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Passivation and Control of Partially Known SISO Nonlinear Systems via Dynamic Neural Networks

J. REYES-REYES, W. YU and A.S. POZNYAK*

CINVESTAV-IPN, Automatic Control Department, A.P. 14-740, CP 07360, Mexico D.F., Mexico

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In this paper, an adaptive technique is suggested to provide the passivity property for a class of partially known SISO nonlinear systems. A simple Dynamic Neural Network (DNN), containing only two neurons and without any hidden-layers, is used to identify the unknown nonlinear system. By means of a Lyapunov-like analysis the new learning law for this DNN, guarantying both successful identification and passivation effects, is derived. Based on this adaptive DNN model, an adaptive feedback controller, serving for wide class of nonlinear systems with an *a priori* incomplete model description, is designed. Two typical examples illustrate the effectiveness of the suggested approach.

Keywords: Passivity; Dynamic Neural Networks; Adaptive stabilization

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1 INTRODUCTION

Passivity is one of the important properties of dynamic systems which provides a special relation between the input and the output of a system and is commonly used in the stability analysis and stabilization of a wide class of nonlinear systems [4,12]. Roughly speaking, if a nonlinear system is passive it can be stabilized by any negative linear

* Corresponding author. Visiting Professor, Department of Mathematics, University of Kansas, 405 Snow Hall, Lawrence, KS 66045. Tel.: 785 864 5162. Fax: 765 864 5255. E-mail: apoznyak@hotmail.com; apoznyak@ctrl.cinvestav.mx.

feedback even under the lack of a detailed description of its mathematical model [19,20]. This property seems to be very attractive in different physical applications. In view of this, the following approach for designing a feedback controller for nonlinear systems is widely used [17]: first, a special internal nonlinear feedback is introduced to passify the given nonlinear system; second, a simple external negative linear feedback is applied to provide a stability property for the obtained closed-loop system. The detailed analysis of this method and the corresponding synthesis of passivating nonlinear feedbacks represent the foundation of *Passivity Theory* [1,12].

In general, Passivity Theory deals with controlled systems whose nonlinear properties are poorly defined (usually by means of sector bounds). Nevertheless, it offers an elegant solution to the problem of absolute stability of such systems. The passivity framework can lead to general conclusions on the stability of broad classes of nonlinear control systems, using only some general characteristics of the input–output dynamics of the controlled system and the input–output mapping of the controller. For example, if the system is passive and it is zero-state detectable, any output feedback stabilizes the equilibrium of the nonlinear system [12].

When the system dynamics are totally or partially unknown, the passivity feedback equivalence turns out to be an important problem. This property can be provided by a special design of robust passivating controllers (adaptive [7,8] and non-adaptive [11,18] passivating control). But all of them require more detailed knowledge on the system dynamics. So, to be realized successfully, an adaptive passivating control needs the structure of the system under consideration as well as the unknown parameters to be linear. If we deal with the non-adaptive passivating control, the nominal part (without external perturbations) of the system is assumed to be completely known.

If the system is considered as a "black-box" (only some general properties are assumed to be verified to guarantee the existence of the solution of the corresponding ODE-models), the learning-based control using *Neural Networks* has emerged as a viable tool [6]. This model-free approach is presented as a nice feature of Neural Networks, but the lack of model for the controlled plant makes hard to obtain theoretical results on the stability and performance of a nonlinear system closed by a designed neuro system [10,13]. In the engineering practice, it is very important to have some theoretical guarantees that the neuro controller can stabilize a given system before its application to a real industrial or mechanical plant. That's why neuro controller design can be considered as a challenge to a modern control community.

Most publications in nonlinear system identification and control use *static* (*feedforward*) *neural networks*, for example, Multilayer Perceptrons (MLP) [16], which are implemented for the approximation of nonlinear function in the right-hand side of dynamic model equations [3]. The main drawback of these neural networks is that the weight updates do not use any information on a local data structure and the applied function approximation is sensitive to the training data [6]. *Dynamic Neural Networks* (DNN) can successfully overcome this disadvantage as well as provide an adequate behavior in the presence of unmodelled dynamics, since their structure incorporate feedback. They have powerful representation capabilities. One of the best known DNN was introduced by Hopfield [5].

For this reason, the framework of neural networks is very convenient for passivation of unknown nonlinear systems. Based on the static neural networks, an adaptive passifying control for unknown nonlinear systems is suggested in [2]. As we stated before, there are many drawbacks on using static neural networks for the control of dynamic systems.

In this paper, we use DNN to passify the unknown nonlinear system. A special storage function is defined in such a way that the aims of the identification and the passivation can be reached simultaneously. It is shown in [9,14,15] that the Lyapunov-like method turns out to be a good instrument to generate a learning law and to establish error stability conditions. By means of this technique we derive a weight adaptation procedure to verify the passivity conditions for the given closed-loop system. Two numerical examples are considered to illustrate the effectiveness of the adaptive passivating control.

2 PARTIALLY UNKNOWN SYSTEM AND APPLIED DNN

2.1 Partially Unknown SISO System

As in [1,2], let us consider a single input–single output (SISO) nonlinear system given by

$$\begin{aligned} \dot{z} &= f_0(z) + p(z,y)y \\ \dot{y} &= a(z,y) + b(z,y)u \\ z(0) &= z_0, \quad y(0) = y_0 \end{aligned} \tag{1}$$

where $\zeta := [z^T, y]^T \in \mathfrak{R}^n$ is the state of the system at time $t \geq 0$, $u \in \mathfrak{R}$ is the input and $y \in \mathfrak{R}$ is the output of this system.

The following interpretation of this structure can be done: the scalar y is "the main controlled state" and the vector z is "the internal dynamic feedback uncontrolled variables" or states (see Fig. 1). A broad enough class of controlled SISO systems has such structure.

The functions $f_0(\cdot)$ and $p(\cdot)$ are assumed to be C^1-vector fields and the functions $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$ are C^1-scalar functions ($b(z,y) \neq 0$ for any z and y). Let it be

$$f_0(0) = 0$$

We also assume that the set U_{ad} of admissible inputs u consists of all $\mathfrak{R}$-valued piecewise continuous functions defined on $\mathfrak{R}$, and satisfying the following property: for any initial conditions $\zeta^0 = \zeta(0) \in \mathfrak{R}^n$ the corresponding output

$$y(t) = h(\Phi(t, \zeta^0, u))$$

of this system (1) satisfies†

$$\int_0^t |y(s)u(s)|\,\mathrm{d}s < \infty, \quad \text{for all } t \geq 0$$

that is, the "energy" stored in the system (1) is bounded at each time t.

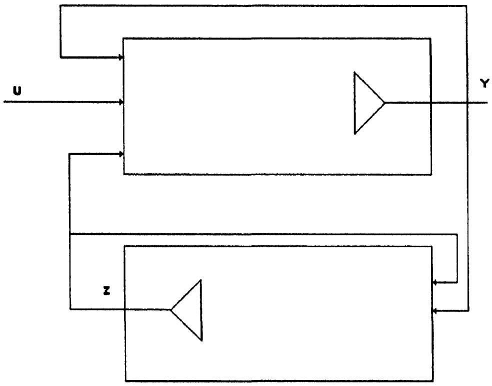

FIGURE 1 The general structure of the controlled system.

† $\Phi(t, \zeta^0, u)$ denotes the flow of $[f_0(z) + p(z,y), a(z,y) + b(z,y)u]^T$ corresponding to the initial condition $\zeta^0 = [(z^0)^T, y^0]^T \in R^n$ and to $u \in U_{ad}$.

DEFINITION 1 Zero dynamics *of the given nonlinear system* (1) *describes those internal dynamics which are consistent with the external constraint* $y = 0$, *that is, the zero dynamics verifies the following ODE*

$$\dot{z} = f_0(z) \tag{2}$$

2.2 Passivity Property and Storage Function

DEFINITION 2 [1,4] *A system* (1) *is said to be* C^r-passive *if there exists a* C^r-*nonnegative function* $V : \mathfrak{R}^n \to \mathfrak{R}$, *called* a storage function, *with* $V(0) = 0$, *such that, for all* $u \in U_{\text{ad}}$, *all initial conditions* ζ^0 *and all* $t \geq 0$ *the following inequality holds*:

$$\dot{V}(\zeta) \leq yu \tag{3}$$

If

$$\dot{V}(\zeta) = yu \tag{4}$$

then the given system (1) *is said to be* C^r-lossless. *If, furthermore, there exists a positive definite function* $S : \mathfrak{R}^n \to \mathfrak{R}$ *such that*

$$\dot{V}(\zeta) = yu - S \tag{5}$$

then the system is said to be C^r-strictly passive.

2.3 Basic Assumptions

For the nonlinear system (1) considered in this paper, the following assumptions are assumed to be fulfilled:

H1: *The zero dynamics* $f_0(z)$ *and the function* $b(z, y)$ *are completely known.*

H2: $f_0(\cdot)$ *satisfies the global Lipschitz condition, that is, for any* $z_1, z_2 \in \mathfrak{R}^{n-1}$

$$\|f_0(z_1) - f_0(z_2)\| \leq L_{f_0}\|z_1 - z_2\|, \quad L_{f_0} > 0$$

H3: *The zero dynamics in* (1) *is Lyapunov stable, that is, there exists a function* $W_0 : \mathfrak{R}^{n-1} \to \mathfrak{R}^+$, *with* $W_0(0) = 0$, *such that for all*

$z \in \Re^{n-1}$

$$\frac{\partial W_0(z)}{\partial z} f_0(z) \leq 0$$

H4: *The unknown part of the system* (1) *is related to the functions* $a(z, y)$ *and* $p(z, y)$ *with an* a priori *known upper bounds, that is,*

$$\|a(z, y)\| \leq \bar{a}(z, y), \qquad \|p(z, y)\| \leq \bar{p}(z, y)$$

where $\bar{a}(z, y)$ *and* $\bar{p}(z, y)$ *are Lipschitz functions (selected by the designer) and satisfying the following "strip conditions":*

$$\bar{a}(z, y) \leq a_0 + a_1(\|z\| + |y|)$$
$$\bar{p}(z, y) \leq p_0 + p_1(\|z\| + |y|).$$

2.4 Dynamic Neural Network

Following to [14,15], to identify this partially unknown nonlinear system we propose DNN having the structure as follows:

$$\begin{cases} \dot{\hat{z}} = f_0(\hat{z}) + [W_1\varphi_1(\hat{z}, \hat{y}) + \psi_1]y \\ \dot{\hat{y}} = W_2\varphi_2(\hat{z}, \hat{y}) + \psi_2 + b(z, y)u \\ \hat{z}(0) = \hat{z}_0, \quad \hat{y}(0) = \hat{y}_0 \end{cases} \tag{6}$$

Here $(\hat{z}, \hat{y}) \in \Re^n$ is the state vector of the DNN, $W_1 \in \Re^{(n-1)\times n}$, $W_2 \in \Re^{1\times n}$ are the weight matrix and vector, correspondingly, the functions $\psi_1 \in \Re^{n\times 1}$ and $\psi_2 \in \Re^{n\times 1}$ are the "output thresholds" of each neuron, the activation functions $\varphi_1(\cdot,\cdot) \in \Re^{n\times 1}$ and $\varphi_2(\cdot,\cdot) \in \Re^{n\times 1}$ are defined as follows:

$$\varphi_i(\alpha, \beta) = [\tanh(k_i \cdot \alpha_1), \ldots, \tanh(k_i \cdot \alpha_{n-1}), \tanh(k_i \cdot \beta)]^{\mathrm{T}}$$
$$k_i \in R, \quad i = 1, 2$$

As it is seen from (6), the structure of the DNN is constructed using the known part $f_0(z)$, $b(z, y)$ and the unknown part is identified by only two neurons (without any hidden layers):

the neuron $[W_1\varphi_1(\hat{z}, \hat{y}) + \psi_1]$ corresponding to the function $p(z, y)$ and $[W_2\varphi_2(\hat{z}, \hat{y}) + \psi_2]$ corresponding to the function $a(z, y)$.

Sure, that in the general case, the unknown nonlinear system (1) does not necessarily belong to the class of systems which can be exactly modelled by Eq. (6). Hence, for any $t \geq 0$ and for any fixed initial weights of the DNN, denoted by W_1^* and W_2^*, there exists, so called, *the identification error* (ν_1, ν_2) defined as

$$\begin{aligned} \nu_1 &:= \dot{z} - f_0(z) + [W_1^* \varphi_1(z,y) + \psi_1] y \\ \nu_2 &:= \dot{y} - W_2^* \varphi_2(z,y) + \psi_2 + b(z,y)u \end{aligned} \tag{7}$$

In view of (1) and (7), we can get the algebraic presentation for the identification error:

$$\begin{aligned} \nu_1 &= B_1(z,y)y - \psi_1 y \\ \nu_2 &= B_2(z,y) - \psi_2 \end{aligned} \tag{8}$$

where

$$\begin{aligned} B_1(z,y) &:= p(z,y) - W_1^* \varphi_1(z,y) \\ B_2(z,y) &:= a(z,y) - W_2^* \varphi_2(z,y) \end{aligned}$$

From H4 and the boundness of the functions φ_1 and φ_2 we can conclude that B_1 and B_2 are also bounded and satisfy

$$\begin{aligned} \|B_1(z,y)\| &\leq \bar{p}(z,y) + \|W_1^*\| \cdot \|\varphi_1\| =: \bar{B}_1(z,y) \\ \|B_2(z,y)\| &\leq \bar{a}(z,y) + \|W_2^*\| \cdot \|\varphi_2\| =: \bar{B}_2(z,y) \end{aligned} \tag{9}$$

Now we are ready to discuss the following problem: how to incorporate this DNN into the internal feedback of the given nonlinear system such that the obtained closed-loop system possesses the passivity property.

3 PASSIVATION OF PARTIALLY UNKNOWN NONLINEAR SYSTEM VIA DNN

In order to simplify the notation, the following expressions will be used:

$$\begin{aligned} &\varphi_i := \varphi_i(z,y), \quad \hat{\varphi}_i := \varphi_i(\hat{z},\hat{y}) \\ &\tilde{\varphi}_i := \varphi_i(\hat{z},\hat{y}) - \varphi_i(z,y) \quad (i = 1,2) \end{aligned}$$

$$\Delta := \begin{bmatrix} \Delta_z \\ \Delta_y \end{bmatrix} = \begin{bmatrix} \hat{z} - z \\ \hat{y} - y \end{bmatrix} \tag{10}$$

For any vector $\omega \in \Re^m$ and any positive integer $m = 1, 2, \ldots$, we denote

$$|\omega| := [\,|\omega_1| \quad |\omega_2| \quad \cdots \quad |\omega_m|\,]^{\mathrm{T}}$$
$$\mathrm{diag}(\omega) := \mathrm{diag}\{\omega_1, \omega_2, \ldots, \omega_m\} \in \Re^{m\times m}$$

and for a scalar $\kappa \in \Re$ and a positive integer $l = 1, 2, \ldots$, we will write

$$\underset{l}{\mathrm{vec}}(\kappa) := [\,\kappa \quad \kappa \quad \cdots \quad \kappa\,]^{\mathrm{T}} \in \Re^l$$

Under the assumptions H1–H4, the original nonlinear system (1) can be represented as

$$\begin{aligned} \dot{z} &= f_0(z) + [W_1^* \varphi_1(z, y) + \psi_1]\, y + \nu_1 \\ \dot{y} &= W_2^* \varphi_2(z, y) + \psi_2 + b(z, y)u + \nu_2 \end{aligned} \tag{11}$$

where the unmodelled dynamics (ν_1, ν_2) are defined by (8).

The following theorem gives the main result on the passivation of this partially unknown nonlinear system via DNN.

THEOREM 3 *Let the assumptions* H1–H4 *hold and the nonlinear system* (11) *be identified by DNN* (6) *with the following differential learning law*

$$\begin{aligned} \dot{W}_1^{\mathrm{T}} &= \eta_1 \left(-2\hat{\varphi}_1 y \Delta_z^{\mathrm{T}} P_z + \varphi_1 y \frac{\partial W_0(z)}{\partial z} \right), \quad W_1(0) = W_1^* \\ \dot{W}_2^{\mathrm{T}} &= \eta_2(-2\hat{\varphi}_2 \Delta_y P_y + \varphi_2 y), \quad W_2(0) = W_2^* \end{aligned} \tag{12}$$

where $P_z \in R^{(n-1)\times(n-1)}$ *is a positive solution to the following matrix Riccati equation*

$$P_z A + A P_z + P_z \Lambda_{\tilde{f}'}^{-1} P_z + I_z \cdot L_{\tilde{f}'}^2 \|\Lambda_{\tilde{f}'}\| = 0 \tag{13}$$

and P_y is a positive constant. Let also the output thresholds of DNN nodes are adjusted according to

$$\begin{aligned}\psi_1 &= -\text{sign}(\text{diag}(\Delta_z P_z))\left[\|W_1^*\||\tilde{\varphi}_1| + \underset{n-1}{\text{vec}}(\bar{B}_1)\right] \text{sign}(y)\\ \psi_2 &= -\text{sign}(\Delta_y P_y)\left[\|W_2^*\| \cdot \|\tilde{\varphi}_2\| + \bar{B}_2\right]\end{aligned} \tag{14}$$

If the control law is constructed as follows:

$$\begin{aligned}u = b^{-1}(z,y)\Bigg[v &- \frac{\partial W_0(z)}{\partial z} W_1\varphi_1\\ &-\left[\bar{B}_2 + \left\|\frac{\partial W_0(z)}{\partial z}\right\|\bar{B}_1\right] \text{sign}(y) - W_2\varphi_2\Bigg]\end{aligned} \tag{15}$$

then such a closed-loop system is passive (with respect to the new input v) with the storage function

$$\begin{aligned}V &= \Delta^{\mathrm{T}} P\Delta + \frac{1}{2}y^2 + W_0(z) + \frac{\eta_2^{-1}}{2}\tilde{W}_2\tilde{W}_2^{\mathrm{T}} + \frac{1}{2}\text{tr}\{\tilde{W}_1\eta_1^{-1}\tilde{W}_1^{\mathrm{T}}\}\\ P &= \begin{bmatrix} P_z & 0\\ 0 & P_y\end{bmatrix} \in R^{n\times n}, \quad 0 < \eta_1 \in R^{n\times n}, \quad 0 < \eta_2 \in R\end{aligned} \tag{16}$$

that is, for any $t \geq 0$

$$\dot{V} \leq vy$$

Proof Let us define

$$\tilde{W}_i := W_i - W_i^*, \quad i = 1,2 \tag{17}$$

Start with the calculation of the derivative of the storage function (16) along the trajectory of the systems (11) and (6):

$$\dot{V} = 2\Delta^{\mathrm{T}} P\dot{\Delta} + \frac{\partial W_0(z)}{\partial z}\dot{z} + y\dot{y} + \eta_2^{-1}\cdot\tilde{W}_2\dot{\tilde{W}}_2^{\mathrm{T}} + \text{tr}\left\{\tilde{W}_1\eta_1^{-1}\dot{\tilde{W}}_1^{\mathrm{T}}\right\} \tag{18}$$

Next, calculate the upper bounds for $\dot{V}$ in such a way that this bound is going to be a function of known data. To do this, we should use the assumptions H1–H4. Start with the term

$$2\Delta^{\mathrm{T}} P\dot{\Delta} = 2\Delta_z^{\mathrm{T}} P_z\dot{\Delta}_z + 2\Delta_y^{\mathrm{T}} P_y\dot{\Delta}_y$$

From (11), (6), (17) and (10) it follows

$$
\begin{aligned}
2\Delta_z^{\mathrm{T}} P_z \dot{\Delta}_z &= \hat{f}_0 - f_0 + (\tilde{W}_1 \hat{\varphi}_1 + W_1^* \tilde{\varphi}_1) y - \nu_1 \\
2\Delta_y^{\mathrm{T}} P_y \dot{\Delta}_y &= \tilde{W}_2 \hat{\varphi}_2 + W_2^* \tilde{\varphi}_2 - \nu_2
\end{aligned}
\tag{19}
$$

Equation (19) can be rewritten as

$$
\begin{aligned}
2\Delta_z^{\mathrm{T}} P_z \dot{\Delta}_z &= 2\Delta_z^{\mathrm{T}} P_z [f_0(\hat{z}) - f_0(z) + (A\Delta_z - A\Delta_z) \\
&\quad + (\tilde{W}_1 \hat{\varphi}_1 + W_1^* \tilde{\varphi}_1) y - \nu_1] \\
&= 2\Delta_z^{\mathrm{T}} P_z [\tilde{f}' + A\Delta_z + (\tilde{W}_1 \hat{\varphi}_1 + W_1^* \tilde{\varphi}_1) y - \nu_1]
\end{aligned}
$$

where

$$
\tilde{f}' := f_0(\hat{z}) - f_0(z) - A(\hat{z} - z)
$$

is the function, which in view of H2, verifies the following Lipschitz condition:

$$
\|\tilde{f}'\| \le L_{\tilde{f}'} \|\Delta_z\|, \quad L_{\tilde{f}'} > 0 \tag{20}
$$

Equation (8) implies that

$$
\begin{aligned}
2\Delta_z^{\mathrm{T}} P \dot{\Delta}_z &= 2\Delta_z^{\mathrm{T}} P_z [\tilde{f}' + A\Delta_z] + 2\Delta_z^{\mathrm{T}} P_z \tilde{W}_1 \hat{\varphi}_1 y \\
&\quad + 2\Delta_z^{\mathrm{T}} P_z [W_1^* \tilde{\varphi}_1 - B_1 + \psi_1] y
\end{aligned}
$$

and, taking into account the inequality (20), the first term from the right-hand side of the term $2\Delta_z^{\mathrm{T}} P_z [\tilde{f}' + A\Delta_z]$ can be estimated as

$$
2\Delta_z^{\mathrm{T}} P_z [\tilde{f}' + A\Delta_z] \le \Delta_z^{\mathrm{T}} [P_z A + A P_z + P_z P_z + I_z \cdot L_{\tilde{f}'}^2 \|\Lambda_{\tilde{f}'}\|] \Delta_z
$$

Also the following estimations hold:

$$
\begin{aligned}
&2\Delta_z^{\mathrm{T}} P_z [W_1^* \tilde{\varphi}_1 - B_1 + \psi_1] y \\
&\quad \le 2|\Delta_z^{\mathrm{T}} P_z| (\|W_1^*\| |\tilde{\varphi}_1| + \underset{n-1}{\mathrm{vec}}(\bar{B}_1)) |y| + 2\Delta_z^{\mathrm{T}} P_z \psi_1 y \\
&\quad = 2\Delta_z^{\mathrm{T}} P_z [\mathrm{sign}(\mathrm{diag}(\Delta_z^{\mathrm{T}} P_z)) \left(\|W_1^*\| |\tilde{\varphi}_1| + \underset{n-1}{\mathrm{vec}}(\bar{B}_1) \right) \mathrm{sign}(y) + \psi_1] y
\end{aligned}
$$

The upper bound for $2\Delta_z^T P\dot{\Delta}_z$ is

$$\begin{aligned}2\Delta_z^T P\dot{\Delta}_z \leq\ & \Delta_z^T[P_zA + AP_z + P_zP_z + I_z \cdot L_{\tilde{f}'}^2\|\Lambda_{\tilde{f}'}\|]\Delta_z \\ & + 2\Delta_z^T P_z\Big[\text{sign}(\text{diag}(\Delta_z^T P_z)) \\ & \times \Big(\|W_1^*\||\tilde{\varphi}_1| + \underset{n-1}{\text{vec}}(\bar{B}_1)\Big)\text{sign}(y) + \psi_1\Big]y \\ & + \text{tr}\{\tilde{W}_1[2\hat{\varphi}_1 y\Delta_z^T P_z]\}\end{aligned} \tag{21}$$

Using (8), analogously to the previous calculations, for the second term in (19) it follows:

$$\begin{aligned}2\Delta_y P_y\dot{\Delta}_y \leq\ & \tilde{W}_2\hat{\varphi}_2 2\Delta_y P_y + 2\Delta_y P_y \\ & \times [\text{sign}(\Delta_y P_y)(\|W_2^*\|\tilde{\varphi}_2 + \bar{B}_2) + \psi_2]\end{aligned} \tag{22}$$

The upper bound of the term $((\partial\mathcal{W}_0(z))/\partial z)\dot{z}$ is

$$\begin{aligned}\frac{\partial\mathcal{W}_0(z)}{\partial z}\dot{z} \leq\ & \frac{\partial\mathcal{W}_0(z)}{\partial z}f_0 + \text{tr}\left\{\tilde{W}_1\left[-\varphi_1 y\frac{\partial\mathcal{W}_0(z)}{\partial z}\right]\right\} \\ & + \left[\left\|\frac{\partial\mathcal{W}_0(z)}{\partial z}\right\|\bar{B}_1\text{sign}(y) + \frac{\partial\mathcal{W}_0(z)}{\partial z}W_1\varphi_1\right]y\end{aligned} \tag{23}$$

The term $y\dot{y}$ is estimated in the following way:

$$y\dot{y} \leq [W_2\varphi_2 + bu + \bar{B}_2\ \text{sign}(y)]\,y + \tilde{W}_2[-\varphi_2 y] \tag{24}$$

Combining (18) with the estimations (21)–(24), finally, the upper bound for the derivative of the storage function (16) can be presented in the following form:

$$\begin{aligned}& 2\Delta^T P\dot{\Delta} \\ & \leq \Delta_z^T[P_zA + AP_z + P_z\Lambda_{\tilde{f}'}^{-1}P_z + I_z \cdot L_{\tilde{f}'}^2\|\Lambda_{\tilde{f}'}\|]\Delta_z \\ & \quad + 2\Delta_z^T P_z\Big[\text{sign}(\text{diag}(\Delta_z^T P_z))\Big(\|W_1^*\||\tilde{\varphi}_1| + \underset{n-1}{\text{vec}}(\bar{B}_1)\Big)\text{sign}(y) + \psi_1\Big]y \\ & \quad + 2\Delta_y P_y\big[\text{sign}(\Delta_y P_y)\big(\|W_2^*\| \cdot \|\tilde{\varphi}_2\| + \bar{B}_2\big) + \psi_2\big] \\ & \quad + \text{tr}\left\{\tilde{W}_1\left[\eta_1^{-1}\dot{\tilde{W}}_1^T + 2\hat{\varphi}_1 y\Delta_z^T P_z - \varphi_1 y\frac{\partial\mathcal{W}_0(z)}{\partial z}\right]\right\} \\ & \quad + \tilde{W}_2\Big[\eta_2^{-1}\dot{\tilde{W}}_2^T + \hat{\varphi}_2 2\Delta_y P_y - \varphi_2 y\Big] \\ & \quad + \frac{\partial\mathcal{W}_0(z)}{\partial z}f_0 + \left[\left\|\frac{\partial\mathcal{W}_0(z)}{\partial z}\right\|\bar{B}_1\text{sign}(y) + \frac{\partial\mathcal{W}_0(z)}{\partial z}W_1\varphi_1\right]y \\ & \quad + [W_2\varphi_2 + bu + \bar{B}_2\text{sign}(y)]\,y\end{aligned} \tag{25}$$

From this expression, it follows that:

- *the first right-hand term* contains the algebraic Riccati equation inside of the brackets and, hence, is equal to zero identically,
- *the second* and *third terms* are cancelled by the output thresholds ψ_1 and ψ_2 selected as in (14),
- since $\dot{\tilde{W}} = \dot{W}_i$ $(i=1,2)$, the learning law for the weights adjusting cancels *the fourth* and *fifth terms* of (25).

By the assumption H3 and imposing the control law as in (15), finally, it follows that

$$\dot{V} \leq vy$$

The theorem is proved.

To clarify the main contribution of this paper, formulated in the proven theorem, the following remarks seems to be useful.

3.1 Structure of Storage Function

The storage function (16) consists of the following three parts:

- the first one $\Delta^T P \Delta$ makes the identification error smaller;
- the second part

$$y^2 + W_0(z)$$

  are the terms related to the passivity property;
- the third one

$$\tfrac{1}{2}\operatorname{tr}\{\tilde{W}_1 \eta_1^{-1} \tilde{W}_1^T\} + \tfrac{1}{2}\eta_2^{-1} \tilde{W}_2 \tilde{W}_2^T$$

  is used to generate the learning law for DNN.

3.2 Thresholds Properties

The output thresholds ψ_1 and ψ_2 are introduced to cancel the influence of the uncertain terms. These thresholds are functions of the bounding functions $\bar{B}_1$ and $\bar{B}_2$, hence, it is preferable to select these functions in such a way that they be sharp (reachable). Such selection can significantly improve the performance of the interconnected system.

3.3 Stabilizing Robust Linear Feedback Control

The control law u is constructed based on the information received from the updated neurons of DNN ($W_1\varphi_1(z,y)$, $W_2\varphi_2(z,y)$). The neural part of the control law identifies the uncertain terms of the system. So, the control law passifies the system and also cancels the influence of the uncertain terms simultaneously. The selection of a feedback as a negative linear one, that is,

$$v = -ky, \quad k > 0$$

that leads to the following inequality:

$$\dot{V} \leq vy = -ky^2 \leq 0$$

So, the closed-loop system turns out to be stable for any negative linear feedback.

3.4 Situation with Complete Information

If the nonlinear functions $f_0(z)$, $p(z,y)$, $a(z,y)$ and $b(z,y)$ are known, then the control law (15) can be constructed by replacing the neuron part $W_1\varphi_1$ with the known function $p(z,y)$, and $W_2\varphi_2$ replaced with $a(z,y)$. The functions $\bar{B}_1$ and $\bar{B}_2$ may be replaced by zeros since these functions are related to the uncertainty (unmodelled dynamics). The control law, making the system with such known model passive from input v to output y, now is

$$u = b^{-1}(z,y)\left[v - \frac{\partial \mathcal{W}_0(z)}{\partial z}p(z,y) - a(z,y)\right]$$

The corresponding storage function is as follows:

$$V_c = \tfrac{1}{2}y^2 + \mathcal{W}_0(z)$$

These facts coincide with the results in [1].

3.5 Two Coupled Subsystems Interpretation

We can consider the given nonlinear system (1) as a collection of two coupled subsystems (see Fig. 1). One of them (say, subsystem F) is

$$\dot{y} = a(z,y) + b(z,y)u$$

with the input u and the output y. The other one (subsystem G) is

$$\dot{z} = f_0(z) + p(z,y)y$$

with the corresponding input y and output z related to the internal dynamics of the whole system. The input of the entire system (1), containing F coupled with G, is u and its output is y. So, in this sense we can say that the function $p(z,y)$ is the "coupling term" of this nonlinear system. In view of this remark, we can say that the uncertainties, described before, are related to the coupling term $p(z,y)$.

3.6 Some Other Uncertainty Descriptions

3.6.1 Case 1: Uncertainty in the Term $p(z,y)$

If the functions of nonlinear system (1) $f_0(z)$, $a(z,y)$, $b(z,y)$ are known and the function $p(z,y)$ is unknown but bounded, that is,

$$\|p(z,y)\| \leq \bar{p}(z,y)$$

then the particular DNN for such uncertain system can be selected as

$$\dot{\hat{z}} = f_0(\hat{z}) + [W_1\varphi_1(\hat{z},y) + \psi_1]y$$

with the learning law

$$\begin{aligned} \dot{W}_1^{\mathrm{T}} &= \eta_1\left(-2\varphi_1(\hat{z},y)\Delta_z^{\mathrm{T}}P_z + \varphi_1(z,y)\frac{\partial \mathcal{W}_0(z)}{\partial z}\right)y \\ W_1(0) &= W_1^* \end{aligned} \tag{26}$$

and the output threshold given by

$$\begin{aligned} \psi_1 = &-\mathrm{sign}(\mathrm{diag}(\Delta_z P_z))\big[\|W_1^*\| \cdot |\varphi_1(\hat{z},y) - \varphi_1(z,y)| \\ &+ \underset{n-1}{\mathrm{vec}}(\bar{B}_1)\big]\,\mathrm{sign}(y) \end{aligned} \tag{27}$$

The passifying control law is

$$\begin{aligned} u = b^{-1}(z,y)\Bigg[v &- \frac{\partial W_0(z)}{\partial z} W_1\varphi_1(z,y) \\ &- \left[\left\|\frac{\partial W_0(z)}{\partial z}\right\|\bar{B}_1\right]\mathrm{sign}(y) - a(z,y)\Bigg] \end{aligned} \tag{28}$$

with the storage function equalled to

$$V_p = \Delta_z^{\mathrm{T}} P_z \Delta_z + W_0(z) + \tfrac{1}{2} y^2 + \mathrm{tr}\{ \tilde{W}_1 \eta_1^{-1} \dot{\tilde{W}}_1^{\mathrm{T}} \} \tag{29}$$

On the other hand, the coupling term $p(z,y)$ can be expressed as

$$p(z,y) = p_0(z,y) + \delta_p(z,y)$$

where $p_0(z,y)$ is a known part and $\delta_p(z,y)$ is an unknown one, satisfying the constraint

$$\|\delta_p(z,y)\| \le \bar{\delta}_p(z,y)$$

In this case the corresponding DNN can be constructed as

$$\dot{\hat{z}} = f_0(\hat{z}) + [p_0(z,y) + W_1 \varphi_1(\hat{z},y) + \psi_1]\, y$$

with the function $\bar{B}_1$ changed to

$$\bar{B}_1 = \bar{\delta}_p(z,y) + \|W_1^*\| \cdot \|\varphi_1\|$$

The control and learning laws, as well as the threshold and the storage function, remain as in (26)–(29). So, we have two alternatives for the uncertainty description in the coupling term $p(z,y)$. But in both cases, the suggested passifying control law (28) turns out to be robust with respect to the uncertainty in this coupling term.

3.6.2 *Case 2: Uncertainty in the Term* $a(z,y)$

The main result of this paper, formulated in the theorem given above, concerns the uncertainty in the terms $p(z,y)$ and $a(z,y)$. As a partial case, we can formulate the main result for the situation when the uncertainties are involved only in the term $a(z,y)$. If the functions $f_0(z)$, $p(z,y)$ of the nonlinear system (1) and $b(z,y)$ are known and only the term $a(z,y)$ is unknown but it is bounded as

$$\|a(z,y)\| \le \bar{a}(z,y)$$

then DNN, identifying the unknown part, can be constructed as follows:

$$\dot{\hat{y}} = W_2\varphi_2(z,\hat{y}) + \psi_2 + b(z,y)u$$

with the weights adjusting according to

$$\begin{aligned} \dot{W}_2^{\mathrm{T}} &= \eta_2(-2\varphi_2(z,\hat{y})\Delta_y P_y + \varphi_2(z,y)y) \\ W_2(0) &= W_2^* \end{aligned} \tag{30}$$

and with the output thresholds tuned as

$$\psi_2 = -\mathrm{sign}(\Delta_y P_y)[\|W_2^*\| \cdot \|\varphi_2(z,\hat{y}) - \varphi_2(z,y)\| + \bar{B}_2]. \tag{31}$$

The control law

$$u = b^{-1}(z,y)1\left[v - \frac{\partial W_0(z)}{\partial z}p(z,y) - \bar{B}_2\mathrm{sign}(y) - W_2\varphi_2\right] \tag{32}$$

passifies the NLS with the storage function

$$V_a = \Delta_y^{\mathrm{T}} P_y \Delta_y + \frac{1}{2}y^2 + \mathcal{W}_0(z) + \frac{\eta_2^{-1}}{2}\tilde{W}_2\tilde{W}_2^{\mathrm{T}} \tag{33}$$

As in the former case, we can present the uncertainty of the function $a(z,y)$ as

$$a(z,y) = a_0(z,y) + \delta_a(z,y)$$

where $a_0(z,y)$ is the known part of $a(z,y)$ and $\delta_a(z,y)$ is the unknown one, satisfying

$$\|\delta_a(z,y)\| \leq \bar{\delta}_a(z,y)$$

Then the DNN can be constructed using only $a_0(z,y)$ by the following way:

$$\dot{\hat{y}} = a_0(z,y) + W_2\varphi_2(z,\hat{y}) + \psi_2 + b(z,y)u \tag{34}$$

The function $\bar{B}_2$ is changed to

$$\bar{B}_2 = \bar{\delta}_a(z, y) + \|W_2^*\| \cdot \|\varphi_2\|$$

The control law, the learning law, the threshold and the storage functions remain as in (30)–(33). For example, for a single link manipulator we can assume that the friction term is only single uncertainty of the corresponding model. So, the friction term $\delta_a(z, y)$ can be identified by DNN given by (34). More details, concerning this example, will be discussed in the next section.

4 NUMERICAL EXPERIMENTS

4.1 Single Link Manipulator

Let us consider the following nonlinear system (single link manipulator)

$$\begin{bmatrix} \dot{z}_1 \\ \dot{y} \end{bmatrix} = \begin{bmatrix} y \\ -(g/a)\cos(z_1) - \lambda(y) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ma^2 \end{bmatrix} u \tag{35}$$

where m is the mass and a is the length of the link, $\lambda(y)$ is the friction of the joint, $z_1 \in \mathfrak{R}$ is the joint variable, $y \in \mathfrak{R}$ is the velocity of z_1 and u is a torque control.

It can be easily seen that the system (35) can be rewritten in the form (1), i.e.,

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y) \\ \dot{y} &= a(z, y) + b(z, y) \end{aligned}$$

with

$$\begin{aligned} f_0(z_1) = 0, \quad & p(z, y) = 1 \\ a(z, y) &= -ga^{-1}\cos(z_1) - \lambda(y) \\ b(z, y) &= (ma^2)^{-1} z_1 \end{aligned}$$

The zero dynamics of the system (35) is stable and the Lyapunov function for this dynamics is

$$W_0(z_1) = \tfrac{1}{2} z_1^2$$

Now, assume that the terms $a(z, y)$ and $b(z, y)$ are unknown. Choose

$$v = -y$$

The initial conditions are selected as

$$[z_1(0), y(0)]^T = [1, 1]^T$$
$$[\hat{z}_1(0), \hat{y}(0)]^T = [5, 0]^T$$

To realize the numerical simulations the following parameters were selected:

$$m = \frac{0.5}{9.8}, \quad a = 0.3, \quad \lambda(y) = 0.1(y + \tanh(50y))$$

$$A = -2, \quad W_1^* = [9.8521, 11.8528], \quad W_2^* = [3.6141, 3.5260]$$

$$P_z = 0.5359, \quad \Lambda_{\tilde{f}'} = 1, \quad L_{\tilde{f}'} = 2$$

$$\bar{B}_1 = 15.4127\|\varphi_1\| + 3, \quad \bar{B}_2 = 5.0492\|\varphi_2\| + |y| + 4$$

$$\eta_1 = \mathrm{diag}\{2, 2\}, \quad \eta_2 = 2, \quad k_1 = 1, \quad k_2 = 1$$

The corresponding simulation results are depicted in Figs. 2–4.

As it is seen from above, the given system reaches to the point $y = 0$ around 0.7 s that correspond to a very quick stabilization process.

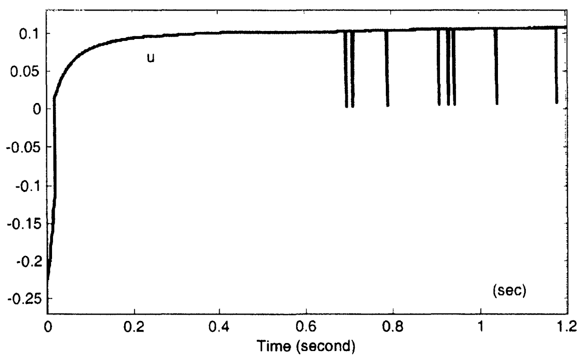

FIGURE 2 The control input u.

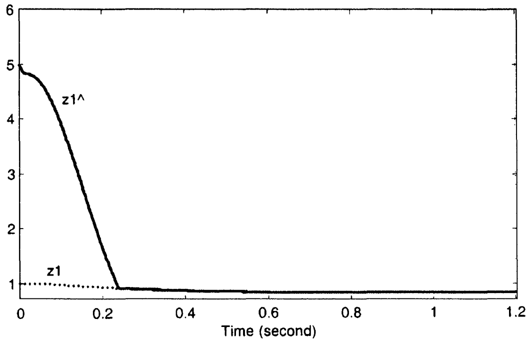

FIGURE 3 The state z_1 and its estimate $\hat{z}_1$.

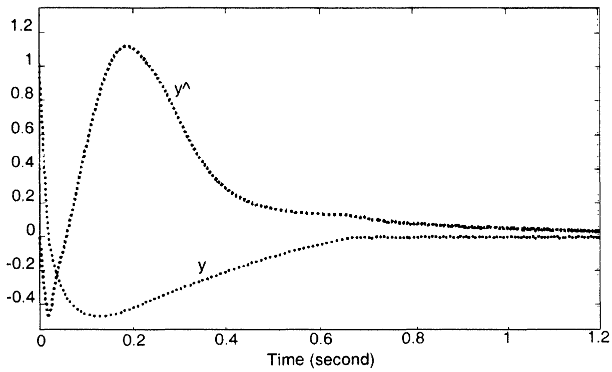

FIGURE 4 The output y and $\hat{y}$.

4.2 Benchmark Passivation

Consider the following benchmark nonlinear system [8]

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -z_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -5z_1^2 \\ -1.5 \\ 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \tag{36}$$

The passifying controller for this system was derived in [2,8,18]. We can rewrite this system (36) in the form (1) with

$$a(z,y) = 0, \quad b(z,y) = 1, \quad f_0 = [-z_1 \quad 0]^{\mathrm{T}}$$
$$p(y,z) = [-5z_1^2 - 1.5]^{\mathrm{T}}$$

The zero dynamics of this system (36) is stable with the corresponding Lyapunov function equal to

$$W_0(z) = (z_1^2 + z_2^2)/2$$

One assumes that $f_0(z)$ and $b(y, z)$ are known and that $p(y, z)$, jointly with $a(y, z)$, are unknown. The initial conditions are selected as follows:

$$[z_1(0), z_2(0), y(0)]^{\mathrm{T}} = [-1, 1, -2]^{\mathrm{T}}$$
$$[\hat{z}_1(0), \hat{z}_2(0), \hat{y}(0)]^{\mathrm{T}} = [1, -1, 1]^{\mathrm{T}}$$

Take the feedback as

$$v = -y$$

The corresponding simulation results are presented in Figs. 5–8.
The parameters were selected as follows:

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}, \quad W_1^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad W_2^* = [0, 0, 0],$$
$$P_z = \begin{bmatrix} 2.2918 & 0 \\ 0 & 2.2918 \end{bmatrix}, \quad \Lambda_{\tilde{f}'} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad L_{\tilde{f}'} = 2$$
$$\bar{B}_1 = 1.5\sqrt{25 * z_1^2 + 2.25 + .01y^2}, \quad \bar{B}_2 = \sqrt{0.01 \cdot z_2^2 + 0.001(z_1^2 + y^2)}$$
$$\eta_1 = \mathrm{diag}\{20, 20, 20\}, \quad \eta_2 = 20, \quad k_1 = 1, \quad k_2 = 1$$

The corresponding stabilizing process is finished around 1.5 s.

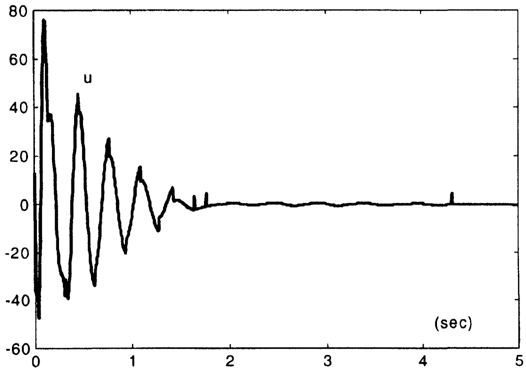

FIGURE 5 The control input u.

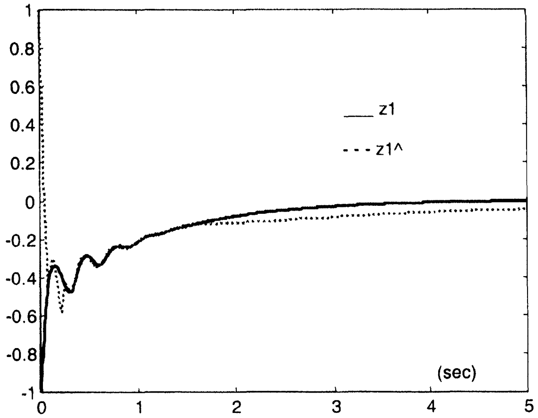

FIGURE 6 The state z_1 and its estimate $\hat{z}_1$.

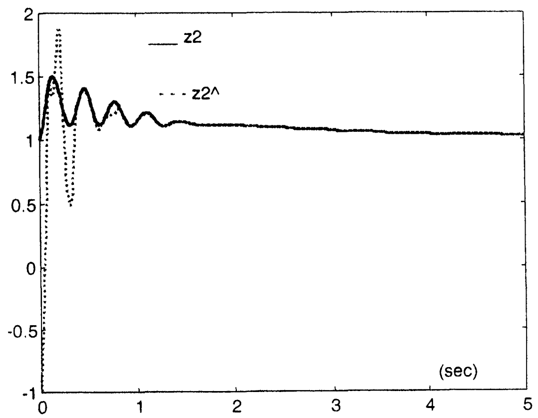

FIGURE 7 The state z_2 and its estimate $\hat{z}_2$.

As it follows from the examples presented above, the suggested approach provides nice stability property for the partially known non-linear systems closed by the simplest linear negative feedback.

5 CONCLUSIONS

The methodology, proposed in this paper, can be considered as an alternative approach to the existing ones dealing with a passivity feedback

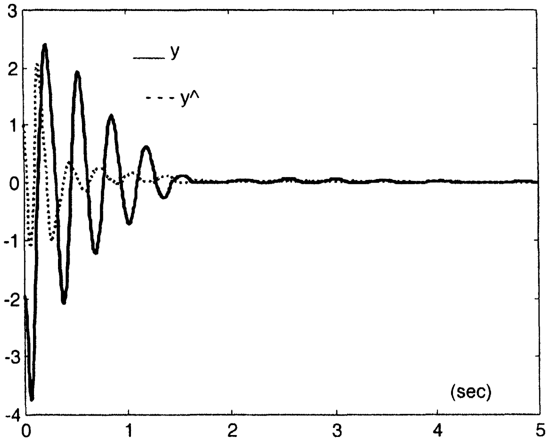

FIGURE 8 The state y and $\hat{y}$.

equivalence within a class of uncertain nonlinear systems. The suggested structure of the DNN is constructed using only the known part of the uncertain nonlinear system. The corresponding output thresholds are adjusted in such a way to compensate the uncertainty influence. A learning law is derived by means of a Lyapunov-like analysis. The passivating feedback control law, as well as the learning law for the dynamical neural network, contain some design parameters which with an adequate selection can improve the performance of the corresponding closed-loop system.

Future research will be devoted to the generalization of this approach to the class of MIMO nonlinear systems with incomplete information.

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