

Research Article

Hölder Scales of Sea Level

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The statistics of sea level is essential in the field of geosciences, ranging from ocean dynamics to climates. The fractal properties of sea level, such as long-range dependence (LRD) or long memory, $1/f$ noise behavior, and self-similarity (SS), are known. However, the description of its multiscale behavior as well as local roughness with the Hölder exponent $h(t)$ from a view of multifractional Brownian motion (mBm) is rarely reported, to the best of our knowledge. In this research, we will exhibit that there is the multiscale property of sea level based on $h(t)$ s of sea level data recorded by the National Data Buoy Center (NDBC) at six stations in the Florida and Eastern Gulf of Mexico. The contributions of this paper are twofold as follows. (i) Hölder exponent of sea level may not change with time considerably at small time scale, for example, daily time scale, but it varies significantly at large time scale, such as at monthly time scale. (ii) The dispersion of the Hölder exponents of sea level may be different at different stations. This implies that the Hölder roughness of sea level may be spatial dependent.

1. Introduction

The study of sea level fluctuations plays a role in geosciences [1–3]. There are two categories of time scales of sea level. One is for yearly data with time scales in one yr, or 10 yr, or more; see, for example, [4–16]. The other is about data with time scales hourly, daily, weekly, or monthly; see, for example, [17–39]. The former generally relates to the study of trend of relative mean sea level with respect to global and Earth or planetary changes, for example, in the field of climates, while the latter is usually associated with the research of local dynamics of sea level in the aspects of navigations, coastal engineering, tide power production, ship design, and so forth. Our research uses the hourly sea level data recorded by NDBC [40].

Since the pioneering work of Hurst on time series with long-range dependence (LRD) is observed in the Nile Basin [41], the LRD property of time series in geosciences has been

widely observed; see, for example, [42–59]. By LRD, one means that the covariance function $C(\tau)$ of time series $x(t)$ decays so slowly such that

$$\int_0^{\infty} C(\tau) d\tau = \infty, \quad (1.1)$$

where τ is time lag and $C(\tau) = E[x(t + \tau)x(t)]$. Therefore, LRD is a global property of time series [60–66].

In addition to LRD, there is another essential property of processes in geosciences, called self-similarity (SS); see, for example, [67–84]. By SS, we mean that a random function $x(t)$ satisfies the property given by

$$x(t) \triangleq a^H x(at), \quad \forall a > 0, t > 0, \quad (1.2)$$

where \triangleq is the equality in distribution, $H \in (0, 1)$ is the Hurst parameter that measures SS, and a is a scale [61, 83–86]. Note that the term SS implies the roughness or irregularity of a random function [86]. If $x(t)$ satisfies (1.2), it is globally self-similar. That is, its irregularity characterized by H keeps the same for all $t > 0$ [87], corresponding the case of monofractal [88, 89].

Since the global SS implies the same value of H for all t , it may be too restrictive to describe real data in engineering and sciences to use a monofractal model. Therefore, multifractal models are desired in various fields of sciences and engineering; see, for example, [85–92] and references therein, including those in geosciences; see, for example, [93–110], just citing a few. From a view of multifractal, a random function that is not self-similar may be of local self-similarity (LSS).

There are several ways of describing multifractality of a random function based on various definitions of dimensions, such as the Minkowski dimension, the Rényi dimension, the Hausdorff dimension, the packing dimension, the box-counting dimension, and the correlation dimension [86, 89, 90, 111–114]. In this paper, we adopt the Hölder exponent $0 < h(t) < 1$ in multifractional Brownian motion (mBm) introduced by Peltier and Levy-Vehel [115]. Taking into account $h(t)$ in mBm, therefore, one may use the following:

$$x(t) \triangleq a^{h(t)} x(at), \quad \forall a > 0, t > 0, \quad (1.3)$$

to characterize the LSS property of a locally self-similar random function $x(t)$ on a point-by-point basis. We call the LSS or local roughness characterized by $h(t)$ the Hölder roughness in this paper. The applications of $h(t)$ attract increasing interests of researchers in sciences and technologies, ranging from teletraffic to geophysics; see, for example, [116–134], simply mentioning a few.

This paper aims at investigating the Hölder multiscales (Hölder scales for short) of sea level. By Hölder scales, we mean the time scales described by the Hölder exponents in mBm. The contributions of this paper are in two aspects. On the one hand, we will reveal that variations of $h(t)$ of sea level may be indistinctively at small time scale, for example, daily time scale, but $h(t)$ of sea level varies significantly at large time scale, such as at monthly time scale. On the other hand, we will exhibit that the dispersion of the Hölder exponents of sea level may usually be spatial dependent.

Table 1: Measured data at LKWF1.

Series name	Record date and time	L (record length)
$x_{\text{lkwf1}_{1996}}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1996	8208
$x_{\text{lkwf1}_{1997}}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1997	7776
$x_{\text{lkwf1}_{1998}}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1998	8736
$x_{\text{lkwf1}_{1999}}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1999	8760
$x_{\text{lkwf1}_{2000}}(t)$	0:00, 1 Jan.–17:00, 26 Feb. 2000	1362
$x_{\text{lkwf1}_{2001}}(t)$	17:00, 8 Aug.–23:00, 31 Dec. 2001	2972
$x_{\text{lkwf1}_{2002}}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 2002	8740
$x_{\text{lkwf1}_{2003}}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 2003	8582
$x_{\text{lkwf1}_{2004}}(t)$	0:00, 1 Jan.–14:00, 5 Oct. 2004	6655

The remaining paper is organized as follows. Data used in this research are briefed in Section 2. The method for describing the Hölder exponent in mBm is explained in Section 3. Results of data processing and discussions are given in Section 4, which is followed by conclusions.

2. Data

NDBC is a part of the US National Weather Service (NWS) [135]. It provides scientists with data for their scientific research, including significant wave height and water level [136]. We use the data measured at stations named LKWF1, LONF1, SAUF1, SMKUF1, SPGF1, and VENF1, respectively. In terms of the names of measurement stations, LKWF1 implies the station at Lake Worth, FL [137]; the station LONF1 is the one at Long Key, FL [138]; the station SAUF1 is at St. Augustine, FL [139]; SMKUF1 is the station at Sombrero Key, FL [140]; SPGF1 is at Settlement Point, GBI [141]; and VENF1 is at Venice, FL [142]. They are located in the Florida and Eastern Gulf of Mexico. The data are under the directory of Water Level, which are publicly accessible [143], referring Gilhousen [144] as an instance of research using the data by NDBC.

All data were hourly recorded with ten separate devices indexed by TGn ($n = 01, 02, \dots, 10$). Without losing generality, this research utilizes the data from the device $TG01$. Denote the data series by $x_{s_yyyy}(t)$, where s is the name of the measurement station and $yyyy$ stands for the index of year. Denote by $h_{s_yyyy}(t)$ its corresponding $h(t)$ at the station s in the year of $yyyy$. For example, $x_{\text{lkwf1}_{2002}}(t)$ and $h_{\text{lkwf1}_{2002}}(t)$, respectively, represent the measured sea level time series and its $h(t)$ at the station LKWF1 in 2002.

If the recorded data are labeled by 99, they are taken as outliers, which are not involved in the computations. In this case, they are replaced with the mean of that series. NDBC suggests that 10 ft should be subtracted from every level series $x_{s_yyyy}(t)$ [145]. By taking into account this suggestion in the computation of $h(t)$, we modify $x_{s_yyyy}(t)$ by subtracting 10 ft and denote $y_{s_yyyy}(t)$ modified data of sea level. That is,

$$y_{s_yyyy}(t) = x_{s_yyyy}(t) - 10. \quad (2.1)$$

Tables 1, 2, 3, 4, 5 and 6 list those data.

Table 2: Measured data at LONF1.

Series name	Record date and time	<i>L</i> (record length)
<i>x_lonf1_1998(t)</i>	0:00, 3 Nov.–23:00, 31 Dec. 1998	1416
<i>x_lonf1_1999(t)</i>	0:00, 1 Jan.–21:00, 31 Dec. 1999	8757
<i>x_lonf1_2000(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2000	8484
<i>x_lonf1_2001(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2001	8760
<i>x_lonf1_2002(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2002	8760
<i>x_lonf1_2003(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2003	8697
<i>x_lonf1_2004(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2004	8758
<i>x_lonf1_2005(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2005	8750
<i>x_lonf1_2006(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2006	8735
<i>x_lonf1_2007(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2007	8692
<i>x_lonf1_2008(t)</i>	0:00, 1 Jan.–21:00, 19 Jan. 2008	444

Table 3: Measured data at SAUF1.

Series name	Record date and time	<i>L</i> (record length)
<i>x_sauf1_1996(t)</i>	0:00, 1 Jan.–14:00, 10 Aug. 1996	5511
<i>x_sauf1_1997(t)</i>	0:00, 25 Feb.–23:00, 31 Dec. 1997	6240
<i>x_sauf1_1998(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 1998	8736
<i>x_sauf1_1999(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 1999	8136
<i>x_sauf1_2000(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2000	8715
<i>x_sauf1_2001(t)</i>	0:00, 1 Jan.–21:00, 31 Dec. 2001	8758
<i>x_sauf1_2002(t)</i>	20:00, 6 Feb.–23:00, 20 Aug. 2002	4684

Table 4: Measured data at SMKF1.

Series name	Record date and time	<i>L</i> (record length)
<i>x_smkf1_1998(t)</i>	0:00, 3 Nov.–23:00, 31 Dec. 1998	1416
<i>x_smkf1_1999(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 1999	7775
<i>x_smkf1_2000(t)</i>	0:00, 1 Aug.–23:00, 31 Dec. 2000	3542
<i>x_smkf1_2001(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2001	5776
<i>x_smkf1_2002(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2002	8742
<i>x_smkf1_2003(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2003	5851
<i>x_smkf1_2004(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2004	8439
<i>x_smkf1_2005(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2005	8667
<i>x_smkf1_2006(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2006	8623
<i>x_smkf1_2007(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2007	8702
<i>x_smkf1_2008(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2008	8679
<i>x_smkf1_2009(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2009	8109
<i>x_smkf1_2010(t)</i>	0:00, 1 Jan.–23:00, 31 July 2010	5074
<i>x_smkf1_2011(t)</i>	0:00, 1 Jan.–23:00, 31 Dec. 2011	8759

Table 5: Measured data at SPGF1.

Series name	Record date and time	<i>L</i> (record length)
<i>x_spgf1_1996(t)</i>	0:00, 1 Jan.–23:00, 15 Dec. 1996	8616
<i>x_spg1_1997(t)</i>	0:00, 6 Mar.–23:00, 15 Dec. 1997	7080
<i>x_spg1_1998(t)</i>	0:00, 1 Jan.–23:00, 7 Jan. 1998	168

Table 6: Measured data at VENF1.

Series name	Record date and time	L (record length)
$x_{\text{venf1}_2002}(t)$	0:00, 1 Oct.–23:00, 31 Dec. 2002	2208
$x_{\text{ven1}_2003}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 2003	8760
$x_{\text{ven1}_2004}(t)$	0:00, 1 Jan.–16:00, 7 Jan. 2004	634
$x_{\text{ven1}_2006}(t)$	14:00, 22 July–23:00, 31 Dec. 2006	3882
$x_{\text{ven1}_2007}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 2007	8663
$x_{\text{ven1}_2008}(t)$	0:00, 1 Jan.–23:00, 31 Oct. 2008	7189

3. Methodology

Let $B(t)$ be the standard Brownian motion. Then, $B(t)$ satisfies the following properties.

(i) The increments $B(\tau + t) - B(t)$ are Gaussian.

(ii) $E[B(\tau + t) - B(t)] = 0$ and

$$\text{Var}[B(t + \tau) - B(t)] = \sigma^2 \tau, \quad (3.1)$$

where $\sigma^2 = E\{[B(t + 1) - B(t)]^2\} = E\{[B(1) - B(0)]^2\} = E\{[B(1)]^2\}$.

(iii) In nonoverlapping intervals $[t_1, t_2]$ and $[t_3, t_4]$, the increments $B(t_4) - B(t_3)$ and $B(t_2) - B(t_1)$ are independent.

(iv) $B(0) = 0$ and $B(t)$ is continuous at $t = 0$.

Kolmogorov introduced a class of random functions the covariance function of which is now recognized as the one of fractional Brownian motion (fBm) [146, Theorem 6]. Note that, for a random function $x(t)$, the function $f(\tau)$ expressed by

$$f(\tau) = \text{Var}[x(t + \tau) - x(t)] = E\{[x(t + \tau) - x(t)]^2\} \quad (3.2)$$

is termed serial variation function; see, for example, Matérn [147, page 51]. It is usually called variogram in geosciences [148–157]. In the field of fluid mechanics, it is named structure function [158–161]. Yaglom derived fBm based on the theory of structure functions [162]. In this paper, we use the fBm introduced by Bandelbrot and van Ness based on fractional calculus [163].

It is well known that $B(t)$ is nondifferentiable in the domain of ordinary functions [164–166]. In the domain of generalized functions, however, it is differentiable [167, 168].

Denote the fBm by $B_H(t)$. Based on the Weyl's fractional derivative or integral [163], it is expressed by

$$B_H(t) - B_H(0) = \frac{1}{\Gamma(H + 1/2)} \left\{ \int_{-\infty}^0 [(t - u)^{H-0.5} - (-u)^{H-0.5}] dB(u) + \int_0^t (t - u)^{H-0.5} dB(u) \right\}. \quad (3.3)$$

If the first item on the right hand of (3.3) is taken as the zero-input response of the system that generates $B_H(t)$ for $t > 0$, we may regard the fBm as the convolution of the impulse function $t^{H-1/2}/\Gamma(H+1/2)$ and $dB(t)/dt$ [169]. Therefore, (3.3) may be rewritten by

$$B_H(t) - B_H(0) = B^0(u) + \frac{t^{H-0.5}}{\Gamma(H+1/2)} * \frac{dB(t)}{dt}, \quad (3.4)$$

where $*$ is the operator of convolution and

$$B^0(u) = \frac{1}{\Gamma(H+1/2)} \int_{-\infty}^0 \left[(t-u)^{H-0.5} - (-u)^{H-0.5} \right] dB(t). \quad (3.5)$$

It may be interesting to note that $t^{H-1/2}/\Gamma(H+1/2)$ is a special case of the operators of fractional order discussed by Mikusinski [170, Equation (59.1)].

The function $B_H(t)$ has the following properties.

- (i) $B_H(0) = 0$.
- (ii) The increments $B_H(t+t_0) - B_H(t_0)$ are Gaussian.
- (iii) Its structure function is given by

$$\text{Var}[B_H(t+\tau) - B_H(t)] = \sigma^2 \tau^{2H}, \quad (3.6)$$

where $\sigma^2 = E\{[B_H(t+1) - B_H(t)]^2\} = E\{[B_H(1) - B_H(0)]^2\} = E\{[B_H(1)]^2\}$.

In addition, it satisfies the self-similarity expressed by (1.2), which implies that $B_H(t)$ is globally self-similar. Consequently, there is a limitation that its self-similarity or roughness keeps the same for all $t > 0$. To release such a limitation, one may adopt the tool of the mBm equipped with the Hölder exponent $h(t)$; see, for example, [115, 119, 133]. In fact, the mBm is a generalization of fBm by replacing the Hurst parameter H in (3.3) with a continuous function $h(t)$ that satisfies $H : [0, \infty) \rightarrow (0, 1)$; see [87, 115–134, 171–182]. Denote the mBm by $X(t)$. Then,

$$X(t) = \frac{1}{\Gamma(h(t)+1/2)} \left\{ \begin{array}{l} \int_{-\infty}^0 \left[(t-u)^{h(t)-0.5} - (-u)^{h(t)-0.5} \right] dB(u) \\ + \int_0^t (t-u)^{h(t)-0.5} dB(u) \end{array} \right\}. \quad (3.7)$$

Considering the local growth of the increment process of $X(t)$, one may write a sequence given by

$$S_k(j) = \frac{m}{N-1} \sum_{j=0}^{j+k} \left| X(i+1) - X(i) \right|, \quad 1 < k < N, \quad (3.8)$$

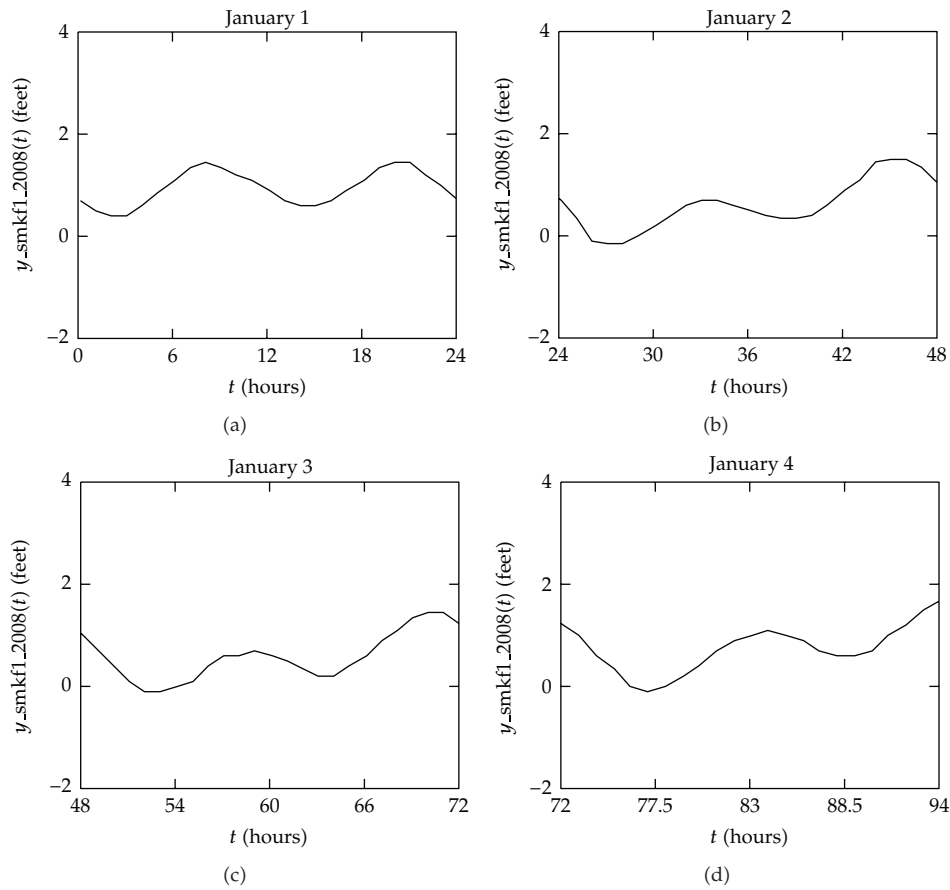


Figure 1: Daily sea level at the station SMKUF1 from January 1 to Jan. 4 in 2008. (a). $x_smkf1_2008(t)$ on Jan. 1, 2008. (b). $x_smkf1_2008(t)$ on Jan. 2, 2008. (c). $x_smkf1_2008(t)$ on Jan. 3, 2008. (d). $x_smkf1_2008(t)$ on Jan. 4, 2008.

where m is the largest integer not exceeding N/k . Then, $h(t)$ at point $t = j/(N - 1)$ is given by

$$h(t) = -\frac{\log\left(\sqrt{\pi/2}S_k(j)\right)}{\log(N - 1)}. \quad (3.9)$$

The above is the expression of applying mBm to investigate $h(t)$ of sea level time series, which measures the Hölder roughness of sea level on a point-by-point basis.

4. Observations and Discussions

We demonstrate $h(t)$ s of sea level series $x_smkf1_2008(t)$ at the time scales of day, week, and month, respectively.

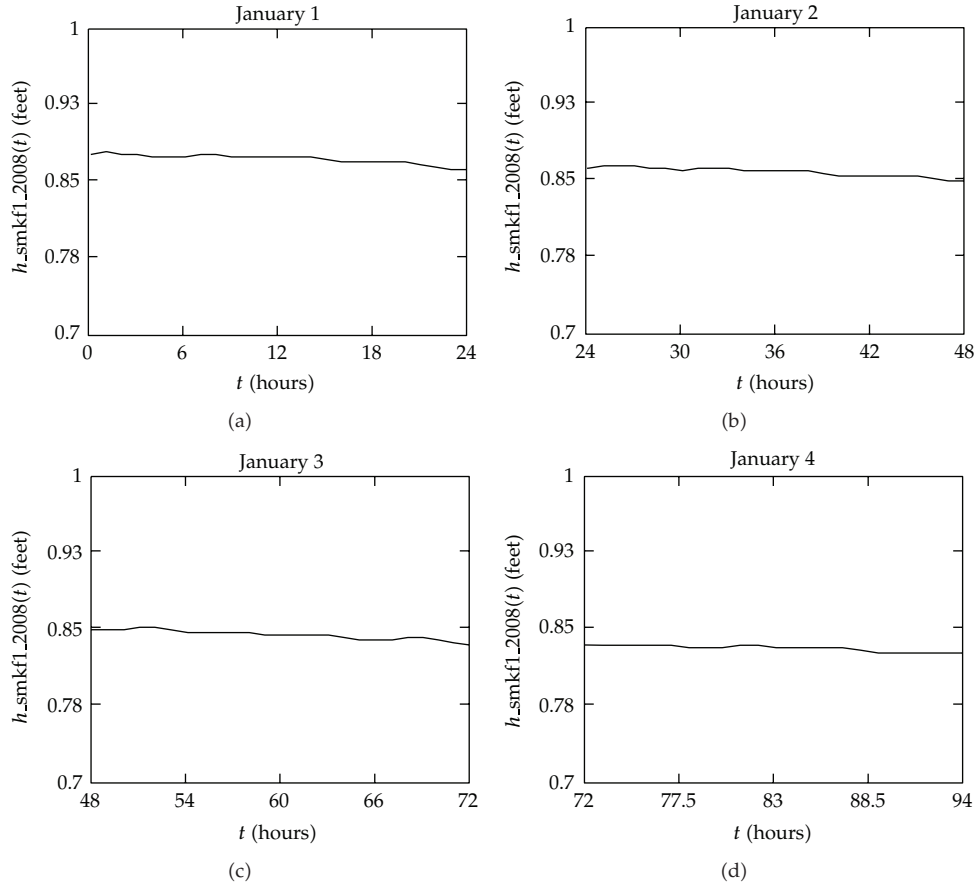


Figure 2: Hölder exponents of daily sea level at the station SMKUF1 from Jan. 1 to Jan. 4 in 2008. (a). $h_{\text{smkf1_2008}}(t)$ on Jan. 1, 2008. (b). $h_{\text{smkf1_2008}}(t)$ on Jan. 2, 2008. (c). $h_{\text{smkf1_2008}}(t)$ on Jan. 3, 2008. (d). $h_{\text{smkf1_2008}}(t)$ on Jan. 4, 2008.

4.1. Hölder Roughness at Daily Time Scale

Figure 1 indicates 4 daily series of sea level at the station SMKUF1 from Jan. 1 to Jan. 4 in 2008. Figure 2 demonstrates their corresponding Hölder exponents. From Figure 2, we see that $4h(t)$ s of daily series of sea level vary with time insignificantly. Therefore, we obtain the remark below.

Remark 4.1. The Hölder exponents of sea level at the daily time scale, that is, 24 hours, may not vary significantly. This may imply that $h(t) \approx h(t + \tau)$ if $\tau \leq 24$ hours.

4.2. Hölder Roughness at Weekly Time Scale

Four weekly series of sea level at the station SMKUF1 in Jan. 2008 are shown in Figure 3. Their corresponding Hölder exponents are plotted in Figure 4. They appear monotonically increase, see Figures 4(b) and 4(d), or decrease, see Figures 4(a) and 4(c). In general, they imply the following remark.

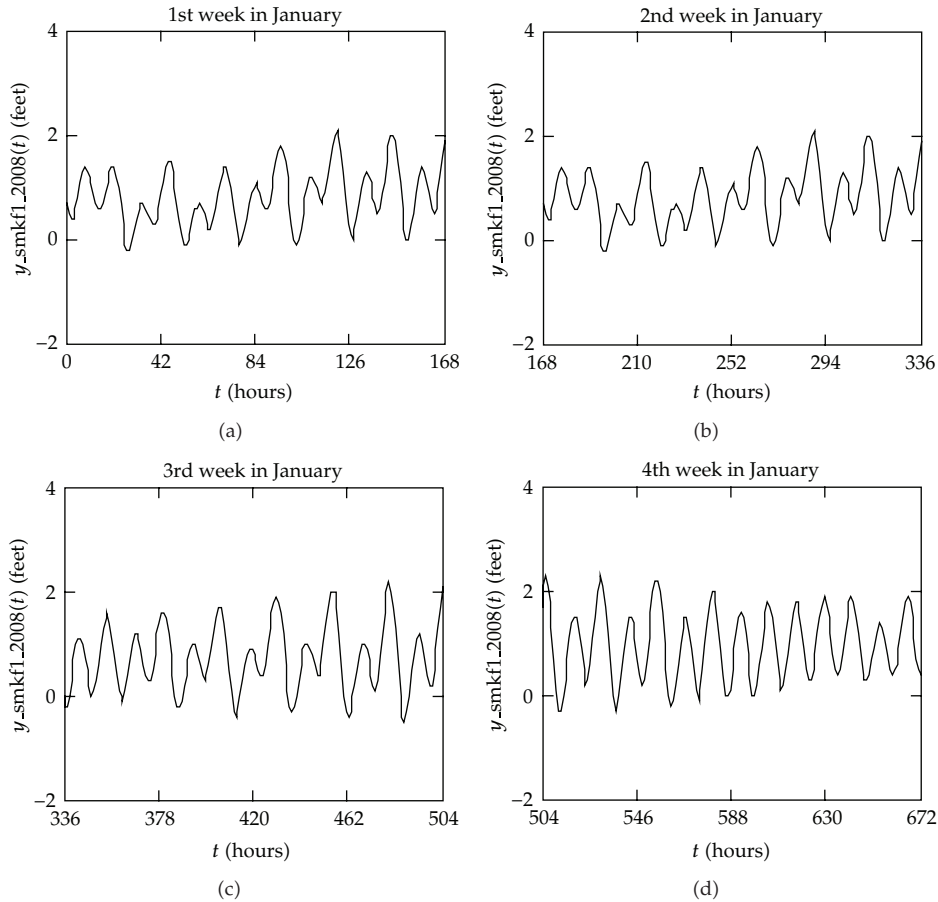


Figure 3: Weekly sea level at the station SMKUF1 on January 2008. (a). $x_{\text{smkf1_2008}}(t)$ in the 1st week in Jan. 2008. (b). $x_{\text{smkf1_2008}}(t)$ in the 2nd week in Jan. 2008. (c). $x_{\text{smkf1_2008}}(t)$ in the 3rd week on January 2008. (d). $x_{\text{smkf1_2008}}(t)$ in the 4th week in Jan. 2008.

Remark 4.2. The Hölder exponents of sea level at the weekly time scale, that is, 168 hours, may not vary considerably enough.

4.3. Hölder Roughness at Monthly Time Scale

Figure 5 illustrates 4 monthly series of sea level at the station SMKUF1 in 2008. Their corresponding Hölder exponents are indicated in Figure 6. From Figure 6, we see the following.

Remark 4.3. The Hölder exponents of sea level at the monthly time scale vary with time significantly.

4.4. Variation of Hölder Roughness at Large Time Scale

We now investigate the Hölder exponents of sea level at large time scale. By large time scale, we mean that the scale is around month or larger. Figure 7(a) indicates the sea level series

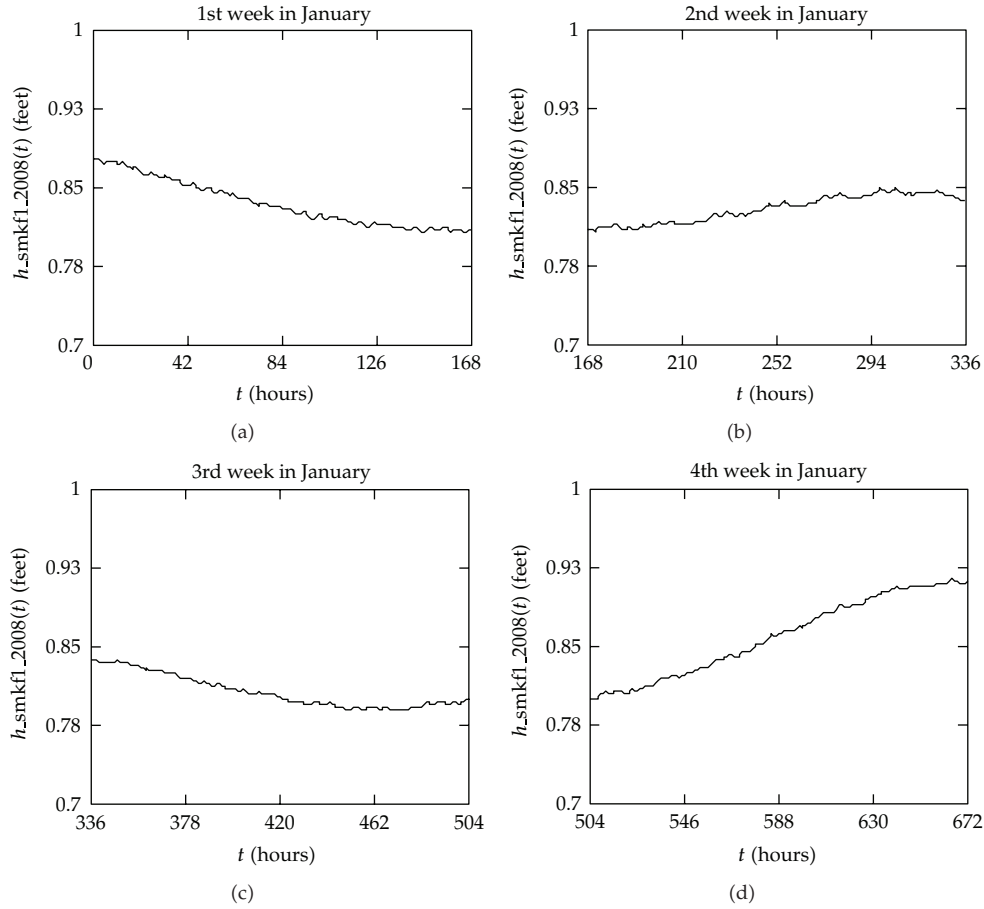


Figure 4: Hölder exponents of weekly sea level at the station SMKUF1 in January 2008. (a). $h_smkf1_2008(t)$ in the 1st week on January 2008. (b). $h_smkf1_2008(t)$ in the 2nd week in Jan. 2008. (c). $h_smkf1_2008(t)$ in the 3rd week in Jan. 2008. (d). $h_smkf1_2008(t)$ in the 4th week in Jan. 2008.

$x_smkf1_2008(t)$, Figure 7(b) shows its Hölder exponent, and Figure 7(c) the histogram of its Hölder exponent.

One thing worth noting is that variances of Hölder exponents of sea level at different stations may be considerably different. For instance,

$$\begin{aligned} \text{Var}[h_smkf1_2008(t)] &= 1.203 \times 10^{-3}, \\ \text{Var}[h_lonf112005(t)] &= 6.425 \times 10^{-4}. \end{aligned} \tag{4.1}$$

The above implies that the variance of $h_smkf1_2008(t)$ is larger than that of $h_lonf112005(t)$ in one magnitude of order. Consequently, comes the following remark.

Remark 4.4. The variances of the Hölder exponents of sea level at different observation stations may be considerably different.

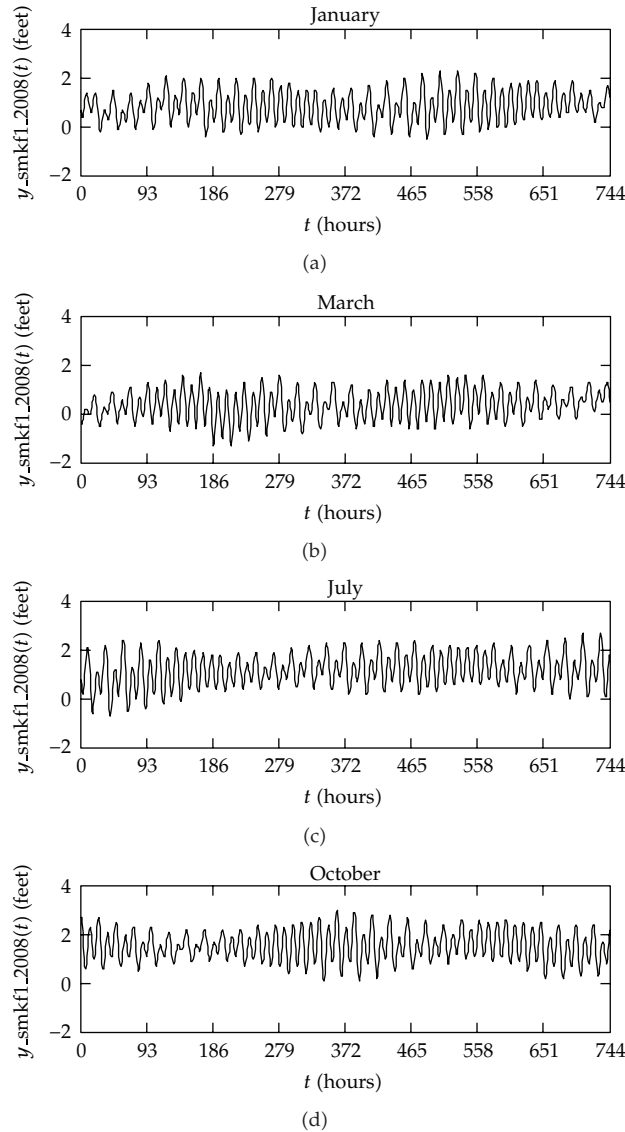


Figure 5: Monthly sea level at the station SMKUF1 in 2008. (a). $x_{\text{smkf1_2008}}(t)$ on January 2008. (b). $x_{\text{smkf1_2008}}(t)$ in March 2008. (c). $x_{\text{smkf1_2008}}(t)$ on July 2008. (d). $x_{\text{smkf1_2008}}(t)$ on October 2008.

We summarize the variances of the Hölder exponents of test data in Tables 7, 8, 9, 10, 11 and 12.

4.5. Discussions

Generally, the Hölder exponents of sea level series are time varying. They are considerably at large time scales but insignificantly at small time scales. In addition, their variations are in general spatial dependent as the Tables 7–12 exhibit. For instance, in 2002, $\text{Var}[h(t)]$ varies, in the form of magnitude of order, from 10^{-3} to 10^{-4} at different stations. This motivates us to

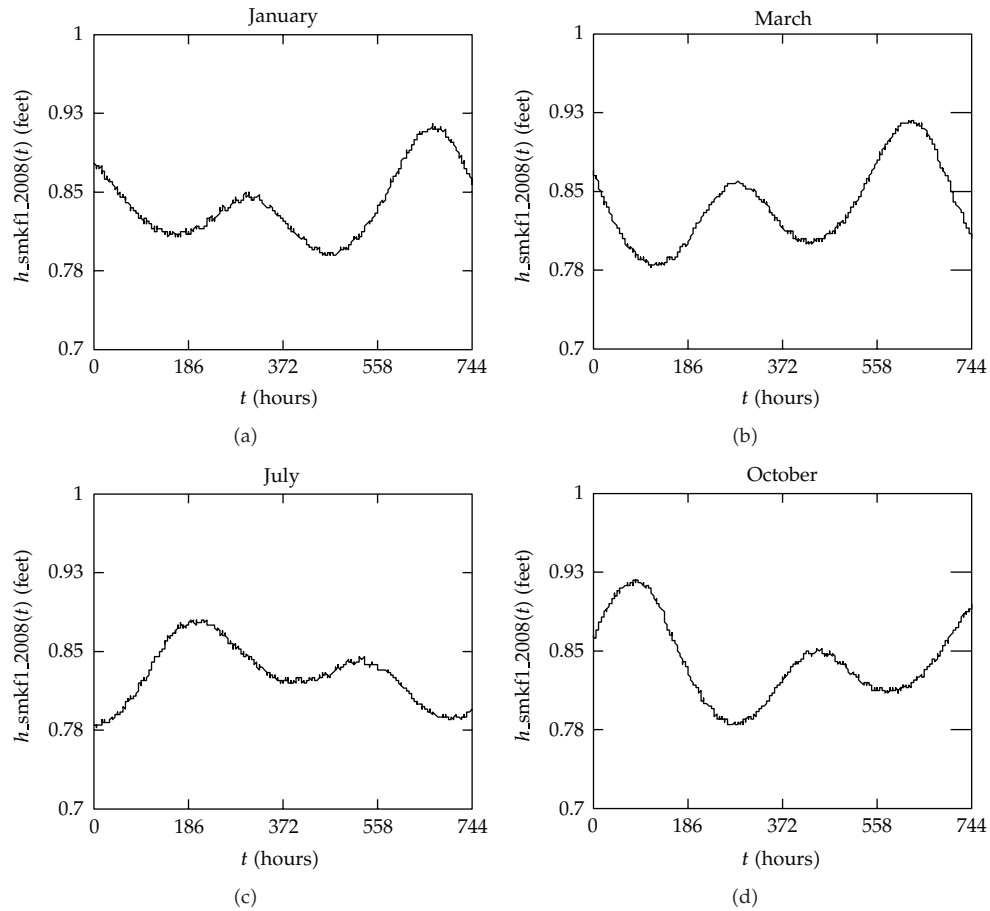


Figure 6: Hölder exponents of month sea level at the station SMKUF1 in 2008. (a). $h_smkf1_2008(t)$ on Jan. 2008. (b). $h_smkf1_2008(t)$ on March 2008. (c). $h_smkf1_2008(t)$ in July 2008. (d). $h_smkf1_2008(t)$ in October 2008.

take the spatial-time modeling of Hölder roughness of sea level as our possible future work. Finally, we note that the meaning of the term of local roughness of a random function is the same as that of local self-similarity [60, 65, 86]. Thus, according to (1.2), Remarks 4.1–4.3 exhibit the self-similarity of sea level at small and large time scales, respectively.

5. Conclusions

We have presented our results in the Hölder exponents of sea level in the Florida and Eastern Gulf of Mexico. The present results reveal an interesting phenomenon of time scales of sea level. To be precise, the Hölder exponents of sea level may not vary considerably at small time scales, such as daily time scale, but vary with time significantly at large time scale, such as monthly time scale. Moreover, our research exhibits that variations of the Hölder exponents of sea levels may be spatial dependent. Though the research is with the data in Florida and Eastern Gulf of Mexico, the results may be useful for further exploring general properties of the Hölder scales and roughness of sea level.

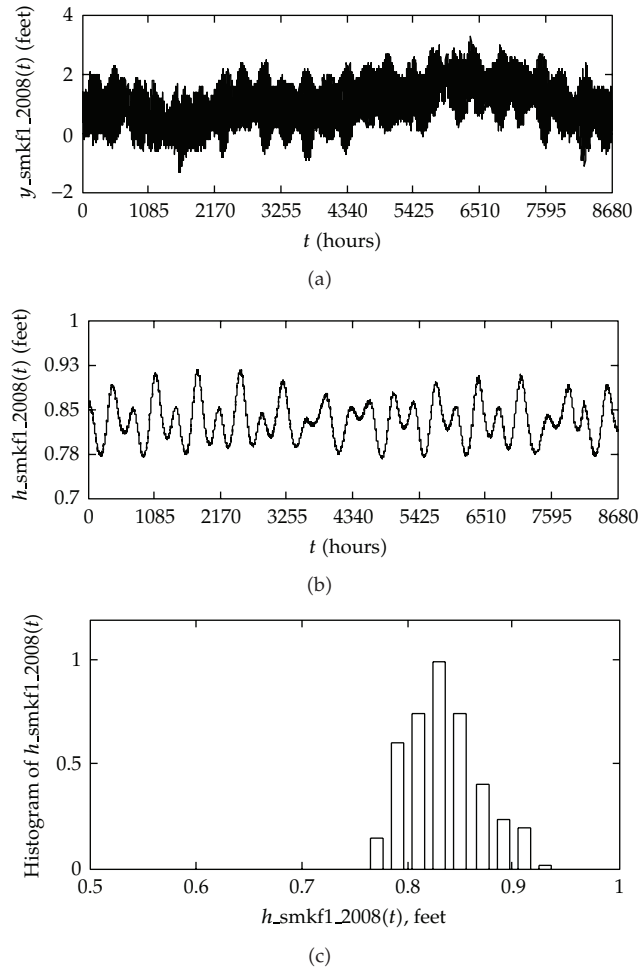


Figure 7: Illustrations of $h_{\text{smkf1_2008}}(t)$ and its Hölder exponent. (a). $x_{\text{smkf1_2008}}(t)$. (b) Hölder exponent $h_{\text{smkf1_2008}}(t)$. (c). Histogram of $h_{\text{smkf1_2008}}(t)$.

Table 7: Variances of the Hölder exponents at LKWF1.

Series name	$\text{Var}[h(t)]$
$x_{\text{lkwf1_1996}}(t)$	1.217×10^{-3}
$x_{\text{lkwf1_1997}}(t)$	1.006×10^{-3}
$x_{\text{lkwf1_1998}}(t)$	9.499×10^{-4}
$x_{\text{lkwf1_1999}}(t)$	1.164×10^{-3}
$x_{\text{lkwf1_2000}}(t)$	5.901×10^{-4}
$x_{\text{lkwf1_2001}}(t)$	1.169×10^{-3}
$x_{\text{lkwf1_2002}}(t)$	8.939×10^{-4}
$x_{\text{lkwf1_2003}}(t)$	9.710×10^{-4}
$x_{\text{lkwf1_2004}}(t)$	9.361×10^{-4}

Table 8: Variances of the Hölder exponents at LONF1.

Series name	Var[$h(t)$]
$h_{\text{lonf1}}_{1998}(t)$	3.978×10^{-3}
$h_{\text{lonf1}}_{1999}(t)$	4.123×10^{-4}
$h_{\text{lonf1}}_{2000}(t)$	1.570×10^{-3}
$h_{\text{lonf1}}_{2001}(t)$	1.135×10^{-3}
$h_{\text{lonf1}}_{2002}(t)$	1.407×10^{-3}
$h_{\text{lonf1}}_{2003}(t)$	2.359×10^{-3}
$h_{\text{lonf1}}_{2004}(t)$	9.493×10^{-4}
$h_{\text{lonf1}}_{2005}(t)$	6.425×10^{-4}
$h_{\text{lonf1}}_{2006}(t)$	1.245×10^{-3}
$h_{\text{lonf1}}_{2007}(t)$	2.142×10^{-3}
$h_{\text{lonf1}}_{2008}(t)$	8.245×10^{-5}

Table 9: Variances of the Hölder exponents at SAUF1.

Series name	Var[$h(t)$]
$x_{\text{sauf1}}_{1996}(t)$	1.083×10^{-3}
$x_{\text{sauf1}}_{1997}(t)$	1.355×10^{-3}
$x_{\text{sauf1}}_{1998}(t)$	8.766×10^{-4}
$x_{\text{sauf1}}_{1999}(t)$	1.324×10^{-3}
$x_{\text{sauf1}}_{2000}(t)$	7.272×10^{-4}
$x_{\text{sauf1}}_{2001}(t)$	6.961×10^{-4}
$x_{\text{sauf1}}_{2002}(t)$	3.992×10^{-3}

Table 10: Variances of the Hölder exponents at SMKf1.

Series name	Var[$h(t)$]
$x_{\text{smkf1}}_{1998}(t)$	9.501×10^{-4}
$x_{\text{smkf1}}_{1999}(t)$	1.144×10^{-3}
$x_{\text{smkf1}}_{2000}(t)$	1.310×10^{-3}
$x_{\text{smkf1}}_{2001}(t)$	1.520×10^{-3}
$x_{\text{smkf1}}_{2002}(t)$	1.181×10^{-3}
$x_{\text{smkf1}}_{2003}(t)$	1.176×10^{-3}
$x_{\text{smkf1}}_{2004}(t)$	1.243×10^{-3}
$x_{\text{smkf1}}_{2005}(t)$	1.210×10^{-3}
$h_{\text{smkf1}}_{2006}(t)$	1.101×10^{-3}
$h_{\text{smkf1}}_{2007}(t)$	1.164×10^{-3}
$h_{\text{smkf1}}_{2008}(t)$	1.203×10^{-3}
$h_{\text{smkf1}}_{2009}(t)$	1.242×10^{-3}
$h_{\text{smkf1}}_{2010}(t)$	1.084×10^{-3}
$h_{\text{smkf1}}_{2011}(t)$	1.176×10^{-3}

Table 11: Variances of the Hölder exponents at SPGF1.

Series name	Var[$h(t)$]
$x_{\text{spgf1}}_{1996}(t)$	1.018×10^{-3}
$x_{\text{spgf1}}_{1997}(t)$	8.803×10^{-4}
$x_{\text{spgf1}}_{1998}(t)$	2.659×10^{-4}

Table 12: Variances of the Hölder exponents at VENF1.

Series name	Var[$h(t)$]
$x_{\text{venf1_2002}}(t)$	1.069×10^{-3}
$x_{\text{venf1_2003}}(t)$	1.271×10^{-3}
$x_{\text{venf1_2004}}(t)$	8.863×10^{-4}
$x_{\text{venf1_2006}}(t)$	2.268×10^{-3}
$x_{\text{venf1_2007}}(t)$	2.454×10^{-3}
$x_{\text{venf1_2008}}(t)$	2.930×10^{-3}

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