Research Article

Parallel-Distributed Model Deformation in the Fingertips for Stable Grasping and Object Manipulation

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The study on the human grip has inspired to the robotics over the past decades, which has resulted in performance improvements of robotic hands. However, current robotic hands do not have the enough dexterity to execute complex tasks. Recognizing this fact, the soft fingertips with hemispherical shape and deformation models have renewed attention of roboticists. A high-friction contact to prevent slipping and the rolling contribution between the object and fingers are some characteristics of the soft fingertips which are useful to improve the grasping stability. In this paper, the parallel distributed deformation model is used to present the dynamical model of the soft tip fingers with n-degrees of freedom. Based on the joint angular positions of the fingers, a control scheme that fuses a stable grasping and the object manipulation into a unique control signal is proposed. The force-closure conditions are defined to guarantee a stable grasping and the boundedness of the closed-loop signals is proved. Furthermore, the convergence of the contact force to its desired value is guaranteed, without any information about the radius of the fingertip. Simulation results are provided to visualize the stable grasping and the object manipulation, avoiding the gravity effect.

1. Introduction

From physiological point of view, the human hands are considered as a powerful tool whereby the human brain interacts with the world, that is, how it perceives and acts with the environment [1]. In order to increase the dexterity in robotic hands, some intelligent human-like functions have been imitated.

In general, the dexterous manipulation in robotics, to emulate pinching motions, have been formulated in terms of the object, that is, the forces/torques exerted on it to produce the desired movements and how they behave [2]. The grasping and the object manipulation are

based on the assumption that the contact between the object and fingers is frictionless, so the finger can only exert a force along the common normal axis at the contact point [3]. Then, to grasp the object without slipping the standard friction cone is used, generating a complex motion control since their evolution is governed by the laws of Coulomb friction, which is nonlinear and imposes constraints on the system. On the other hand, some authors have considered fingers with a very sharp curvature assuming that the contact point between the fingers and object does not change significantly. Although, some manipulation tasks are executed by robotic fingers successfully, this assumption is not valid for several manipulation tasks because the rolling between object and fingers is essential in the human manipulation tasks. Moreover, some attempts to determine the best grasp configuration and manipulation tasks are presented in 2D and 3D [4–11]. Unfortunately, a lot of them require an exact knowledge of the system parameters and the object localization [12–15]. Some authors, to reproduce more characteristics of the human fingertips, have considered the use of deformation models with hemispherical soft tips. Many hemispherical soft tip fingers have been designed and constructed to execute several manipulation tasks [16–19] where a high-contact friction to prevent slipping and the rolling of the finger tip on the object surface are some characteristics [3, 20, 21]. Nevertheless, in these approaches the contribution of the fingertip deformation on the manipulation tasks in a dynamic sense is not evident, considering that the rolling constraint is defined in kinematic or semidynamics sense [21–24].

On the basis that human hands have fingers with soft tips, in this paper the grasping and the object manipulation is presented, using a pair of robotic fingers with n-degree of freedoms and deformable tips. The parallel deformation model is based on a virtual spring with infinitesimal section, where the normal and tangential deformations are taking into account [25]. So, a tangential movement of the object without slipping and a dependency of the relative orientation between the object and the finger are considered. Inclusion of the normal and tangential deformations in the deformation model, contribute to reproduce some intrinsic characteristics of the deformable material and a better grip. The key of our approach is to introduce the parallel deformation model to grasp an object using a pair of robotic fingers with n-degrees of freedom. Moreover, the grasping controller guaranteed that the contact force converges to the desired value, avoiding a direct dependence with the radius of the tip [26, 27]. An approximation of the object angle based on the joint angular position of the fingers is used to control orientation of the object. Finally, a control signal for translation of the object is defined. To carry out the grasping and the object manipulation, the superposition principle is used [28, 29], which allows us to separate a complex task into a set of basic tasks, where each task has a unique stationary point which represents the desired action [27]. Boundedness of all closed-loop signals is proved, while the asymptotic stability is guaranteed using the stability on the manifold [27, 28, 30]. It is important to notice that forces-closure conditions, to grasp firmly an object, are satisfied dynamically during manipulation task execution, rather than a static equilibrium. The proposed approach is validated by numerical simulations through a pair of robotic fingers with soft tips in the horizontal plane.

This paper is organized as follows. Section 2 presents dynamical equations of the fingers-object system. The blind control law is proposed in Section 3. Simulation results to confirm the validity of our approach are presented in Section 4. Finally, the conclusions are presented in Section 5.

2. Dynamical Equations

Consider a pair of soft tip fingers, with three degree of freedom each one, grasping an object in the horizontal plane, as shown in Figure 1. In the fingers-object system O is the origin for
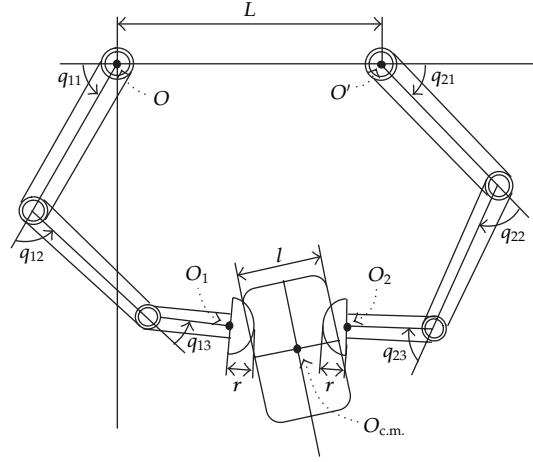
Figure 1: Rigid object grasped by a pair soft tip fingers.

the left finger and it is considered as the reference frame, O' is the origin for the right finger, and L is the distance between the origins of each finger. In addition, $q_i = [q_{i1}, q_{i2}, q_{i3}]^T$ is the joint angular positions of the finger i, r_i is the radius of the soft tip finger i, l is the length of the object, l_{ij} is the length of the link j for the finger i, $O_i = (x_i, y_i)$ is the center position of the deformable fingertips i with $i = 1, 2$, $O_{c.m.} = (x, y)$ is the center of mass of the object, and θ is the orientation of the object. Unlike deformation model proposed by [22, 28], where the force applied to the object produce a distribute pressure and it is parametrized as a normal force f_i with respect to the object surface. In this paper we use the deformation model proposed in [25], where a virtual spring inside the soft tip finger allows us to known the normal and tangential movements in the deformable material. In such a way, the elastic energy induced by the soft fingertip i is given as

$$P_i(d_{ni}, d_{ti}, \phi_i) = \pi E \left[\frac{d_{ni}^3}{3\cos^2(\phi_i)} + d_{ni}^2 d_{ti} \tan(\phi_i) + d_{ni} d_{ti}^2 \right], \tag{2.1}$$

where d_{ni} is the maximum radial deformation, ϕ_i is the object relative orientation angle, d_{ti} is the contact tangential displacement of the object, and E is the Young's modulus of the finger tip material, as shown in Figure 2. Thus, the total potential energy of the deformable fingertips is expressed as

$$P = P_1(d_{n1}, d_{t1}, \phi_1) + P_2(d_{n2}, d_{t2}, \phi_2). \tag{2.2}$$

The constraint between the radial deformation of the fingertip i and the object is given as

$$C_{ni} = -\left[(r_i - d_{ni}) + l_i + (-1)^i \left((x - x_i)\cos(\theta) - (y - y_i)\sin(\theta) \right) \right] = 0, \tag{2.3}$$

which guarantees that exists a distance that limits the grasping on the object in normal direction. A particular case of (2.3) is considered when $d_{ni} = 0$ which represents the normal
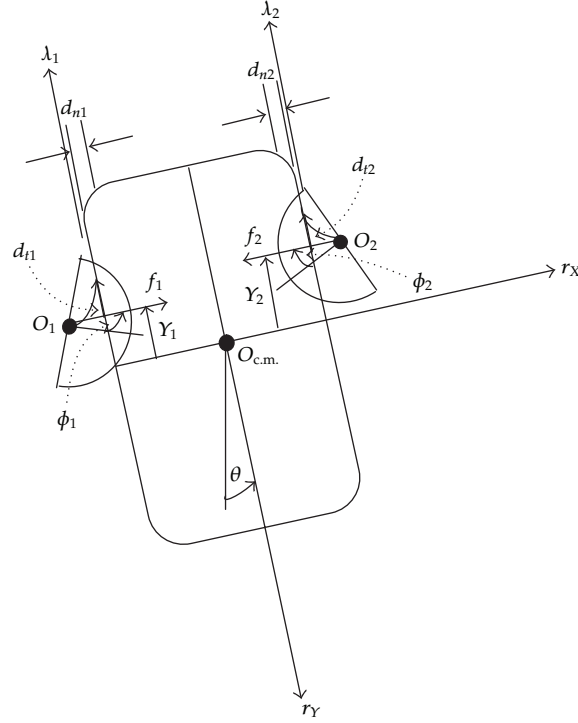
Figure 2: Deformation Model proposed by [25].

constraint for a rigid fingertip as reported in [30]. Accordingly, the normal constrained for the fingers-object system is defined as

$$C_n = \sum_i f_i C_{ni},$$ (2.4)

where f_i is the Lagrange multiplier and represents the contact force i.

On the other hand, taking into account the curvature effects of the fingertip, the rolling of the object on the soft tip finger will be defined as movement of the contact area, where the relative velocity of the contact area between the finger tips and the object is zero. Assuming that the normal deformation d_{ni} is smaller than the radius of the tip i [25], the angular displacement of the contact area and the projection displacement of the center of mass of the object is defined as

$$\dot{Y}_i = -r\dot{\phi}_i,$$ (2.5)

where

$$Y_i = (x_i - x)\sin(\theta) + (y_i - y)\cos(\theta),$$
$$\phi_i = \pi - (-1)^i\theta - e_i^T q_i,$$ (2.6)

and $e_i = [1, \ldots, 1]^T$ of the same size as the vector q_i, for $i = 1, 2$. Moreover, according to the deformation model, a tangential displacement d_{ti} on the deformable fingertip i arises when the object is rolling on the fingertip. Then, the rolling constraint between finger tip i and the object surface is given as

$$C_{ti} = Y_i - c_{Si} + r_i \phi_i + d_{ti} = 0, \tag{2.7}$$

where c_{Si} is the integration constant with respect to the initial conditions of contact. To avoid initial conditions in (2.7) a velocity constraint is defined as [25]

$$\dot{C}_{ti} = \dot{Y}_i + r_i \dot{\phi}_i + \dot{d}_{ti} = 0. \tag{2.8}$$

The Lagrangian of the system with holonomic constraint is described by

$$L = K - P + C_n, \tag{2.9}$$

where P and C_n are defined in (2.2) and (2.4), respectively, and K is the kinetic energy of the system defined as

$$K = \Sigma_i \frac{1}{2} \left[\dot{q}_i^T H_i(q_i) \dot{q}_i + m_{ni} \dot{d}_{ni}^2 + m_{ti} \dot{d}_{ti}^2 \right] + \frac{1}{2} \dot{p}^T H_0 \dot{p}, \tag{2.10}$$

with $H_i(q_i)$ is the inertia matrix of the finger i, $H_0 = \mathrm{diag}(m, m, I)$, $p = [x, y, \theta]^T$, and m_{ni}, m_{ti} are the normal and tangential mass deformations, respectively. Applying the Lagrangian variational principle, incorporating the rolling velocity constraint, the equations of motion are expressed for each component of the vector as follows:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{z}} \right) - \frac{\partial L}{\partial z} = \frac{\partial}{\partial \dot{z}} \left(\lambda_1 \dot{C}_{t1} + \lambda_2 \dot{C}_{t2} \right) \tag{2.11}$$

where $z = [q_1^T, q_2^T, x, y, \theta, d_{n1}, d_{n2}, d_{t1}, d_{t2}]^T$ is the vector of generalized coordinates and λ_i is the Lagrange multiplier which represents the tangential force exerted for the finger i on the object surface. Note that treatment of the velocity constraint should not be done in the Lagrangian (2.9), but rather in the equations of motion [31].

Thus, the equations of motion for the fingers are given as

$$H_i(q_i) \ddot{q}_i + \frac{1}{2} \dot{H}_i(q_i) \dot{q}_i + S_i(q_i, \dot{q}_i) \dot{q}_i - (-1)^i f_i J_i^T r_X - \lambda_i \left(J_i^T r_Y - r e_i \right)$$

$$- \frac{\pi E d_{ni}^2}{\cos^2(\phi_i)} \left(d_{ti} + \frac{2}{3} d_{ni} \tan(\phi_i) \right) e_i = u_i, \tag{2.12}$$

where $r_X = [\cos(\theta), \sin(\theta)]^T$, $r_Y = [\sin(\theta), \cos(\theta)]^T$, J_i is the Jacobian of the point (x_i, y_i) with respect to the joint variables q_{ij}, $e_i = [1, 1, 1]^T$, $(1/2) \dot{H}_i(q_i) + S_i(q_i, \dot{q}_i)$ represent the matrix of Coriolis and centripetal forces, and u_i stands for the torque input. It is important to notice that

the moments induced by the tangential and normal forces contribute to grasp and manipulate an object more securely. In addition, the terms of deformation model give us information about the behavior of the deformable material. Furthermore, the movement equations of the object are given as

$$m\ddot{x} - (f_1 - f_2)\cos(\theta) + (\lambda_1 + \lambda_2)\sin(\theta) = 0, \tag{2.13}$$

$$m\ddot{y} + (f_1 - f_2)\sin(\theta) + (\lambda_1 + \lambda_2)\cos(\theta) = 0, \tag{2.14}$$

$$I\ddot{\theta} - \Upsilon_1 f_1 + \Upsilon_2 f_2 - \lambda_1(d_{n1} - l_1) + \lambda_2(d_{n2} - l_2) + \frac{\pi E d_{n1}^2}{\cos^2(\phi_1)}\left(d_{t1} + \frac{2}{3}d_{n1}\tan(\phi_1)\right)$$
$$- \frac{\pi E d_{n2}^2}{\cos^2(\phi_2)}\left(d_{t2} + \frac{2}{3}d_{n2}\tan(\phi_2)\right) = 0. \tag{2.15}$$

The last two terms of (2.15) represent the contribution of the deformable fingertips to assure a stable grasping through induced forces/moments. Finally, dynamical equations related to the normal d_{ni} and tangential d_{ti} movements on the fingertips are defined as

$$m_{ni}\ddot{d}_{ni} + \pi E\left(\frac{d_{ni}^2}{\cos^2(\phi_i)} + 2d_{ni}d_{ti}\tan(\phi_i) + d_{ti}^2\right) - f_i = 0, \tag{2.16}$$

$$m_{ti}\ddot{d}_{ti} + \pi E\left(d_{ni}^2\tan(\phi_i) + 2d_{ni}d_{ti}\right) - \lambda_i = 0. \tag{2.17}$$

Summing the products between $\dot{q}_i^T$ with (2.12), $\dot{x}$ with (2.13), $\dot{y}$ with (2.14), $\dot{\theta}$ with (2.15), $\dot{d}_{ni}$ with (2.16), and $\dot{d}_{ti}$ with (2.17) yields

$$\sum_{i=1,2}\int_0^t\left(\dot{q}_i^T u_i\right)d\tau = E(t) - E(0) \geq -E(0), \tag{2.18}$$

where $E = K + P$ corresponds to the total energy of the system.

3. Controller Design

3.1. Immobilization on the Object

As first step before to execute manipulation tasks, a stable grasping must be established. To grasp stably an object, the force-closure is used to guarantee that the object should be held securely by the fingers. This mean that maintaining the contact between the fingers and

the object, that is, $f_1 > 0$ and $f_2 > 0$ for any $t > 0$, the forces and torques applied on the object should immobilize it. Let the forces and torques applied on the object be defined as

$$-(f_1 - f_2)\cos(\theta) + (\lambda_1 + \lambda_2)\sin(\theta) = 0,$$

$$(f_1 - f_2)\sin(\theta) + (\lambda_1 + \lambda_2)\cos(\theta) = 0,$$

$$-\Upsilon_1 f_1 + \Upsilon_2 f_2 + \lambda_1(d_{n1} - l_1) - \lambda_2(d_{n2} - l_2) + \frac{\pi E d_{n1}^2}{\cos^2(\phi_1)}\left(d_{t1} + \frac{2}{3}d_{n1}\tan(\phi_1)\right)$$

$$-\frac{\pi E d_{n2}^2}{\cos^2(\phi_2)}\left(d_{t2} + \frac{2}{3}d_{n2}\tan(\phi_2)\right) = 0.$$

(3.1)

Then, if we choose that

$$f_1 = f_2 = f_d, \qquad \lambda_1 + \lambda_2 = 0,$$

(3.2)

the first two equations are equal to zero, while the third equation is given as

$$-f_d(\Upsilon_1 - \Upsilon_2) + \lambda_1(d_{n1} - l_1 + d_{n2} - l_2) + \frac{\pi E d_{n1}^2}{\cos^2(\phi_1)}\left(d_{t1} + \frac{2}{3}d_{n1}\tan(\phi_1)\right)$$

$$-\frac{\pi E d_{n2}^2}{\cos^2(\phi_2)}\left(d_{t2} + \frac{2}{3}d_{n2}\tan(\phi_2)\right) = 0.$$

(3.3)

Thus, a force-closure can be established, if the forces acting on the object are defined as,

$$f_i \longrightarrow f_d, \qquad \Upsilon_1 - \Upsilon_2 \longrightarrow 0, \qquad \lambda_i \longrightarrow 0,$$

$$\frac{\pi E d_{n1}^2}{\cos^2(\phi_1)}\left(d_{t1} + \frac{2}{3}d_{n1}\tan(\phi_1)\right) - \frac{\pi E d_{n2}^2}{\cos^2(\phi_2)}\left(d_{t2} + \frac{2}{3}d_{n2}\tan(\phi_2)\right) \longrightarrow 0 \quad \text{for } i = 1, 2,$$

(3.4)

where f_d is the desired normal force. Once fingers grasp an object and hold it securely, we are in conditions to execute manipulation tasks on the object as orientation and move it at $x - y$ coordinates.

Inspired that humans can execute some manipulation tasks without any object information, in this paper a stable grasping and the object manipulation based only on the center position of the soft tip finger (x_i, y_i) are presented, so that the orientation θ and the object parameters are avoided. Using the superposition principle, the joint torque u_i applied to each finger can be defined as

$$u_i = u_{fci} + u_{\theta ci}$$

$$= -c_i \dot{q}_i + (-1)^i \frac{f_d}{l} J_i^T \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix} + (-1)^i \beta \frac{\overline{\Delta \tan(\theta)}}{x_2 - x_1} J_i^T \begin{pmatrix} \overline{\tan(\theta)} \\ 1 \end{pmatrix}, \quad i = 1, 2,$$

(3.5)

where c_i is a diagonal symmetric positive definite matrix, $\beta > 0$, $f_d > 0$ is the desired contact force, $\Delta \overline{\tan(\theta)} = \overline{\tan(\theta)} - \tan(\theta_d)$, θ_d is the desired angle of rotation, and

$$\overline{\tan(\theta)} = \frac{y_1 - y_2}{x_2 - x_1},$$

$$\overline{l}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = l_w^2 + (Y_1 - Y_2)^2 \quad \text{where } l_w = r_1 + r_2 + l - d_{n1} - d_{n2}. \tag{3.6}$$

Notice that u_{fci} refers to the stable grasp exerted on the object, while $u_{\theta ci}$ indicates the orientation control of the object. In the former case, a stable grasp is achieved minimizing the distance between the centers of the fingertips through the normal and tangential forces, which guarantee the control objectives $\Delta f_i \to 0$, $\Delta Y \to 0$. In the latter case, to avoid the measurement of θ, an approximation of θ is proposed by the trigonometric tangent function $\overline{\tan(\theta)}$. This approximation has the feature that the convergence of $\overline{\tan(\theta)}$ to $\tan(\theta)$ is guaranteed once $\Delta Y \to 0$ is satisfied. This implies that the conditions of the stable grasping $\Delta f_i \to 0$, $\Delta Y \to 0$, and $\overline{\tan(\theta)}$ should be satisfied.

To start the analysis and to ensure a stable grasping, it is considered that the joint torque is defined as $u_i = u_{fci}$. Substituting u_i in (2.12), the closed-loop system equations are defined as

$$H_i(q_i)\ddot{q}_i + \frac{1}{2}\dot{H}_i(q_i)\dot{q}_i + S_i(q_i, \dot{q}_i)\dot{q}_i - (-1)^i \Delta f_i J_i^T r_x - \Delta \lambda_i \left(J_i^T r_y - r_i e_i \right)$$

$$- \frac{\pi E d_{ni}^2}{\cos^2(\phi_i)}\left(d_{ti} + \frac{2}{3}d_{ni}\tan(\phi_i) \right)e_i - (-1)^i \frac{f_d}{\overline{l}}\Delta Y r_i e_i = -c_i \dot{q}_i,$$

$$m\ddot{x} - (\Delta f_1 - \Delta f_2)\cos(\theta) + (\Delta \lambda_1 + \Delta \lambda_2)\sin(\theta) = 0,$$

$$m\ddot{y} + (\Delta f_1 - \Delta f_2)\sin(\theta) + (\Delta \lambda_1 + \Delta \lambda_2)\cos(\theta) = 0,$$

$$I\ddot{\theta} - \Delta f_1 Y_1 + \Delta f_2 Y_2 - \Delta \lambda_1(d_{n1} - l_1) + \Delta \lambda_2(d_{n2} - l_2) \tag{3.7}$$

$$+ \sum_{i=1}^{2}(-1)^i \frac{\pi E d_{ni}^2}{\cos^2(\phi_i)}\left(d_{ti} + \frac{2}{3}d_{ni}\tan(\phi_i) \right) - \frac{f_d}{\overline{l}}\Delta Y(r_1 + r_2) = 0,$$

$$m_{ni}\ddot{d}_{ni} + \pi E \left(\frac{d_{ni}^2}{\cos^2(\phi_i)} + 2d_{ni}d_{ti}\tan(\phi_i) + d_{ti}^2 \right) - \Delta f_i - \frac{l_w}{\overline{l}}f_d = 0,$$

$$m_{ti}\ddot{d}_{ti} + \pi E \left(d_{ni}^2 \tan(\phi_i) + 2d_{ni}d_{ti} \right) - \Delta \lambda_i + (-1)^i \frac{f_d}{\overline{l}}\Delta Y = 0,$$

where $\Delta f = f_i - f_d \frac{l_w}{\overline{l}}$, $\Delta Y = Y_1 - Y_2$, and $\Delta \lambda_i = \lambda_i + (-1)^i \frac{f_d}{\overline{l}}\Delta Y$.

Expressing the closed-loop system equations (3.7) in a vector-matrix equation we have that

$$H\ddot{z} + C\dot{z} + P(z) - A\Delta\lambda - D\Delta Y + F\dot{z} = 0, \tag{3.8}$$

where

$$H = \text{diag}(H_1(q_1), H_2(q_2), m, m, I, m_{n1}, m_{n2}, m_{t1}, m_{t2}),$$

$$C = \text{diag}\left(\frac{1}{2}\dot{H}_1(q_1) + S_1(q_1, \dot{q}_1), \frac{1}{2}\dot{H}_2(q_2) + S_2(q_2, \dot{q}_2), 0, 0, 0, 0, 0, 0, 0\right),$$

$$F = \text{diag}(c_1, c_2, 0, 0, 0, 0, 0, 0, 0),$$

$$D = \left[-\frac{f_d}{l}r_1e_1, \frac{f_d}{l}r_2e_2, 0, 0, \frac{f_d}{l}(r_1 + r_2), \frac{l_w}{l\Delta Y}f_d, \frac{l_w}{l\Delta Y}f_d, \frac{f_d}{l}, -\frac{f_d}{l}\right]^T,$$

$$\Delta\lambda = \left[\Delta f_1, \Delta f_2, \Delta\lambda_1, \Delta\lambda_2\right]^T,$$

$$P(z) = \begin{bmatrix} -\dfrac{\pi E d_{n1}^2}{\cos^2(\phi_1)}\left(d_{t1} + \dfrac{2}{3}d_{n1}\tan(\phi_1)\right)e_1 \\[2ex] -\dfrac{\pi E d_{n2}^2}{\cos^2(\phi_2)}\left(d_{t2} + \dfrac{2}{3}d_{n2}\tan(\phi_2)\right)e_2 \\[2ex] 0 \\[1ex] 0 \\[1ex] \sum_{i=1}^{2}(-1)^i\dfrac{\pi E d_{ni}^2}{\cos^2(\phi_i)}\left(d_{ti} + \dfrac{2}{3}d_{ni}\tan(\phi_i)\right) \\[2ex] \pi E\left(\dfrac{d_{n1}^2}{\cos^2(\phi_1)} + 2d_{n1}d_{t1}\tan(\phi_1) + d_{t1}^2\right) \\[2ex] \pi E\left(\dfrac{d_{n2}^2}{\cos^2(\phi_2)} + 2d_{n2}d_{t2}\tan(\phi_2) + d_{t2}^2\right) \\[2ex] \pi E(d_{n1}^2\tan(\phi_1) + 2d_{n1}d_{t1}) \\[1ex] \pi E(d_{n2}^2\tan(\phi_2) + 2d_{n2}d_{t2}) \end{bmatrix},$$

(3.9)

$$A = \begin{bmatrix} -J_1^T r_X & 0_{3\times 1} & J_{S_{q1}} & 0_{3\times 1} \\ 0_{3\times 1} & J_2^T r_X & 0_{3\times 1} & J_{S_{q2}} \\ \cos(\theta) & -\cos(\theta) & -\sin(\theta) & -\sin(\theta) \\ -\sin(\theta) & \sin(\theta) & -\cos(\theta) & -\cos(\theta) \\ Y_1 & -Y_2 & (d_{n1} - l_1) & -(d_{n2} - l_2) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

with $J_{S_{qi}} = J_i^T r_Y - r_i e_i$, for $i = 1, 2$.

Taking the inner product between $\dot{z}$ and (3.8) yields

$$\frac{d}{dt}(K(z,\dot{z}) + P(z) + P_{\mathrm{art}}(z)) = -\dot{z}^T F\dot{z},$$

$$\frac{d}{dt}E_T(z,\dot{z}) = -\sum_{i=1,2}\left(\dot{q}_i^T c_i \dot{q}_i\right),$$

(3.10)

where $P_{\mathrm{art}}(z) = (f_d/2\bar{l})\Delta Y^2 - \sum_i \int_0^{d_{ni}} f_d d(\xi)$ is called artificial potential energy. Then, the system satisfys the passivity condition in closed loop.

Although $\dot{E}_T(z,\dot{z})$ is negative definite along the solution trajectories of fingers-object system, $E(z,\dot{z})$ cannot play a role of a Lyapunov function for the closed-loop system. Because it is neither define positive in the 26-dimensional state space $(z,\dot{z})$ nor in 18-dimensional constrained manifold M_{18} defined by

$$M_{18} = \left\{(z,\dot{z}) : C_{ti} = 0,\ \dot{C}_{ti} = 0,\ C_{ni} = 0,\ \dot{C}_{ni} = 0\right\}, \quad \text{for } i = 1,2.$$

(3.11)

In order to define a constrained manifold where the trajectories of the system guarantees a stable grasping; a first step is to show the boundedness of solutions in the closed loop. Considering that $C_{ni} = 0$ and $C_{ti} = 0$ we have that $(d/dt)C_{ni} = \dot{z}^T(\partial/\partial z)C_{ni} = 0$ and $(d/dt)C_{ti} = \dot{z}^T(\partial/\partial z)C_{ti} = 0$, respectively. In fact, if the matrix $A = [(\partial/\partial z)C_{n1}, (\partial/\partial z)C_{n2}, (\partial/\partial z)C_{t1}, (\partial/\partial z)C_{t2}]$ from previous expressions we have that $0 = A^T\dot{z}$. Then,

$$0 = \frac{d}{dt}\left(A^T\dot{z}\right) = A^T\ddot{z} + \dot{A}^T\dot{z}.$$

(3.12)

Now, if we multiply (3.8) by $A^T H^{-1}$ we obtain that

$$\Delta\lambda = \left(A^T H^{-1} A\right)^{-1}\left[-\dot{A}^T\dot{z} + A^T H^{-1}(C\dot{z} + P(z) - D\Delta Y + F\dot{z})\right],$$

(3.13)

where $A^T\ddot{z} = -\dot{A}^T\dot{z}$ from (3.12). Taking into account that $\dot{z}$, ΔY, d_{ni}, d_{ti}, and ϕ_i are bounded due to $E_T(z,\dot{z}) \leq 0$; we have that $\Delta\lambda$ is bounded under assumption that the matrix A in nondegenerate. Then, from (3.8) we have that $\ddot{z}$ is uniformly bounded which implies that $\dot{z}$ is uniformly continuous. Now, given that $\dot{q} \in L_2$ from (3.10) by Barbalat lemma [32] we have that $\dot{q} \to 0$ as $t \to 0$ which implies that $\dot{z} \to 0$ as $t \to 0$. Due to $\dot{z}$ and ΔY are uniformly continuous, we have that $\ddot{z}$ is uniformly continuous too. This in turn implies that $\ddot{z} \to 0$ at $t \to \infty$ and (3.8) will be defined as

$$P(z) = [A, D]\begin{bmatrix}\Delta\lambda \\ \Delta Y\end{bmatrix}.$$

(3.14)

Assuming that the matrix $[A, D]$ is nondegenerate, a point that minimizes the right side of the (3.14) on M_{18} exists. The unique critical point (z^*) that minimizes (3.14) can be defined as

$$\Delta\lambda = 0_{4\times1}, \quad \Delta Y = 0.$$

(3.15)

This means that the closed-loop trajectories converge to critical point in the equilibrium point manifold (EP) defined as

$$M_4 = \{(z) : C_{ti} = 0, \ C_{ni} = 0, \ \Delta Y \rightarrow 0, \ \Delta f_i \rightarrow 0, \ \Delta \lambda_i \rightarrow 0\}, \quad \text{for } i = 1, 2. \qquad (3.16)$$

Hence, the total potential energy is minimizing and the object is held securely. It is important to notice that initial conditions of the system and the length of the object are very important parameters to guarantee the convergence on M_4. Furthermore, the matrix $[A, D]$ is non-degenerate if J_1 and J_2 are full rank matrices, that is, the matrix is degenerate if $q_{i2} = q_{i3} = 0$ or $q_{i2} = q_{i3} = \pi$ for $i = 1, 2$. These joint values represent a special configuration of the fingers which can be excluded as a possible configuration in the manipulation tasks. At the same time, it is possible to show that the matrix A is nondegenerate if the J_1 and J_2 are full rank matrices.

Now, we are in conditions to define the following.

(i) Let the neighborhood $N_{26}(z^*, r_0)$ singularity-free be with a radius $r_0 > 0$ around the critical point $(z^*, 0)$ defined as

$$N_{26}(z^*, r_0) = \left\{ (z, \dot{z}) : \frac{1}{2} \Delta p^T H_0 \Delta p + \sum_{i=1,2} \frac{1}{2} \Delta q_i^T H_i(q_i) \Delta q_i, \ \leq r_0^2 \right\}, \qquad (3.17)$$

where $\Delta p = p - p_d$, $\Delta q = q_i - q_d$. That is, singular configurations of the soft finger tips are avoided inside the neighborhood $N_{26}(z^*, r_0)$ provide that r_0 is chosen adequately.

(ii) Let the neighborhood $N_{18}(z^*, r_0)$ around the critical point be where the closed-loop trajectories remain with $\delta > 0$, that is,

$$N_{18}(z^*, r_0) = \{(z, \dot{z}) : E_T \leq \delta, \ (z, \dot{z}) \in M_{18}\}. \qquad (3.18)$$

(iii) Definition (see [26]) If for any given $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ and another constant $r_1 > 0$ being less than r_0 and independent of ε such that the solution tracking trajectories starting from any initial condition $(z(0), \dot{z}(0))$ lying on $N_{18}(\delta(\varepsilon)) \cap N_{26}(r_1)$ remains on $N_{18}(\varepsilon) \cap N_{26}(r_0)$ and converges asymptotically to the set $M_4 \cap N_{18}(\varepsilon)$ as $t \rightarrow \infty$, then it is called that the state $(z^*, 0)$ is stable.

Finally, we have the following result.

Theorem 3.1. *Considering that the desired reference state $(z^*, 0)$ and initial state $(z(0), \dot{z}(0))$ lying on M_4. The trajectories in the closed-loop system remains on $N_{18}(\varepsilon) \cap N_{26}(r_0)$ and converges asymptotically to the set $M_4 \cap N_{26}(\varepsilon)$ as $t \rightarrow \infty$ under assumption that the matrix $[A, D_\Theta]$ is nondegenerate in a neighborhood $N_{26}(r_0)$. Thus, is assured that $\Delta f_i \rightarrow 0$, $Y_1 - Y_2 \rightarrow 0$, $\Delta \lambda_i \rightarrow 0$ as $t \rightarrow \infty$.*

Remark 3.2. Once the forces applied to the object have been compensated to hold it stably, the object can be rotated to the desired angle using the superposition principle, that is, the control law now is defined as

$$u_i = u_{fci} + u_{\theta ci}. \tag{3.19}$$

Premultiplying $\dot{q}_i^T$ by $u_{\theta ci}$ we have that

$$\sum_{i=1}^{2} \dot{q}_i^T u_{\theta ci} = \frac{d}{dt}(E_0), \tag{3.20}$$

where $E_0 = (1/2)\beta(\overline{\Delta \tan(\theta)})^2$. This means that there exists a constant η such that

$$\int_0^t \left(\sum_{i=1}^{2} \dot{q}_i^T u_{\theta ci} \right) d\psi = E_0(t) - E_0(0) \geq \eta, \tag{3.21}$$

where $\eta = -E_0(0)$.

Now, taking the inner product between $\dot{z}$ and the closed-loop system equations with u_i defined in (3.19) we have that

$$\frac{d}{dt}E_{T1} = -\sum_{i=1,2} \left(\dot{q}_i^T c_i \dot{q}_i \right), \tag{3.22}$$

where $E_{T1} = K + P + P_{art1}$ and $P_{art1} = P_{art} + (1/2)\beta(\overline{\Delta \tan(\theta)})^2$. As in stable grasping, the closed-loop trajectories converge to critical point on a constrained manifold where $\Delta \lambda_i \rightarrow 0$, $\Delta f_i \rightarrow 0$, $\Delta Y \rightarrow 0$ and $\overline{\Delta \tan(\theta)} \rightarrow 0$ as $t \rightarrow \infty$.

Remark 3.3. Now, when the stable grasping and object orientation tasks has been established, the objective will be to move the object to desired coordinates x_d by the following control law

$$u_{xi} = -\frac{\gamma_x}{2}(\overline{x} - x_d)\frac{\partial x_i}{\partial q_i}, \tag{3.23}$$

where $\gamma_x > 0$, $x_d > 0$, x_i is the Cartesian coordinates and $\overline{x}$ is the estimated position of the object which is defined as an average distance between center positions of the deformable fingertips, that is,

$$\overline{x} = \frac{x_1 + x_2}{2}. \tag{3.24}$$

Using the superposition principle, the control law for the stable grasping and object manipulation is defined as

$$u_i = u_{fci} + u_{\theta ci} + u_{xi} \quad \text{for } i = 1, 2. \tag{3.25}$$

As in the previous case, if we multiplying u_{xi} by $\dot{q}^T$ we have that

$$\sum_{i=1}^{2} \dot{q}_i^T u_{xi} = \frac{d}{dt}(E_1),$$

(3.26)

where $E_1 = (1/2)\gamma_x(\overline{x} - x_d)^2$ is the shifting energy to move the object in x-coordinates. Consequently, exists a constant η_1 such that

$$\int_0^t \left(\sum_{i=1}^{2} \dot{q}_i^T u_{xi} \right) d\psi = E_1(t) - E_1(0) \geq \eta_1,$$

(3.27)

where $\eta_1 = -E_1(0)$.

Now, taking the inner product between $\dot{z}$ and the closed-loop system equations with u_i defined in (3.25) we have

$$\frac{d}{dt} E_{T2} = -\sum_{i=1,2} \left(\dot{q}_i^T c_i \dot{q}_i \right),$$

(3.28)

where $E_{T2} = K + P + P_{art2}$ and $P_{art2} = P_{art1} + (1/2)\gamma_x(\overline{x} - x_d)^2$. Then, the passivity condition is satisfy in the closed-loop. Hence, the closed-loop system trajectories converge to critical point on a constrained manifold where $\Delta\lambda_i \to 0$, $\Delta f_i \to 0$, $\Delta Y \to 0$, $\overline{\Delta \tan(\theta)} \to 0$, and $\overline{x} \to x_d$ as $t \to \infty$.

4. Simulation Results

In order to demonstrate usefulness of our scheme for stable grasping and object orientation, numerical simulations were carried out on a pair of deformable fingertips in the horizontal plane, see Figure 1. The simulations were implemented on stiff numerical solver on Matlab R2007b, under 1 ms sampling time. Additionally, to approximate the holonomic constraints was used the Constrained Stabilization Method (CSM) [33].

The physical parameters of the fingers and object are shown in Table 1 where l_{ij}, m_{ij} and I_{ij} are the length, mass and moment of inertia of the link $j = 1, 2, 3$ for the finger $i = 1, 2$, and M, I, l are the mass, moment of inertia and the length of the object, respectively (Table 2). Moreover, $L = 0.64$ [m] is the distance between fingers, $E = 50000$ [N/m^2] is the Young's modulus of the fingertips and $r_i = 0.01$ [m] is the radius of the hemispherical finger tip for $i = 1, 2$.

The simulation study is divided in three steps. As first step before any manipulation task is necessary to guarantee that the stable grasp is achieved through the control law defined as $u_i = u_{fci}$. The initial conditions used in the simulations are $q_1(0) = (30, 91.61, 71.24)^T$, $q_2(0) = (30, 91.61, 71.24)^T$, $(x(0), y(0)) = (0.032, 0.047)$ [m], $\theta(0) = 0$, and $(d_{n1}, d_{n2}, d_{t1}, d_{t2}) = (0.0025, 0.0025, 0, 0)$ [m] which establish the following conditions: $f_1(0) = 1.1$ [N], $f_2(0) = 1.1$ [N]$\Delta Y(0) = 0$ [m]. For reference, these initial conditions are called normal initial conditions (CI$_N$).

Table 1: Physical parameters.

Parameter	Value
$l_{11} = l_{21}$	0.05 (m)
$l_{12} = l_{22}$	0.04 (m)
$l_{13} = l_{23}$	0.03 (m)
$m_{11} = m_{21}$	0.05 (kg)
$m_{12} = m_{22}$	0.03 (kg)
$m_{13} = m_{23}$	0.02 (kg)
$I_{11} = I_{21}$	1.4167×10^{-5} (kg m^2)
$I_{12} = I_{22}$	6.25×10^{-6} (kg m^2)
$I_{13} = I_{23}$	3×10^{-6} (kg m^2)

Table 2: Parameters of the object.

Parameter	Value
M	0.02 (kg)
I	5.67×10^{-6} (kg m^2)
l	0.03 (m)

In Figure 3 we observe that the forces on the object Δf_i and $\Delta \lambda_i$ converge rapidly to the desired values using the normal initial conditions defined previously. At the same time, the fast convergence of ΔY to zero is shown in Figure 3. On the other hand, the Figures 4, 5, and 6 show the performance of Δf_i, $\Delta \lambda_i$, and ΔY for different values of damping gains c_i. Notice that the damping gains are related with the convergence velocity towards equilibrium point that minimizes the potential energy of the system. However, the system takes longer to converge and several oscillations arise when the damping gains are small. The control parameters used in this step are $f_d = 1$ [N] and $c_1 = c_2 = 0.01$ [Nms/rad].

Finally, the Figures 7, 8, and 9 show the convergence of Δf_i and ΔY to zero under more extreme initial conditions. The initial conditions used in these simulations are CI$_1$ and CI$_2$ which are defined as $q_1(0) = (30, 95.65, 64.93)^T$, $q_2(0) = (30, 100.45, 76.09)^T$, $\Delta Y(0) = 0.01$ [m], and $q_1(0) = (30, 93.73, 68.32)^T$, $q_2(0) = (30, 102.16, 72.78)^T$, $(d_{n1}, d_{n2}, d_{t1}, d_{t2}) = (0.0013, 0.0037, 0, 0)$ [m], $\Delta Y(0) = 0.01$ [m], respectively.

Notice that the convergence to zero of Δf_i and ΔY present a transient responses for different initial conditions, but still converge to the desired values in few seconds. Additionally, the control objectives converge in different times. Specially, the convergence of ΔY takes more time for establishing a stable grasping when the initial conditions are more extreme, CI$_2$. Thus, it is possible to describe the stable grasping in two phases. In the first phase, the stabilization of the normal and tangential forces on object is performed while the stabilization of the rotational moments to stop the angular motion of the object is carried out in a second phase.

Once the all forces applied to the object have been compensated, it is possible to execute any manipulation task. In this case the rotation of the object, to a desired angle θ_d, will be realized. Using the superposition principle the control law is given as $u_i = u_{fci} + u_{\theta ci}$. The convergence to zero of Δf_i, $\Delta \lambda_i$, ΔY, and $\Delta \theta$ using the CI$_N$ initial conditions are shown in Figures 10 and 11. It is important to notice the special role of $Y_1 - Y_2$ as a parameter to increase the dexterity. In this case, the convergence of ΔY is closely associated to object orientation
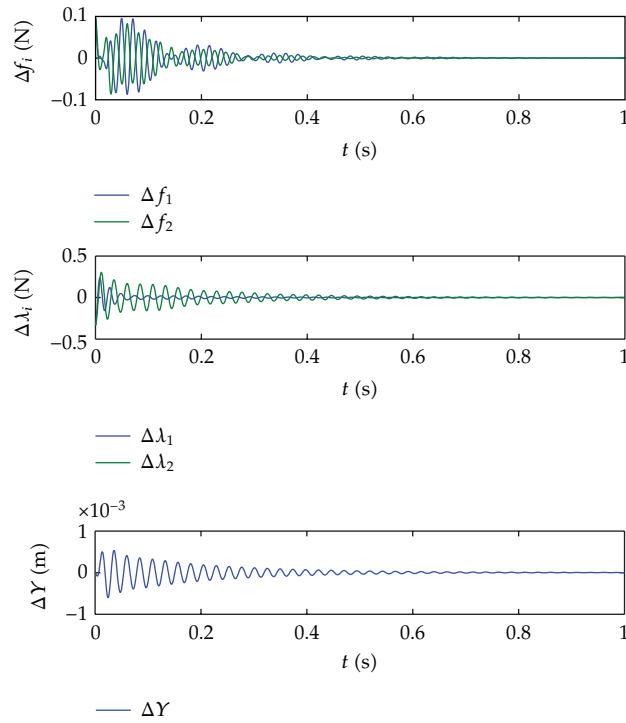
Figure 3: Convergence of Δf_i, $\Delta \lambda_i$, and ΔY for $i = 1, 2$.
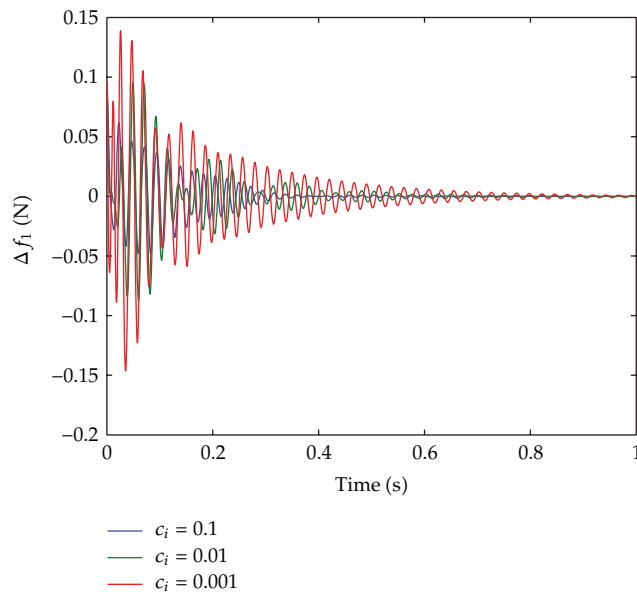

Figure 4: Convergence of Δf_i considering several values of c_i.
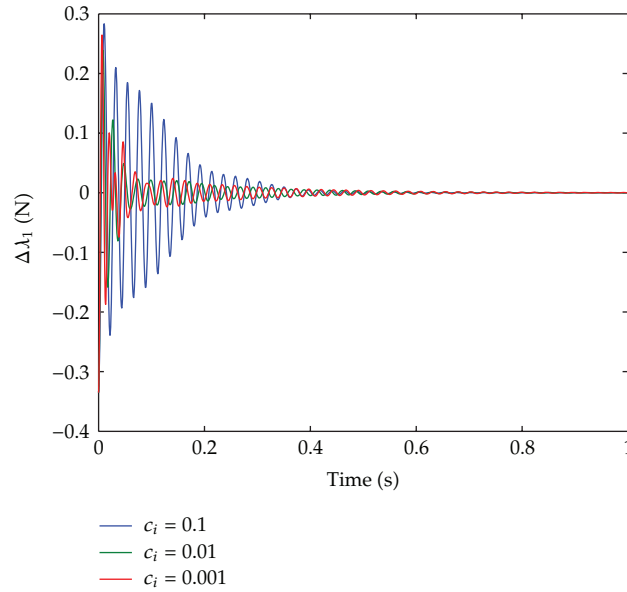
Figure 5: Convergence of $\Delta\lambda_i$ considering several values of c_i.
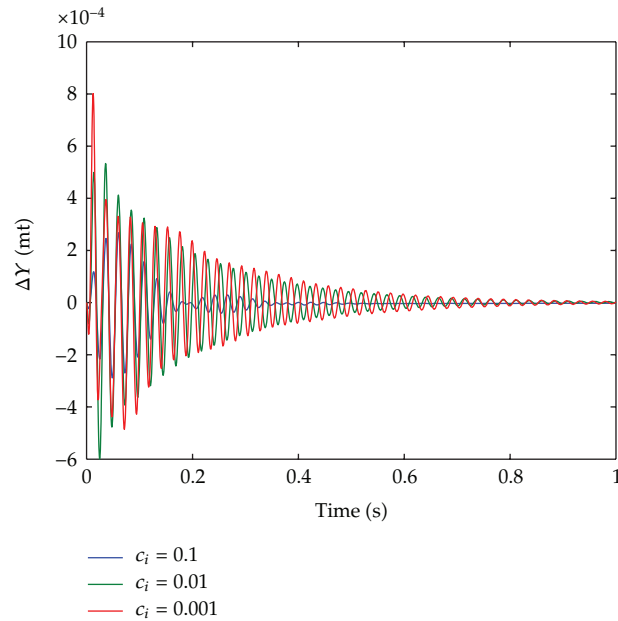

Figure 6: Convergence of ΔY considering several values of c_i.

through $\overline{\tan(\theta)}$. The control parameters using in this second step are $f_d = 1$ [N], $c_1 = c_2 = 0.01$ [Nms/rad], $\beta = 0.1$ [N/rad], and $\theta_d = -10(\pi/180)$ [rad].

As the final step of this study, a stable grasping and object manipulation which include the orientation and translation of the object to a desired reference is presented. Using a super-position principle, the control law for this step is defined as $u = u_{fci} + u_{\theta ci} + u_{\text{tras}}$.
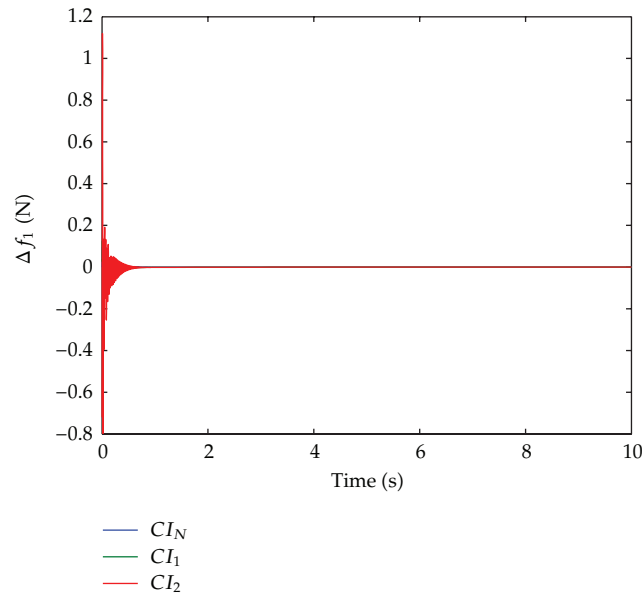
Figure 7: Convergence of Δf_i considering different initial conditions.
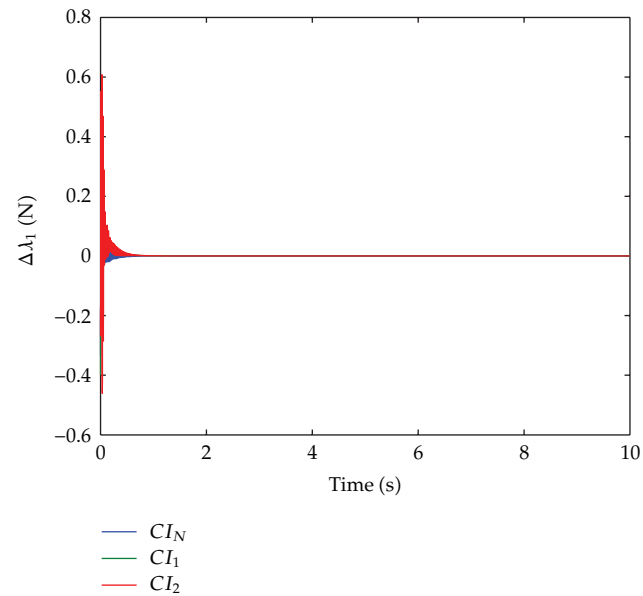
Figure 8: Convergence of $\Delta \lambda_i$ considering different initial conditions.

The Figures 12, 13, and 14 show the convergence of Δf_i, $\Delta \lambda_i$, ΔY, and $\Delta \theta$ to zero using CI_N initial conditions. The control parameters used in this step are $f_d = 1$ [N], $c_1 = c_2 = 0.01$ [Nms/rad], $\beta = 0.1$ [N/rad], $\theta_d = -10(\pi/180)$ [rad], $\gamma_x = 50$ [N/m], and $\overline{x}_d = \overline{x}_0 + 0.01$ [m].
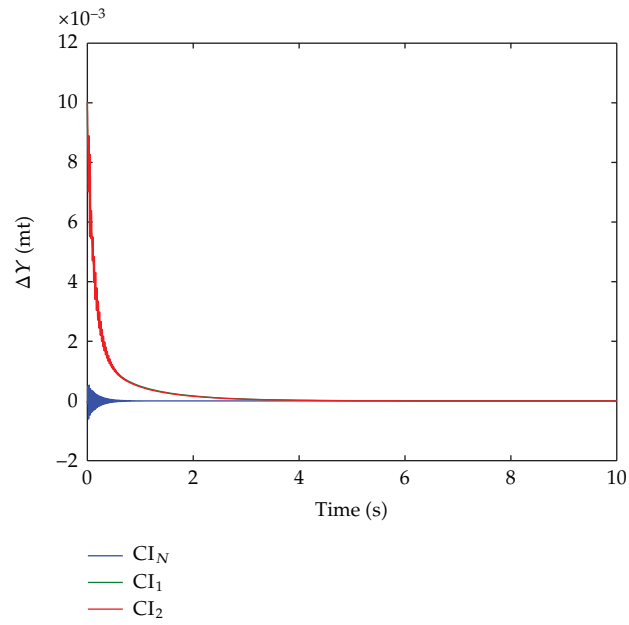
Figure 9: Convergence of ΔY considering different initial conditions.
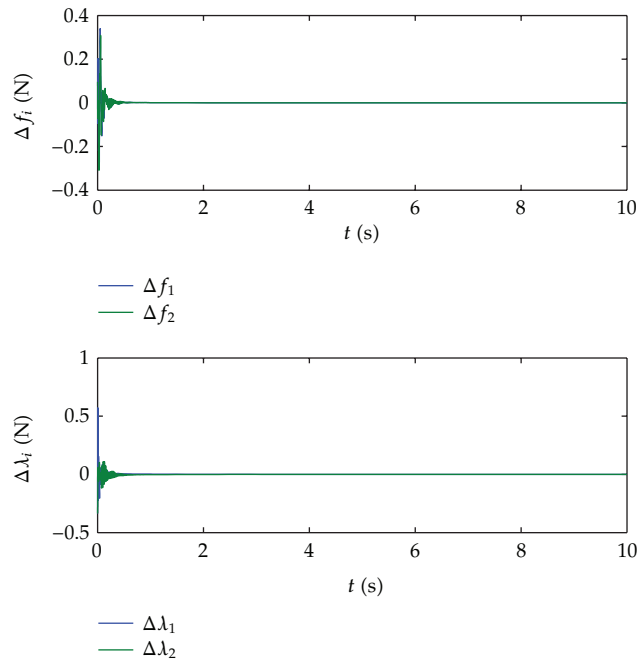
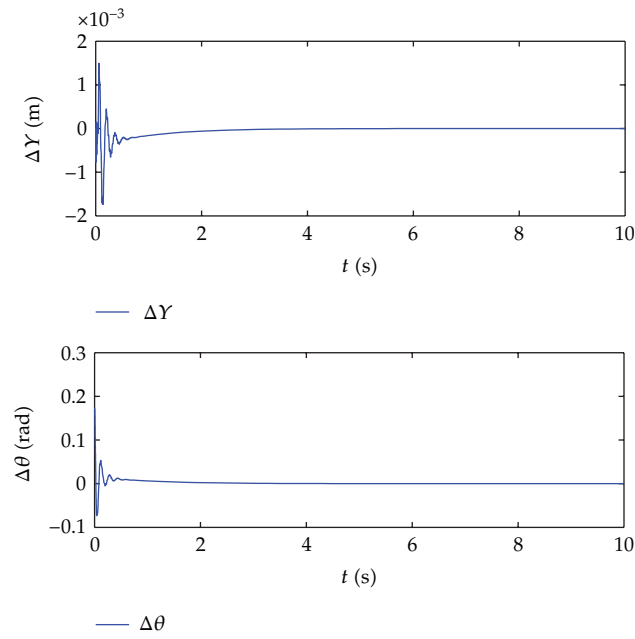**Figure 10:** Convergence of Δf_i and $\Delta \lambda_i$ for $i = 1, 2$.

Figure 11: Convergence of ΔY and $\Delta \theta$ for $i = 1, 2$.
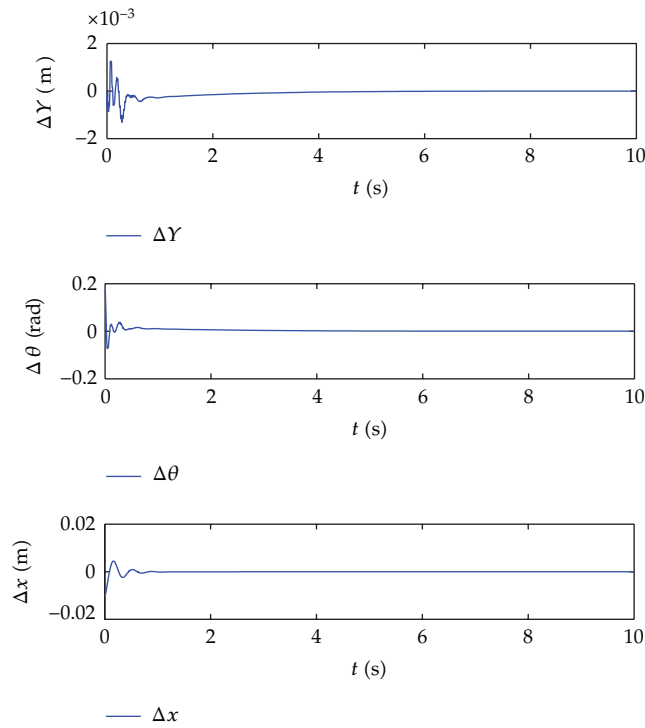
Figure 12: Convergence of ΔY, $\Delta \theta$, and Δx for $i = 1, 2$.
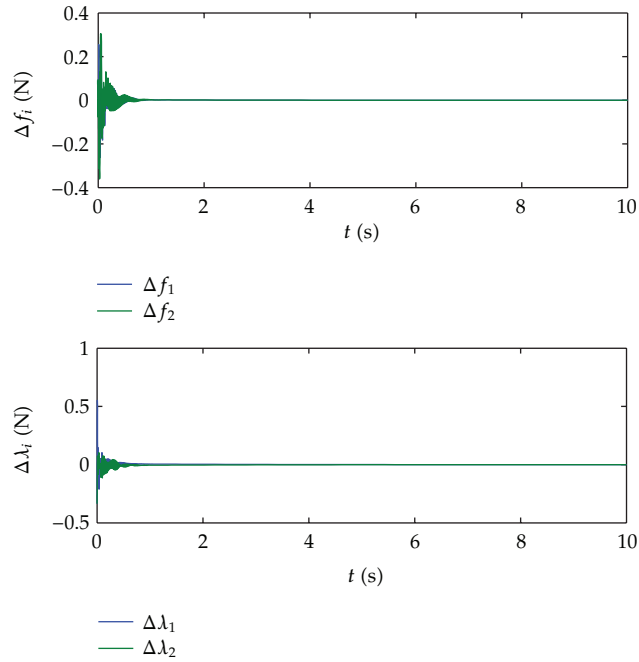
Figure 13: Convergence of Δf_i and $\Delta \lambda_i$ for $i = 1, 2$.
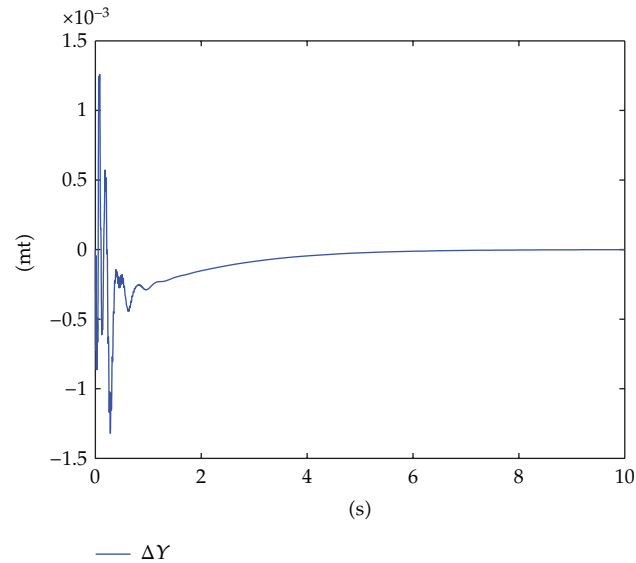

Figure 14: Convergence of ΔY for $i = 1, 2$.

5. Conclusions

A scheme to grasp and manipulate an object using soft tip fingers is presented. In order to include more characteristics of the human fingertips a parallel deformation model is used. The control law proposed ensures stability on a constrained manifold. Furthermore,

the control law avoids information of the radius of the tips and the convergence to the desired force value is guaranteed.

Numerical simulations, on a pair of deformable fingertips in horizontal plane, allow us to visualize the convergence of the closed-loop trajectories to the desired point. In addition, the special role of ΔY in the manipulation task and the effects of the superposition principle that have been observed.

Acknowledgments

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