^T(t)T_{i_k}w(t) \leq X^T(t)\Xi_k X(t), \tag{3.27}$$

where  $X^T(t) = (x^T(t)x^T(t-h)w^T(t))$ ,

$$\Xi_k = \begin{bmatrix} \Delta_k & \tilde{P}_{i_k}^{-1}A_{di_k} & \tilde{P}_{i_k}^{-1}B_{2i_k} \\ * & -e^{\alpha h}\tilde{Q}_{i_{k-m}}^{-1} & 0 \\ * & * & -T_{i_k} \end{bmatrix}, \tag{3.28}$$

$$\Delta_k = A_{i_k}^T\tilde{P}_{i_k}^{-1} + \tilde{P}_{i_k}^{-1}A_{i_k} + \rho^*\tilde{Q}_{i_k}^{-1} - \alpha\tilde{P}_{i_k}^{-1} + U_{i_k}^TU_{i_k} + \tilde{P}_{i_k}^{-1}\tilde{P}_{i_k}^{-1}.$$

Using Schur complement, we obtain from (3.19) that

$$\begin{bmatrix} O_i & \tilde{P}_i^{-1}A_{di} & \tilde{P}_i^{-1}B_{2i} \\ * & -e^{\alpha h}\tilde{Q}_j^{-1} & 0 \\ * & * & -T_i \end{bmatrix} < 0, \tag{3.29}$$

where

$$O_i = A_i^T \tilde{P}_i^{-1} + \tilde{P}_i^{-1} A_i + \rho^* \tilde{Q}_i^{-1} - \alpha \tilde{P}_i^{-1} + U_i^T U_i + \tilde{P}_i^{-1} \tilde{P}_i^{-1}. \quad (3.30)$$

Noticing that the above inequality holds for all  $i, j \in \bar{N}$ , then we have  $\Xi_k < 0$  for  $i_k, i_{k-1} \in \bar{N}$ . Thus,

$$\dot{V}(x(t)) - \alpha V(x(t)) - w^T(t) T_{i_k} w(t) < 0. \quad (3.31)$$

When  $t \in (t_k, t_{k+1}]$ , according to Lemma 3.3, we can obtain (3.12) from condition (3.20).

Combining (3.31) and (3.12), we can obtain that

$$\begin{aligned} V(t) &< e^{\alpha(t-t_k)} V(t_k^+) + \int_{t_k}^t e^{\alpha(t-s)} w^T(s) T_{i_k} w(s) ds \\ &< e^{\alpha(t-t_k)} e^{\alpha h} \rho^* V(t_k) + \int_{t_k}^t e^{\alpha(t-s)} w^T(s) T_{i_k} w(s) ds \\ &< e^{\alpha(t-t_k)} e^{\alpha h} \rho^* \left[ e^{\alpha(t_k-t_{k-1})} V(t_{k-1}^+) + \int_{t_{k-1}}^{t_k} e^{\alpha(t_k-s)} w^T(s) T_{i_{k-1}} w(s) ds \right] \\ &\quad + \int_{t_k}^t e^{\alpha(t-s)} w^T(s) T_{i_k} w(s) ds \\ &< \dots \\ &< e^{\alpha(t-t_0)} \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} V(t_0) + \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} \int_{t_0}^{t_1} e^{\alpha(t-s)} w^T(s) T_{i_0} w(s) ds \\ &\quad + \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_1,t)} \int_{t_1}^{t_2} e^{\alpha(t-s)} w^T(s) T_{i_1} w(s) ds \\ &\quad + \dots + e^{\alpha h} \rho^* \int_{t_{k-1}}^{t_k} e^{\alpha(t-s)} w^T(s) T_{i_{k-1}} w(s) ds + \int_{t_k}^t e^{\alpha(t-s)} w^T(s) T_{i_k} w(s) ds \\ &= e^{\alpha(t-t_0)} \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} V(t_0) + \int_{t_0}^t e^{\alpha(t-s)} \left( e^{\alpha h} \rho^* \right)^{N_\sigma(s,t)} w^T(s) T_{i_k} w(s) ds \\ &< e^{\alpha t} \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} V(t_0) + \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} e^{\alpha t} \int_{t_0}^t w^T(s) T_{i_k} w(s) ds \\ &< e^{\alpha T_f} \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} \left[ V(t_0) + \int_{t_0}^{T_f} w^T(s) T_{i_k} w(s) ds \right] \\ &< e^{\alpha T_f} \left( e^{\alpha h} \rho^* \right)^{N_\sigma(t_0,t)} \left[ V(t_0) + \lambda_{\max}(T_{i_k}) d^2 \right]. \end{aligned} \quad (3.32)$$

Noticing that  $N_\sigma(t_0, T_f) < T_f/\tau_a$  and according to (3.21), we have

$$\begin{aligned} V(t) &< e^{(\alpha+\alpha h/\tau_a)T_f} (\rho^*)^{T_f/\tau_a} \left[ V(t_0) + \lambda_3 d^2 \right], \\ V(t) &\geq x^T(t) \tilde{P}_{i_k}^{-1} x(t) = x^T(t) R^{1/2} P_{i_k}^{-1} R^{1/2} x(t) \\ &\geq \lambda_{\min}(P_{i_k}^{-1}) x^T(t) R x(t) = \frac{1}{\lambda_{\max}(P_{i_k})} x^T(t) R x(t). \end{aligned} \quad (3.33)$$

Because  $\lambda_1 R^{-1} < \tilde{P}_i < R^{-1}$ , we have

$$V(t) > x^T(t) R x(t). \quad (3.34)$$

According to the Lyapunov function that we have chosen, we have

$$\begin{aligned} V(t_0) &= x^T(t_0) \tilde{P}_i^{-1} x(t_0) + \int_{t_0-h}^{t_0} v(s) x^T(s) e^{-\alpha(t_0-s)} \tilde{Q}_i^{-1} x(s) ds \\ &\leq \max_{i \in \bar{N}} \lambda_{\max}(P_i^{-1}) x^T(t_0) R x(t_0) \\ &\quad + h e^{\alpha h} \rho^* \max_{i \in \bar{N}} \lambda_{\max}(Q_i^{-1}) \sup_{t_0-h \leq \theta \leq t_0} x^T(\theta) R x(\theta) \\ &\leq \left( \frac{1}{\min_{i \in \bar{N}} \lambda_{\min}(P_i)} + \frac{\rho^* h e^{\alpha h}}{\min_{i \in \bar{N}} \lambda_{\min}(Q_i)} \right) \sup_{t_0-h \leq \theta \leq t_0} x^T(\theta) R x(\theta). \end{aligned} \quad (3.35)$$

According to (3.21), the following inequality is derived:

$$V(t_0) < \left( \frac{1}{\lambda_1} + \frac{\rho^* h e^{\alpha h}}{\lambda_2} \right) c_1^2. \quad (3.36)$$

Combining (3.33), (3.34), and (3.36), we can obtain that

$$x^T(t) R x(t) < V(t) < e^{(\alpha+\alpha h/\tau_a)T_f} (\rho^*)^{T_f/\tau_a} \left[ \left( \frac{1}{\lambda_1} + \frac{\rho^* h e^{\alpha h}}{\lambda_2} \right) c_1^2 + \lambda_3 d^2 \right]. \quad (3.37)$$

Using Schur complement, (3.22) is equivalent to

$$\left( \frac{1}{\lambda_1} + \frac{\rho^* h e^{\alpha h}}{\lambda_2} \right) c_1^2 + \lambda_3 d^2 < c_2^2 e^{-\alpha T_f}. \quad (3.38)$$

From (3.38), we can obtain that  $\tau_a > 0$ .

Substituting (3.23) into (3.37) leads to

$$x^T(t)Rx(t) < c_2^2. \quad (3.39)$$

Thus, system (3.1a)–(3.1c) is finite-time bounded with respect to  $(c_1^2, c_2^2, T_f, d^2, R, \sigma(t))$ .

The proof is completed.  $\square$

**Corollary 3.5.** *R is a positive definite matrix, let  $w(t) \equiv 0$ ,  $\tilde{P}_i = R^{-1/2}P_iR^{-1/2}$ ,  $\tilde{Q}_i = R^{-1/2}Q_iR^{-1/2}$  for all  $i \in \bar{N}$ . If there exist positive scalars  $\rho_i \geq 1$ ,  $i \in \bar{N}$ ,  $\rho^* = \max\{\rho_i, i \in \bar{N}\}$ ,  $\alpha, \lambda_1, \lambda_2$  and symmetric positive matrices  $P_i, P_j, Q_i$  for all  $i, j \in \bar{N}$  with appropriate dimensions such that*

$$\begin{aligned} & \frac{1}{\rho^*}(I + E_i)^{-1}\tilde{Q}_j(I + E_i)^{-T} - \tilde{Q}_i \leq 0, \quad \forall i, j \in \bar{N} \\ & \begin{bmatrix} \tilde{P}_i A_i^T + A_i \tilde{P}_i - \alpha \tilde{P}_i + I & A_{di} \tilde{Q}_i & \tilde{P}_i \\ * & -e^{ah} \tilde{Q}_j & 0 \\ * & * & -(\rho^{*-1} \tilde{Q}_i + U_{i_k}^T U_{i_k}) \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \\ & \begin{bmatrix} -e^{ah} \rho^* \tilde{P}_j & \tilde{P}_j (I + E_j^T) & \tilde{P}_j E_j^T \\ * & -\tilde{P}_i & 0 \\ * & * & -e^{-ah} (\rho^*)^{-1} \tilde{Q}_i \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \\ & \lambda_1 R^{-1} < \tilde{P}_i < R^{-1}, \quad \lambda_2 R^{-1} < \tilde{Q}_i, \quad \forall i \in \bar{N} \\ & \begin{bmatrix} -c_2^2 e^{-\alpha T_f} & c_1 & c_1 \\ * & -\lambda_1 & 0 \\ * & * & -\frac{1}{\rho^* h} e^{-ah} \lambda_2 \end{bmatrix} < 0 \end{aligned} \quad (3.40)$$

hold with average dwell time

$$\tau_a > \tau_a^* = \frac{T_f(\alpha h + \ln \rho^*)}{\ln(c_2^2 e^{-\alpha T_f}) - \ln[(1/\lambda_1 + \rho^* h e^{\alpha h} / \lambda_2) c_1^2]}. \quad (3.41)$$

System (3.1a)–(3.1c) with  $w(t) \equiv 0$  is finite-time stable with respect to  $(c_1^2, c_2^2, T_f, R, \sigma(t))$ .

### 3.2. $H_\infty$ Performance Analysis

In this subsection,  $H_\infty$  performance of the following system is investigated:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-h) + f_{\sigma(t)}(x(t)) + B_{2\sigma(t)}w(t), \quad t \neq t_k \quad (3.42a)$$

$$\Delta x = E_{\sigma(t)}x(t), \quad t = t_k, \quad k = 1, 2, 3, \dots \quad (3.42b)$$

$$z(t) = C_{\sigma(t)}x(t), \quad (3.42c)$$

$$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0] \quad (3.42d)$$

**Theorem 3.6.**  $R$  is a positive definite matrix. Let  $\tilde{P}_i = R^{-1/2}P_iR^{-1/2}$ ,  $\tilde{Q}_i = R^{-1/2}Q_iR^{-1/2}$  for all  $i \in \bar{N}$ . Suppose that there exist positive scalars  $\rho_i \geq 1, i \in \bar{N}$ ,  $\rho^* = \max\{\rho_i, i \in \bar{N}\}$ ,  $\alpha, \gamma, \varepsilon$  and symmetric positive matrices  $P_i, P_j, Q_i$  for all  $i, j \in \bar{N}$  such that

$$\frac{1}{\rho^*}(I + E_i)^{-1}\tilde{Q}_j(I + E_i)^{-T} - \tilde{Q}_i \leq 0, \quad \forall i, j \in \bar{N} \quad (3.43)$$

$$\begin{bmatrix} \tilde{P}_i A_i^T + A_i \tilde{P}_i - \alpha \tilde{P}_i + I & A_{di} \tilde{Q}_i & B_{2i} & \tilde{P}_i & \tilde{P}_i C_i^T \\ * & -e^{ah} \tilde{Q}_j & 0 & 0 & 0 \\ * & * & -\gamma^2 & 0 & 0 \\ * & * & * & -(\rho^{*-1} \tilde{Q}_i + U_{i_k}^T U_{i_k}) & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \quad (3.44)$$

$$\begin{bmatrix} -e^{ah} \rho^* \tilde{P}_j & \tilde{P}_j (I + E_j^T) & \tilde{P}_j E_j^T \\ * & -\tilde{P}_i & 0 \\ * & * & -e^{-ah} (\rho^*)^{-1} \tilde{Q}_i \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \quad (3.45)$$

$$\tilde{P}_i < R^{-1}, \quad \forall i \in \bar{N} \quad (3.46)$$

$$-c_2^2 + e^{\alpha T_f} \gamma^2 d^2 < 0 \quad (3.47)$$

hold with average dwell time

$$\tau_a > \tau_a^* = \max \left\{ \frac{T_f(\alpha h + \ln \rho^*)}{\ln(c_2^2) - \ln(e^{\alpha T_f} \gamma^2 d^2)}, \frac{h}{\varepsilon} \right\}. \quad (3.48)$$

Then, system (3.42a)–(3.42d) is finite-time bounded and has  $H_\infty$  performance with respect to  $(0, c_2^2, T_f, d^2, \bar{\gamma}, R, \sigma(t))$ , where  $\bar{\gamma}^2 = e^{(1+\varepsilon)\alpha T_f} (\rho^*)^{\varepsilon T_f/h} \gamma^2$ .

*Proof.* When  $t \in (t_k, t_{k+1}]$ ,  $\sigma(t) = i_k$ ,  $i_k \in \bar{N}$ ,  $k = 0, 1, 2, 3, \dots$ . Choose the following Lyapunov functional candidate for system (3.42a)–(3.42d)

$$V(t) = x^T(t) \tilde{P}_{\sigma(t)}^{-1} x(t) + \int_{t-h}^t v(s) x^T(s) e^{\alpha(t-s)} \tilde{Q}_{\sigma(s)}^{-1} x(s) ds. \quad (3.49)$$

When  $t \in (t_k, t_{k+1}]$ ,

$$\dot{V}(x(t)) - \alpha V(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \leq X^T(t)\Psi_k X(t), \quad (3.50)$$

where  $X^T(t) = (x^T(t) \ x^T(t-h) \ w^T(t))$ ,

$$\Psi_k = \begin{bmatrix} \Delta_k & \tilde{P}_{i_k}^{-1} A_{di_k} & \tilde{P}_{i_k}^{-1} B_{2i_k} \\ * & -e^{ah} \tilde{Q}_{i_k-m}^{-1} & 0 \\ * & * & -\gamma^2 I \end{bmatrix}, \quad (3.51)$$

$$\Delta_k = A_{i_k}^T \tilde{P}_{i_k}^{-1} + \tilde{P}_{i_k}^{-1} A_{i_k} + \rho^* \tilde{Q}_{i_k}^{-1} - \alpha \tilde{P}_{i_k}^{-1} + U_{i_k}^T U_{i_k} + \tilde{P}_{i_k}^{-1} \tilde{P}_{i_k}^{-1} + C_{i_k}^T C_{i_k}.$$

Using Schur complement, we obtain from (3.44) that

$$\begin{bmatrix} E_i & \tilde{P}_i^{-1} A_{di} & \tilde{P}_i^{-1} B_{2i} \\ * & -e^{ah} \tilde{Q}_j^{-1} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (3.52)$$

where  $E_i = A_i^T \tilde{P}_i^{-1} + \tilde{P}_i^{-1} A_i + \rho^* \tilde{Q}_i^{-1} - \alpha \tilde{P}_i^{-1} + U_i^T U_i + \tilde{P}_i^{-1} \tilde{P}_i^{-1} + C_i^T C_i$ .

Noticing that the above inequality holds for all  $i, j \in \bar{N}$ , then we have  $\Psi_k < 0$ , for  $i_k, i_{k-m} \in \bar{N}$ .

Thus,

$$\dot{V}(x(t)) - \alpha V(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) < 0, \quad (3.53)$$

Let  $\gamma^2 w^T(s)w(s) - z^T(s)z(s) = \Delta(s)$ , from (3.32), we have

$$V(t) < e^{\alpha(t-t_0)} \left( e^{ah} \rho^* \right)^{N_\sigma(t_0,t)} V(t_0) + \int_{t_0}^t e^{\alpha(t-s)} \left( e^{ah} \rho^* \right)^{N_\sigma(s,t)} \Delta(s) ds. \quad (3.54)$$

Under zero initial condition, we have

$$0 < \int_{t_0}^t e^{\alpha(t-s)} \left( e^{ah} \rho^* \right)^{N_\sigma(s,t)} \Delta(s) ds, \quad (3.55)$$

that is,

$$\int_{t_0}^t e^{\alpha(t-s)} \left( e^{ah} \rho^* \right)^{N_\sigma(s,t)} z^T(s)z(s) ds < \int_{t_0}^t e^{\alpha(t-s)} \left( e^{ah} \rho^* \right)^{N_\sigma(s,t)} \gamma^2 w^T(s)w(s) ds. \quad (3.56)$$

Noticing that

$$\int_{t_0}^t e^{\alpha(t-s)} \left( e^{ah} \rho^* \right)^{N_\sigma(s,t)} z^T(s)z(s) ds > \int_{t_0}^t z^T(s)z(s) ds. \quad (3.57)$$

Then, we have

$$\int_{t_0}^t e^{\alpha(t-s)} \left( e^{\alpha h} \rho^* \right)^{N_{\sigma}(s,t)} \gamma^2 w^T(s) w(s) ds < e^{\alpha t} \left( e^{\alpha h} \rho^* \right)^{N_{\sigma}(t_0,t)} \int_{t_0}^t \gamma^2 w^T(s) w(s) ds. \quad (3.58)$$

Let  $t = T_f$ , because  $\tau_a > h/\varepsilon$ , we have

$$\int_{t_0}^{T_f} z^T(s) z(s) ds < e^{(1+\varepsilon)\alpha T_f} (\rho^*)^{\varepsilon T_f/h} \gamma^2 \int_{t_0}^{T_f} w^T(s) w(s) ds, \quad (3.59)$$

then

$$\int_{t_0}^{T_f} z^T(s) z(s) ds < \bar{\gamma}^2 \int_{t_0}^{T_f} w^T(s) w(s) ds. \quad (3.60)$$

Thus, system (3.42a)–(3.42d) is finite-time bounded and has  $H_{\infty}$  performance with respect to  $(0, c_2^2, T_f, d^2, \bar{\gamma}, R, \sigma(t))$ , where  $\bar{\gamma}^2 = e^{(1+\varepsilon)\alpha T_f} (\rho^*)^{\varepsilon T_f/h} \gamma^2$ .

The proof is completed.  $\square$

*Remark 3.7.* When  $\rho^* = 1$ , Theorem 3.6 degenerates to the result of [41], which cannot guarantee the finite-time boundedness of the addressed system if  $\rho^* > 1$ .

### 3.3. Robust Finite-Time $H_{\infty}$ Control

Consider system (2.1a)–(2.1d), under the switching controller  $u_1(t) = K_{\sigma(t)} x(t)$ ,  $t \neq t_k$  and impulsive controller  $u_2(t_k) = \bar{K}_{\sigma(t)} x(t_k)$ ,  $t = t_k$ , the corresponding closed-loop system is given by

$$\dot{x}(t) = \left( \hat{A}_{\sigma(t)} + \hat{B}_{1\sigma(t)} K_{\sigma(t)} \right) x(t) + \hat{A}_{d\sigma(t)} x(t-h) + f_{\sigma(t)}(x(t)) + B_{2\sigma(t)} w(t), \quad t \neq t_k \quad (3.61a)$$

$$\Delta x = \left( E_{\sigma(t)} + \bar{K}_{\sigma(t)} \right) x(t), \quad t = t_k, \quad k = 1, 2, 3, \dots \quad (3.61b)$$

$$z(t) = \left( C_{\sigma(t)} + D_{\sigma(t)} K_{\sigma(t)} \right) x(t), \quad (3.61c)$$

$$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0]. \quad (3.61d)$$

**Theorem 3.8.** Consider impulsive switched system (2.1a)–(2.1d), let  $\tilde{P}_i = R^{-1/2} P_i R^{-1/2}$ ,  $\tilde{Q}_i = R^{-1/2} Q_i R^{-1/2}$  for all  $i \in \bar{N}$ . If there exist positive scalars  $\rho_i \geq 1$ ,  $i \in \bar{N}$ ,  $\rho^* = \max\{\rho_i, i \in \bar{N}\}$ ,  $\alpha, \gamma, \varepsilon, \delta_i$  and positive definite symmetric matrices  $P_i, Q_i$ , and matrices  $Y_i$ ,  $i \in \bar{N}$ , with appropriate dimensions, such that the following inequalities hold



$$\frac{1}{\rho^*} (I + E_i)^{-1} \tilde{Q}_j (I + E_i)^{-T} - \tilde{Q}_i \leq 0, \quad \forall i, j \in \bar{N} \quad (3.62)$$

$$\begin{bmatrix} \Gamma_i & A_{di} \tilde{Q}_i & B_{2i} & \tilde{P}_i & \tilde{P}_i C_i^T + Y_i^T D_i^T & Y_i^T E_{Bi}^T + \tilde{P}_i E_{Ai}^T \\ * & -e^{\alpha h} \tilde{Q}_j & 0 & 0 & 0 & \tilde{Q}_i E_{A_{di}}^T \\ * & * & -\gamma^2 & 0 & 0 & 0 \\ * & * & * & -(\rho^{*-1} \tilde{Q}_i + U_{i_k}^T U_{i_k}) & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\delta_i \end{bmatrix} < 0, \quad \forall i, j \in \bar{N}, \quad (3.63)$$

where

$$\begin{aligned} \Gamma_i &= \tilde{P}_i A_i^T + Y_i^T B_{1i}^T + A_i \tilde{P}_i + B_{1i} Y_i - \alpha \tilde{P}_i + I + \delta_i H_i H_i^T, \\ &\begin{bmatrix} -e^{\alpha h} \rho^* \tilde{P}_j & \tilde{P}_j \\ * & -\tilde{P}_i \end{bmatrix} < 0, \quad \forall i, j \in \bar{N} \end{aligned} \quad (3.64)$$

$$\tilde{P}_i < R^{-1}, \quad i \in \bar{N}, \quad (3.65)$$

$$-c_2^2 + e^{\alpha T_f} \gamma^2 d^2 < 0. \quad (3.66)$$

Then, under the controller  $K_i = Y_i \tilde{P}_i^{-1}$ ,  $\bar{K}_i = -E_i$ , and the following average dwell time scheme

$$\tau_a > \tau_a^* = \max \left\{ \frac{T_f (\alpha h + \ln \rho^*)}{\ln(c_2^2) - \ln(e^{\alpha T_f} \gamma^2 d^2)}, \frac{h}{\varepsilon} \right\}, \quad (3.67)$$

the corresponding closed-loop system is finite-time bounded with  $H_\infty$  performance with respect to  $(0, c_2^2, T_f, d^2, \bar{\gamma}, R, \sigma(t))$  and  $\bar{\gamma}^2 = e^{(1+\varepsilon)\alpha T_f} (\rho^*)^{\varepsilon T_f / h} \gamma^2$ .

*Proof.* According to Assumption 2.1, we have

$$\hat{A}_i + \hat{B}_{1i} K_i = (A_i + B_{1i} K_i) + H_i F_i (E_{Ai} + E_{Bi} K_i), \quad \hat{A}_{di} = A_{di} + H_i F_i E_{A_{di}}. \quad (3.68)$$

Now replacing  $A_i, A_{di}, C_i$  in the left side of (3.44) with  $\hat{A}_i + \hat{B}_{1i} K_i, \hat{A}_{di}, C_i + D_i K_i$ , we can obtain that

$$\Theta_{ij} = \begin{bmatrix} \Omega_i & (A_{di} + H_i F_i E_{A_{di}}) \tilde{Q}_i & B_{2i} & \tilde{P}_i & \tilde{P}_i (C_i + D_i K_i)^T \\ * & -e^{\alpha h} \tilde{Q}_j & 0 & 0 & 0 \\ * & * & -\gamma^2 & 0 & 0 \\ * & * & * & -(\rho^{*-1} \tilde{Q}_i + U_{i_k}^T U_{i_k}) & 0 \\ * & * & * & * & -I \end{bmatrix}, \quad (3.69)$$

where

$$\Omega_i = [(A_i + B_{1i}K_i) + H_iF_i(E_{Ai} + E_{Bi}K_i)]\tilde{P}_i + \tilde{P}_i[(A_i + B_{1i}K_i) + H_iF_i(E_{Ai} + E_{Bi}K_i)]^T - \alpha\tilde{P}_i + I. \quad (3.70)$$

From (3.69), we know that

$$\Theta_{ij} = \Pi_{1ij} + \Pi_{2ij}, \quad (3.71)$$

where

$$\Pi_{1ij} = \begin{bmatrix} \Upsilon_{1i} & A_{di}\tilde{Q}_i & B_{2i} & \tilde{P}_i & \tilde{P}_i(C_i + D_iK_i)^T \\ * & -e^{ah}\tilde{Q}_j & 0 & 0 & 0 \\ * & * & -\gamma^2 & 0 & 0 \\ * & * & * & -(\rho^{*-1}\tilde{Q}_i + U_{ik}^T U_{ik}) & 0 \\ * & * & * & * & -I \end{bmatrix}, \quad (3.72)$$

$$\Pi_{2ij} = \begin{bmatrix} \Upsilon_{2i} & H_iF_iE_{Adi}\tilde{Q}_i & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

with

$$\begin{aligned} \Upsilon_{1i} &= \tilde{P}_i(A_i + B_{1i}K_i)^T + (A_i + B_{1i}K)\tilde{P}_i - \alpha\tilde{P}_i + I, \\ \Upsilon_{2i} &= \tilde{P}_i(E_{Ai} + E_{Bi}K_i)^T F_i^T H_i^T + H_iF_i(E_{Ai} + E_{Bi}K_i)\tilde{P}_i, \end{aligned} \quad (3.73)$$

let  $Y_i = K_i\tilde{P}_i$ , then

$$\begin{aligned} \Upsilon_{1i} &= \tilde{P}_iA_i^T + Y_i^T B_{1i}^T + A_i\tilde{P}_i + B_{1i}Y_i - \alpha\tilde{P}_i + I, \\ \Upsilon_{2i} &= (Y_i^T E_{Bi}^T + \tilde{P}_i E_{Ai}^T) F_i^T H_i^T + H_iF_i(E_{Ai}\tilde{P}_i + E_{Bi}Y_i). \end{aligned} \quad (3.74)$$

From Lemma 2.11, we can obtain that

$$\begin{aligned}
\Theta_{ij} &= \Pi_{1ij} + \begin{bmatrix} H_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} E_{Ai} \tilde{P}_i + E_{Bi} Y_i & E_{Adi} \tilde{Q}_i & 0 & 0 & 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} Y_i^T E_{Bi}^T + \tilde{P}_i E_{Ai}^T \\ \tilde{Q}_i E_{Adi}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i \begin{bmatrix} H_i^T & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\leq \Pi_{1ij} + \delta_i \begin{bmatrix} H_i \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} H_i^T & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\quad + \frac{1}{\delta_i} \begin{bmatrix} Y_i^T E_{Bi}^T + \tilde{P}_i E_{Ai}^T \\ \tilde{Q}_i E_{Adi}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_{Ai} \tilde{P}_i + E_{Bi} Y_i & E_{Adi} \tilde{Q}_i & 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{3.75}$$

Using Schur complement lemma, we get from (3.63) that

$$\Theta_{ij} < 0. \tag{3.76}$$

Now we choose  $\bar{K}_i = -E_i$ , and replacing  $E_i$  in (3.45) with  $E_i + \bar{K}_i$ , we know that

$$\begin{bmatrix} -e^{\alpha h} \rho^* \tilde{P}_j & \tilde{P}_j & 0 \\ * & -\tilde{P}_i & 0 \\ * & * & -e^{-\alpha h} (\rho^*)^{-1} \tilde{Q}_i \end{bmatrix} < 0, \tag{3.77}$$

by (3.64), we know that the condition(3.45) hold.

Then, system (2.1a)–(2.1d) is robust finite-time bounded with  $H_\infty$  performance with respect to  $(0, c_2^2, T_f, d^2, \bar{\gamma}, R, \sigma(t))$ , and  $\bar{\gamma}^2 = e^{(1+\varepsilon)\alpha T_f} (\rho^*)^{\varepsilon T_f/h} \gamma^2$ .

The proof is completed.  $\square$

*Remark 3.9.* In order to eliminate the impulsive jump, we design an impulsive feedback controller  $\bar{K}_i = -E_i$ ,  $t = t_k$ . Then the system becomes a switched system with continuous states.

#### 4. Numerical Examples

In this section, we present two examples to illustrate the effectiveness of the proposed approach.

*Example 4.1.* Consider system (2.1a)–(2.1d) with the following parameters.

*Subsystem 1*

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -8 & 1 \\ 2 & -7 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} -1.3 & 0.1 \\ 0.2 & -1 \end{bmatrix}, & H_1 &= \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix}, & E_{Ad1} &= \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & -0.2 \end{bmatrix}, \\
 E_1 &= \begin{bmatrix} 0.43 & 0 \\ 0 & 0.15 \end{bmatrix}, & U_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, & B_{11} &= \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix}, \\
 E_{B1} &= \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.3 \end{bmatrix}, & B_{21} &= \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \\
 C_1 &= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, & D_1 &= \begin{bmatrix} 0.8 & 0 \\ 0 & 0.2 \end{bmatrix}, & E_{A1} &= \begin{bmatrix} -0.3 & -0.1 \\ 0.2 & -0.1 \end{bmatrix},
 \end{aligned} \tag{4.1}$$

$f_1(x(t)) = 0.1 \sin x(t)$ , where  $\|f_1(x(t))\| < \|U_1 x(t)\|$ .

*Subsystem 2*

$$\begin{aligned}
 A_2 &= \begin{bmatrix} -7 & 2 \\ 1 & -6 \end{bmatrix}, & A_{d2} &= \begin{bmatrix} -1.2 & 0.1 \\ 0.3 & -1.1 \end{bmatrix}, & H_2 &= \begin{bmatrix} -0.1 & 0.2 \\ -0.2 & -0.1 \end{bmatrix}, & E_{Ad2} &= \begin{bmatrix} -0.3 & 0.1 \\ 0.2 & -0.3 \end{bmatrix}, \\
 E_2 &= \begin{bmatrix} 0.15 & 0 \\ 0 & 0.4 \end{bmatrix}, & U_2 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.18 \end{bmatrix}, & B_{12} &= \begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix}, & E_{B2} &= \begin{bmatrix} -0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \\
 B_{22} &= \begin{bmatrix} -1 & 0 \\ 2 & 0.8 \end{bmatrix}, & C_2 &= \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.8 & 0 \\ 1 & -1 \end{bmatrix}, & E_{A2} &= \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & -0.2 \end{bmatrix},
 \end{aligned} \tag{4.2}$$

$f_2(x(t)) = 0.18 \cos x(t)$ , where  $\|f_2(x(t))\| < \|U_2 x(t)\|$ .

Choosing  $T_f = 12, h = 0.2, d^2 = 10, R = I, \alpha = 0.1, C_2^2 = 2, \varepsilon = 0.1, \gamma^2 = 0.5441, \rho^* = 1$ , solving the LMIs in (3.62)–(3.66) leads to

$$\begin{aligned}
\tilde{Q}_1 &= \begin{bmatrix} 1.3506 & -0.1265 \\ -0.1265 & 0.7891 \end{bmatrix}, & \tilde{Q}_2 &= \begin{bmatrix} 0.5042 & 0.0525 \\ 0.0525 & 0.3221 \end{bmatrix}, & \Upsilon_1 &= \begin{bmatrix} 0.0234 & -0.3577 \\ 0.1631 & 0.2680 \end{bmatrix}, \\
\Upsilon_2 &= \begin{bmatrix} -0.0001 & -0.5221 \\ 0.1109 & 0.0371 \end{bmatrix}, & \tilde{P}_1 &= \begin{bmatrix} 0.9887 & 0.0011 \\ 0.0011 & 0.9921 \end{bmatrix}, & \tilde{P}_2 &= \begin{bmatrix} 0.9995 & -0.0001 \\ -0.0001 & 1.0006 \end{bmatrix}, \\
K_1 &= \begin{bmatrix} 0.0241 & -0.3605 \\ 0.1647 & 0.2699 \end{bmatrix}, & K_2 &= \begin{bmatrix} -0.0001 & -0.5218 \\ 0.1109 & 0.0371 \end{bmatrix}, & & (4.3) \\
\tilde{Q}_1^{-1} - (I + E_1)^T \tilde{Q}_1^{-1} (I + E_1) &\leq 0, \\
\tilde{Q}_2^{-1} - (I + E_2)^T \tilde{Q}_2^{-1} (I + E_2) &\leq 0, \\
\tilde{Q}_2^{-1} - (I + E_2)^T \tilde{Q}_2^{-1} (I + E_2) &\leq 0, \\
\tilde{Q}_1^{-1} - (I + E_1)^T \tilde{Q}_2^{-1} (I + E_1) &\leq 0,
\end{aligned}$$

$\tau_a > \tau_a^* = 1.2049$ , we choose  $\tau_a = 2$ ,  $\bar{\gamma}^2 = e^{(1+\varepsilon)\alpha T_f} (\rho^*)^{\varepsilon \alpha T_f} \gamma^2 = 2.0368$ , then the system is finite-time bounded according to [41, Theorem 3].

*Example 4.2.* Consider system (2.1a)–(2.1d) with the following parameters.

#### Subsystem 1

$$\begin{aligned}
A_1 &= \begin{bmatrix} -8 & 1 \\ 2 & -7 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} -1.3 & 0.1 \\ 0.2 & -1 \end{bmatrix}, & H_1 &= \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix}, & E_{Ad1} &= \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & -0.2 \end{bmatrix}, \\
E_1 &= \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, & U_1 &= \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, & B_{11} &= \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix}, & E_{B1} &= \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.3 \end{bmatrix}, & (4.4) \\
B_{21} &= \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, & C_1 &= \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, & D_1 &= \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}, & E_{A1} &= \begin{bmatrix} -3 & -0.1 \\ 0.2 & -1 \end{bmatrix},
\end{aligned}$$

$$f_1(x(t)) = 0.01 \sin x(t).$$

#### Subsystem 2

$$\begin{aligned}
A_2 &= \begin{bmatrix} -7 & 2 \\ 1 & -6 \end{bmatrix}, & A_{d2} &= \begin{bmatrix} -1.2 & 0.1 \\ 0.3 & -1.1 \end{bmatrix}, & H_2 &= \begin{bmatrix} -0.1 & 0.2 \\ -0.2 & -0.1 \end{bmatrix}, & E_{Ad2} &= \begin{bmatrix} -0.3 & 0.1 \\ 0.2 & -0.3 \end{bmatrix}, \\
E_2 &= \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, & U_2 &= \begin{bmatrix} 0.02 & 0 \\ 0 & 0.08 \end{bmatrix}, & B_{12} &= \begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix}, & E_{B2} &= \begin{bmatrix} -0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \\
B_{22} &= \begin{bmatrix} -1 & 0 \\ 2 & 0.8 \end{bmatrix}, & C_2 &= \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}, & D_2 &= \begin{bmatrix} 8 & 0 \\ 1 & 8 \end{bmatrix}, & E_{A2} &= \begin{bmatrix} -1 & 0.3 \\ 0.2 & -2 \end{bmatrix}, & (4.5)
\end{aligned}$$

$$f_2(x(t)) = 0.02 \cos x(t).$$

(1) Let  $h = 0.2, T_f = 12, d^2 = 10, R = I, \alpha = 0.001, C_2^2 = 21, \rho^* = 1.3, \gamma^2 = 0.9344$ . By solving the LMIs in (3.62)–(3.66), we can get

$$\begin{aligned} \tilde{Q}_1 &= \begin{bmatrix} 0.4252 & 0.0387 \\ 0.0387 & 1.2272 \end{bmatrix}, & \tilde{Q}_2 &= \begin{bmatrix} 0.4352 & 0.0470 \\ 0.0470 & 1.2369 \end{bmatrix}, & Y_1 &= \begin{bmatrix} 0.0866 & -0.4834 \\ -0.0863 & 0.5554 \end{bmatrix}, \\ Y_2 &= \begin{bmatrix} 0.1064 & -0.2575 \\ -0.1260 & 0.2934 \end{bmatrix}, & \tilde{P}_1 &= \begin{bmatrix} 0.4606 & 0.0418 \\ 0.0418 & 0.9965 \end{bmatrix}, & \tilde{P}_2 &= \begin{bmatrix} 0.5364 & -0.0611 \\ -0.0611 & 0.9884 \end{bmatrix}, & (4.6) \\ K_1 &= \begin{bmatrix} 0.2329 & -0.4949 \\ -0.2389 & 0.5673 \end{bmatrix}, & K_2 &= \begin{bmatrix} 0.1699 & -0.2500 \\ -0.2024 & 0.2844 \end{bmatrix}, \end{aligned}$$

and  $\tau_a > \tau_a^* = 3.8340$ . We choose  $\tau_a = 4, \varepsilon = 0.05, \bar{\gamma}^2 = 0.9464$ , the initial condition  $x(t) = 0, t \in [-h, 0]$ , the switching signal is shown in Figure 1, and state trajectories of the closed-loop system are shown in Figure 2.

We can see from Figure 2 that the states of the system are continuous due to the feedback  $\bar{K}_i$  in impulsive instants.

(2) Let  $h = 0.2, T_f = 12, d^2 = 10, R = I$ , and  $\alpha = 0.001$ . By solving the LMIs of [41, Theorem 3], we can get

$$\begin{aligned} \tilde{Q}_1 &= \begin{bmatrix} 0.4015 & 0.0359 \\ 0.0359 & 1.0563 \end{bmatrix}, & \tilde{Q}_2 &= \begin{bmatrix} 0.5224 & 0.1104 \\ 0.1104 & 1.0717 \end{bmatrix}, & Y_1 &= \begin{bmatrix} 0.1245 & -0.6523 \\ -0.0998 & 0.5952 \end{bmatrix}, \\ Y_2 &= \begin{bmatrix} 0.1279 & -0.2380 \\ -0.1006 & 0.2699 \end{bmatrix}, & \tilde{P}_1 &= \begin{bmatrix} 0.5577 & 0.0099 \\ 0.0099 & 0.9992 \end{bmatrix}, & \tilde{P}_2 &= \begin{bmatrix} 0.5577 & 0.0099 \\ 0.0099 & 0.9992 \end{bmatrix}, \\ K_1 &= \begin{bmatrix} 0.2349 & -0.6552 \\ -0.1896 & 0.5976 \end{bmatrix}, & K_2 &= \begin{bmatrix} 0.2336 & -0.2405 \\ -0.1852 & 0.2720 \end{bmatrix}, & (4.7) \end{aligned}$$

$$\tilde{Q}_1^{-1} - (I + E_1)^T \tilde{Q}_1^{-1} (I + E_1) > 0,$$

$$\tilde{Q}_2^{-1} - (I + E_2)^T \tilde{Q}_1^{-1} (I + E_2) > 0,$$

$$\tilde{Q}_2^{-1} - (I + E_2)^T \tilde{Q}_2^{-1} (I + E_2) > 0,$$

$$\tilde{Q}_1^{-1} - (I + E_1)^T \tilde{Q}_2^{-1} (I + E_1) > 0.$$

Obviously, the above inequalities do not satisfy the conditions of [41, Theorem 3]. Thus, we cannot draw the conclusion that the closed-loop system is finite-time bounded from Theorem 3 in [41].

## 5. Conclusions

This paper has investigated robust finite-time  $H_\infty$  control for a class of impulsive switched nonlinear systems with time-delay. Based on piecewise Lyapunov function, sufficient conditions which guarantee finite-time boundedness of the impulsive switched system are

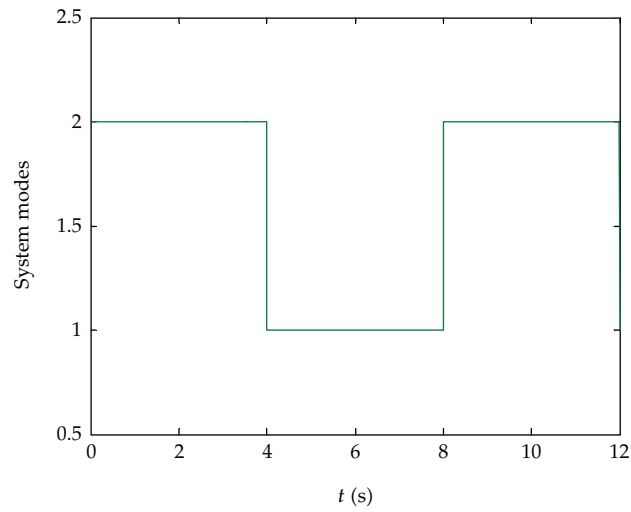


Figure 1: Switching signal.

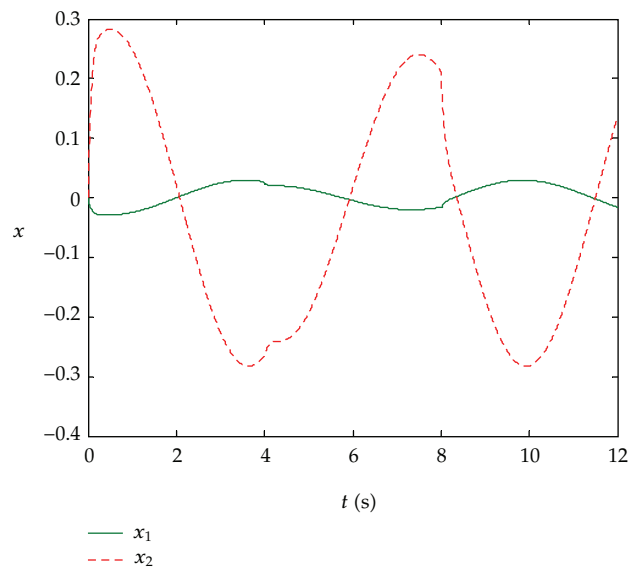


Figure 2: State trajectories of the closed-loop system.

derived. Then, a feedback control scheme consisting of an impulsive feedback controller and a switching controller is proposed, and the proposed control strategy can guarantee that the closed-loop system is finite-time bounded with  $H_\infty$  disturbance attenuation level. Finally, the results are illustrated by means of two numerical examples.

## Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grant nos. 61074023, 60974027, and 61273120.

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