

Fundamental Problems for Infinite Plate with a Curvilinear Hole Having Finite Poles

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(Received 3 December 2000; Revised 14 May 2001)

In the present paper Muskhelishvili's complex variable method of solving two-dimensional elasticity problems has been applied to derive exact expressions for Goursat's functions for the first and second fundamental problems of the infinite plate weakened by a hole having many poles and arbitrary shape which is conformally mapped on the domain outside a unit circle by means of general rational mapping function. Some applications are investigated. The interesting cases when the shape of the hole takes different shapes are included as special cases.

Keywords: Fundamental problem; An infinite plate; Complex variable method; Rational mapping

1. INTRODUCTION

Problems dealing with isotropic homogeneous perforated infinite plate have been investigated by several authors [1–6]. Some of them [1–3] used Laurent's theorem to express each complex potential as a power series, others [4–6] used complex variable method of Cauchy type integrals.

It is known that [4] the first and second fundamental problems in the plane theory of elasticity are equivalent to finding two analytic

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functions, $\phi_1(z)$ and $\psi_1(z)$ of one complex argument $z = x + iy$. These functions must satisfy the boundary conditions

$$k_1 \phi_1(t) - t \overline{\phi_1'(t)} - \overline{\psi_1(t)} = f(t), \quad (1.1)$$

where for the first fundamental problem $k_1 = -1$, $f(t)$ is a given function of stresses; while for the second fundamental problem $k_1 = \chi = \lambda + 3\mu/\lambda + \mu > 1$, $f(t) = 2\mu g(t)$ is a given function of the displacement, λ, μ are called the Lamé's constants, χ is called Muskhelishvili's constant and with t denoting the affix of a point on the boundary. In terms of the rational mapping function $z = cw(\zeta)$, $c > 0$, $w'(\zeta)$ does not vanish or become infinite for $|\zeta| > 1$, the infinite region outside a closed contour conformally mapped outside the unit circle γ . The two complex functions of potential $\phi_1(z)$ and $\psi_1(z)$ take the form

$$\phi_1(z) = -\frac{X + iY}{2\pi(1 + \chi)} \ln \zeta + c\Gamma\zeta + \phi(\zeta), \quad (1.2)$$

$$\psi_1(z) = \frac{\chi(X + iY)}{2\pi(1 + \chi)} \ln \zeta + c\Gamma^*\zeta + \psi(\zeta), \quad (1.3)$$

where X, Y are the components of the resultant vector of all external forces acting on the boundary, and Γ, Γ^* are complex constants. Generally, the complex functions $\phi(\zeta), \psi(\zeta)$ are single-valued analytic functions within the region outside the unit circle and $\phi(\infty) = 0$, it will be assumed that $\Gamma = \bar{\Gamma}$ and $X = Y = 0$ for the first fundamental problem.

The rational mapping $z = cw(\zeta)$ maps the boundary C of the given region occupied by the middle plane of the plate in the z -plane onto the unit circle γ in the ζ -plane. Curvilinear coordinates (ρ, θ) are thus introduced into the z -plane which are the maps of the polar coordinates in the ζ -plane as given by $\zeta = \rho e^{i\theta}$.

Substituting $w(\zeta)$ into Eq. (1.1), we have

$$\phi_1(cw(\zeta)) - \frac{w(\zeta)}{w'(\zeta)} \overline{\phi_1'(cw(\zeta))} - \overline{\psi_1(cw(\zeta))} = f(cw(\zeta)). \quad (1.4)$$

Muskhelishvili [4] used the transformation $z = cw(\zeta) = c(\zeta + m\zeta^{-1})$ in Eq. (1.4) for solving the problem of stretching of an infinite plate

weakened by an elliptic hole. This transformation conformally maps the infinite domain bounded internally by an ellipse onto the domain outside the unit circle $|\zeta| = 1$ in the ζ -plane. Also the application of the Hilbert problem for a stretched infinite plate weakened by a circular cut is discussed in [4]. The two rational mapping functions

$$z = c \frac{\zeta + m\zeta^{-1}}{1 - n\zeta^{-1}}, \quad (c > 0, |n| < 1), \quad [\text{see Ref. 5}], \quad (1.5)$$

and

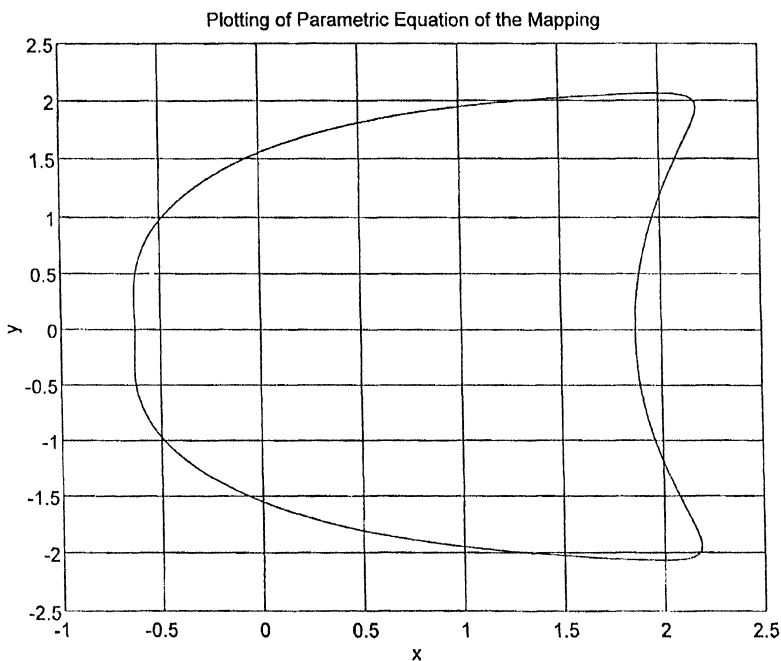
$$z = c \frac{\zeta + m\zeta^{-\ell}}{1 - n\zeta^{-\ell}}, \quad (c > 0, |n| < 1; \ell = 1, \dots, P), \quad [\text{see Ref. 6}], \quad (1.6)$$

where $c > 0, m, n$ are real parameters restricted such that $z'(\zeta)$ does not vanish or become infinite outside γ , are used by El-Sirafy and Abdou [5], Abdou and Kar-Eldin [6] respectively in Eq. (1.4) to solve the first and second fundamental problems of the infinite plate with a curvilinear hole C in the same previous domain.

In this paper, the complex variable method has been applied to solve the first and second fundamental problems for the same previous domain of the infinite plate with a general curvilinear hole C having finite poles conformally mapped on the domain outside a unit circle γ by the rational functions

$$z = cw(\zeta) = c \frac{\zeta + \sum_{j=1}^P m_j \zeta^{-1}}{\prod_{j=1}^P (1 - n_j \zeta^{-1})}, \quad (|n_j| < 1), \quad (1.7)$$

where $c > 0, m$'s and n 's are real parameters restricted such that $w'(\zeta)$ does not vanish or become infinite outside γ . None of the authors discussed Eq. (1.4) with several poles. The interesting cases when the shape of the hole is an ellipse, hypotrochoidal, a crescent or a cut having the shape of a circular arc are included as special ones. Holes corresponding to certain combinations of the parameters m 's and n 's are sketched (see Figs. 1–6). Some applications of the first and second fundamental problems of the infinite plate with a curvilinear hole having several poles are investigated.



$$z = \frac{2(\zeta - 0.2\zeta^{-1} + 0.31\zeta^{-2} + 0.01\zeta^{-3})}{(1+0.4\zeta^{-1})(1-0.4\zeta^{-1})(1-0.7\zeta^{-1})}$$

FIGURE 1

2. METHOD OF SOLUTION

The expression $w(\zeta^{-1})/w'(\zeta)$ can be written in the form

$$\frac{w(\zeta^{-1})}{w'(\zeta)} = \alpha(\zeta^{-1}) + \beta(\zeta), \quad (2.1)$$

where

$$\alpha(\zeta) = \sum_{k=1}^P \frac{h_k}{\zeta - n_k},$$

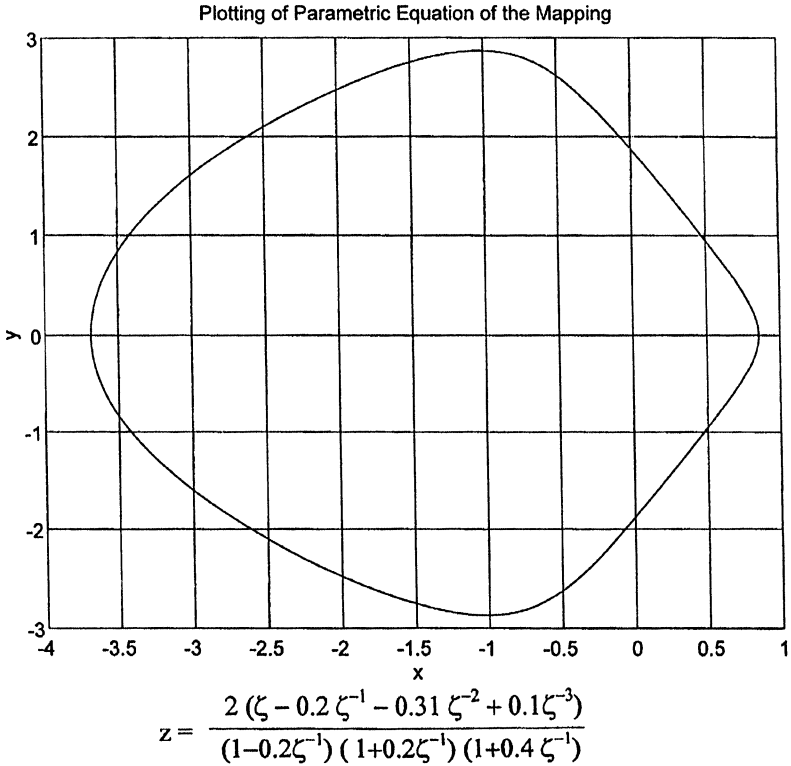
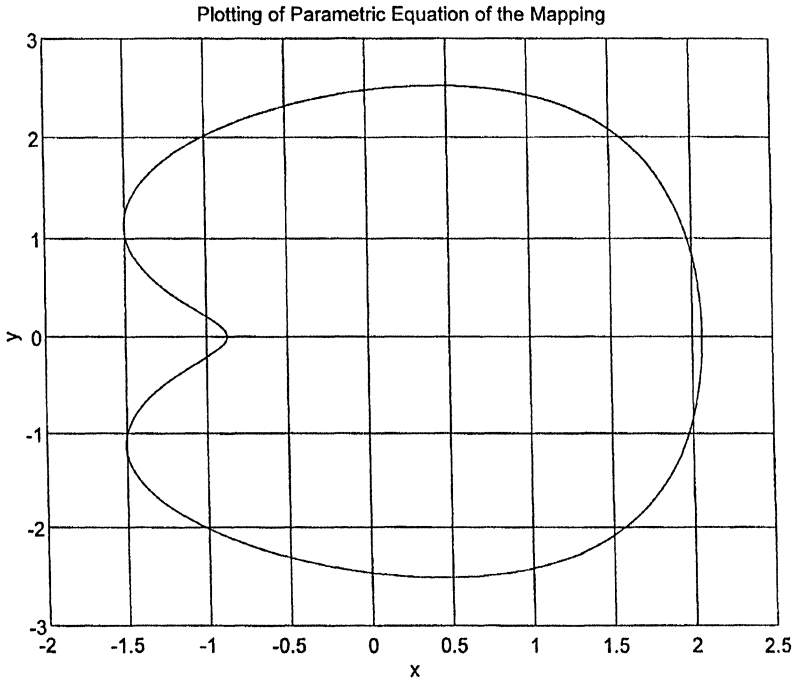


FIGURE 2

$$h_k = \frac{\left[n_k^{P+1} + \sum_{j=1}^P m_j n_k^{P-j} \right] \left(\prod_{j=1}^P (1 - n_j n_k) \right) \left(\prod_{j=1, k \neq j}^P (n_k - n_j) \right)^{-1}}{\left[(1 + \ell) + \sum_{j=1}^P (P - j) m_j n_k^{1+j} - \sum_{j=1}^P 1/(1 - n_j n_k) \right.}$$

$$\left. - \left(\sum_{j=1}^P m_j n_k^{1+j} \right) \sum_{j=1}^P (1/1 - n_j n_k) \right], \tag{2.2}$$

and $\beta(\zeta)$ is a regular function for $|\zeta| > 1$.



$$z = \frac{2(\zeta - 0.2\zeta^{-1} + 0.31\zeta^{-2} + 0.05\zeta^{-3})}{(1 - 0.1\zeta^{-1})(1 + 0.4\zeta^{-1})(1 - 0.3\zeta^{-1})}$$

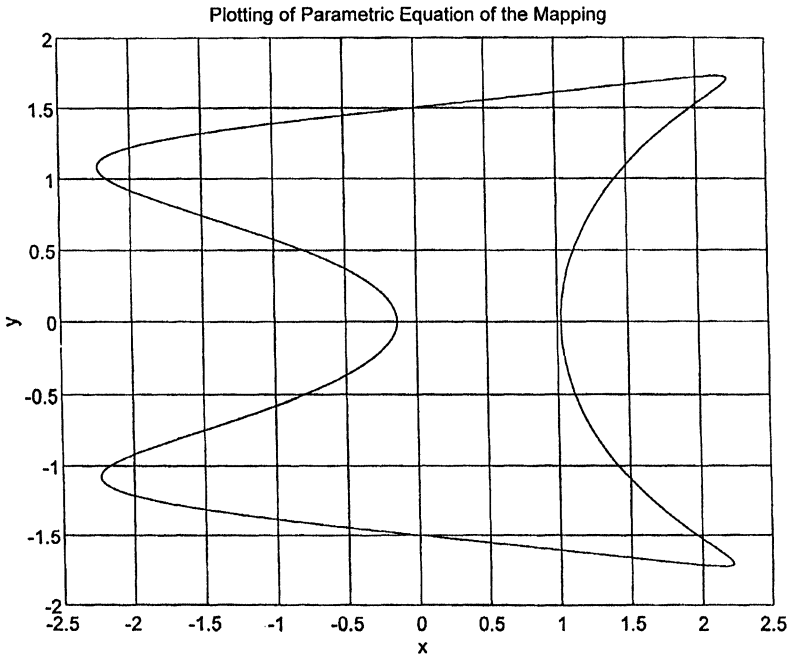
FIGURE 3

Using Eqs. (1.2), (1.3) and (2.1), the boundary condition Eq. (1.4) can be written in the form

$$k_1\phi(\sigma) - \alpha(\sigma)\overline{\phi'(\sigma)} - \overline{\psi_*(\sigma)} = f_*(\sigma), \tag{2.3}$$

where $\sigma = e^{i\theta}$ denotes the value of ζ on the boundary of the unit circle γ , while

$$\begin{aligned} \psi_*(\zeta) &= \psi(\zeta) + \beta(\zeta)\phi'(\zeta), \\ f_*(\zeta) &= F(\zeta) - ck_1\Gamma\zeta + c\overline{\Gamma^*}\zeta^{-1} + N(\zeta)(\alpha(\zeta) + \overline{\beta(\zeta)}), \\ N(\zeta) &= c\overline{\Gamma} - \frac{X - iY}{2\pi(1 + \chi)}\zeta, \end{aligned}$$



$$z = \frac{2(\zeta - 0.2\zeta^{-1} + 0.31\zeta^{-2} - 0.4\zeta^{-3})}{(1 - 0.7\zeta^{-1})(1 + 0.6\zeta^{-1})(1 - 0.01\zeta^{-1})}$$

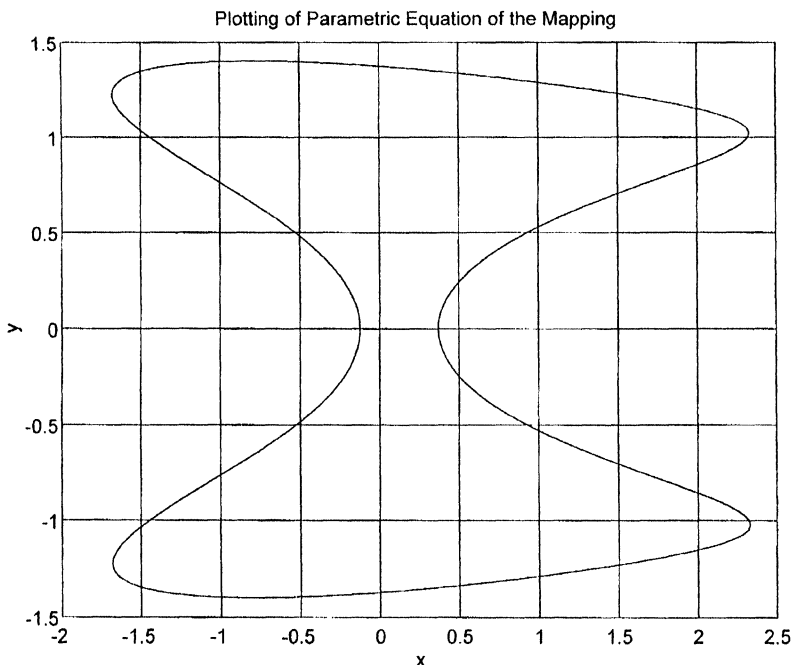
FIGURE 4

and

$$F(\zeta) = f(t). \tag{2.4}$$

Assume that the derivatives of $F(\sigma)$ must satisfy the Hölder condition.

Our aim is to determine the function $\phi(\zeta)$ and $\psi(\zeta)$, for the various fundamental problems, from Eq. (2.3). For this multiplying both sides of Eq. (2.3) by $(1/2\pi i)d\sigma/(\sigma - \zeta)$ then integrating the result around



$$z = c \frac{\zeta - 0.2\zeta^{-1} - 0.3\zeta^{-2} - 0.4\zeta^{-3}}{(1 - 0.9\zeta^{-1})(1 + 0.6\zeta^{-1})(1 - 0.1\zeta^{-1})}$$

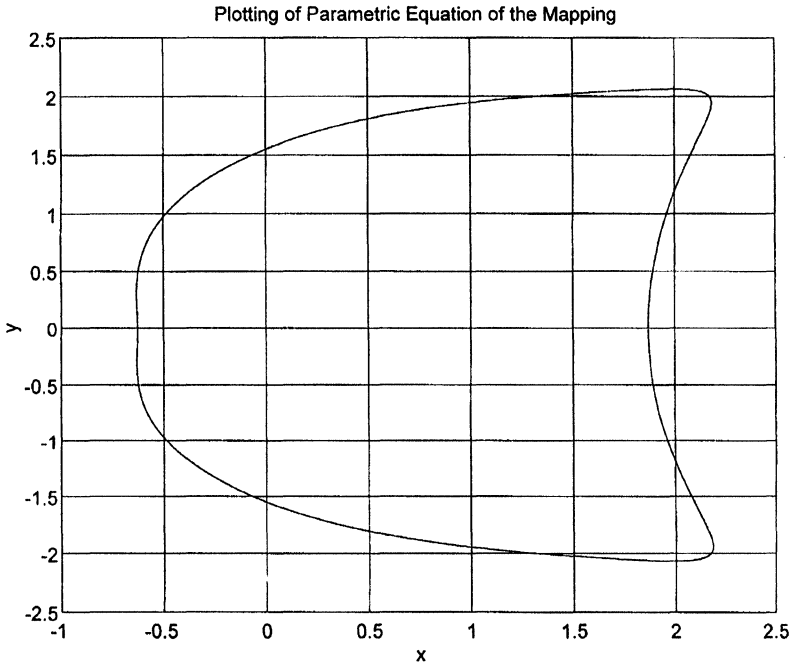
FIGURE 5

the unit circle γ and evaluating the integrals thus formulated by residue theorems, one has

$$k_1 \phi(\zeta) + \frac{1}{2\pi i} \int_{\gamma} \frac{\alpha(\sigma) \overline{\phi'(\sigma)}}{\sigma - \zeta} = c \overline{\Gamma^*} \zeta^{-1} - A(\zeta) + \sum_{j=1}^P \frac{h_j N(n_j)}{\zeta - n_j}, \quad (2.5)$$

where

$$A(\zeta) = -\frac{1}{2\pi i} \sum_{v=0}^{\infty} \frac{1}{\zeta^{1+v}} \int \sigma^v F(\sigma) d\sigma, \quad (|\zeta| > 1). \quad (2.6)$$



$$z = c \frac{\zeta - 0.2\zeta^{-1} + 0.3\zeta^{-2} + 0.1\zeta^{-3}}{(1 + 0.4\zeta^{-1})(1 - 0.4\zeta^{-1})(1 - 0.7\zeta^{-1})}$$

FIGURE 6

Using Eq. (2.1) we have

$$\frac{1}{2\pi i} \int_{\gamma} \frac{\alpha(\sigma) \overline{\phi'(\sigma)}}{\sigma - \zeta} = c \sum_{j=1}^P \frac{h_j b_j}{n_j - \zeta}, \tag{2.7}$$

where b 's are complex constant to be determined. Hence

$$-k_1 \phi(\zeta) = A(\zeta) - c \overline{\Gamma^*} \zeta^{-1} + \sum_{j=1}^P \frac{h_j}{n_j - \zeta} (c b_j + N(n_j)). \tag{2.8}$$

Differentiating Eq. (2.8) with respect to ζ , and using the result of $\overline{\phi'(\sigma)}$ in Eq. (2.7), we obtain

$$ck_1b_j + cn_j^2\Gamma^* + d_jh_j(c\bar{b}_j + \overline{N(n_j)}) = -\overline{A'(n_j)},$$

where

$$d_{j,k} = n_j^2(1 - n_jn_k)^{-2}, \quad (k = 1, \dots, P).$$

Hence

$$b_j = \frac{k_1E_j - h_jd_{j,k}\bar{E}_j}{c(k_1^2 - h_j^2d_{j,k}^2)},$$

where

$$E_j = -\overline{A'(n_j)} - c\Gamma^*n_j^2 - h_jd_{j,k}\overline{N(n_j)}. \tag{2.9}$$

Also, from Eq. (2.3), $\psi(\zeta)$ can be determined in the form

$$\psi(\zeta) = \frac{ck_1\bar{\Gamma}}{\zeta} - \frac{w(\zeta^{-1})}{w'(\zeta)}\phi_*(\zeta) + \sum_{j=1}^P \frac{h_j\zeta}{1 - n_j\zeta} \phi_*(n_j^{-1}) + B(\zeta) - B, \tag{2.10}$$

where

$$\phi_*(\zeta) = \phi'(\zeta) + \overline{N(\zeta)}, \quad B(\zeta) = \frac{1}{2\pi i} \int_{\gamma} \frac{\overline{F(\sigma)}}{\sigma - \zeta} d\sigma,$$

and

$$B = \frac{1}{2\pi i} \int_{\gamma} \overline{F(\sigma)} \frac{d\sigma}{\sigma}. \tag{2.11}$$

3. SPECIAL CASES

- (i) For $n_j's = 0, 0 \leq m_j \leq 1, j = 1, 2, \dots, P$, we get the rational mapping function

$$z = c(\zeta + m_1\zeta^{-1} + m_2\zeta^{-2} + \dots + m_P\zeta^{-P}). \tag{3.1}$$

The physical interest of the mapping (3.1) comes from the following

1. A circle of radius $c : m_j = 0, j = 1, 2, \dots, P$.
2. An ellipse $m_j = 0, j \geq 2$.
3. A square with rounded corners with diagonals parallel to the x - and y -axis $m_1 = m_j = 0, j \geq 3, m_2 =$ about 0.1. The same square with its sides parallel to the axis $m_1 = m_j = 0, j \geq 3, m_2 =$ about -0.1 .
4. An ovaloid $m_j = 0, j \geq 3, m_1 =$ about 0.3, $m_2 =$ about -0.05 .
5. A triangle $m_j = 0, j \neq 3$.

More information and applications on technology for the special cases of the mapping (3.1) are found in [1, 2].

- (ii) For $m_j = 0, j \geq 2, m_1 = -1$ the boundary C degenerate into a circular cut with P poles (see Fig. 1, $P = 1$). And for m_1 takes values near $-1, P = 1$, the edge of the hole resembles the shape of a crescent (see Fig. 2).

Many interesting cases the reader can be derived and used according to the technology work.

4. EXAMPLES

4.1. Curvilinear Hole for an Infinite Plate Subjected to a Uniform Tensile Stress

For $k_1 = -1, \Gamma = p/4, \Gamma^* = -(p/2)e^{-2i\theta}, 0 \leq \theta \leq 2\pi$ and $X = Y = f = 0$, we have an infinite plate stretched at infinity by the application of a uniform tensile stress of intensity P , making an angle θ with the x -axis. The plate weakened by a curvilinear hole C having finite poles which is free from stress.

The Goursat's functions (3.8)–(3.10) take the form

$$\phi(\zeta) = \frac{cP}{2} \left[\exp(2i\theta)\zeta^{-1} + \sum_{j=1}^P \frac{h_j Q_j}{n_j - \zeta} \right], \tag{4.1}$$

$$\psi(\zeta) = -\frac{cP}{4}\zeta^{-1} - \frac{w(\zeta^{-1})}{w'(\zeta)} \phi_*(\zeta) + \sum_{j=1}^P \frac{h_j \phi_*(n_j^{-1})\zeta}{1 - n_j \zeta}, \tag{4.2}$$

where

$$Q_j = \left[\frac{\frac{1}{2} - n_j^2 \cos 2\theta}{1 - h_j d_j} + \frac{n_j^2 \sin 2\theta}{1 + h_j d_j} \right],$$

and

$$\phi_*(\zeta) = \phi'(\zeta) + \frac{cP}{4}. \quad (4.3)$$

4.2. Curvilinear Hole Having Finite Poles the Edge of Which is Subject to a Uniform Pressure

For $k_1 = -1$, $X = Y = \Gamma = \Gamma^* = 0$ and $f(t) = Pt$, where P is a real constant. The formulae (3.8)-(3.10) become

$$\phi(\zeta) = \sum_{k,j=1}^P \frac{(n_k^{P+1} + m_j n_k^{P-j})(1 - h_j d_{j,k})}{(n_k - \zeta)(1 + h_j d_{j,k})} P, \quad (4.4)$$

and

$$\psi(\zeta) = -\frac{w(\zeta^{-1})}{w'(\zeta)} \phi'(\zeta) - cP \sum_{j=1}^P (n_j + \zeta^{-1}) + \sum_{j=1}^P \frac{h_j \zeta}{1 - n_j \zeta} \phi'(n_j^{-1}). \quad (4.5)$$

Hence Eqs. (4.4) and (4.5) give the solution of the first fundamental problem when the edge of the hole is subject to uniform pressure P . Putting in Eqs. (4.4) and (4.5) $-iT$ instead of P , we have the first fundamental problem when the edge of the hole is subject to a uniform tangential stress T .

4.3. Uni-directional Tension of an Infinite Plate with a Rigid Curvilinear Centre

For $k_1 = \chi, \Gamma = p/4, \Gamma^* = -p/2e^{-2i\theta}, X = Y = 0, f(t) = 2iP\epsilon t$, we have the two complex functions

$$-\chi\phi(\zeta) = \frac{cP}{2}e^{2i\theta}\zeta^{-1} + 2i\epsilon\mu\epsilon \sum_{j,k=1}^P \frac{\chi(n_k^{p+1} + m_j n_k^{p-j})}{(n_k - \zeta)(\chi + h_j d_{j,k})} + \frac{cP}{2} \sum_{j,k=1}^P \frac{h_j Q_{j,k}^{(2)}}{n_j - \zeta}, \tag{4.6}$$

$$\begin{aligned} \psi(\zeta) &= 2i\epsilon\mu\epsilon \sum_{j=1}^P n_j + c\left(\frac{\chi p}{4} + 2\mu\epsilon i\right)\zeta^{-1} - \frac{w(\zeta^{-1})}{w'(\zeta)}\phi_*(\zeta) \\ &+ \sum_{j=1}^P \frac{h_j \zeta}{1 - n_j \zeta} \phi_*(n_j^{-1}), \end{aligned} \tag{4.7}$$

where

$$Q_{j,k}^{(2)} = \frac{(\chi + 2n_j^2 \cos 2\theta)}{2(\chi + h_j d_{j,k})} - i \frac{n_j^2 \sin 2\theta}{\chi - h_j d_{j,k}},$$

and

$$\phi_*(\zeta) = \phi'(\zeta) + \frac{cP}{4}. \tag{4.8}$$

Therefore, we have the case of uni-directional tension of an infinite plate with a rigid curvilinear centre. The constant ϵ can be determined from the condition that the resultant moment of the forces acting on the curvilinear centre from the surrounding material must vanish *i.e.*

$$M = \text{Re} \left\{ \iint \left[\psi(\zeta) - \frac{cP}{2}e^{-2i\theta}\zeta \right] w'(\zeta) d\zeta \right\} = 0. \tag{4.9}$$

Hence, we have

$$\varepsilon = \frac{P(1 + \chi) \left(\sum_{j=1}^P n_j + N \right) \sin 2\theta}{4\mu \left[1 + \sum_j n_j + L \right]},$$

where

$$\begin{aligned} N &= \sum_{j,k=1}^P \frac{h_j n_j^2 n_k^2}{(1 - n_j n_k)^2 (\chi - h_j d_{j,k})}, \\ L &= \sum_{j,k=1}^P \frac{n_k^{P+3} + m_j n_k^{P-j+2}}{(1 - n_j n_k)^2 (\chi + h_j d_{j,k})}. \end{aligned} \tag{4.10}$$

Case 1 Bi-axial tension with $k = \chi, X = Y = 0, \Gamma = \bar{\Gamma} = P/2, \Gamma^* = 0$ and $f(t) = 2\mu g(t)$ under the same condition of example (4.3), one obviously will have $\varepsilon = 0$ and the two complex functions are

$$\phi(\zeta) = \frac{cP}{2} \sum_{j,k=1}^P \frac{h_j}{(\zeta - n_j)(\chi + h_j d_{j,k})}, \tag{4.11}$$

$$\psi(\zeta) = \frac{c\chi P}{2} \zeta^{-1} - \frac{w(\zeta^{-1})}{w'(\zeta)} \phi_*(\zeta) + \sum_{j=1}^P \frac{h_j \zeta}{1 - n_j \zeta} \phi_*(n_j^{-1}), \tag{4.12}$$

where

$$\phi_*(\zeta) = \phi' + \frac{cP}{2}. \tag{4.13}$$

Case 2 When the curvilinear centre is not allowed to rotate:

Under the condition of the preceding example (4.3) the rigid curvilinear kernel is restrained in its original position by a couple which is not sufficient to rotate the kernel, then $\varepsilon = 0$.

Hence the two complex functions are

$$-\chi\phi(\zeta) = \frac{cP}{2} e^{2i\theta} \zeta^{-1} + \frac{cP}{2} \sum_{j,k=1}^P \frac{h_j Q_{j,k}^{(2)}}{n_j - \zeta}, \tag{4.14}$$

and

$$\psi(\zeta) = \frac{c\chi P}{4}\zeta^{-1} - \frac{w(\zeta^{-1})}{w'(\zeta)}\phi_*(\zeta) + \sum_{j=1}^P \frac{h_j\zeta}{1-n_j\zeta}\phi_*(n_j^{-1}), \tag{4.15}$$

where $Q_{j,k}^{(2)}$ and $\phi_*(\zeta)$ are given by Eq. (4.8).

The resultant moment is given by

$$M = \frac{cP\pi(1+\chi)}{\chi} \left[\sum_{j=1}^P n_j + \sum_{j,k=1}^P \frac{h_j n_j^2 n_k}{(1-n_j n_k)^2 (\chi - h_j d_{j,k})} \right] \sin 2\theta. \tag{4.16}$$

Case 3 When a couple with a given moment acts on the curvilinear hole.

We assume that the stresses vanish at infinity. Under the same conditions of example (4.3), the complex functions take the following form

$$\phi(\zeta) = 2ic\mu\varepsilon \sum_{j,k=1}^P \frac{(n_k^{P+1} + m_j n_k^{P-j})}{(\zeta - n_k)(\chi + h_j d_{j,k})}, \tag{4.17}$$

$$\psi(\zeta) = 2ic\mu\varepsilon \left(\sum_{j=1}^P n_j + \zeta^{-1} \right) - \frac{w(\zeta^{-1})}{w'(\zeta)}\phi' + \sum_{j=1}^P \frac{h_j\zeta}{1-n_j\zeta}\phi'(n_j^{-1}), \tag{4.18}$$

where

$$\varepsilon = \frac{M\chi}{4\pi c\mu[1 + \sum_{j=1}^P n_j + L]}, \tag{4.19}$$

and L is given by Eq. (4.10).

4.4. The Force Acts on the Centre of the Curvilinear Hole

In this case, it will be assumed that the stresses vanish at infinity. It is easily seen that the kernel does not rotate. In general, the kernel remains in its original position.

Hence, one assumes $\Gamma = \Gamma^* = f(t) = 0$ and $k = \chi$. The Goursat's functions are

$$\phi(\zeta) = \frac{c}{2\pi\chi(1+\chi)} \sum_{j,k=1}^P \frac{ch_j}{\zeta - n_j} \left[\frac{\chi h_j d_{j,k}(X + iY)}{c(\chi^2 - h_j d_{j,k})} - \left\{ 1 + \frac{h_j^2 d_{j,k}^2}{c(\chi^2 - h_j^2 d_{j,k}^2)} \right\} (X - iY) \right], \quad (4.20)$$

$$\psi(\zeta) = \sum \frac{h_j \zeta}{1 - n_j \zeta} \phi_*(n_j^{-1}) - \frac{w(\zeta^{-1})}{w'(\zeta)} \phi_*(\zeta), \quad (4.21)$$

where

$$\phi_*(\zeta) = \phi'(\zeta) - \frac{X + iY}{2\pi(1+\chi)\zeta}. \quad (4.22)$$

Therefore, we have the solution of the second fundamental problem in the case, when a force (X, Y) acts on the centre of the curvilinear kernel.

5. CONCLUSION

From the above results and discussions the following may be concluded:

1. In the theory of two dimensional linear elasticity one of the most useful techniques for the solution of boundary value problems for a wkwardly shaped region is to transformation the region into one simpler shape.
2. The mapping function (1.7) maps the curvilinear hole C in the z -plane onto the domain outside a unit circle ζ -plane under the condition $w'(\zeta)$ does not vanish or become infinite outside γ .

3. The physical interest of the mapping (1.7) comes from its strong special cases which discussed here, moreover many new cases can be obtained according to the technology of the work, where the fundamental problems of the infinite plate with a curvilinear hole having finite poles are not discussed before.
4. The complex variable method (Cauchy method) is considered as one of the best method for solving the integro-differential equations (2.3) taken on closed contour γ , and obtained the two complex potential functions $\phi(z)$ and $\psi(z)$ directly.
5. This paper can be considered as a generalization of the work of the infinite plate with a curvilinear hole under certain conditions [1–6].

6. FUTURE WORK

The influence of a small hole having finite poles of rigid inclusion on the transverse flexure of thin plates will be discussed.

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