Mathematical Problems in Engineering
Volume 4, pp. 393–421
Reprints available directly from the publisher
Photocopying permitted by license only

Torsional and Longitudinal Vibration of Suspension Bridge Subject to Aerodynamic Forces

N.U. AHMED* and H. HARBI

Department of Electrical Engineering and Mathematics, University of Ottawa, 161 Louis Pasteur Street, PO Box 450 Stn. A, Ottawa, Ontario K1N 6N5, Canada

(Received 27 January 1998)

In this paper we consider a dynamic model of suspension bridge governed by a pair of coupled partial differential equations which describe both torsional and longitudinal vibration of the road bed. The vertical and torsional motions are coupled through a nonlinear operation with the nonlinearity arising from loss of tension in the vertical cables supporting the decks. We study the impact of wind forces on the stability of motion of this system both in the absence and presence of viscous and structural damping. The results are illustrated by numerical simulation.

Keywords: Suspension bridge; Dynamic models; Stability; Asymptotic stability; Aerodynamic damping; Structural damping; Numerical results

AMS subject classification: 34K2O; 35Q72; 93D05; 93D20

1. INTRODUCTION

Since the collapse of the Tacoma Narrows bridge, extensive studies on the dynamics of suspension bridges, and their stability properties, oscillation, and occurrence of traveling waves were carried out by many workers in the field [1–11]. See also the references therein. In a series of papers Lazer and McKenna [1,6], McKenna and Walter [1,5,6] presented a PDE model for suspension bridges taking into

* Corresponding author. Tel.: 613-562–5738. Fax: 613-562-5175.
E-mail: ahmed@site.uottawa.ca.

account the coupling provided by the stays (vertical cables) connecting the suspension cable to the deck of the road bed. This coupling is fundamentally nonlinear. Recently in [8–10] the authors of this paper developed rigorous analysis of suspension bridge based on the PDE models and also studied their stability. However, these studies do not include the torsional motion. In a later paper, Jacover and McKenna [7] presented a model that includes torsional motion. However, a complete analysis of the system was not available. In this paper we use Jacover–McKenna model combined with the model for aerodynamic forces as developed in the excellent book by Roseau [11] and present a complete mathematical analysis. We also add some stochastic terms in the model to represent random fluctuation of the aerodynamic forces. We present both deterministic and stochastic analysis and illustrate our theoretical analysis with numerical simulation results.

The rest of the paper is organized as follows. In Section 2, a brief account of the function spaces used is presented. In Section 3, dynamic models of suspension bridges and questions of their stability are studied including conservative and damped systems. In Section 4, we present the mathematical analysis of the system subject to deterministic forces. In Section 5, we study its stochastic counterpart allowing random fluctuation of wind velocity. In Section 6, we present and discuss the numerical results for the deterministic part.

2. SOME RELEVANT FUNCTION SPACES

Let $\Sigma \subset R^n$ be an open bounded set with smooth boundary $\partial\Sigma$ and let $L_2(\Sigma)$ denote the space of equivalence classes of Lebesgue measurable and square integrable functions with the standard norm topology. Let $H^m(\Sigma) \equiv H^m$, $m \in N$, denote the standard Sobolev space with the usual norm topology

$$\|\psi\|_{H^m} \equiv \sum_{|\alpha| \le m} \|D^\alpha \psi\|_{L_2(\Sigma)}, \ \alpha = (\alpha_1, \alpha_2, \dots, \alpha_n), \ \alpha_i \ge 0, \ |\alpha| \equiv \sum \alpha_i$$

and $H_0^m(\Sigma) \equiv H_0^m \subset H^m$ denote the completion in the topology of H^m of C^∞ functions on Σ with compact support. From classical results on Sobolev spaces it is well known that the elements of H_0^m

are those of H^m which, along with their conormal derivatives up to order $m-1$, vanish on the boundary $\partial\Sigma$. The dual of H_0^m is H^{-m} which is a subspace of the space of distributions $\mathcal{D}'(\Sigma)$.

3. DYNAMIC MODELS OF SUSPENSION BRIDGE

In this section we present a dynamic model of suspension bridge combining the aerodynamic forces caused by wind as presented by Roseau [11] and the nonlinear couplings arising from loss of tension in the vertical cables as suggested by McKenna and Jacover [7]. Thus an approximate model that describes both longitudinal (vertical) and torsional vibration of the road bed is given by the following system of partial differential equations:

$$\begin{aligned} &mz_{tt} + \beta_1 D^4 z - \gamma_1 D^2 z + KF_1(z,\theta) = f_1, \quad x \in \Sigma \equiv (0,L), \ \ t \geq 0, \\ &I\theta_{tt} + \beta_2 D^4 \theta - \gamma_2 D^2 \theta + K\ell F_2(z,\theta) = f_2, \quad x \in \Sigma \equiv (0,L), \ \ t \geq 0, \end{aligned} \tag{3.1}$$

along with the suspension cable positions given by

$$\begin{aligned} y_1 &= F_0(z + \ell \sin\theta), \\ y_2 &= F_0(z - \ell \sin\theta). \end{aligned} \tag{3.2}$$

The first equation describes the displacement of the deck in the vertical plain from the rest position and the second equation describes the roll angle or the angle of twist of the road bed around the longitudinal axis from the horizontal plain. This is known as torsional motion. Here m is the mass per unit length of the bridge, $\mathcal{I} = 2m\ell^2$ is the mass moment of inertia (per unit length of the bridge) along the longitudinal axis, $\{\beta_1 = EI,\ \beta_2 = 2\ell^2\beta_1\}$ are the flexural rigidities, $\{\gamma_1 = (mg/8S)L^2,\ \gamma_2 = (KG + 2\ell^2\gamma_1)\}$ are the coefficients of elasticity of the suspension cables and S is its sag. The function

$$F_0(\xi) = \begin{cases} \xi & \text{if } \xi < 0, \\ 0 & \text{otherwise}, \end{cases} \tag{3.3}$$

signifying that if the vertical cables were loose, restraining force is zero and the suspension cables are free from the deck. The functions

F_1 and F_2 are given by

$$\begin{aligned} F_1(z,\theta) &\equiv F_0(z+\ell\sin\theta)+F_0(z-\ell\sin\theta), \\ F_2(z,\theta) &\equiv (F_0(z+\ell\sin\theta)-F_0(z-\ell\sin\theta))\cos\theta. \end{aligned} \tag{3.4}$$

These functions take into account the nonlinearity that arises due to loss of tension in the vertical cables tied to the girders. The functions $\{f_1, f_2\}$ represent all nonconservative forces including aerodynamic forces. The aerodynamic components of forces f_1 and f_2 are given approximately by

$$\begin{aligned} f_{1a} &= 2\pi\rho\ell v^2(\theta-(z_t/|v|)), \\ f_{2a} &= \pi\rho\ell^2 v^2(\theta-(z_t/|v|)-\ell(\theta_t/|v|)), \end{aligned} \tag{3.5}$$

where ρ is the air density, v is the wind velocity with angle of attack ν, with respect to the y-axis, and 2ℓ is the width of the bridge. For detailed derivation of these forces, the reader is referred to Roseau [11].

In order to solve the system of Eqs. (3.1), one must provide the boundary and initial conditions for z and θ. The boundary conditions are given by

$$\begin{aligned} &z(t,0)=z(t,L)=0, \qquad Dz(t,0)=Dz(t,L)=0, \\ &\theta(t,0)=\theta(t,L)=0, \qquad D\theta(t,0)=D\theta(t,L)=0, \end{aligned} \tag{3.6a}$$

in case the ends are clamped. In case they are hinged at both ends the boundary conditions are given by

$$\begin{aligned} &z(t,0)=z(t,L)=0, \qquad D^2z(t,0)=D^2z(t,L)=0, \\ &\theta(t,0)=\theta(t,L)=0, \qquad D^2\theta(t,0)=D\theta(t,L)=0. \end{aligned} \tag{3.6b}$$

These boundary conditions must hold for all $t \geq 0$. The initial conditions are given by

$$\begin{aligned} &z(0,x)=z_0(x), \qquad z_t(0,x)=z_1(x), \\ &\theta(0,x)=\theta_0(x), \qquad \theta_t(0,x)=\theta_1(x), \end{aligned} \tag{3.7}$$

where $\{z_0, z_1, \theta_0, \theta_1\}$ are suitable functions satisfying some conditions compatible with the given boundary conditions.

Conservative System

Consider the system (3.1) in the absence of nonconservative forces, that is, $f_1 = 0$, $f_2 = 0$, and subject to the boundary conditions (3.6). Considering all kinetic and potential energies of the system, the total energy functional is given by

$$E(t) \equiv (1/2)\int_0^L \Big\{[mz_t^2 + \beta_1 (D^2 z)^2 + \gamma_1 (Dz)^2] + [\mathcal{I}\theta_t^2 + \beta_2 (D^2\theta)^2 + \gamma_2 (D\theta)^2] + 2K[G_0(z + \ell \sin\theta) + G_0(z - \ell \sin\theta)]\Big\} \mathrm{d}x, \tag{3.8}$$

where

$$G_0(\eta) \equiv -\int_{-\eta}^{0} F_0(\xi)\,\mathrm{d}\xi = -\int_0^{\eta} F_0(-\xi)\,\mathrm{d}\xi.$$

Using the boundary conditions (3.6a)/(3.6b) and the system of Eq. (3.1) it is easy to verify that

$$\dot{E}(t) \equiv 0, \quad t \geq 0. \tag{3.9}$$

Thus we have the following result.

THEOREM 3.1 *In the absence of external forces, a suspension bridge, linear or nonlinear, is conservative. The total energy functionals given by* (3.8) *is a Lyapunov function for the systems* (3.1) *and* (3.6) *and hence it is stable in the Lyapunov sense.*

From engineering point of view Lyapunov stability is not good enough, one must have asymptotic stability with reasonably satisfactory decay rate.

Damped system First note that the external forces (and torques) f_1 and f_2 can be written as

$$\begin{aligned} f_1 &\equiv f_{1a} + f_{1v} + f_{1s}, \\ f_2 &\equiv f_{2a} + f_{2v} + f_{2s} \end{aligned} \tag{3.10}$$

where $f_{i,a}$, $i = 1, 2$ represent the aerodynamic forces, $f_{i,v}$, $i = 1, 2$ represent the viscous forces and $f_{i,s}$, $i = 1, 2$ the forces exerted by the structural elastic resistance. In case the wind velocity $v = 0$, the forces $f_{1a} = 0$ and $f_{2a} = 0$. Thus in the presence of only viscous and structural dampings we have

$$\begin{aligned} f_1 &= f_{1v} + f_{1s} \equiv -k_{11}z_t + k_{12}D^2 z_t - k_{13}D^4 z_t, \\ f_2 &= f_{2v} + f_{2s} \equiv -k_{21}\theta_t + k_{12}D^2\theta_t - k_{23}D^4\theta_t. \end{aligned} \tag{3.11}$$

For the system (3.1) with the forces $\{f_1, f_2\}$ as defined above, the existence question is somewhat subtle. The question of existence and regularity properties of solutions are studied in Section 4. Here we are interested in stability. Taking the time derivative of the expression (3.8) and using the system of Eq. (3.1) and the boundary conditions (3.6) we have

$$\begin{aligned} \dot{E}(t) &= \int_0^L \{f_1 z_t + f_2\theta_t\}\,\mathrm{d}x \\ &= -\int_0^L (\{k_{11}(z_t)^2 + k_{12}(Dz_t)^2 + k_{13}(D^2 z_t)^2\}. \\ &\qquad + \{k_{21}(\theta_t)^2 + k_{22}(D\theta_t)^2 + k_{23}(D^2\theta_t)^2\})\,\mathrm{d}x \le 0. \end{aligned} \tag{3.12}$$

By using (3.1) and (3.12) and the boundary conditions (3.6), it is easy to show that $\dot{E}(t) < 0$, for all $t > 0$, and that the system is asymptotically stable. More precisely we have the following result.

THEOREM 3.2 *Consider the system* (3.1) *and* (3.6) *and suppose the forces* $\{f_1, f_2\}$ *are as given by* (3.11). *Then the system is asymptotically stable if any one of the pairs* $\{(k_{1i}, k_{2j}),\ i, j = 1, 2, 3\}$ *is strictly positive.*

Remark Another proof is given by use of semigroup theory (see Corollary 4.4).

4. ABSTRACT MODEL

In this section we wish to present an abstract model of the system (3.1) subject to the boundary conditions (3.6). This is useful in (i) the study of existence of solutions and their regularity properties, (ii) modeling stochastic wind forces and carrying out mathematical analysis of the stochastic system, (iii) study of stability. For this we introduce the Hilbert spaces

$$\begin{aligned} H &\equiv L_2(\Sigma) \times L_2(\Sigma), \\ V &\equiv H_0^2 \times H_0^2, \end{aligned} \tag{4.1}$$

with the natural scalar products:

$$\begin{aligned} (\xi, \eta)_H &= (\xi_1, \eta_1)_{L_2(\Sigma)} + (\xi_2, \eta_2)_{L_2(\Sigma)}, \quad \text{for } \xi, \eta \in H, \\ (\xi, \eta)_V &= (\xi_1, \eta_1)_{H^2(\Sigma)} + (\xi_2, \eta_2)_{H^2(\Sigma)}, \quad \text{for } \xi, \eta \in V. \end{aligned} \tag{4.2}$$

By virtue of Poincare inequality, the scalar product for V as defined above is equivalent to simply

$$(\xi, \eta)_V = (D^2\xi_1, D^2\eta_1)_{L_2(\Sigma)} + (D^2\xi_2, D^2\eta_2)_{L_2(\Sigma)}, \quad \text{for } \xi, \eta \in V. \tag{4.3}$$

Define

$$\begin{aligned} a_1 &\equiv (\beta_1/m), \quad b_1 \equiv (\gamma_1/m), \\ a_2 &\equiv (\beta_2/\mathcal{I}) \quad \text{and} \quad b_2 \equiv (\gamma_2/\mathcal{I}) \end{aligned} \tag{4.4}$$

and let $A(D)$ denote the formal partial differential operator given by

$$A(D)\psi \equiv \begin{pmatrix} a_1 D^4\psi_1 - b_1 D^2\psi_1 \\ a_2 D^4\psi_2 - b_2 D^2\psi_2 \end{pmatrix} \quad \text{for } \psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}; \tag{4.5}$$

and A its realization, with the boundary condition (3.6), as an unbounded closed operator in the Hilbert space H with domain

$$D(A) = (H^4 \cap H_0^2) \times (H^4 \cap H_0^2) \subset H$$

which is dense in H. Since the coefficients $\{a_1, b_1, a_2, b_2\}$ are all positive it is easy to verify that A is a positive selfadjoint operator in H. On the other hand identifying H with its own dual H^* we have the Gelfand triple $\{V, H, V^*\}$ with the injections $V \hookrightarrow H \hookrightarrow V^*$ being continuous and dense. Then we consider the operator A as a bounded operator from V to its dual V^*, and again since the coefficients are positive it is easy to verify that A is coercive and there exists a number $\delta > 0$, such that

$$\langle Av, v \rangle_{V^*, V} \geq \delta \|v\|^2, \quad \forall v \in V.$$

Let F_c denote the coupling operator, and F_n the input operator, that includes all nonconservative forces such as viscous and aerodynamic dampings, as defined below:

$$\begin{aligned} F_c(\psi) &\equiv \begin{pmatrix} -(K/m)F_1(\psi_1,\psi_2) \\ -(K\ell/\mathcal{I})F_2(\psi_1,\psi_2) \end{pmatrix}, \\ F_n(\psi,\dot{\psi}) &\equiv \begin{pmatrix} (1/m)f_1(t,.,\psi,D\psi,D^2\psi,\psi_t) \\ (1/\mathcal{I})f_2(t,.,\psi,D\psi,D^2\psi,\psi_t) \end{pmatrix}. \end{aligned} \tag{4.6}$$

Combining these we can write the system (3.1) as an ordinary second order differential equation in the Hilbert space H given by

$$\ddot{\psi} + A\psi = F_c(\psi) + F_n(\psi,\dot{\psi}), \;\; t \geq 0, \quad \psi(0) = \psi_0. \tag{4.7}$$

To deal with stochastic counter parts of this system it is convenient to write this system as a first order differential equation in an appropriate Hilbert space. Towards this goal we introduce the Hilbert space $E \equiv V \times H$ with the scalar product and the associated norms given by

$$\begin{aligned} \langle \phi,\psi \rangle_E \equiv {} & \beta_1(D^2\phi_1,D^2\psi_1) + \gamma_1(D\phi_1,D\psi_1) + \beta_2(D^2\phi_2,D^2\psi_2) \\ & + \gamma_2(D\phi_2,D\psi_2) + m(\phi_3,\psi_3) + \mathcal{I}(\phi_4,\psi_4), \end{aligned} \tag{4.8}$$

and

$$\begin{aligned} \|\phi\|_E^2 \equiv {} & \beta_1\|D^2\phi_1\|^2_{L_2(\Sigma)} + \gamma_1\|D\phi_1\|^2 + \beta_2\|D^2\phi_2\|^2_{L_2(\Sigma)} \\ & + \gamma_2\|D\phi_2\|_{L_2(\Sigma)} + m\|\phi_3\|^2_{L_2(\Sigma)} + \mathcal{I}\,\|\phi_4\|^2_{L_2(\Sigma)}, \end{aligned} \tag{4.9}$$

respectively where we have used $(\,,)$ to denote the scalar product in $L_2(\Sigma)$. Note that E is actually the physical energy space and (4.9) denotes the sum of elastic potential energies and the kinetic energies. We consider E for the state space and

$$\phi \equiv \{\phi_1,\phi_2,\phi_3,\phi_4\}' \equiv \{z,\theta,z_t,\theta_t\}' \tag{4.10}$$

for the state. Define the operator $\mathcal{A}$ as the realization of the formal differential operator

$$\mathcal{A}(D)\phi \equiv \{\phi_3,\phi_4,-a_1D^4\phi_1 + b_1D^2\phi_1,-a_2D^4\phi_2 + b_2D^2\phi_2\}' \tag{4.11}$$

with either one of the boundary conditions (3.6), say. Note that the domain of the operator $\mathcal{A}$ is given by

$$D(\mathcal{A}) \equiv (H^4 \cap H_0^2) \times (H^4 \cap H_0^2) \times H_0^2 \times H_0^2, \tag{4.12}$$

with

$$\mathcal{A}\phi = \mathcal{A}(D)\phi, \quad \text{for } \phi \in D(\mathcal{A}).$$

Clearly both the domain and the range of the operator $\mathcal{A}$ are in E.

Define the operators $\mathcal{F}_\mathrm{c}$ and $\mathcal{F}_\mathrm{n}$ on E as follows:

$$\begin{aligned} \mathcal{F}_\mathrm{c}(\phi) &\equiv \{0, 0, -(K/m)F_1(\phi), -(K\ell/\mathcal{I})F_2(\phi)\}', \\ \mathcal{F}_\mathrm{n}(t,\phi) &\equiv \{0, 0, (1/m)f_1, (1/\mathcal{I})f_2\}', \end{aligned} \tag{4.13}$$

where $\{f_i, \ i = 1, 2\}$ are as in (4.6). Dependence of these functions on time are due to the presence of wind velocity as a parameter which may vary with time. Using (4.10), (4.11) and (4.13) we can rewrite system (4.7) as a first order evolution equation on the Hilbert space E as follows:

$$\begin{aligned} \dot{\phi} &= \mathcal{A}\phi + \mathcal{F}_\mathrm{c}(\phi) + \mathcal{F}_\mathrm{n}(t,\phi), \\ \phi(0) &\equiv \phi_0, \end{aligned} \tag{4.14}$$

where $\phi(0) \equiv \{\phi_1(0), \phi_2(0), \phi_3(0), \phi_4(0)\}' \equiv \phi_0$ denotes the initial state. From here on we shall deal with this first order system. The following result is fundamental.

LEMMA 4.1 *The operator $\mathcal{A}$ is the infinitesimal generator of a C_0-semigroup of contractions $S(t), t \geq 0$ in the Hilbert space E.*

Proof The operator $\mathcal{A}$ is closed and $D(\mathcal{A})$ is dense in E. For each $\phi \in D(\mathcal{A})$, one can verify that $\langle \mathcal{A}\phi, \phi \rangle = 0 = \langle \mathcal{A}^*\phi, \phi \rangle$. Thus both $\mathcal{A}$ and $\mathcal{A}^*$ are dissipative in E. Hence it follows from Theorem 2.2.18 [12, p. 36] that $\mathcal{A}$ is the infinitesimal generator of a C_0-semigroup $S(t)$, $t \geq 0$, of contractions in E.

Remark It is easy to verify that $i\mathcal{A} = (i\mathcal{A})^*$ or equivalently $\mathcal{A} = -\mathcal{A}^*$. Thus it follows from Stone's theorem [12, Theorem 3.1.4] that actually $S(t)$, $t \in R$, is a unitary group of linear operators in E. Using this result and the variation of constants formula we can rewrite the evolution equation (4.14) as an integral equation on E,

$$\phi(t) = S(t)\phi_0 + \int_0^t S(t-s)(\mathcal{F}_c(\phi(s)) + \mathcal{F}_n(s,\phi(s)))\,ds, \quad t \geq 0 \quad (4.15)$$

THEOREM 4.2 *Suppose the functions* f_i, $i=1,2$ *are measurable in* $(t,x)\in I\times\Sigma$ *and continuous in the rest of the variables satisfying, for some* $g \in L_\infty^+(I\times\Sigma, R)$, *the Lipschitz and growth conditions*

$$\begin{aligned} |f_i(t,x,\xi) - f_i(t,x,\eta)| &\leq g(t,x)\|\xi-\eta\|_{R^8}, \\ |f_i(t,x,\xi)| &\leq g(t,x)(1+\|\xi\|_{R^8}), \end{aligned} \quad (4.16)$$

for all $\xi,\eta\in R^8$ where R^n $(n=8)$ *is equipped with the standard Euclidean norm. Then for each* $\phi_0\in E$ *the system* (4.14) *has a unique mild solution* $\phi\in C(I,E)$ *on any finite interval* $I\equiv[0,T]$.

Proof Define the operator

$$\mathcal{F}(t,\phi) \equiv \mathcal{F}_c(\phi) + \mathcal{F}_n(t,\phi)$$

and the operator $\mathcal{G}$ with values $(\mathcal{G}\phi)(t)$ as follows:

$$(\mathcal{G}\phi)(t) \equiv S(t)\phi_0 + \int_0^t S(t-s)\mathcal{F}(s,\phi(s))\,ds, \quad t\in I.$$

By use of assumption (4.16) and the properties of the functions F_1, F_2 given by Eqs. (3.3) and (3.4), it is easy to verify that $\mathcal{F}(t,v)$ is strongly measurable in $t\in I$ and continuous in $v\in E$ satisfying the Lipschitz and the growth conditions

$$\begin{aligned} \|\mathcal{F}(t,u) - \mathcal{F}(t,v)\|_E &\leq \tilde{K}\|u-v\|_E, \\ \|\mathcal{F}(t,u)\|_E &\leq \tilde{K}(1+\|u\|_E), \end{aligned} \quad (4.17)$$

where the constant $\tilde{K}$ is dependent on the L_∞ norm of g on the set $I\times\Sigma$ and the stiffness coefficient K of the vertical cables and the width ℓ of the decks. By use of the growth condition of the operator $\mathcal{F}$ and the C_0 property of the semigroup $S(t), t\geq 0$, one can easily verify that $\mathcal{G}$ maps $C(I,E)$ into itself. Using the Lipschitz property one can verify that for large enough $n \in N$, the nth iterate of $\mathcal{G}$ denoted by $\mathcal{G}^n$ is a contraction in $C(I, E)$. Thus by Banach fixed point theorem, $\mathcal{G}^n$ and hence $\mathcal{G}$ has one and the same fixed point in $C(I, E)$. This proves that the system (4.14) has a unique mild solution $\phi \in C(I, E)$.

In view of the aerodynamic forces given by the expressions in (3.5) and the nonlinear coupling given by (3.4), it is clear that the Lipschitz and the growth assumptions of Theorem 4.1 are sufficient conditions for the existence of solution of the partial differential equation (3.1) subject to any one of the boundary conditions in (3.6) and initial condition (3.7). In addition to containing aerodynamic components f_{1a} and f_{2a} given by Eq. (3.5), the functions $\{f_1, f_2\}$ can also absorb viscous damping terms under the same assumptions. Problem arises if f_1 and f_2 are to take care of higher order structural damping components. For example, suppose these functions are given by

$$\begin{aligned} f_1 &= f_{1a} + f_{1v} + f_{1s}, \\ f_2 &= f_{2a} + f_{2v} + f_{2s}, \end{aligned} \tag{4.18}$$

with $\{f_{1v}, f_{2v}\}$ denoting the viscous damping components while the structural damping components $\{f_{1s}, f_{2s}\}$ are given by

$$\begin{aligned} f_{1s}(\psi, \dot{\psi}) &\equiv \gamma_{11} D^2 \psi_{1,t} - \gamma_{12} D^4 \psi_{1,t}, \\ f_{2s}(\psi, \dot{\psi}) &\equiv \gamma_{21} D^2 \psi_{2,t} - \gamma_{22} D^4 \psi_{2,t}. \end{aligned} \tag{4.19}$$

The parameters $\{\gamma_{i,j},\ 1 \leq i,\ j \leq 2\}$ are nonnegative and they are dependent only on the material properties of the road bed and the girders. Define the operator $\mathcal{D}$ as

$$\mathcal{D}\phi \equiv \{0, 0, (\gamma_{11}/m) D^2 \phi_3 - (\gamma_{12}/m) D^4 \phi_3, (\gamma_{21}/\mathcal{I}) D^2 \phi_4 - (\gamma_{22}/\mathcal{I}) D^4 \phi_4\}' \tag{4.20}$$

with domain

$$D(\mathcal{D}) = (H^4 \cap H_0^2) \times (H^4 \cap H_0^2) \times (H^4 \cap H_0^2) \times (H^4 \cap H_0^2).$$

The operator $\mathcal{F}_n(t, \phi)$ is again given by Eq. (4.13) with f_1 and f_2 replaced by $f_{1a} + f_{1v}$ and $f_{2a} + f_{2v}$ respectively. Now we have the following system generalizing the evolution equation (4.14):

$$\dot{\phi} = \mathcal{A}\phi + \mathcal{D}\phi + \mathcal{F}_c(\phi) + \mathcal{F}_n(t,\phi) \equiv \mathcal{A}\phi + \mathcal{D}\phi + \mathcal{F}(t,\phi), \quad \phi(0) \equiv \phi_0, \tag{4.21}$$

THEOREM 4.3 *Let the operators* $\{\mathcal{A}, \mathcal{F}_c\}$ *and* $\mathcal{F}_n$ *be as in Theorem* 4.2 *satisfying the assumptions given there and let* $\mathcal{D}$ *be the operator as defined above. Then for each* $\phi_0 \in E$ *the system* (4.21) *has a unique mild solution* $\phi \in C(I,E)$ *on any finite interval* $I \equiv [0,T]$.

Proof Define the operator

$$\hat{\mathcal{A}} \equiv \mathcal{A} + \mathcal{D}$$

with domain given by

$$D(\hat{\mathcal{A}}) \equiv (H^4 \cap H_0^2) \times (H^4 \cap H_0^2) \times (H^4 \cap H_0^2) \times (H^4 \cap H_0^2).$$

Clearly $D(\hat{\mathcal{A}})$ is dense in E and it is easy to verify that the graph of $\hat{\mathcal{A}}$ denoted by $\Gamma(\hat{\mathcal{A}})$ is weakly closed and hence a closed subset of $E \times E$. Thus $\hat{\mathcal{A}}$ is also a closed operator. Using integration by parts, it follows from (4.19) and the definition of the operator $\mathcal{D}$ that $\langle \mathcal{D}\phi, \phi \rangle_E \leq 0$ for all $\phi \in D(\mathcal{D})$. Thus both $\hat{\mathcal{A}}$ and $\hat{\mathcal{A}}^*$ are dissipative. Thus by Lemma 4.1, $\hat{\mathcal{A}}$ is also the infinitesimal generator of a C_0-semigroup of contractions in the Hilbert space E. Let $\hat{S}(t), t \geq 0$, denote the corresponding semigroup. Using this semigroup we can write the evolution equation (4.21) as an integral equation on E like (4.15) giving

$$\phi(t) = \hat{S}(t)\phi_0 + \int_0^t \hat{S}(t-s)\mathcal{F}(s,\phi(s))\,\mathrm{d}s, \quad t \geq 0. \tag{4.22}$$

Then the conclusion of this theorem follows from the same arguments as in Theorem 4.2. This completes the proof.

Remark Note that unlike $\mathcal{A}, \hat{\mathcal{A}}$ is not a generator of a unitary group. This is due to the fact that the operator $\mathcal{D}$ is not skew adjoint while $\mathcal{A}$ is.

COROLLARY 4.4 *The semigroup $\hat{S}(t), t \geq 0$, is asymptotically stable in the Hilbert space E.*

Proof By Theorem 4.3, $\hat{S}$ is a contraction semigroup in E, but it is not a unitary group. This means that $\|\hat{S}(t)\phi_0\|_E \leq \|\phi_0\|_E$ and that $\|\hat{S}(t)\phi_0\|_E \not\equiv \|\phi_0\|_E$ for all $t \geq 0$. Hence there exists a number t_0 with $0 < t_0 < \infty$ and a number $\rho \in (0, 1)$ such that $\|\hat{S}(t)\| \leq \rho < 1$ for all $t \geq t_0$. Thus for any $\phi_0 \in E$ and $t \geq nt_0$, we have

$$\|\hat{S}(t)\phi_0\| = \|\hat{S}(t - nt_0)\hat{S}(nt_0)\phi_0\| \leq \|\hat{S}(nt_0)\phi_0\| \leq (\rho)^n \|\phi_0\|.$$

Hence letting $n \to \infty$ we have $\lim_{t\to\infty} \hat{S}(t)\phi_0 = 0$. This proves the assertion.

5. STOCHASTIC MODEL

In Section 3 we have seen the expressions for aerodynamic forces given by Eq. (3.5). It is natural to think that there are random variations of these forces due to random fluctuation of the wind velocity and its direction. To model the impact of this on the dynamics of the structure, one must determine the flow dynamics of the wind around the exterior of the bridge which itself is in state of motion. This can be done only by solving the Navier–Stokes equation for compressible fluid (air) in the exterior domain around the bridge which is coupled with the equations of vibration given by Eq. (3.1). This is certainly a very complex situation. We wish to present here something that is mathematically tractable. Let v denote the mean velocity of wind, with ν being the angle of attack measured with respect to the horizontal plane, and let $\tilde{v}$ be its random fluctuation giving the instantaneous wind velocity

$$V = v + \tilde{v}. \tag{5.1}$$

We denote the random fluctuation by

$$\mathrm{d}\tilde{v} = \sigma\,\mathrm{d}w \text{ or equivalently } (\mathrm{d}/\mathrm{d}t)\tilde{v} = \sigma(\mathrm{d}/\mathrm{d}t)w, \tag{5.2}$$

where σ^2 stands for the instantaneous wind fluctuation energy and w stands for one dimensional standard Wiener process with its generalized derivative $(\mathrm{d}/\mathrm{d}t)w$ being the white noise. We assume that σ is a bounded measurable function of t for all $t \geq 0$. Let A_e denote the effective area of the deck per unit length of the span as seen by the wind, ρ the density of air. Then the instantaneous momentum per unit length of the bridge is given by

$$M = \rho A_e (v + \tilde{v})^2 \tag{5.3}$$

Assuming the time rate of change of the effective area A_e to be negligible, the Ito derivative of the momentum M is given by

$$\begin{aligned} dM(t) &\equiv \rho A_e \, d(v + \tilde{v})^2 \\ &= 2\rho A_e (v + \tilde{v})(a \, dt + d\tilde{v}) \\ &= 2\rho A_e (v + \tilde{v}) a \, dt + 2\rho A_e (v + \tilde{v}) \, d\tilde{v}, \end{aligned} \tag{5.4}$$

where $a = (d/dt)v$ denotes the mean acceleration. Assuming $v \gg \tilde{v}$ we approximate this by

$$\begin{aligned} dM &\cong 2\rho A_e v a \, dt + 2\rho A_e v \, d\tilde{v} \\ &\cong 2\rho A_e v a \, dt + 2\rho A_e v \sigma \, dw. \end{aligned} \tag{5.5}$$

Thus the random fluctuation of the wind forces acting on the structure in the direction of the wind can be approximately modeled by the second component of the above expression. Since the angle of attack is given by ν, the vertical component of the fluctuating force is given by

$$2(\rho A_e v \sigma) \sin \nu \, (dw/dt).$$

Adding this fluctuation to the aerodynamic forces given by the expression (3.5), we have

$$\begin{aligned} \tilde{f}_{1a} &\equiv f_{1a} + 2(\rho A_e v \sigma) \sin \nu \, (dw/dt) \\ \tilde{f}_{2a} &\equiv f_{2a} + (\rho \ell A_e v \sigma) \sin \nu \, (dw/dt). \end{aligned} \tag{5.6}$$

Letting $\theta = \theta(t, x)$ denote the angle of inclination of the surface of the deck measured with respect to the horizontal plane at time t and at the position $x \in (0, L)$, the effective area A_e, per unit length of the span as defined earlier, is given by

$$A_e \equiv 2\ell \sin(\theta - \nu). \tag{5.7}$$

Using this in Eq. (5.6), the full aerodynamic forces acting on the bridge is given by

$$\begin{aligned} \tilde{f}_{1a} &\equiv f_{1a} + 4\rho\ell \sin(\theta - \nu) v\sigma \sin\nu \, (\mathrm{d}w/\mathrm{d}t) \\ \tilde{f}_{2a} &\equiv f_{2a} + 2\rho\ell^2 \sin(\theta - \nu) v\sigma \sin\nu \, (\mathrm{d}w/\mathrm{d}t). \end{aligned} \tag{5.8}$$

Define the functions

$$\begin{aligned} \sigma_1(t,\theta) &\equiv 4\rho\ell \sin(\theta - \nu) v\sigma \sin\nu \\ \sigma_2(t,\theta) &\equiv 2\rho\ell^2 \sin(\theta - \nu) v\sigma \sin\nu, \end{aligned} \tag{5.9}$$

and the operator

$$\mathcal{N}(t,\phi) \equiv \{0, 0, \sigma_1(t,\phi_2), \sigma_2(t,\phi_2)\}'. \tag{5.10}$$

Now adding the fluctuating component of forces to the expression (4.18) is equivalent to adding the expression $\mathcal{N}(t,\phi)(\mathrm{d}w/\mathrm{d}t)$ to the evolution equation (4.21) giving

$$\begin{aligned} \dot{\phi} &\equiv (\mathcal{A}\phi + \mathcal{D}\phi + \mathcal{F}(t,\phi)) + \mathcal{N}(t,\phi)(\mathrm{d}w/\mathrm{d}t), \\ \phi(0) &\equiv \phi_0. \end{aligned} \tag{5.11}$$

Equation (5.11) can be rigorously interpreted as a stochastic evolution equation in the sense of Ito if written as

$$\begin{aligned} \mathrm{d}\phi &\equiv (\mathcal{A}\phi + \mathcal{D}\phi + \mathcal{F}(t,\phi))\,\mathrm{d}t + \mathcal{N}(t,\phi)\,\mathrm{d}w, \\ \phi(0) &\equiv \phi_0, \end{aligned} \tag{5.12}$$

It is clear from the expressions (5.9) and (5.10) that the operator $\mathcal{N}$ maps $I \times E$ into E, and trivially satisfies both Lipschitz and (at most linear) growth condition. The explicit dependence of $\mathcal{N}$ on t is intended to signify the variation of the mean wind velocity v and σ with time. Using the semigroup $\{\hat{S}(t),\ t \geq 0\}$ of Theorem 4.3, we can write Eq. (5.12) as an stochastic integral equation

$$\begin{aligned} \phi(t) = \hat{S}(t)\phi_0 &+ \int_0^t \hat{S}(t-s)\mathcal{F}(s,\phi(s))\,\mathrm{d}s \\ &+ \int_0^t \hat{S}(t-s)\mathcal{N}(s,\phi(s))\,\mathrm{d}w(s), \quad t \geq 0. \end{aligned} \tag{5.13}$$

Let $(\Omega, G, G_t\uparrow, P)$ denote a filtered probability space where Ω denotes the space of elementary events, G denotes the sigma algebra of subsets of the space Ω and $\{G_t,\ t\geq 0,\}\subset G$ denotes an increasing family of subsigma algebras of the sigma algebra G so that the Wiener process $\{w(t),\ t\geq 0,\}$ is G_t adapted. Let $L_2(\Omega, E)$ denote the space of E-valued square integrable random variables. This is a Hillbert space with the scalar product and norms given by

$$(f,g)\equiv\int_\Omega (f(w),g(w))_E P(\mathrm{d}w)\equiv E\{(f,g)_E\}$$

$$\|f\|_{L_2(\Omega,E)}\equiv\left(\int_\Omega \|f\|_E^2 P(\mathrm{d}w)\right)^{1/2}\equiv\left(E\|f\|_E^2\right)^{1/2}.$$

Let $M_\infty(I,E)$ denote the space of G_t adapted random processes defined on the interval $I\subset R$, taking values in E and having finite second moments. This space is furnished with the norm topology given by

$$\|\phi\|_{M_2}=\text{ess-sup}\{(E\,\|\phi(t)\|_E^2)^{1/2},\ t\in I\}.$$

It is easy to verify that $M_\infty(I,E)$ is a closed subspace of the Banach space $L_\infty(I,\ L_2(\Omega,\ E))$ and hence it is also a Banach space with respect to the norm topology given above.

DEFINITION 5.1 The system (5.12) is said to have a mild solution $\phi\in M_\infty(I,E)$ if $\phi(0)=\phi_0$ a.s. and ϕ satisfies the integral equation (5.13) in the Banach space $M_\infty(I,E)$.

THEOREM 5.2 *Suppose the basic assumptions of Theorem* 4.3 *hold and that the mean velocity* $v\in L_\infty(I)$ *and* $\sigma\in L_2(I)$. *Then for each* G_0*-measurable initial state* $\phi_0\in L_2(\Omega,E)$, *the stochastic evolution equation* (5.12) *has a unique mild solution* $\phi\in M_\infty(I,E)$.

Proof Again this can be proved by use of Banach fixed point theorem. We present an outline of the proof. For the existence of a solution to Eq. (5.13), it suffices to show that the operator $\mathcal{G}$ as defined below:

$$(\mathcal{G}\phi)(t)\equiv\hat{S}(t)\phi_0+\int_0^t\hat{S}(t-s)\mathcal{F}(s,\phi(s))\,\mathrm{d}s + \int_0^t\hat{S}(t-s)\mathcal{N}(s,\phi(s))\,\mathrm{d}w(s), \tag{5.14}$$

has a fixed point in $M_\infty(I, E)$. Under the stated assumptions it is easy to verify that $\mathcal{G}$ maps $M_\infty(I, E)$ to $M_\infty(I, E)$ and for sufficiently large $n \in N$, the n-th power of $\mathcal{G}$, denoted by $\mathcal{G}^n$, is a contraction in M_∞ (I, E). Thus by Banach fixed point theorem, both $\mathcal{G}^n$ and $\mathcal{G}$ have one and the same fixed point in $M_\infty(I, E)$.

Remark Since here the noise process is given simply by a one dimensional Brownian motion, it is trivial to verify that the solution process $\phi \in C(I, E)$ with probability one. This is true even for more general situation as considered below.

In the preceding analysis we have assumed that the wind fluctuation energy is uniformly distributed throughout the span of the bridge. This assumption may not be true for long bridges. In this case we can use distributed white noise or, equivalently, distributed Wiener process, for example,

$$W(t, x) = \beta(x)w(t). \tag{5.15}$$

The function β is chosen so that the noise energy is spatially distributed along the span of the bridge as required by the environment. We can choose for β any bounded measurable function on $(0, L) \equiv \Sigma$. Thus the function β can be used to represent localized wind activities. Associating β with the functions σ_1, σ_2 as defined by the expression (5.9) giving

$$\begin{aligned} \sigma_1(t, \theta) &\equiv 4\rho\ell\beta \sin(\theta - \nu)v\sigma \sin\nu \\ \sigma_2(t, \theta) &\equiv 2\rho\ell^2\beta \sin(\theta - \nu)v\sigma \sin\nu, \end{aligned} \tag{5.16}$$

we have the evolution equation (5.12) with $\mathcal{N}$ replaced by $\tilde{\mathcal{N}}$ to reflect the modification. Again the analysis is carried out using the integral equation

$$\begin{aligned} \phi(t) = \hat{S}(t)\phi_0 &+ \int_0^t \hat{S}(t-s)\mathcal{F}(s, \phi(s))\,\mathrm{d}s \\ &+ \int_0^t \hat{S}(t-s)\,\tilde{\mathcal{N}}(s, \phi(s))\,\mathrm{d}w(s), \quad t \geq 0, \end{aligned} \tag{5.17}$$

or equivalently the fixed point problem $\phi = \tilde{\mathcal{G}}\phi$ in the Banach space $M_\infty(I, E)$, where $\tilde{\mathcal{G}}$ denotes the operator representing the right hand expression of (5.17). Mathematical analysis of this model is similar to that of the model (5.13). First note that the nonlinear diffusion operator $\tilde{\mathcal{N}}$ maps $I \times E$ to E. Indeed

$$\begin{aligned}\|\tilde{\mathcal{N}}(t,\phi)\|_E^2 &= \|\sigma_1(t,\phi_2)\|_F^2 + \|\sigma_2(t,\phi_2)\|_F^2 \\ &= \int_0^L |4\rho\ell v(t)\sigma(t)(\sin(\phi_2-\nu)\sin\nu)\beta(x)|^2\,\mathrm{d}x \\ &\quad + \int_0^L |2\rho\ell^2 v(t)\sigma(t)(\sin(\phi_2-\nu)\sin\nu)\beta(x)|^2\,\mathrm{d}x.\end{aligned}$$

Since the mean wind velocity v is bounded and $\beta \in L_\infty(0,L)$, there exists a constant c such that

$$\begin{aligned}\|\tilde{\mathcal{N}}(t,\phi)\|_E^2 &\leq \left\{\left((4\rho\ell)^2 + (2\rho\ell^2)^2\right)(\sin^2\nu)v^2(t)\right\}\sigma^2(t)\|\beta\|_F^2 \\ &\leq c^2\sigma^2(t)\|\beta\|_F^2 \leq Lc^2\sigma^2(t)\|\beta\|_\infty^2, \end{aligned} \tag{5.18}$$

where $\|\beta\|_\infty \equiv \text{ess-sup}\{|\beta(x)|, x \in [0,L]\}$. Clearly this bound holds for all $\phi \in E$, and therefore the operator $\tilde{\mathcal{N}}$ is uniformly bounded in E for almost all $t \in I$. Similarly one can verify that $\tilde{\mathcal{N}}$ satisfies the Lipschitz condition

$$\|\tilde{\mathcal{N}}(t,\phi) - \tilde{\mathcal{N}}(t,\psi)\|_E^2 \leq K(t)\|\phi-\psi\|_E^2, \tag{5.19}$$

where $K(t)$ is an integrable function given by

$$K(t) \equiv \tilde{c}L\left((4\rho\ell)^2 + (2\rho\ell^2)^2\right)(\sin^2\nu)\|\beta\|_\infty^2 v^2(t)\sigma^2(t),$$

where $\tilde{c}$ denotes the embedding constant $H_0^2 \hookrightarrow F \equiv L_2(\Sigma)$. Since the first two terms of the integral equations (5.13) and (5.17) are the same, it suffices to consider only the stochastic convolution term. We show that $\sup_{t\in I} E\|Z(t,\phi)\|_E^2 < \infty$ for any $\phi \in M_\infty(I,E)$, where Z denotes the stochastic convolution

$$Z(t,\phi) \equiv \int_0^t \hat{S}(t-s)\tilde{\mathcal{N}}(s,\phi(s))\,\mathrm{d}w(s). \tag{5.20}$$

Letting $\{e_k, k \in N\}$ denote any orthonormal basis of the Hilbert space E we have

$$E\langle Z(t,\phi), e_k\rangle^2 = E\int_0^t \langle \hat{S}(t-s)\tilde{\mathcal{N}}(s,\phi(s)), e_k\rangle^2\,\mathrm{d}s.$$

Summing this with respect to the indices k and using (5.18) we have

$$\begin{aligned} E\|Z(t,\phi)\|_E^2 &= E\int_0^t \sum_k \langle \hat{S}(t-s)\tilde{\mathcal{N}}(s,\phi(s)), e_k\rangle^2 \, \mathrm{d}s \\ &= \int_0^t E\|\hat{S}(t-s)\tilde{\mathcal{N}}(s,\phi(s))\|_E^2 \, \mathrm{d}s \\ &\le M^2 \int_0^t E\|\tilde{\mathcal{N}}(s,\phi(s))\|_E^2 \, \mathrm{d}s \\ &\le (Mc)^2 L\|\beta\|_\infty \int_0^t \sigma^2(s)\, \mathrm{d}s < \infty. \end{aligned} \tag{5.21}$$

Similarly for $\phi,\psi \in M_\infty(I,E)$, using (5.19) we have

$$E\,\|Z(t,\phi) - Z(t,\psi)\|_E^2 \le M^2 \int_0^t K(s) E\,\|\phi(s) - \psi(s)\|_E^2 \, \mathrm{d}s. \tag{5.22}$$

Using (5.21) one can easily verify that $\tilde{\mathcal{G}}$ maps $M_\infty(I,E)$ into itself and using (5.22) one can verify that for sufficiently large $n \in N$, the nth iterate of $\tilde{\mathcal{G}}$ denoted by $\tilde{\mathcal{G}}^n$, is a contraction. Thus $\tilde{\mathcal{G}}$ has a unique fixed point which solves Eq. (5.17). Hence the conclusion of Theorem 5.2 also remains valid for distributed noise.

6. NUMERICAL RESULTS AND DISCUSSIONS

The results of Sections 3 and 4 are illustrated by numerical simulation. First note that the aerodynamic forces as given by Eq. (3.5), which is reproduced here for the convenience of the reader,

$$\begin{aligned} f_{1a} &= 2\pi\rho\ell v^2(\theta - (z_t/|v|)), \\ f_{2a} &= \pi\rho\ell^2 v^2(\theta - (z_t/|v|) - \ell(\theta_t/|v|)), \end{aligned} \tag{3.5}$$

provide viscous damping through the second term in the first equation, and second and third terms in the second equation. The other two terms have destabilizing effect. Two sets of graphs are presented.

For the first set we assume normal viscous damping negligible, except for the damping components arising from the wind activities as mentioned above. Figures 1–4 provide the results on total energy and vertical and torsional displacements for increasing mean wind velocity v. It is clear from these figures that even though initially

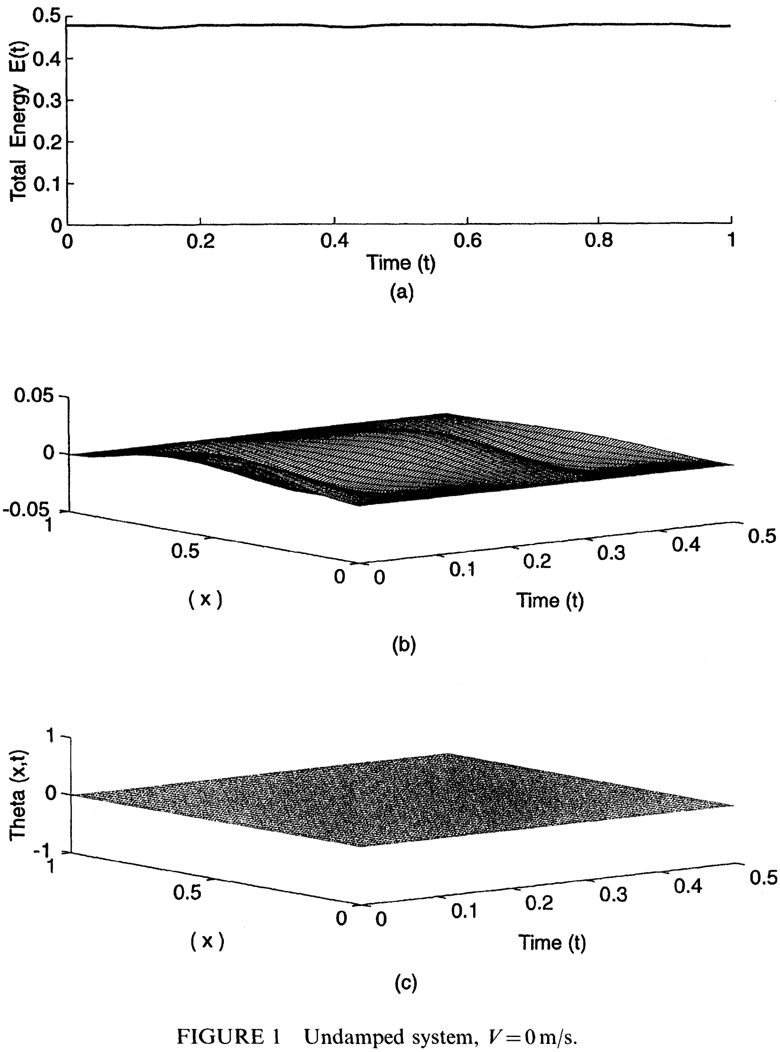

FIGURE 1 Undamped system, $V = 0\,\mathrm{m/s}$.

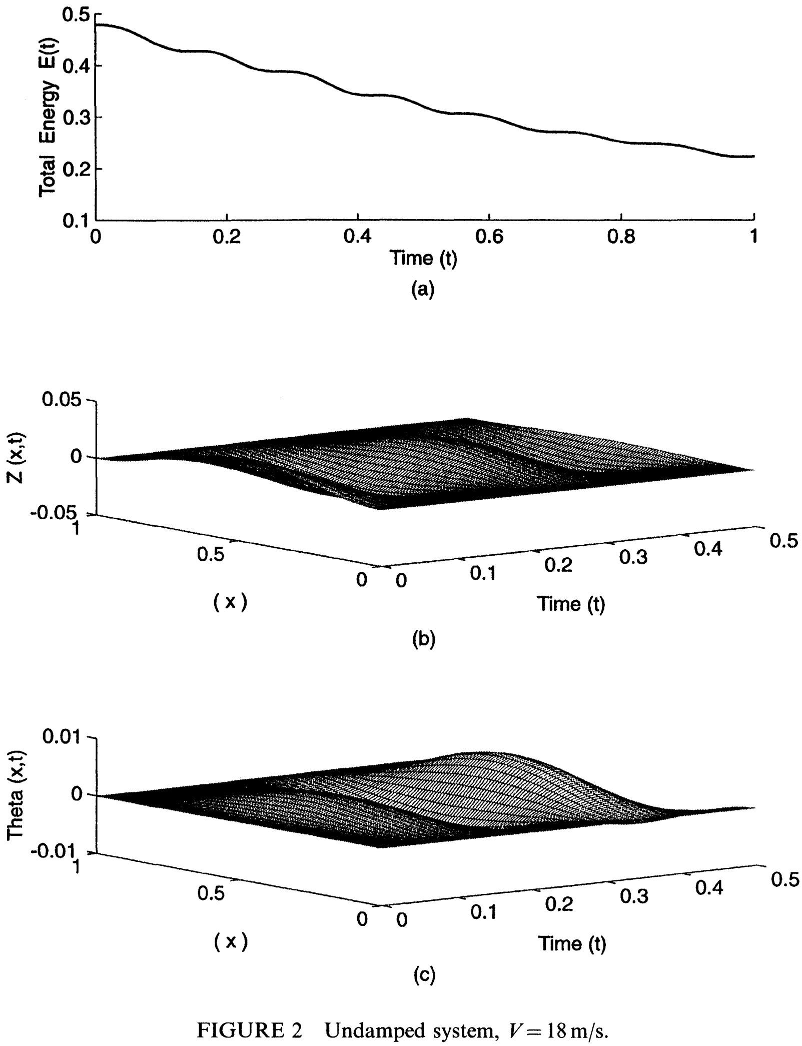

FIGURE 2 Undamped system, $V = 18\,\mathrm{m/s}$.

$\theta(0,x) = \theta_t(0,x) \equiv 0$, the vertical motion induces torsional motion (see Figs. 2(c), 3(c) and 4(c)). Further, at low wind velocities, the aerodynamic damping components are predominant leading to decay of initial energy (see Figs. 1(a), 2(a) and 3(a)). With further increase of wind velocity (see Fig. 4(a)) the destabilizing factors, as mentioned

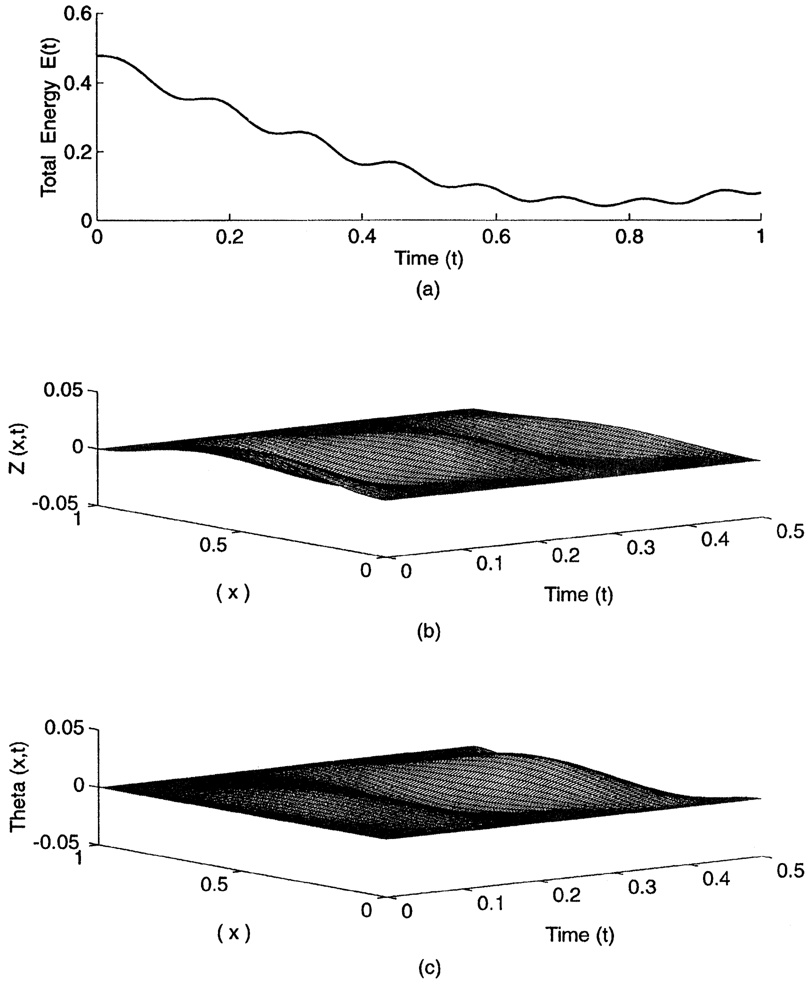

FIGURE 3 Undamped system, $V = 40\,\mathrm{m/s}$.

above, dominate (over the aerodynamic damping forces) and lead to catastrophic increase of energy.

The second set of results repeat those of the first set for naturally damped system. That is, here we assume that the normal viscous

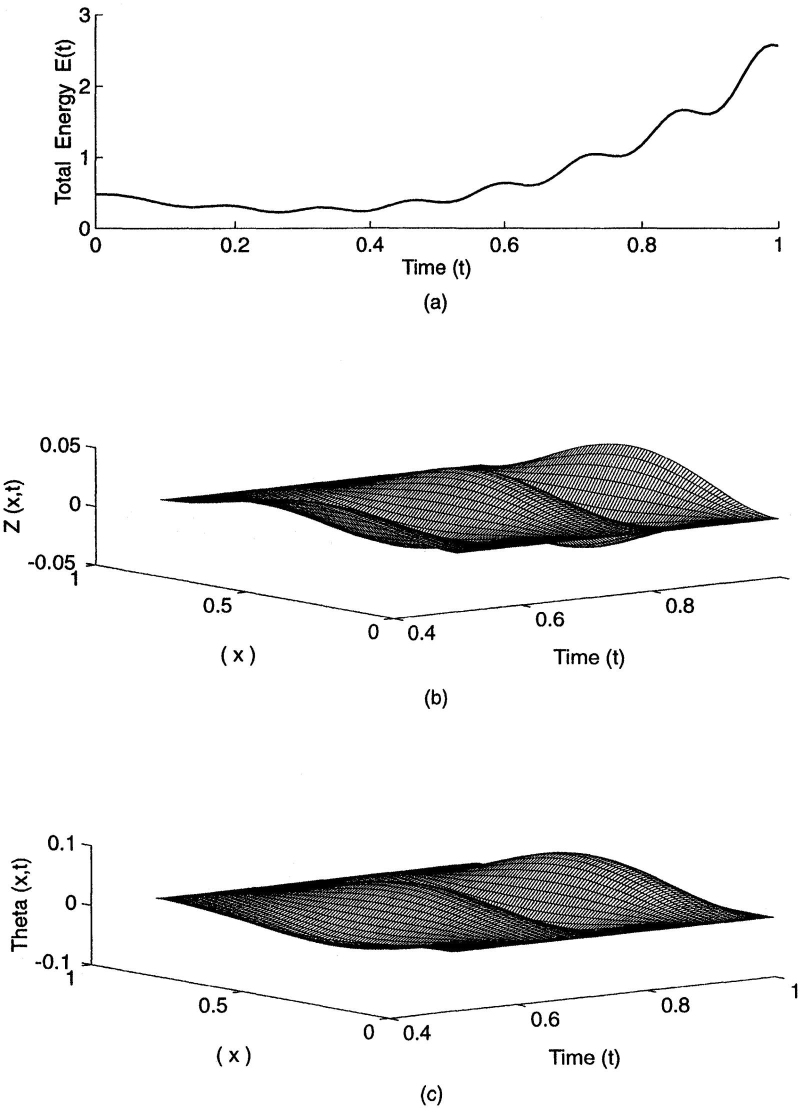

FIGURE 4 Undamped system, $V = 50\,\mathrm{m/s}$.

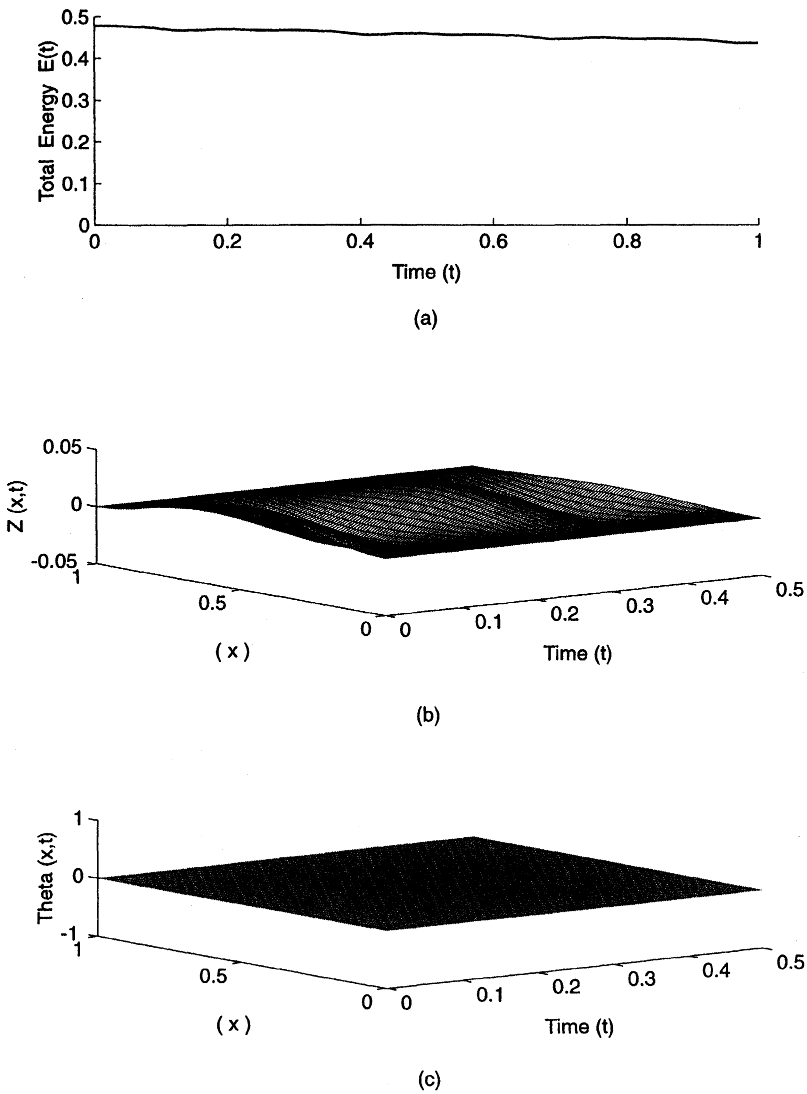

FIGURE 5 Damped system, $V = 0\,\text{m/s}$.

damping component provided by the surrounding atmosphere (at normal temperature and pressure) at zero wind velocity is not negligible. The results are plotted in Figs. 5–9. Comparing the energy plots of Figs. 1(a), 2(a), 3(a) and 4(a) with those of Figs. 5(a), 6(a),

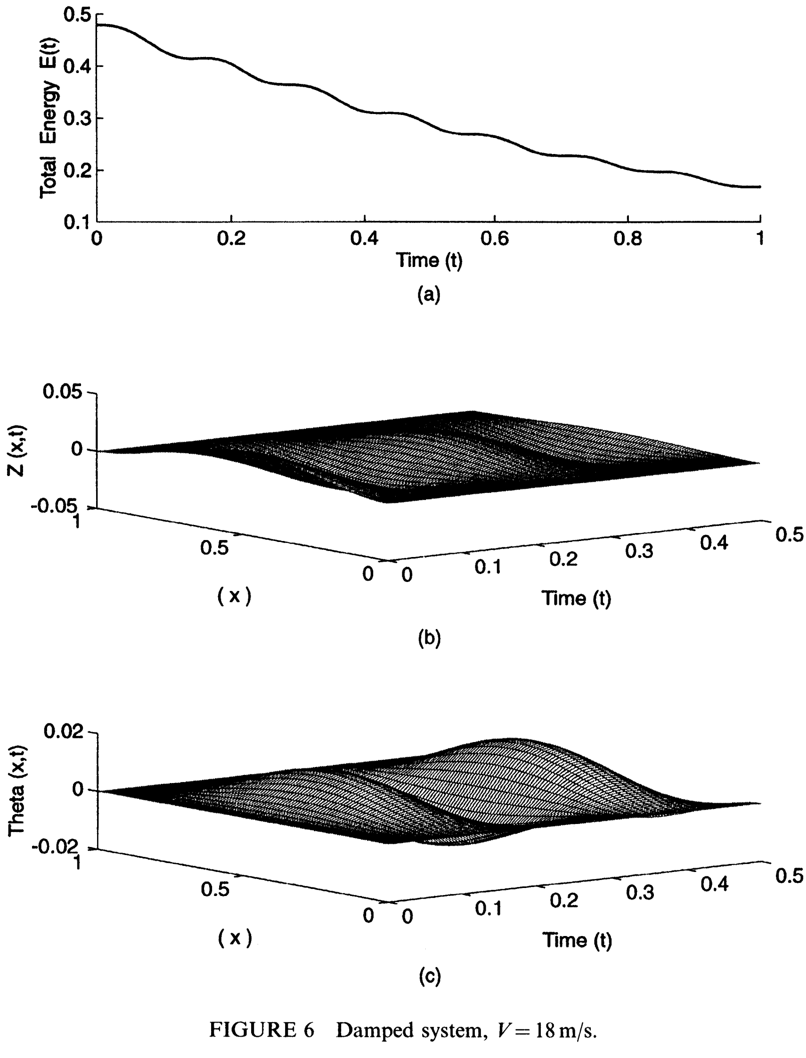

FIGURE 6 Damped system, $V = 18$ m/s.

7(a) and 8(a) it is clear that the presence of small natural damping has some stabilizing effect up to a larger wind velocity. For the undamped system (Fig. 4(a)) the bridge experiences catastrophic oscillations at $v = 50$ while for the naturally damped system (Fig. 8(a)), at the same wind velocity, the bridge experiences oscillation

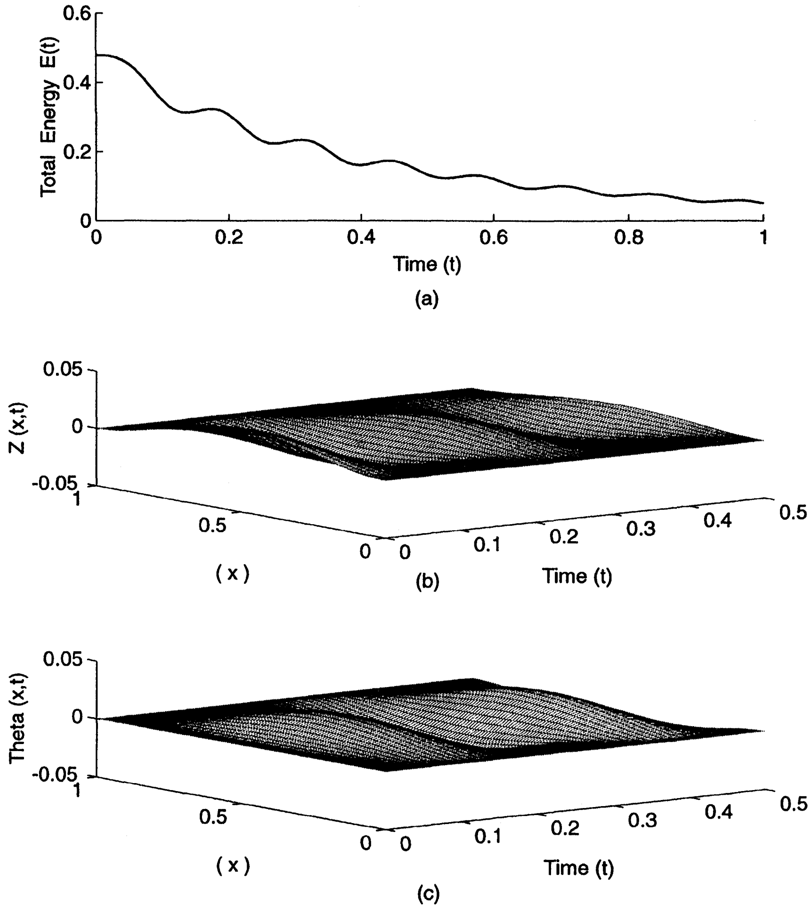

FIGURE 7 Damped system, $V = 40\,\mathrm{m/s}$.

but not so catastrophic. However, for $v = 60$ catastrophic oscillation sets in as shown in Fig. 9.

We have not yet computed solutions in the presence of stochastic perturbation. We plan to do it in a forthcoming paper.

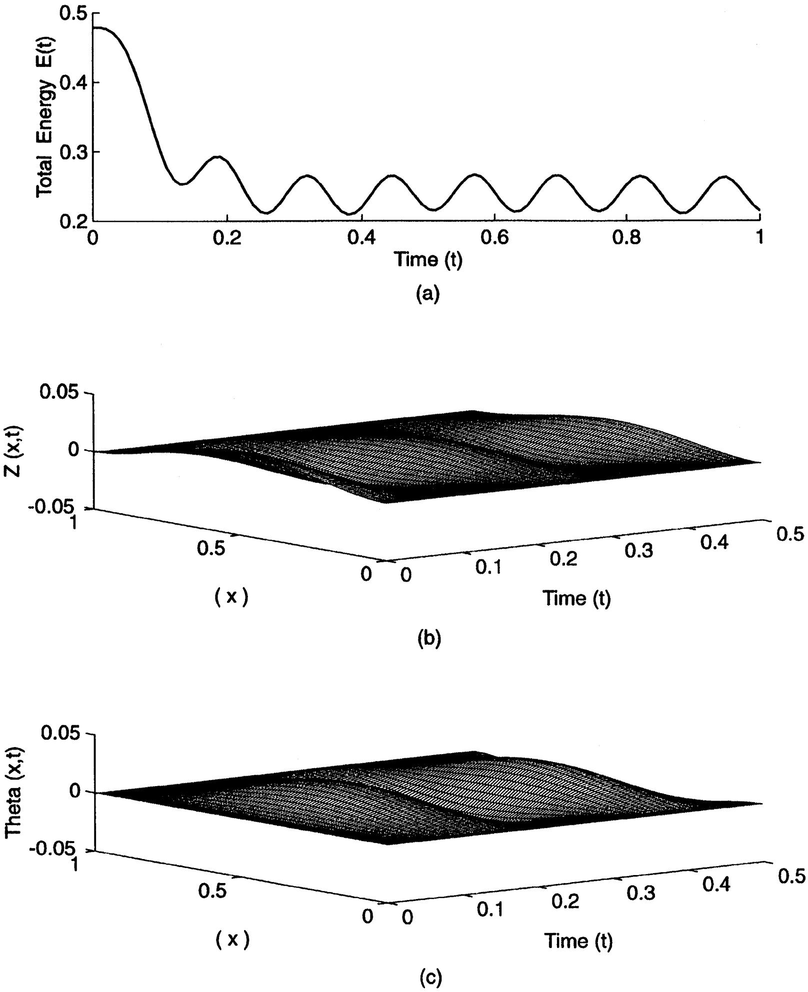

FIGURE 8 Damped system, $V = 50\,\text{m/s}$.

SUN-SPARC station no. 5 was used for all the computations. The data used for the simulation are as follows: $m = 10$, $\beta_1 = 10$, $\gamma_1 = 0.035$, $K = 10$, $\rho = 1.18$, $L = 1$, $\ell = 0.05$, $\mathcal{I} = 2m\ell^2 = 0.05$, $\beta_2 = 2\ell^2\beta_1 = 0.05$, $G = 0.18$, $\gamma_2 = (KG + 2\ell^2\gamma_1) = 1.8$. For numerical stability we have chosen the ratio $(\Delta t/\Delta x) = 2 \times 10^{-3}$.

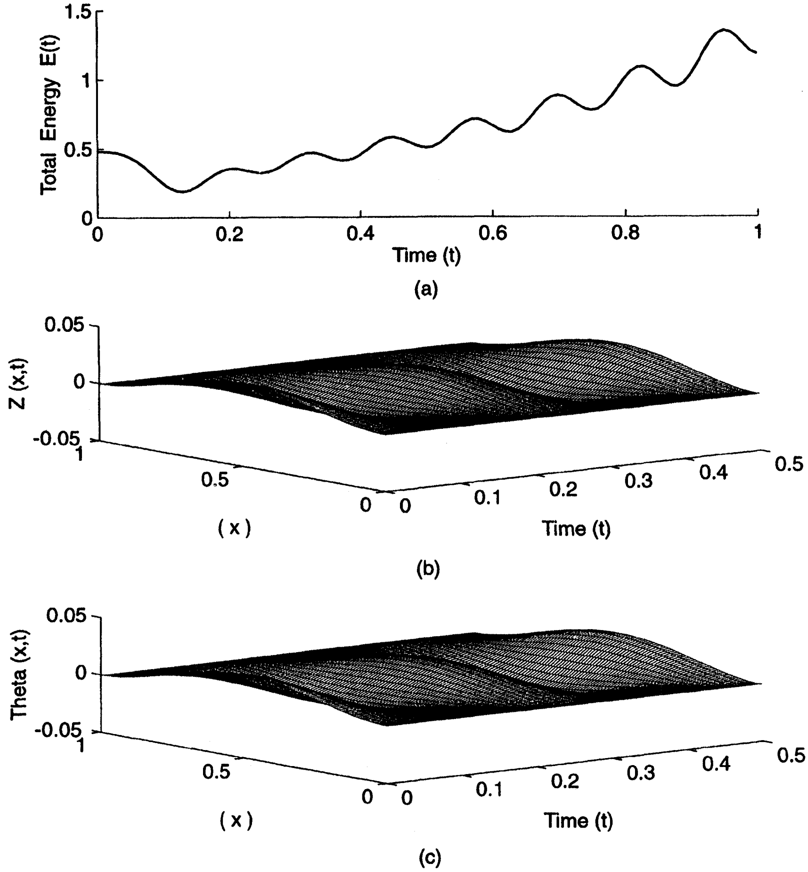

FIGURE 9 Damped system, $V = 60\,\text{m/s}$.

References

[1] A.C. Lazer and P.J. McKenna, Large-amplitude periodic oscillations in suspension bridge: Some new connections with nonlinear analysis. *SIAM Rev.* **32**, 537–78, 1990.

[2] O.H. Amann, T. Von Karman and G.B. Woodruff, The failure of the Tacoma Narrows bridge, Federal Works Agency, 1941.

[3] F. Bleich, C.B. McCullough, R. Rosecrans and G.S. Vincent, *The Mathematical Theory of Suspension Bridges*, US Dept. of Commerce, Bureau of Public Roads, 1950.

[4] A.M. Abdel-Ghaffer, Suspension bridge vibration: Continuum formation, *J. Engrg. Mech.* **108**, 1215–1232, 1982.

[5] P.J. McKenna and W. Walter, Nonlinear oscillation in a suspension bridge, *Arch. Rational Mech. Anal.* **98**, 167–177, 1987.

[6] A.C. Lazer and P.J. McKenna, Large scale oscillation behavior in loaded asymmetric systems. *Ann. Inst. H. Poincare, Analyse Nonlineaire* **4**, 244–274, 1987.

[7] Daniela Jacover and P.J. McKenna, Nonlinear torsional flexings in a periodically forced suspended beam, *J. Computational and Applied Mathematics* **52**, 241–265, 1994.

[8] N.U. Ahmed and H. Harbi, Mathematical analysis of dynamic models of suspension bridge, *SIAM J. Applied Mathematics* **58**(3), 853–874, 1998.

[9] N.U. Ahmed and H. Harbi, Dynamic models of suspension bridge and their stability, *Proc. of the IASTED International Conference, Control 97*, Cancun, Mexico, May 28–31, 1997, 386–390.

[10] N.U. Ahmed and H. Harbi, Stability of suspension bridge and aerodynamic vs structural damping, *J. Mathematical Problems in Engineering* **4**, 73–98, 1998.

[11] M. Roseau, *Vibrations in Mechanical Systems*, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1984.

[12] N.U. Ahmed, *Semigroup Theory with Applications to Systems and Control*, Pitman Research Notes in Mathematics Series, 246, Longman Scientific and Technical, UK, John Wiley & Sons, Inc., New York, 1991.