Mathematical Problems in Engineering
Volume 5, pp. 479–498
Reprints available directly from the publisher
Photocopying permitted by license only

© 2000 OPA (Overseas Publishers Association) N.V.
Published by license under
the Gordon and Breach Science
Publishers imprint.
Printed in Singapore.

Bottlenecks with Respect to Due-Time Performance in Pull Serial Production Lines*

JINGSHAN LI[‡] and SEMYON M. MEERKOV[†]

Department of Electrical Engineering and Computer Sciences, The University of Michigan, 1301 Beal Ave., Ann Arbor, MI 48109-2122, USA

(Received 10 September 1999)

In this work, serial production lines with finished goods buffers operating in the pull regime are considered. The machines are assumed to obey Bernoulli reliability model. The problem of satisfying customers demand is addressed. The level of demand satisfaction is quantified by the due-time performance (DTP), which is defined as the probability to ship to the customer a required number of parts during a fixed time interval. Within this scenario, the definitions of DTP bottlenecks are introduced and a method for their identification is developed.

Keywords: Production systems; Due-time performance; Bottleneck; Blockage; Starvation

1 INTRODUCTION

Serial production lines are sets of machines and buffers arranged in a consecutive order as shown in Fig. 1. Here the circles represent the machines (m_i, $i = 1, \dots, M$) and rectangles are the buffers ($B_1, \dots, B_{M-1}$ are referred to as in-process buffers and B_M is called the finished goods buffer – FGB). In practice, the machines are not absolutely reliable and experience random breakdowns. In this situation,

* This work was supported by NSF Grant No. DMI-9820580.
[†] Corresponding author. Tel.: +1(734)763-6349. Fax: +1(734)763-8041.
E-mail: smm@eecs.umich.edu.
[‡] E-mail: jsli@eecs.umich.edu.

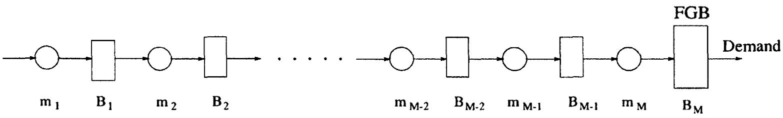


FIGURE 1 Pull serial production line.

the number of parts produced by the last machine during a fixed interval of time is a random variable. Its distribution characterizes both production volume (throughput or production rate – PR) and production variability. The throughput has been studied in production systems literature for a long period of time (see [1–5] for thorough reviews and exposition). In contrast, the production variability has been addressed in just a few relatively recent publications, [6–11]. Most of them study the issue of production variance. Although quite useful, this variability measure is not of immediate practical importance since it does not directly characterize the variability measure of interest – the due-time performance (DTP). This measure is defined in [12] as the probability to meet the customer demand, i.e., to ship a required number of parts during a fixed time interval. Several results concerning DTP have been described in [12–15].

Current manufacturing literature classifies production systems as operating in one of two regimes – push or pull [5,16–19]. Although a number of interpretations of these terms is available, the present work views push (respectively, pull) systems as those operating so as to maximize the throughput (respectively, DTP). The emphasis of this paper is on the pull serial production lines.

In any regime, push or pull, improving performance is an important task of production line management and control. In practice, it is often accomplished by identifying the bottleneck machine (BN-M) and improving its operation. In push production system, the bottleneck is often understood as the machine with the worst throughput in isolation, [20,21]. However, as it was shown in [22], such a machine is not necessarily the most impeding, as far the *system* throughput is concerned: Relatively good machines, or even the best one, may be the bottleneck. Based on this observation, Kuo *et al.* [23] introduced a definition for the production rate bottleneck (PR-BN) as the machine, to the performance of which the system throughput has the largest sensitivity. In Ref. [23–26] a method for PR-BN identification is developed and a number of applications is reported.

The problem of bottleneck machine identification in pull production systems does not seem to have been addressed in the literature. The goal of this paper is to present several results concerning this problem. Specifically, in Section 2, we formulate the model of the pull serial production line under consideration and formally introduce the DTP measure. The notions of DTP bottleneck machine (DTP-BN-M) and DTP bottleneck buffer (DTP-BN-B) are introduced in Section 3. Sections 4–6 present a method for their identification. Finally, Section 7 formulates the conclusions.

2 SYSTEM MODEL

The following model of a pull serial production line is considered throughout this work.

Machines

(i) Each machine, m_i, $i = 1, \ldots, M$, requires a fixed unit of time to process a part. This unit is referred to as the *cycle time*. All machines have identical cycle time. The time axis is slotted with the slot duration equal to the cycle time.

Remark 2.1 The assumption of the fixed processing time is appropriate for many production systems in large volume manufacturing environment. On the other hand, the assumption of identical cycle time may or may not hold, depending on the nature of the production system. Typically, in systems with automated material handling, this assumption is satisfied.

(ii) During a cycle time, each machine can be in one of two states: "up" or "down". When up, the machine could process a part. When down, no processing can take place.

(iii) The status of the machines, i.e., up or down, is determined by the process of Bernoulli trials. In other words, it is assumed that during each slot, machine m_i, $i = 1, \ldots, M$, is up with probability p_i and down with probability $1 - p_i$; the status of the machine is determined at the beginning of each cycle, independent of the status of this machine in the previous cycle.

Remark 2.2 Assumption (iii) defines the Bernoulli statistics of machine breakdowns. The Bernoulli model is appropriate for modular production lines where operators use the push-buttons and stop a module of the operational conveyor in order to accomplish the operation with the highest possible quality. The duration of this "breakdown" is short and of the order of the cycle time and, therefore, the probability to produce a part during a cycle time arises naturally. Another frequent perturbation is pallet jams on the operational conveyor; to correct for this problem also a short period of time is required. In many car and engine assembly lines these are the predominant perturbations. In these situations, the Bernoulli reliability model is appropriate. The literature offers another model of machines reliability – the Markovian model, [1–11]. The Markovian model is appropriate for machining operations where the downtime is typically due to machine breakdowns and the repair time is much larger than the cycle time. We study here only the Bernoulli case and plan to analyze the Markovian model in the future work.

Buffers

(iv) Each buffer B_i, $i=1,\dots,M$, has capacity $1 \leq N_i < \infty$, $i=1,\dots,M$. Buffers $B_1,\dots,B_{M-1}$ are called in-process buffers. Buffer B_M is the FGB. With a slight abuse of notations, the capacity of the FGB will be always denoted as N_M, even when M is specified.

Starvation Rule

(v) If B_i, $i=1,\dots,M-1$, is empty at the beginning of the time slot, then m_{i+1}, $i=1,\dots,M-1$, is starved during this time slot. The first machine is never starved.

Blockage Rule

(vi) If B_i, $i=1,\dots,M$, is full at the beginning of a time slot and m_{i+1}, $i=1,\dots,M$ does not take a part from B_i at the beginning of this slot, then m_i, $i=1,\dots,M$, is blocked during this time slot.

Remark 2.3 Assumptions (iii), (v) and (vi) are formulated in terms of time-dependent failures, i.e., machines can go down even when blocked or starved [3,4]. Another possible model is that of operation-dependent failures, where no breakdowns of starved or blocked machines is possible, [3,4]. Both models are practical, depending on the production system at hand: for automated palletized material handling systems, the time-dependent model is more applicable. In case of manual material handling, operation-dependent failures often take place.

Demand

(vii) From the point of view of the demand, the time axis is divided into "epochs", each consisting of T time slots (Fig. 2).

(viii) At the end of each epoch, a shipment of D parts has to be available for the customer. If PR is the production rate of the system, then

$$D \leq T \cdot \mathrm{PR}. \tag{2.1}$$

Remark 2.4 A method for calculating the production rate in the system defined by (i)–(vi) without FGB has been developed in [22]. Thus, the upper bound of D is readily available.

Demand Satisfaction Policy

(ix) At the beginning of epoch i, parts are removed from the FGB in the amount of $\min(H(i-1), D)$, where $H(i-1)$ is the number of parts in the FGB at the end of $(i-1)$th epoch. If $H(i-1) \geq D$, the shipment is complete; if $H(i-1) < D$, the balance of the shipment, i.e., $D - H(i-1)$ parts, is to be produced by m_M. Parts produced are immediately removed from the FGB and prepared for shipment, until the shipment is complete, i.e., D parts are available. If the shipment is complete before the end of the epoch, the system continues operating, but with the parts being accumulated in the FGB, either until the end of the epoch or until the last machine, m_M, is blocked, whatever occurs

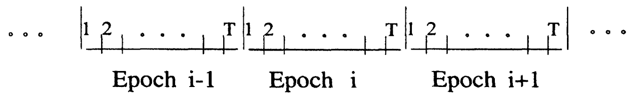


FIGURE 2 Epochs.

first. If the shipment is not complete by the end of the epoch, an incomplete shipment is sent to the customer. No backlog is allowed.

Remark 2.5 In the make-to-order pull production systems literature, [17,18], the demand is random and has to be satisfied immediately, otherwise it is backordered.

Assumptions (i)–(ix) define the system under consideration. In an appropriately defined state space, the system (i)–(ix) is a stationary ergodic Markov process. Only the steady state of this chain (i.e., the invariant measure or the stationary distribution) is analyzed in this work. We refer to this steady state as the "normal system operation". Let $\bar{t}_i$ be the number of parts produced by machine m_M during epoch i. Then the DTP can be defined as the probability that $\bar{t}_i$ plus the number of parts left in the buffer at the end of epoch $(i-1)$, $H(i-1)$, is greater than the shipment size, D, i.e.,

$$\mathrm{DTP} = \Pr(H(i-1) + \bar{t}_i \geq D). \tag{2.2}$$

In the framework of (i)–(ix), DTP is a function of all system parameters. In other words,

$$\mathrm{DTP} = \mathrm{DTP}(p, N, N_M, D, T), \tag{2.3}$$

where N_M, T, and D are defined in (iv), (vii), and (viii), respectively, p and N are vectors of the machines and in-process buffers parameters,

$$p = [p_1, \ldots, p_M], \qquad N = [N_1, \ldots, N_{M-1}].$$

A method for analyzing function (2.3) is presented in [14,15]. This function constitutes the basis for the analysis described in this work.

3 DEFINITIONS AND PROBLEM FORMULATION

Using (2.3), we define DTP bottlenecks as follows:

DEFINITION 3.1 *Machine m_i is the* due-time performance bottleneck machine (DTP-BN-M) *if* $\forall j \neq i$,

$$\frac{\partial \mathrm{DTP}(p_1, \ldots, p_M, N_1, \ldots, N_{M-1}, N_M, D, T)}{\partial p_i} > \frac{\partial \mathrm{DTP}(p_1, \ldots, p_M, N_1, \ldots, N_{M-1}, N_M, D, T)}{\partial p_j}.$$

DEFINITION 3.2 *Buffer* B_i *is the* due-time performance bottleneck buffer (DTP-BN-B) *if* $\forall j \neq i$,

$$\mathrm{DTP}(p_1, \ldots, p_M, N_1, \ldots, N_i + 1, \ldots, N_{M-1}, N_M, D, T)$$
$$> \mathrm{DTP}(p_1, \ldots, p_M, N_1, \ldots, N_j + 1, \ldots, N_{M-1}, N_M, D, T).$$

The goal of this work is to derive a tool for identification of these bottlenecks. Unfortunately, direct identification, using Definitions 3.1 and 3.2, is impossible since the sensitivities involved cannot be either calculated in a closed form or measured on the factory floor during the normal system of operation. Therefore, the tool sought has to be an indirect one. More specifically, we are seeking a DTP-BN identification tool that is based on real time data, which can be measured on the factory floor. We refer to this tool as *DTP-bottleneck indicator*. The problem, then, addressed in this paper is: Given a production system, defined by (i)–(ix), derive bottleneck indicators for DTP-BN-M and DTP-BN-B identification, which are based on real time measurements.

As it will be shown below, the DTP-BN indicators derived in this work depend heavily on the notions of manufacturing blockage and manufacturing starvation. They are defined as follows:

DEFINITION 3.3 *Machine* m_i, $i = 1, \ldots, M-1$, *is said to be* blocked in the manufacturing sense *during a time slot if it is up during this time slot,* B_i *is full at the beginning of this time slot, and* m_{i+1} *fails to take a part from* B_i *at the beginning of this time slot.*

DEFINITION 3.4 *Machine* m_M *is said to be* blocked in the manufacturing sense *during a time slot if it is up during this time slot and* B_M *is full at the beginning of this time slot.*

DEFINITION 3.5 *Machine* m_i, $i = 2, \ldots, M$, *is said to be* starved in the manufacturing sense *during a time slot if it is up during this time slot and* B_{i-1} *is empty at the beginning of this time slot.*

Let mb_i and ms_i denote the probabilities of manufacturing blockage and starvation, respectively. Then, according to Definitions 3.3–3.5,

$$\begin{aligned} mb_i &= \text{Prob}(\{m_i \text{ is up during a time slot}\} \cap \{B_i \text{ is full at the} \\ &\qquad \text{beginning of this slot}\} \cap \{m_{i+1} \text{ fails to take a part from} \\ &\qquad B_i \text{ at the beginning of this slot}\}), \quad i = 1, \dots, M-1, \\ mb_M &= \text{Prob}(\{m_M \text{ is up during a time slot}\} \\ &\quad \cap \{B_M \text{ is full at the beginning of this slot}\}), \\ ms_i &= \text{Prob}(\{m_i \text{ is up during a time slot}\} \\ &\quad \cap \{B_{i-1} \text{ is empty at the beginning of this slot}\}), \quad i = 2, \dots, M. \end{aligned}$$

Remark 3.1 Definitions 3.3 and 3.5 are identical to those introduced in [23] for push production lines (i.e. for lines without FGB). However, since FGB may cause blockage of the last machine, m_M, numerical values of ms_i $(i=2,\dots,M)$ and mb_i $(i=1,\dots,M-1)$ for lines with and without FGB, but otherwise identical, may be quite different.

4 DTP-BN IDENTIFICATION: GENERAL CONSIDERATIONS

In the case of serial production lines without finished goods buffers, the problem of PR-BN identification has been analyzed in [23] (for Bernoulli machines) and in [24–26] (for Markovian machines). For the case of Bernoulli machines, it was shown analytically that the PR-BN, defined through partial derivatives of the production rate with respect to machine parameters p_i, $i=1,\dots,M$, can be identified by measuring ms_i and mb_i. More specifically, it was shown that if $mb_i > ms_{i+1}$, the PR-BN is downstream of machine m_i; if $mb_{i-1} < ms_i$, the PR-BN is upstream of m_i. It turned out that the same indicator may be used for identification of the so-called c-BNs in production lines with Markovian machines [26].

Unfortunately, analytical derivation of similar indicators for DTP-BNs, starting from Definitions 3.1 and 3.2, is unmanageable. Therefore, a heuristic approach is used. Below we outline the rationale for using data, other than ms_i and mb_i, to identify DTP-BNs. Based on these heuristics, two subsequent sections formulate BN indicators and justify them numerically.

The rationale for the heuristics used in this work is as follows:

(a) As it was pointed out above, in two-machine lines without FGB, inequality $mb_1 > ms_2$ (respectively, $mb_1 < ms_2$) implies that machine m_2 (respectively, m_1) is the PR-BN [23]. Unfortunately, in two-machine lines with FGB, numerical experiments indicate that these inequalities do not identify the DTP-BN; a large number of counterexamples have been found. Thus, unlike PR-BNs, relative values of ms_i and mb_i do not identify DTP-BN-M, and new quantities for its identification must be found.

(b) In two-machine lines without the FGB, inequality $p_1 < p_2$ implies that machine m_1 is the PR-BN [23]. In two machine lines with FGB, this inequality does not necessarily mean that m_1 is the DTP-BN. This is true due to the fact that m_2 may be blocked by the FGB and, therefore, "effective p_2" is $p_2 - mb_2$. Thus, even if $p_2 > p_1$, machine m_2 may still be the DTP-BN. (Figures 4 and 7 explained below, exemplify this situation.)

(c) On the other hand, it is reasonable to assume that inequality $p_1 > p_2$ implies that m_2 is indeed the DTP-BN in a two-machine line. This follows from the fact that in both cases, with and without the FGB, m_1 can only be blocked and cannot be starved. Thus, inequality $p_1 > p_2$ may be considered as a candidate for indicating that m_2 is the DTP-BN.

(d) Inequality $p_1 > p_2$ can be re-formulated equivalently, in terms of mb's, ms's and p's, as follows:

PROPOSITION 4.1 *Assume that* $p_1 > p_2$. *Then, under assumptions* (i)–(ix),

$$q_1 mb_1 > (q_2 + mb_2) ms_2, \tag{4.1}$$

where $q_i = 1 - p_i$, $i = 1, 2$.

Proof See the Appendix.

(e) Since, as it follows from the above proposition and argument (c), $q_1 mb_1 > (q_2 + mb_2) ms_2$ indicates that m_2 is the DTP-BN, the inverse of this inequality, i.e., $q_1 mb_1 < (q_2 + mb_2) ms_2$, may indicate that m_1 is the DTP-BN. This is verified in Section 5 below.

To extend the above argument to the M-machine case, we "symmetrize" (4.1) as follows:

$$(q_i + ms_i)mb_i > (q_{i+1} + mb_{i+1})ms_{i+1}, \quad i = 1, \dots, M-1, \tag{4.2}$$

keeping in mind that $ms_i = 0$ for $i = 1$. In analogy with the PR-BN, we expect that this inequality would imply that DTP-BN is downstream of m_i; otherwise it is upstream of m_{i+1}. This is verified in Section 6.

Note that all quantities involved in (4.2), i.e., $q_i = 1 - p_i$, ms_i, and mb_i, can be measured on the factory floor during the normal system operation.

Remark 4.1 In the case of each particular machine, inequalities

$$\begin{aligned} &(q_i + ms_i)mb_i > (q_i + mb_i)ms_i, \quad i = 1, \dots, M, \\ &(q_i + ms_i)mb_i < (q_i + mb_i)ms_i, \quad i = 1, \dots, M, \end{aligned} \tag{4.3}$$

imply, respectively, that

$$\begin{aligned} &mb_i > ms_i, \quad i = 1, \dots, M, \\ &mb_i < ms_i, \quad i = 1, \dots, M. \end{aligned} \tag{4.4}$$

Thus, for each particular machine, (4.4) can be used instead of (4.3). This will be utilized below for DTP-BN-B identification.

5 DTP-BN IDENTIFICATION: TWO-MACHINE CASE

DTP-BOTTLENECK INDICATOR 5.1 *Under assumptions* (i)–(ix) *with* $M = 2$, *machine* m_1 *is the DTP-BN-M and* B_1 *is the DTP-BN-B if*

$$q_1 mb_1 < (q_2 + mb_2)ms_2.$$

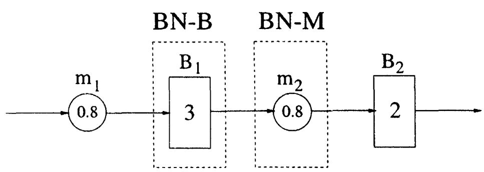
If

$$q_1 mb_1 > (q_2 + mb_2)ms_2,$$

m_2 *is the DTP-BN-M. If, in addition,* $ms_2 < mb_2$, *the FGB is DTP-BN-B; otherwise DTP-BN-B is* B_1.

Numerical Justification The above indicator has been justified using discrete event simulations. The simulation approach used here and throughout this paper is as follows: In each run of the corresponding discrete event model, zero initial conditions for all buffers have been assumed and a 10,000T time slots of warm up period has been carried out, where, as before, T is the length of the epoch. The next 100,000T slots of stationary regime have been used to statistically evaluate the quantities of interest. Fifty simulation runs were performed to determine the confidence interval. The 95% confidence intervals for all statistical estimates have been evaluated according to methodology of [27].

Using this approach, we simulated a large number of two-machine systems defined by assumptions (i)–(ix). Three typical examples are shown in Figs. 3–5. In each of these figures, the four rows of numbers show the values of $(q_i+mb_i)ms_i$ and $(q_i+ms_i)mb_i$, $\partial\text{DTP}/\partial p_i$ and $\text{DTP}(N_i+1)$, along with 95% confidence intervals. The values of $\partial\text{DTP}/\partial p_i$ have been evaluated using finite differences $\Delta\text{DTP}/\Delta p_i$, $\forall i$, with the step $\Delta p_i = 0.0001$. In Figs. 3 and 5, $T=4$, $D=3$, in Fig. 4, $T=3$, $D=2$. The bottlenecks identified by Indicator 5.1 are supported by the values of $\partial\text{DTP}/\partial p_i$ and $\text{DTP}(N_i+1)$. Thus, the DTP-BN-Ms and DTP-BN-Bs in Figs. 3–5, respectively, are machines m_2, m_1 and m_2 and buffers B_1, B_1 and B_2. (Note, for instance, that in Figs. 3 and 5 the largest buffer and the best machine are bottlenecks, respectively.)



$(q_i+mb_i)ms_i$:	0 ± 0	0.0091 ± 0.0003
$(q_i+ms_i)mb_i$:	0.0148 ± 0.0003	0.0089 ± 0.0002
$\frac{\partial DTP}{\partial p_i}$:	0.8567 ± 0.0012	1.2046 ± 0.0013
$DTP(N_i+1)$:	0.9390 ± 0.0011	0.9388 ± 0.0010

FIGURE 3 DTP-BN identification in two-machine case: Example 1.

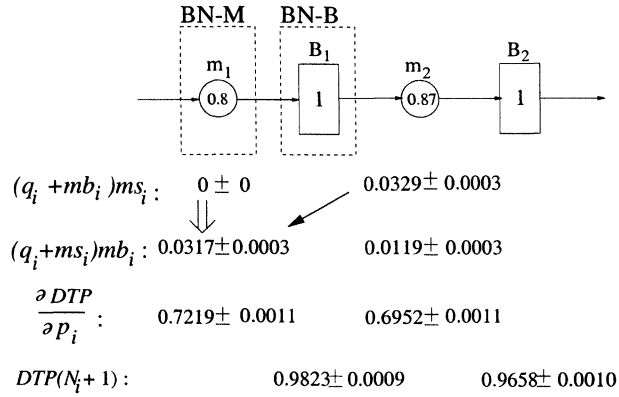


FIGURE 4 DTP-BN identification in two-machine case: Example 2.

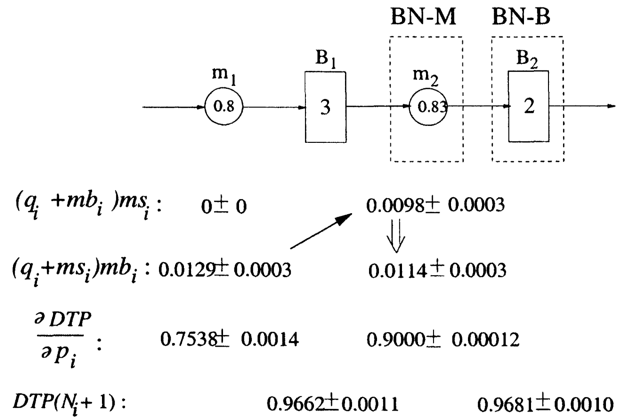


FIGURE 5 DTP-BN identification in two-machine case: Example 3.

In most systems analyzed, Indicator 5.1 resulted in correct BN identification. However, several counter examples have also been discovered. One of them is shown in Fig. 6. Here Indicator 5.1 resulted in m_2, whereas $\partial\mathrm{DTP}/\partial p_i$ gave m_1, as the bottleneck. To illustrate further the region where Indicator 5.1 does not work, we vary parameter p_2 of m_2, keeping all other parameters constant. The result is shown in Fig. 7. When p_2 changes from 0.6–0.8, the bottleneck machine shifts from m_2 to m_1. As it follows from this figure, the range of p_2 where Indicator 5.1 does not work is quite small (from 0.718 to

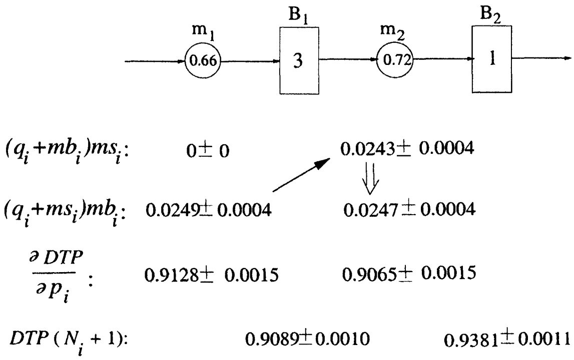


FIGURE 6 Counterexample for DTP-Bottleneck Indicator 5.1.

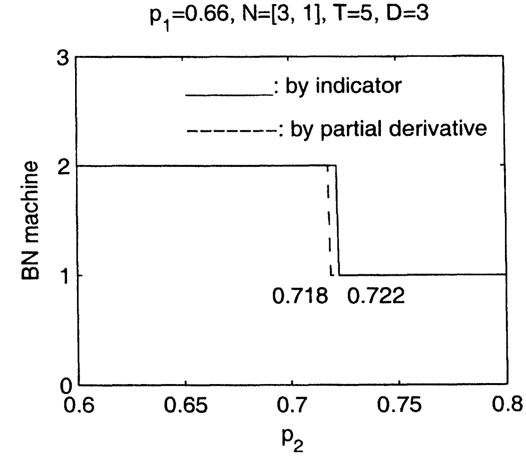


FIGURE 7 Comparison of DTP-Bottleneck Indicator 5.1 with $\max(\partial \mathrm{DTP}/\partial p_i)$.

0.722). Therefore, we conclude that Indicator 5.1 can be used as a tool for DTP-BN identification for most values of system parameters.

Bottleneck Indicator 5.1 can be given a graphical interpretation similar to that of [23–26]. Specifically, arranging thin arrows as shown in Figs. 3–5 (from the larger number to the smaller), we observe that a machine, which has no emanating thin arrows, is the DTP-BN-M. In addition, placing a thick arrow under the BN-M, pointing down if the BN-M is more often blocked than starved and pointing up otherwise, we obtain, using Remark 4.1 and Indicator 5.1, that the BN-B is in front of the BN-M if the thick arrow points up and after the BN-M if it points down. This rule is used in Section 6 for DTP-BNs identification in M-machine lines.

Remark 5.1 In two-machine lines, with $p_1 = p_2$, both machines are "equally" PR-BNs [23]. As it follows from (A.4) of the Appendix and Indicator 5.1, if $p_1 = p_2$, machine m_2 is the DTP-BN. (See Fig. 3 for an example.) This implies that a ramp, rather than a bowl-type, distribution of p_is is optimal in pull production systems. In addition, this implies that, unlike the push case, the pull systems do not possess the property of reversibility [12].

6 DTP-BN IDENTIFICATION: M-MACHINE CASE

Consider a production line shown in Fig. 8. Assume that its operation satisfies assumptions (i)–(ix) and that p_i, ms_i, and mb_i, $i = 1, \ldots, M$ are measured during the normal system operation. Calculate quantities $(q_i + ms_i)mb_i$ and $(q_{i+1} + mb_{i+1})ms_{i+1}$, where $q_i = 1 - p_i$, $i = 1, \ldots, M-1$, and place them under each machine, as shown in Fig. 8. Using these data, assign arrows according to the following

Rule 6.1 If

$$(q_i + ms_i)mb_i > (q_{i+1} + mb_{i+1})ms_{i+1}, \quad i = 1, \ldots, M-1,$$

the arrow is directed from machine m_i to machine m_{i+1}. If

$$(q_{i-1} + ms_{i-1})mb_{i-1} < (q_i + mb_i)ms_i, \quad i = 2, \ldots, M,$$

the arrows is directed from machine m_i to machine m_{i-1}. In addition, under each machine with no emanating arrows, place a thick arrow

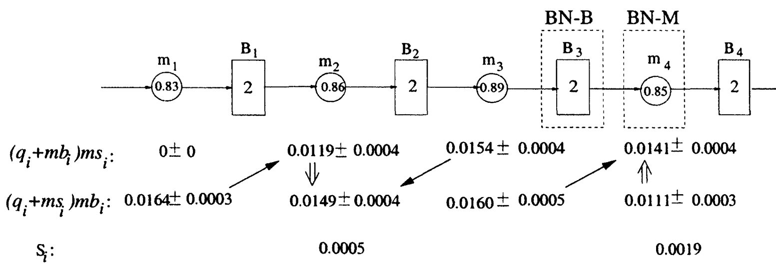


FIGURE 8 Illustration of DTP-BN identification in M-machine line.

pointing down if for this machine mb is larger than ms; otherwise, place a thick arrow pointing up.

Introduce the numbers S_j defined as follows:

$$\begin{aligned} S_1 &= (q_2 + mb_2)ms_2 - q_1 mb_1, \\ S_i &= \min\{(q_{i-1} + ms_{i-1})mb_{i-1} - (q_i + mb_i)ms_i, \\ &\qquad (q_{i+1} + mb_{i+1})ms_{i+1} - (q_i + ms_i)mb_i\}, \quad i = 2, \ldots, M-1. \\ S_M &= (q_{M-1} + ms_{M-1})mb_{M-1} - (q_M + mb_M)ms_M. \end{aligned} \tag{6.1}$$

We refer to these numbers as the *bottleneck severity*.

DTP-BOTTLENECK INDICATOR 6.1 *Consider a serial production line with arrows assigned according to Rule* 6.1. *Then, if there is a single machine with no arrows emanating from it, this machine is the DTP-BN-M. If there are multiple machines with no emanating arrows, the machine with the largest severity is the DTP-BN-M. The DTP-BN-B is the buffer immediately after the DTP-BN-M if the thick arrow under this machine points down; otherwise it is the buffer immediately in front of this machine.*

Thus, according to this indicator, machine m_4 and buffer B_3 are the DTP-BNs of the system shown in Fig. 8.

Numerical Justification We simulated a large number of M-machine systems defined by assumptions (i)–(ix). Three typical examples are shown in Figs. 9–11. In Figs. 9 and 10, $T=3$, $D=2$, in Fig. 11,

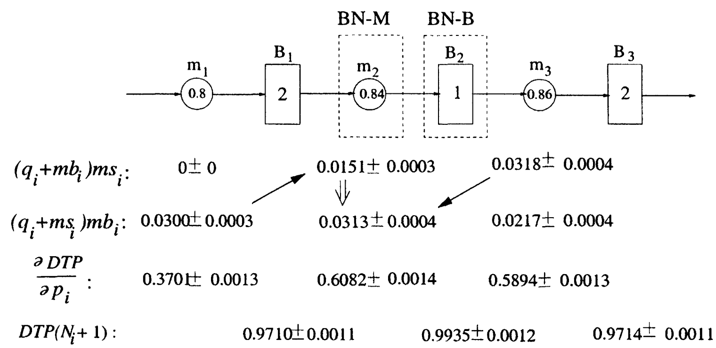


FIGURE 9 DTP-BN identification in M-machine case: example 1.

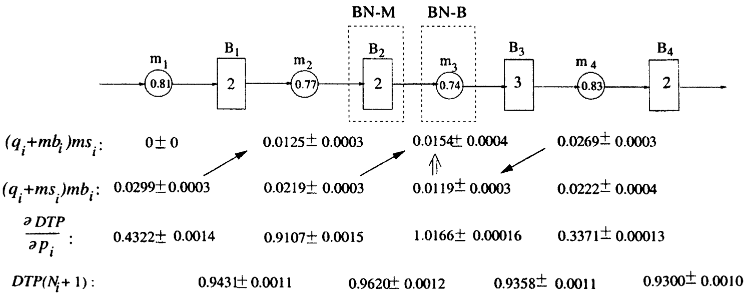


FIGURE 10 DTP-BN identification in M-machine case: example 2.

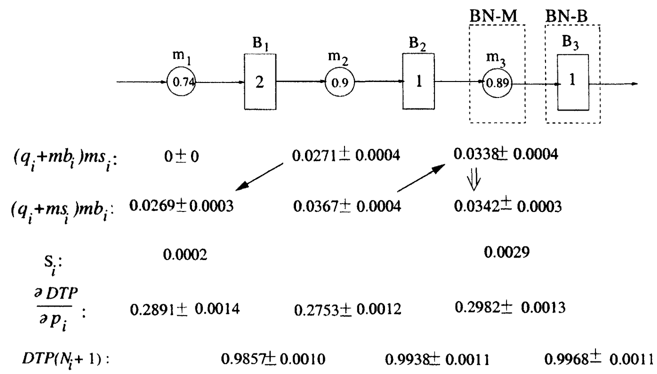


FIGURE 11 DTP-BN identification in M-machine case: example 3.

$T=5$, $D=3$. The bottlenecks identified by Indicator 6.1 are supported by the values of $\partial\text{DTP}/\partial p_i$ and $\text{DTP}(N_i+1)$. Thus, the DTP-BN-Ms and DTP-BN-Bs in Figs. 9–11 are machines m_2, m_3 and m_3 and buffers B_2, B_2 and B_3, respectively.

In most systems considered, the DTP-BNs identified using Indicator 6.1 and $\partial\text{DTP}/\partial p_i$ coincide. However, a few counterexamples have been discovered. One of them is shown in Fig. 12, in which $T=5$, $D=3$. According to Indicator 6.1, the DTP-BN-M is m_2 whereas according to $\partial\text{DTP}/\partial p_i$ the bottleneck is m_1. However, the difference between $\partial\text{DTP}/\partial p_2$ and $\partial\text{DTP}/\partial p_1$ is small. The same situation was observed in all counterexamples discovered. Therefore, we conclude that DTP-Bottleneck Indicator 6.1 can be used for most values of system parameters.

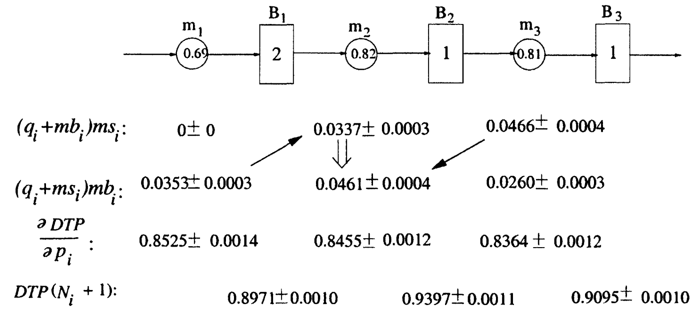


FIGURE 12 Counterexample to DTP-Bottleneck Indicator 6.1.

7 CONCLUSIONS

This paper provides a method for identification of DTP-BNs in pull serial production lines. The advantage of the method is that it is based on data available on the factory floor through real time measurements. These data are machine efficiency (modeled by p_i) and frequency of machine manufacturing blockage and starvation (modeled by mb_i and ms_i). The disadvantage of the method is that it is not proved analytically and only numerical justification is obtained. Proving this method analytically (or, more precisely, providing precise conditions when it works) is a challenging mathematical problem. The importance of this problem, however, is justified by its practical utility. This will be a part of future work of the authors and, hopefully, some interested readers.

References

[1] Y. Dallery and S.B. Gershwin, "Manufacturing flow line systems: A review of models and analytical results", *Queuing Systems*, **12**, 3–94, 1992.
[2] N. Viswanadham and Y. Narahari, *Performance Modeling of Automated Manufacturing Systems*, Prentice Hall, 1992.
[3] J.A. Buzacott and J.G. Shantikumar, *Stochastic Models of Manufacturing Systems*, Prentice Hall, 1993.
[4] S.B. Gershwin, *Manufacturing Systems Engineering*, Prentice Hall, 1994.
[5] T. Altiok, *Performance Analysis of Manufacturing Systems*, Springer, 1996.
[6] G.J. Miltenburg, "Variance of the number of units produced on a transfer line with buffer inventories during a period of length T", *Naval Research Logistics*, **34**, 811–822, 1987.

[7] K.B. Hendrics, "The output process of serial production lines of exponential machines with finite buffers", *Operations Research*, **40**, 1139–1147, 1992.
[8] S.B. Gershwin, "Variance of Output of a tandem production system", in: *Queuing Networks with Finite Capacity*, Eds., R.O. Onvaral and I.F. Akyildi, Elsevier Science Publishers, 1993.
[9] M. Carrascosa, Variance of the output in a deterministic two-machine line, M.S. Thesis, Laboratory for Manufacturing and Productivity, MIT, 1995.
[10] B. Tan, "An efficient method for variance of the output in a production line with a finite buffer", in: *Proceedings of International Workshop on Performance Evaluation and Optimization of Production Lines*, pp. 135–155, Samos Island, Greece, 1997.
[11] P. Ciprut, M.O. Hongler and Y. Salama, "On the variance of the production output of transfer lines", *IEEE Transaction on Robotics and Automation*, **15**, 33–43, 1999.
[12] D.A. Jacobs and S.M. Meerkov, "System-theoretic analysis of due-time performance in production systems", *Mathematical Problems in Engineering*, **1**, 225–243, 1995.
[13] B. Tan, "Effects of variability on the due-time performance of production lines", Koc University Working Paper Series 97-03, Istanbul, Turkey, 1997.
[14] J. Li and S.M. Meerkov, "Production variability in manufacturing systems: Problem formulation and performance bounds", *Proc. of 37th IEEE CDC*, pp. 2730–2735, Tampa, FL, December 1998.
[15] J. Li and S.M. Meerkov, "Production variability in manufacturing systems: Bernoulli reliability model", *Annals of Operations Research*, 2000.
[16] B. Sarker and J. Fitzsimmons, "Performance of push and pull systems: a simulation and comparative study", *International Journal of Production Research*, **27**, 309–352, 1989.
[17] T. Hodgson, "An analytical model of a two-product one machine production/inventory system", *Management Science*, **19**, 391–405, 1972.
[18] T. Altiok and R. Ranjan, "Multi-state pull-type production/inventory systems", *IIE Transactions*, **27**, 190–200, 1995.
[19] Y. Hirakawa, "Performance of a multistage hybrid push/pull production control system", *International Journal of Production Economics*, **44**, 129–135, 1996.
[20] E.M. Goldratt and J. Cox, *The Goal*, North Rivers Press, NY, 1986.
[21] M.S. Spencer and J.F. Cox, "Optimum production technology (OPT) and the theory of constraints (TOC): Analysis and genealogy", *International Journal of Production Research*, **33**, 1495–1504, 1995.
[22] D.A. Jacobs and S.M. Meerkov, "A system-theoretic property of serial production lines: improvability", *International Journal of System Science*, **26**, 755–785, 1995.
[23] C.-T. Kuo, J.T. Lim and S.M. Meerkov, "Bottlenecks in serial production lines", *Mathematical Problems in Engineering*, **2**, 233–276, 1996.
[24] S.-Y. Chiang, C.-T. Kuo and S.M. Meerkov, "Bottlenecks in Markovian production lines: A systems approach", *IEEE Transaction Robotic and Automation*, **14**, 352–359, 1998.
[25] S.-Y. Chiang, C.-T. Kuo and S.M. Meerkov, "Bottlenecks in Markovian production lines: Identification and application", *Proc. of the 37th IEEE CDC*, pp. 4344–4345, Tampa, FL, December 1998.
[26] S.-Y. Chiang, C.-T. Kuo and S.M. Meerkov, "c-Bottlenecks in serial production lines", *Proc. of the 38th IEEE CDC*, Phoenix, AZ, December 1999.
[27] A.M. Law and W.D. Kelton, *Simulation Modeling and Analysis*, McGraw-Hill, 1991.

APPENDIX

Proof of Proposition 4.1 Let

$$X_1(k) = \text{Prob}\{k \text{ parts in buffer } B_1\}, \quad k = 0, 1, \ldots, N_1.$$

Under assumptions (i)–(ix), this probability distribution must satisfy, in the steady state, the following equations:

$$\begin{aligned}
X_1(0) &= q_1 X_1(0) + q_1(p_2 - mb_2)X_1(1), \\
X_1(1) &= p_1 X_1(0) + [p_1(p_2 - mb_2) + q_1(q_2 + mb_2)]X_1(1) \\
&\quad + q_1(p_2 - mb_2)X_1(2), \\
X_1(k) &= p_1(q_2 + mb_2)X_1(k-1) + [p_1(p_2 - mb_2) + q_1(q_2 + mb_2)]X_1(k) \\
&\quad + q_1(p_2 - mb_2)X_1(k+1), \quad k = 2, \ldots, N_1 - 1, \\
X_1(N_1) &= p_1(q_2 + mb_2)X_1(N_1 - 1) \\
&\quad + [q_2 + mb_2 + p_1(p_2 - mb_2)]X_1(N_1). \qquad \text{(A.1)}
\end{aligned}$$

Solving (A.1), we obtain

$$X_1(k) = \frac{\alpha^k}{q_2 + mb_2} X_1(0), \quad k = 1, \ldots, N_1, \qquad \text{(A.2)}$$

where

$$\alpha = \frac{p_1(q_2 + mb_2)}{(p_2 - mb_2)q_1}.$$

Therefore,

$$\begin{aligned}
q_1 mb_1 - (q_2 + mb_2)ms_2 &= q_1 p_1 (q_2 + mb_2) X_1(N_1) - (q_2 + mb_2)p_2 X_1(0) \\
&= p_1 q_1 (q_2 + mb_2) \frac{\alpha^{N_1}}{q_2 + mb_2} X_1(0) \\
&\quad - p_2(q_2 + mb_2)X_1(0) \\
&= X_1(0)\left[p_1 q_1 \alpha^{N_1} - p_2(q_2 + mb_2)\right]. \qquad \text{(A.3)}
\end{aligned}$$

If $p_1 \geq p_2$, then $\alpha > 1$. When $N_1 = 1$, we have

$$\begin{aligned} q_1 m b_1 - (q_2 + m b_2) m s_2 &= X_1(0)(p_1 q_1 \alpha^{N_1} - p_2(q_2 + m b_2) \\ &= X_1(0)\left[p_1 q_1 \frac{p_1(q_2 + m b_2)}{(p_2 - m b_2) q_1} - p_2(q_2 + m b_2)\right] \\ &= \frac{(q_2 + m b_2) X_1(0)}{p_2 - m b_2}(p_1^2 - p_2^2 + p_2 m b_2) \\ &> 0. \end{aligned} \tag{A.4}$$

When $N_1 > 1$,

$$\alpha^{N_1} > \alpha,$$

and it follows again that

$$q_1 m b_1 - (q_2 + m b_2) m s_2 > 0.$$

Proposition 4.1 is proved.